Operation on sets Venn Diagrams

**Exercise 6.1**

1. \[ A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \quad B = \{3, 5, 7, 9, 11\} \quad C = \{0, 5, 10, 20\} \]
   
   \[ i) \quad A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11\} \]
   
   \[ ii) \quad A \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 20\} \]
   
   \[ iii) \quad B \cup C = \{0, 3, 5, 7, 9, 10, 11, 20\} \]
   
   \[ iv) \quad A \cap B = \{3, 5, 7\} \]
   
   \[ v) \quad A \cap C = \{0, 5\} \]
   
   \[ vi) \quad B \cap C = \{5\} \]

2. \[ A = \{0, 1, 4, 7\} \quad \Xi = \{x | x \in \mathbb{N}, x \leq 10\} \]

   \[ \text{Given} \quad \Xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \]

   \[ \text{Complement of} \quad A = A' = \{2, 3, 5, 6, 8, 9, 10\} \]

   \[ ii) \quad A = \{\text{Consonants}\} \quad \Xi = \{\text{alphabets of English}\} \]

   \[ \text{Complement of} \quad A = A' = \{\text{vowels}\} \]

   \[ = \{a, e, i, o, u\} \]

   \[ iii) \quad A = \{\text{boys in Class VIII of all schools in Bengaluru}\} \quad \Xi = \{\text{students in Class VIII of all schools in Bengaluru}\} \]
Complement of $A = A' = \{ \text{girls in Class VIII of all schools in Bengal} \}$

iv. $A = \{ \text{letters of KALKA} \}$ and $\Xi = \{ \text{letters of KOLKATA} \}$

Complement of $A = A' = \{ 0, 1 \}$

v. $A = \{ \text{odd natural numbers} \}$ and $\Xi = \{ \text{whole numbers} \}$

Complement of $A = A' = \{ 0, 2, 4, 6, 8, 10, 12, \ldots \}$

3. $A = \{ x : x \in \mathbb{N} \text{ and } 3 < x < 7 \}$ and $B = \{ x : x \in \mathbb{N} \text{ and } x \leq 4 \}$

$A = \{ 4, 5, 6 \}$ and $B = \{ 0, 1, 2, 3, 4 \}$

i) $A \cup B = \{ 0, 1, 2, 3, 4, 5, 6 \}$
ii) $A \cap B = \{ 4 \}$
iii) $A - B = \{ 5, 6 \}$
iv) $B - A = \{ 0, 1, 2, 3 \}$

4. $P = \{ 0, 1, 2, 3, 4, 5 \}$ and $\Phi = \{ 4, 5, 6, 7, 8 \}$

i) $P \cup \Phi = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8 \}$
ii) $P \cap \Phi = \{ 4, 5 \}$
iii) $P - \Phi = \{ 0, 1, 2, 3 \}$
iv) $\Phi - P = \{ 6, 7, 8 \}$

Yes, $P \cup \Phi$ is a proper superset of $P \cap \Phi$ but Vice versa is not possible since $A$ contains elements not in $B$. 

Since $A$ contains elements not in $B$. 

5. \( A = \{ \text{letters of word INTEGRITY} \} \) \( B = \{ \text{letters of word RECKONING} \} \)
   
   i) \( A \cup B = \{ I, N, T, E, G, R, \emptyset, Y, C, K, O \} \)
   
   ii) \( A \cap B = \{ I, N, E, G, R \} \)
   
   iii) \( A - B = \{ T, Y \} \)
   
   iv) \( B - A = \{ C, K, O \} \)

   a) \( \eta(A) = 7 \quad \eta(B) = 8 \quad \eta(A \cap B) = 5 \quad \eta(A \cup B) = 10 \)
   
   b) \( \eta(A - B) = \eta(B) - \eta(A \cap B) = 8 - 5 = 3 \quad \eta(B - A) = \eta(A) - \eta(A \cap B) = 7 - 5 = 2 \)

   b) \( \eta(A \cup B) - \eta(A) - \eta(B) + \eta(A \cap B) = 10 - 7 - 8 + 5 = 2 \quad \eta(A \cup B) = 10 \)

   c) \( \eta(A \cup B) - \eta(A) + \eta(A \cap B) = 10 - 7 + 5 = 8 \quad \eta(B - A) = \eta(B) - \eta(A \cap B) = 8 - 5 = 3 \)

   d) \( \eta(A - B) + \eta(B - A) + \eta(A \cap B) = 3 + 2 + 5 = 10 = \eta(A \cup B) \)

6. \( \mathcal{S} = \{ 10, 11, 12, 13, 14, \ldots, 90 \} \)

   \( A = \{ 5, 10, 11, 15, 20, 25, 30, 35, 40 \} \)

   \( B = \{ 6, 12, 18, 24, 30, 36, 42 \} \)

   i) \( A \cup B = \{ 5, 6, 10, 11, 12, 13, 15, 18, 20, 24, 25, 30, 34, 35, 36, 40, 42 \} \)

   ii) \( \eta(A) = 8 \quad \eta(B) = 6 \quad \eta(A \cap B) = 1 \quad \eta(A \cup B) = 13 \)

   \( \eta(A) + \eta(B) - \eta(A \cap B) = 8 + 6 - 1 = 13 = \eta(A \cup B) \)
i) \( A' = \{ 5, 9 \} \)

ii) \( B' = \{ 1, 2, 3, 5, 7, 9 \} \)

iii) \( A \cup B = \{ 1, 2, 3, 4, 6, 7, 8 \} \)

iv) \( A \cap B = \{ 4, 6, 8 \} \)

v) \( A - B = A \cap B' = \{ 1, 2, 3, 5, 7, 9 \} \)

vi) \( B - A = B \cap A' = \{ \} \)

vii) \((A \cap B)' = \{ 1, 2, 3, 5, 7, 9 \} \)

viii) \( A' \cup B' = \{ 1, 2, 3, 5, 7, 9 \} \)

\[
(a) \quad (A \cap B)' = A' \cup B' = \{ 1, 2, 3, 5, 7, 9 \} \quad \text{Verified}
\]

\[
(b) \quad \eta(A) = 7 \quad \eta(A') = 2 \quad \eta(B) = 9
\]

\[
\eta(A) + \eta(A') = 7 + 2 = 9 = \eta(B) \quad \text{Verified}
\]

\[
(c) \quad \eta(A \cap B) + \eta((A \cap B)')
\]

\[
\eta(A \cap B) = 3 \quad \eta((A \cap B)') = 6
\]

\[
3 + 6 = 9 = \eta(B) \quad \text{Verified}
\]

\[
(d) \quad \eta(A - B) = 4 \quad \eta(B - A) = 0 \quad \eta(A \cup B) = 9
\]

\[
\eta(A - B) + \eta(B - A) + \eta(A \cup B) = 4 + 0 + 3 = 7 = \eta(A \cup B)
\]

8. \( \Sigma_1 = \{ x: x \in \mathbb{Z}, x \leq 10 \} \), \( A = \{ x: x \geq 5 \} \), \( B = \{ x: 3 \leq x < 8 \} \)

\[
\Sigma_1 = \{ \frac{3}{2}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \} \quad A = \{ 5, 6, 7, 8, 9, 10 \}
\]

\[
B = \{ 3, 4, 5, 6, 7 \}
\]

i) \( A \cup B = \{ 2, 4, 5, 6, 7, 8, 9, 10 \} \)

\[
A' = \{ 0, 1, 2, 3, 4 \}
\]

\[
(A \cup B)' = \{ 0, 1, 2 \} \quad B' = \{ 0, 1, 2, 8, 9, 10 \}
\]

\[
A' \cap B' = \{ 0, 1, 2 \}
\]
Therefore \((A \cup B)' = A' \cap B' = \{0,1,2\}\)

\[\text{ii) } A \cap B = \{5,6,7\} \Rightarrow (A \cap B)' = \{0,1,2,3,4,8,9,10\}\]
\[A' \cup B' = \{0,1,2,3,4,8,9,10\}\]

Therefore \((A \cap B)' = A' \cup B'\)

\[\text{iii) } A - B = \{8,9,10\}\]
\[A \cap B = \{8,9,10\}\]
\[\therefore A - B = A \cap B\]

\[\text{iv) } B - A = \{3,4\}\]
\[B \cap A' = \{3,4\}\]

9. \(\eta(A) = 20, \ \eta(B) = 16, \ \eta(A \cup B) = 30, \ \eta(A \cap B) = ?\)

We know \(\eta(A \cup B) = \eta(A) + \eta(B) - \eta(A \cap B)\)

\[30 = 20 + 16 - \eta(A \cap B)\]
\[\eta(A \cap B) = 76 - 30 = 46 \]
\[\eta(A \cap B) = 46\]

10. \(\eta(S) = 20, \ \eta(A') = 7, \ \eta(A) = ?\)

We know \(\eta(A) + \eta(A') = \eta(S)\)

\[\eta(A) = 20 - 7 = 13\]
\[\eta(A) = 13\]

11. \(\eta(S) = 40, \ \eta(A) = 20, \ \eta(B') = 16, \ \eta(A \cup B) = 32\)

\[\eta(B) + \eta(B') = \eta(S) - \eta(A) - \eta(A \cup B) = 40 - 20 - 32 = 28\]
\[\eta(B) = 28\]
\[ \eta(A \cup B) = \eta(A) + \eta(B) - \eta(A \cap B) \]
\[ 32 = 20 + 24 - \eta(A \cap B) \]
\[ \eta(A \cap B) = 44 - 32 = 12 \]
\[ \eta(B) = 24; \quad \eta(A \cup B) = 12 \]

12. \[ \eta(\Xi) = 32, \quad \eta(A) = 20, \quad \eta(B) = 16, \quad \eta((A \cup B)') = 7 \]

i) \[ \eta(A \cup B) = \eta(\Xi) - \eta((A \cup B)') \]
\[ = 32 - 7 = 25 \]
\[ \boxed{\eta(A \cup B) = 25} \]

ii) \[ \eta(A \cap B) = \eta(A) + \eta(B) - \eta(A \cup B) \]
\[ = 20 + 24 - 25 = 19 \]
\[ \boxed{\eta(A \cap B) = 8} \]

iii) \[ \eta(A - B) = \eta(A) - \eta(A \cap B) \]
\[ = 20 - 8 = 12 \]
\[ \boxed{\eta(A - B) = 12} \]

13. \[ \eta(\Xi) = 40 : \eta(A') = 15, \quad \eta(B) = 12, \quad \eta((A \cap B)') = 32 \]

i) \[ \eta(A) = \eta(\Xi) - \eta(A') = 40 - 15 = 25 \]

ii) \[ \eta(B') = \eta(\Xi) - \eta(B) = 40 - 12 = 28 \]

iii) \[ \eta(A \cap B) = \eta(A) + \eta(B) - \eta(A \cup B) = 28 + 12 \]
\[ \boxed{\eta(A \cap B) = 40 - 22 = 8} \]

iv) \[ \eta(A \cup B) = \eta(A) + \eta(B) - \eta(A \cap B) = 25 + 12 - 8 = 29 \]

v) \[ \eta(A - B) = \eta(A) - \eta(A \cap B) = 25 - 8 = 17 \]

vi) \[ \eta(B - A) = \eta(B) - \eta(A \cap B) = 12 - 8 = 4 \]
14. \( \eta(A - B) = 12 , \eta(B - A) = 16 , \eta(AB) = 5 \)

i) \( \eta(A) \)

ii) \( \eta(B) \)

iii) \( \eta(AB) \)

i) \( \eta(A - B) = \eta(A) - \eta(AB) \)

\[ \eta(A) = 12 + 5 = 17 \]

ii) \( \eta(B - A) = \eta(B) - \eta(AB) \)

\[ \eta(B) = 16 + 5 = 21 \]

iii) \( \eta(AB) = \eta(A) + \eta(B) - \eta(AB) \)

\[ = 17 + 21 - 5 = 33 \]

\[ \eta(AB) = 33 \]
EXERCISE: 6.2

1. From venn diagram, we find that

i) \( A = \{0, 5, 7, 8, 9, 11\} \)

ii) \( B = \{2, 5, 6, 8\} \)

iii) \( \bar{A} = \{0, 1, 2, 4, 5, 6, 7, 8, 9, 11, 12\} \)

iv) \( A' = \{1, 2, 4, 12\} \)

v) \( B' = \{0, 1, 4, 7, 9, 11, 12\} \)

vi) \( A \cup B = \{0, 2, 5, 6, 7, 8, 9, 11\} \)

vii) \( A \cap B = \{5, 8\} \)

viii) \( (A \cup B)' = \{1, 4, 12\} \)

ix) \( (A \cap B)' = \{0, 1, 2, 4, 6, 7, 9, 11, 12\} \)

2.

i) \( P = \{a, b, c, d, e, f, g, h, i\} \)

ii) \( Q = \{b, d, e\} \)

iii) \( \bar{Q} = \{a, b, c, d, e, f, g, h, i\} \)

iv) \( P' = \{c, j\} \)

v) \( Q' = \{a, c, f, g, h, i, j\} \)

vi) \( P \cup Q = \{a, b, d, e, f, g, h, i\} \)

vii) \( P \cap Q = \{b, d, e\} \)

viii) \( (P \cup Q)' = \{c, j\} \)

ix) \( (P \cap Q)' = \{a, c, f, g, h, i, j\} \)
3. i) \( E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)

   ii) \( A \cap B = \{0, 5, 8\} \)

   iii) \( A \cap B \cap C = \{0, 5\} \)

   iv) \( C' = \{2, 7, 8, 9, 10, 11, 12\} \)

   v) \( A - C = A \cap C' = \{8, 10\} \)

   vi) \( B - C = B \cap C' = \{7, 8, 11\} \)

   vii) \( C - B = C \cap B' = \{3, 4, 6\} \)

   viii) \( (A \cup B)' = \{2, 4, 6, 9, 12\} \)

   ix) \( (A \cup B \cup C)' = \{2, 7, 12\} \)

4. i) \( A = \{x \mid x \in N, x = 2n, n \leq 5\} \); \( B = \{x \mid x \in W, x = 4n, n \leq 5\} \)

\[ A = \{2, 4, 6, 8, 10\} \quad B = \{4, 8, 12, 16\} \]

\[ A \bigcap B = \{2, 4, 8\} \]

ii) \( A = \{\text{prime factors of 42}\} \); \( B = \{\text{prime factors of 60}\} \)

\[ A = \{2, 3, 7\} \quad B = \{2, 3, 5\} \]

iii) \( P = \{x \mid x \in W, x < 10\} \); \( Q = \{\text{prime factors of 210}\} \)

\[ P = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \quad Q = \{2, 3, 5\} \]
5. \( \eta(A) = 22 \quad \eta(B) = 18 \quad \eta(\overline{A} \cap \overline{B}) = 5 \)

\[ \begin{array}{c}
A \\
0 & 1 \\
4 & 6 & 2 & 3 & 5 \\
\end{array} \ \\
\begin{array}{c}
B \\
17 & 5 & 13 \\
\end{array} \]

i) \( \eta(A \cup B) = 17 + 5 + 13 = 35 \)

ii) \( \eta(A - B) = \eta(A) - \eta(A \cap B) = 17 - 5 = 12 \)

iii) \( \eta(B - A) = \eta(B) - \eta(A \cap B) = 13 - 5 = 8 \)

6. \( \eta(A) = 25 \quad \eta(B) = 16 \quad \eta(\overline{A} \cap \overline{B}) = 6 \quad \eta((A \cup B)') = 5 \)

\[ \begin{array}{c}
A \\
1 & 2 & 5 \\
\end{array} \ \\
\begin{array}{c}
B \\
9 & 6 & 10 \\
\end{array} \]

i) \( \eta(A \cap B) = 19 + 6 + 10 = 35 \)

ii) \( \eta(\overline{A} \cup B) = \eta(A \cup B) + \eta((A \cup B)') = 35 + 5 = 40 \)

iii) \( \eta(A - B) = \eta(A) - \eta(A \cap B) = 25 - 6 = 19 \)

iv) \( \eta(B - A) = \eta(B) - \eta(A \cap B) = 16 - 6 = 10 \)
7. $\eta(\xi) = 25 \quad \eta(A') = 7 \quad \eta(B) = 10 \quad \text{BCA}$

\[ \eta(A) = \eta(\xi) - \eta(A') = 25 - 7 = 18 \]

\[ \eta(A - B) = \eta(A) - \eta(\text{A} \cap \text{B}) = 18 - 10 = 8 \]

: Cardinal number of set $A - B$ is 8.

8.

\[ \text{No of boys who play atleast one of the two games} = \eta(\text{only cricket}) + \eta(\text{only football}) + \eta(\text{both cricket and football}) \]

\[ = 20 + 12 + 5 \]

\[ = 37 \]

ii) Neither cricket nor football

\[ \eta((\text{A} \cup \text{B})') = \text{Total student} - \eta(\text{A} \cup \text{B}) \]

\[ = 50 - 37 = 13 \]
9.

\[ \begin{array}{c}
\text{From Venn Diagram} \\
\end{array} \]

i) Both orange and banana

\[ n(\text{A} \cap \text{B}) = n(\text{A}) + n(\text{B}) - n(\text{A} \cup \text{B}) \]

\[ n(\text{A}) = 32, \quad n(\text{B}) = 26, \quad n(\text{A} \cup \text{B}) = 40 \]

\[ n(\text{A} \cap \text{B}) = n(\text{B}) - n(\text{B} - \text{A}) = 26 - 6 = 20 \]

ii) \( n(\text{B}) = 29 \)

\[ n(\text{A} \cup \text{B}) = n(\text{A}) + n(\text{B}) - n(\text{A} \cap \text{B}) \]

\[ n(\text{A}) = 40, \quad n(\text{B}) = 29, \quad n(\text{A} \cap \text{B}) = 20 \]

\[ n(\text{A} \cup \text{B}) = 40 - 29 + 20 = 31 \]

\[ n(\text{A} - \text{B}) = 14 - 6 = 8 \]

10.

\[ \begin{array}{c}
\text{(A) Bengali, (B) English} \\
\end{array} \]

\[ n(\text{A} \cup \text{B}) = 60 \]

\[ n(\text{A} \cap \text{B}) = x \\
\]

\[ n(\text{A} \cup \text{B}) = n(\text{A}) + n(\text{B}) - n(\text{A} \cap \text{B}) \]

\[ n(\text{A} \cup \text{B}) = 45 - x + 28 - x = 73 - 2x \]

\[ 60 = 73 - 2x \]

\[ 2x = 13 \]

\[ x = 6.5 \]
\[ 60 = 45 + 28 - \chi \]
\[ \chi = 73 - 60 \]
\[ \chi = 13 \]

\text{So, the number of people who speak both Bengali and English are 13}