

Understanding Quadrilaterals

1. Exercise: 13.1

- a) Simple Curves:- (i), (ii), (iii), (iv), (vi),
- b) Simple closed curves:- (iii), (v), (vi)
- c) Polygon:- (iii), (vi)
- d) Convex polygon:- (iii),
- e) Concave polygon:- vi

2.

a) A convex quadrilateral has two diagonals

b) A regular hexagon has 9 diagonals

3.

i) 8

$$\text{Sum of all interior angles} = (2n-4) \times 90$$

$$\text{Given no. of sides of polygon } (n) = 8$$

$$\begin{aligned}\text{Sum of all interior angles} &= (2 \times 8 - 4) \times 90 \\ &= (16-4) \times 90 \\ &= 12 \times 90 \\ &= 1080^\circ\end{aligned}$$

ii) Given

$$\text{no. of sides of polygon } (n) = 12$$

$$\begin{aligned}\text{Sum of all interior angles} &= (2n-4) \times 90 \\ &= (2 \times 12 - 4) \times 90 \\ &= 1800^\circ\end{aligned}$$

4. given exterior

(i)

exterior angle of polygon = 24°

sum of all exterior angles of polygon = 360°

$$\therefore n \times 24 = 360$$

$$\boxed{n=15}$$

(ii) given

exterior angle of polygon = 60°

sum of all exterior angles of polygon = 360°

$$\therefore n \times 60 = 360$$

$$\boxed{n=6}$$

\therefore no. of sides of given polygon is 6

iii)

given

exterior angle of polygon = 72°

sum of all exterior angles of polygon = 360°

$$\therefore n \times 72^\circ = 360$$

$$\boxed{n=5}$$

\therefore no. of sides of given polygon is 5.

5.

(i) For polygon with 'n' sides,

The each interior angle of polygon is given by

$$= \frac{(2n-4) \times 90}{n}$$

Given interior angle of polygon = 90°

$$\frac{(2n-4) \times 90}{n} = 90$$

$$(2n-4) = n$$

$$2n-n = 4$$

$$\boxed{n=4}$$

 \therefore no. of sides of polygon = 4

(ii) For a polygon with 'n' sides

The each interior angle of polygon is given by

$$= \frac{(2n-4) \times 90}{n}$$

Given interior angle = 108°

$$\frac{(2n-4) \times 90}{n} = 108$$

$$2n-4 = \frac{6}{5} \cdot n \Rightarrow 5(2n-4) = 6n$$

$$10n-20 = 6n$$

$$10n-6n = 20$$

$$4n = 20$$

4

$$n = \frac{20}{4}$$

$$\boxed{n=5}$$

∴ no. of sides of polygon = 5.

(ii) For a polygon with 'n' sides,

The each interior angle of polygon is given by

$$\frac{(2n-4) \times 90}{n}$$

Given interior angle = 165°

$$\frac{(2n-4) \times 90}{n} \approx 165$$

$$\frac{(2n-4)}{n} = \frac{11}{6}$$

$$6(2n-4) = 11 \times n$$

$$12n - 24 = 11n$$

$$12 - 8$$

$$12n - 11n = 24$$

$$\boxed{n=24}$$

∴ no. of sides of polygon = 24.

6.

Given Sum of interior angles of a polygon = 1260°

$$\therefore (2n-4) \times 90 = 1260$$

where n = no. of sides of polygon

$$(2n-4) = \frac{1260}{90}$$

$$2n-4 = 14$$

$$2n = 14+4$$

$$2n = 18$$

$$n = 18/2$$

$$\boxed{n=9}$$

\therefore Given polygon has nine sides.

7.

Given

Ratio of angles of pentagon = $7:8:11:13:15$

Let Angles of pentagon = $7x, 8x, 11x, 13x, 15x$

Sum of angles of polygon = $(2n-4) \times 90$

$$7x + 8x + 11x + 13x + 15x = (2 \times 5 - 4) \times 90$$

$$54x = 6 \times 90$$

$$x = \frac{540}{54}$$

$$\boxed{x = 10^\circ}$$

\therefore Angles of pentagon = $70^\circ, 80^\circ, 110^\circ, 130^\circ, 150^\circ$

8. Given angles of pentagon = x° , $(x-10)^\circ$, $(x+20)^\circ$, $(2x-44)^\circ$ and $(2x-70)^\circ$

Sum of interior angles of polygon = $(2n-4) \times 90^\circ$

$$x + (x-10) + (x+20) + (2x-44) + (2x-70) = (2 \times 5 - 4) \times 90$$

$$7x - 104 = 6 \times 90$$

$$7x = 540 + 104$$

$$7x = 644$$

$$x = \frac{644}{7}$$

$$\boxed{x = 92}$$

∴ Angles of pentagon = 92° , $(92-10)^\circ$, $(92+20)^\circ$,
 $(2 \times 92 - 44)^\circ$, $(2 \times 92 - 70)^\circ$

9. Given

Exterior angles Ratio = 1:2:3:4:5

Let Exterior angles = $x, 2x, 3x, 4x, 5x$

Sum of the exterior angles = 360°

$$x + 2x + 3x + 4x + 5x = 360$$

$$15x = 360$$

$$x = \frac{360}{15}$$

$$\underline{\underline{x = 24}}$$

External angles of pentagon

7

$$24^\circ, 48^\circ, 72^\circ, 96^\circ, 120^\circ$$

Internal angle = $180 - \text{External angle}$

Internal angles of pentagon -

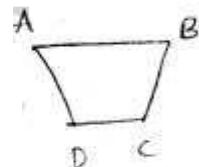
$$= 180 - 24, 180 - 48, 180 - 72, 180 - 96, 180 - 120$$

Interior angles = $156^\circ, 132^\circ, 108^\circ, 84^\circ, 60^\circ$
of pentagon.

10. Given

$$\angle A : \angle D = 2 : 3$$

$$\angle B : \angle C = 7 : 8$$



$$\text{let } \angle A = 2x, \angle D = 3x$$

$$\text{let } \angle B = 7y, \angle C = 8y$$

$$\angle B + \angle C = 180^\circ (\because AB \parallel DC)$$

$$7y + 8y = 180^\circ$$

$$15y = 180^\circ$$

$$\boxed{y = 12}$$

$$\therefore \angle B = 7y = 7 \times 12 = 84^\circ$$

$$\therefore \angle C = 8y = 8 \times 12 = 96^\circ$$

$$\text{by } \angle A + \angle D = 180^\circ$$

$$2x + 3x = 180^\circ$$

$$5x = 180$$

$$x = 36$$

$$\angle A = 2x = 2 \times 36 = 72^\circ$$

$$\angle D = 3x = 3 \times 36 = 108^\circ$$

$$\therefore \angle A = 72^\circ, \angle B = 84^\circ, \angle C = 96^\circ, \angle D = 108^\circ$$

II.

From $\triangle ABC$

$$\angle DBC + \angle C + \angle CBD = 180^\circ$$

$$x + 5x + 8 + \angle CBD = 180^\circ$$

$$6x + 8 + \angle CBD = 180^\circ$$

$$\angle CBD = 180 - 6x - 8$$

$$\angle CBD = 172 - 6x$$

$$\angle CDB + \angle ADB = 3x + 16^\circ$$

$$172 - 6x + \angle ADB = 3x + 16^\circ$$

$$\angle ADB = 3x + 16 + 6x - 172$$

$$\angle ADB = 9x - 162$$

In $\triangle ADB$

$$\angle ADB + \angle DAB + \angle ABD = 180^\circ$$

$$9x - 162 + 3x + 4 + 50 = 180^\circ$$

$$12x - 108 = 180$$

$$12x = 180 + 108 = 288$$

$$x = \frac{288}{12}$$

$$\boxed{x = 24^\circ}$$

$$\begin{aligned}
 \text{(ii)} \quad \angle DAB &= 3x + 4 \\
 &= 3 \times 24 + 4 \\
 &= 72 + 4 \\
 \angle DAB &= 76^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \angle ADB &= 9x - 162 \\
 &= 9 \times 24 - 162 \\
 \angle ADB &= 216 - 162 \\
 \angle ADB &= 54^\circ
 \end{aligned}$$

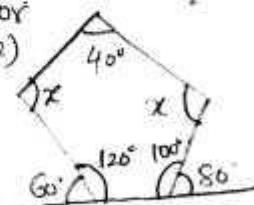
12.

(i) Sum of angles in quadrilateral = 360°

$$\begin{aligned}
 40 + 140 + 100 + x &= 360 \\
 280 + x &= 360 \\
 x &= 360 - 280 \\
 x &= 80
 \end{aligned}$$

(ii) Interior angle = $180 - (\text{exterior angle})$

Sum of interior angles



$$\text{Sum of interior angles of a pentagon} = (2 \times 5 - 4) \times 90$$

$$100 + x + x + 120 + 100 = 6 \times 90$$

$$2x + 260 = 540$$

$$2x = 540 - 260$$

$$2x = 280$$

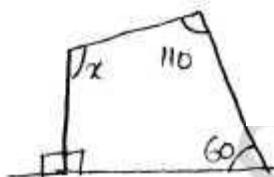
$$x = \frac{280}{2}$$

$$\boxed{x = 140}$$

Q. (iii)

Sum of interior angles
of a quadrilateral = 360°

Given



$$\therefore x + 110 + 60 + 90 = 360$$

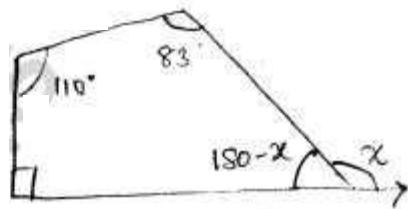
$$x + 260 = 360$$

$$x = 360 - 260$$

$$\boxed{x = 100}$$

Q. (iv)

Sum of interior angles
of a quadrilateral = 360°



$$110 + 83 + 180 - x + 90 = 360$$

$$463 - x = 360$$

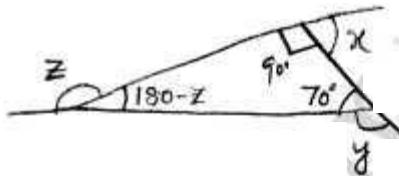
$$x = 463 - 360$$

$$\boxed{x = 103}$$

13.

(i)

Sum of angles in
a triangle = 180°



$$90 + 70 + 180 - z = 180^\circ$$

$$z = 90 + 70$$

$$z = 160$$

$$90 + x = 180 \quad (\because \text{Forms straight line})$$
$$x = 180 - 90$$

$$x = 90$$

$$70 + y = 180 \quad (\because \text{Forms straight line})$$
$$y = 180 - 70$$

$$y = 110$$

$$\therefore x + y + z = 90 + 110 + 160$$

$$\therefore x + y + z = 360^\circ$$

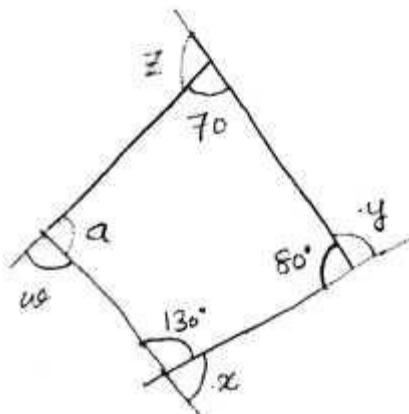
(ii) Sum of interior angles
in a quadrilateral = 360°

$$70 + 80 + 130 + a = 360^\circ$$

$$280 + a = 360$$

$$a = 360 - 280$$

$$a = 80^\circ$$



$$a+w = 180 \quad (\because \text{Forms straight line})$$

$$80+w = 180$$

$$w = 180 - 80$$

$$w = 100$$

$$z+70 = 180 \quad (\because \text{Forms straight line})$$

$$z = 180 - 70$$

$$z = 110$$

$$80+y = 180 \quad (\because \text{Forms straight line})$$

$$y = 180 - 80$$

$$y = 100$$

$$130+x = 180 \quad (\because \text{Forms straight line})$$

$$x = 180 - 130$$

$$x = 50$$

$$\therefore a+y+z+w = 50+100+110+100 = 360^\circ$$

14.

Given

heptagon has three equal angles = $120, 120, 120$

let Remaining four equal angles = x, x, x, x

Sum of interior angles of heptagon = $(2 \times 7 - 4) \times 90$

$$= 10 \times 90$$

$$\therefore 120 + 120 + 120 + x + x + x + x = 900$$

$$360 + 4x = 900$$

$$4x = 900 - 360$$

$$4x = 540$$

$$x = \frac{540}{4}$$

$$\boxed{x = 135^\circ}$$

The other equal angle of heptagon = 135°

15.

Ratio between exterior and interior angles

$$= 1 : 5$$

(i) Let exterior angle = x

$$\text{Interior angle} = 5x$$

$$\text{Exterior angle} + \text{Interior angle} = 180$$

$$x + 5x = 180$$

$$6x = 180$$

$$x = \frac{180}{6}$$

$$\boxed{x = 30}$$

Each exterior angle = $x = 30^\circ$

ii) Each interior angle = $5x = 5 \times 30 = 150^\circ$

iii) no. of sides of polygon = $\frac{360}{\text{Exterior angle}}$
 $= \frac{360}{30} = 12$

\therefore no. of sides of polygon = 12.

16. Given

$$\text{Each interior angle of polygon} = 2 \times \text{Exterior angle}$$

$$\text{Interior angle} + \text{Exterior angle} = 180$$

$$2 \times \text{Exterior angle} + \text{Exterior angle} = 180$$

$$3 \times \text{Exterior angle} = 180$$

$$\text{Exterior angle} = \frac{180}{3}$$

$$\text{Exterior angle} = 60^\circ$$

$$\text{No. of sides of polygon} = \frac{360}{\text{Exterior angle}}$$
$$= \frac{360}{60}$$

$$\text{No. of sides of polygon} = 6.$$

Exercise 13.2

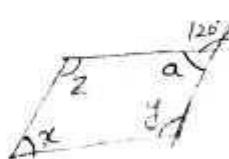
1.

- i. 6cm (\because opposite sides are equal)
- ii. 9cm (\because opposite sides are equal)
- iii. $\angle DCB = 60^\circ$ ($\because \angle DCB, \angle CBA$ are Supplementary)
- iv. $\angle ADC = 120^\circ$ (\because opposite angles are equal)
- v. $\angle DAB = 60^\circ$ (\because Adjacent angles are Supplementary)
- vi. $OC = 7\text{ cm}$ ($\because 'O'$ bisects $DB \& AC$)
- vii. $OB = 5\text{ cm}$ ($\because 'O'$ bisects DB)
- viii. $m\angle DAB + m\angle CDA = 180^\circ$

2.

(i.) Given parallelogram

Let the unknown angle = a



$$\therefore a + 120^\circ = 180^\circ \quad (\because \text{Adjacent angles are Supplementary})$$

$$a = 180 - 120^\circ$$

$$\boxed{a = 60^\circ}$$

$$\therefore a + y = 180^\circ \quad (\because \text{Adjacent angles are Supplementary})$$

$$60^\circ + y = 180^\circ$$

$$y = 180 - 60^\circ$$

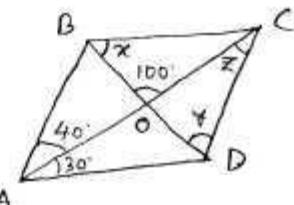
$$\boxed{y = 120^\circ}$$

$$x=60 \quad z=120 \quad (\because \text{opposite angles are equal in parallelogram})$$

(ii) ABCD is a parallelogram

$$z=40^\circ (\because AB \parallel CD)$$

At 'O'



$$100 + \angle COD = 180^\circ \quad (\because \text{forms straight line})$$

$$\angle COD = 180 - 100$$

$$\angle COD = 80^\circ$$

In $\triangle COD$

$$z + \angle COD + y = 180^\circ$$

$$40 + 80 + y = 180$$

$$y + 120 = 180$$

$$y = 180 - 120$$

$$\boxed{y = 60^\circ}$$

In $\triangle ABC$

$$\angle ACB = 30^\circ \quad (\because BC \parallel AD)$$

$$\therefore x + \angle ACB + \angle BOC = 180^\circ$$

$$x + 30 + 100 = 180$$

$$x + 130 = 180$$

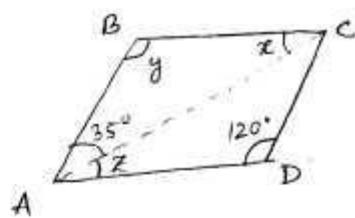
$$x = 180 - 130$$

$$\boxed{x = 50^\circ}$$

$$x = 50^\circ, y = 60^\circ, z = 40^\circ$$

(iii) ABCD is a parallelogram

$y = 120^\circ$ (\because opposite angles
are equal in
parallelogram)



$z + 35^\circ + 180 = 180$ (\because Adjacent angles are
supplementary)
 $z + 155 = 180$

$$z = 180 - 155$$

$$\boxed{z = 25^\circ}$$

$z = y$ $z = x$ ($\because AB \parallel CD$)

$$x = 25^\circ$$

$$\therefore x = 25^\circ, y = 120^\circ, z = 25^\circ$$

(iv) ABCD is a parallelogram

$$\therefore \angle A + \angle D = 180^\circ$$

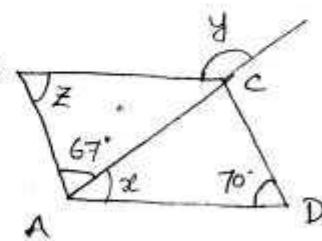
(\because Adjacent angles are
supplementary)

$$67 + x + 70 = 180$$

$$x + 137 = 180$$

$$x = 180 - 137$$

$$x = 43^\circ$$



$Z = 70^\circ$ (\because opposite angles are equal in a parallelogram)

$$\underline{\angle BCA} = \underline{\angle CAD} \quad (\because AD \parallel BC)$$

$$\underline{\angle BCA} = x$$

$$\underline{\angle BCA} = 43^\circ$$

At C'

$$\underline{\angle BCA} + y = 180^\circ \quad (\because \text{forms a straight line})$$

$$43 + y = 180$$

$$y = 180 - 43$$

$$\boxed{y = 137^\circ} \quad \boxed{y = 137^\circ}$$

$$\therefore x = 43^\circ, y = 137^\circ, Z = 70^\circ$$

3. Let x, y be lengths of adjacent sides of parallelogram

i. Given

$$\text{Perimeter} = 72 \text{ cm}$$

$$x:y = 5:7 \Rightarrow \frac{x}{y} = \frac{5}{7} \Rightarrow x = \frac{5}{7}y$$

$$x+y+x+y = 72 \quad (\because \text{opposite sides are equal in length})$$
$$2(x+y) = 72$$

$$2\left(\frac{5}{7}y + y\right) = 72$$

$$\underline{\text{B}} \quad \frac{12}{7}y = 36$$

$$y = \frac{36 \times 7}{12}$$

$$y = 21 \text{ cm}$$

$$x = \frac{5}{7} \cdot y$$

$$x = \frac{5}{7} \times 21$$

$$x = 15 \text{ cm}$$

$$\therefore x = 15 \text{ cm}, y = 21 \text{ cm}.$$

$\therefore 15\text{cm}, 21\text{cm}$ are lengths of sides of parallelogram

4.

Given

Angles of parallelogram are in the ratio of 4:5

Let the angle be $4x, 5x$

$$4x + 5x = 180 \quad (\because \text{Adjacent angles are supplementary})$$

$$9x = 180$$

$$x = 20$$

$$\text{Angle } 4x = 4 \times 20 = 80^\circ$$

$$5x = 5 \times 20 = 100^\circ$$

\therefore Four Angles of parallelogram = $80^\circ, 100^\circ, 80^\circ, 100^\circ$

(\because opposite are equal in a parallelogram)

5.

$$(i) \angle A + \angle C = 180^\circ ?$$

may (or) may not be

$$(\because \angle A = \angle C = 90^\circ)$$

$$(ii) AD = BC = 6\text{cm}, AB = 5\text{cm}, DC = 4.5\text{cm}$$

$$\text{No } (\because AD \neq BC)$$

$$(iii) \angle B = 80^\circ, \angle D = 70^\circ ?$$

No, opposite angles must be equal in parallelogram.

$$(iv) \angle B + \angle C = 180^\circ ?$$

Yes. (\because Contra-adjacent angles are supplementary)

6.

$$y = 40^\circ$$

$$(\because HE \parallel OP)$$

At 'o'

$$\angle HOP + 70^\circ = 180^\circ \quad (\because \text{Forms straight line})$$

$$\angle HOP = 180^\circ - 70^\circ$$

$$\angle HOP = 110^\circ$$

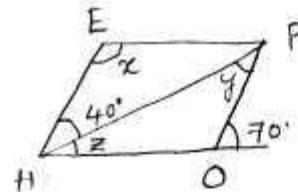
$$\therefore x = \angle HOP = 110^\circ \quad (\because \text{opposite angles are equal})$$

$$40^\circ + z + 110^\circ = 180^\circ \quad (\because \text{Adjacent angles are supplementary})$$

$$z + 150 = 180$$

$$z = 180 - 150$$

$$\boxed{z = 30^\circ}$$



ii)

At 'o'

$$x + 60 + 80 = 180^\circ$$

(\therefore forms straight line)

$$x + 140 = 180^\circ$$

$$x = 180 - 140$$

$$\boxed{x = 40^\circ}$$

$$z = x = 40^\circ \quad (\because RO \parallel EP)$$

$\angle O + \angle P = 180^\circ$ (\because Adjacent angles are supplementary)

$$x + 60 + y = 180^\circ$$

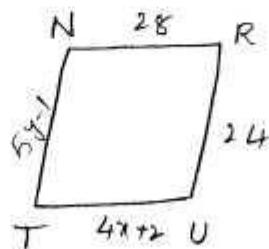
$$40 + 60 + y = 180$$

$$y + 100 = 180$$

$$y = 180 - 100$$

$$\boxed{y = 80^\circ}$$

7.



Opposite sides are equal

$$5y - 1 = 24$$

$$4x + 2 = 28$$

$$5y = 24 + 1$$

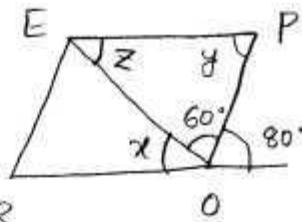
$$4x = 28 - 2$$

$$5y = 25$$

$$4x = 26$$

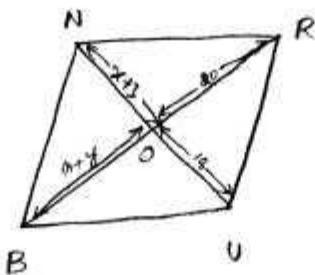
$$\boxed{y = 5}$$

$$x = \frac{26}{4} = \frac{13}{2}$$



22

$$\therefore x = 6.5, y = 5$$



'O' bisects the \overline{BR}

$$\overline{BO} = \overline{OR}$$

$$x + y = 20 \rightarrow ①$$

'O' bisects the \overline{NV}

$$\overline{NO} = \overline{OV}$$

$$x + 3 = 18 \rightarrow ②$$

$$x = 18 - 3$$

$$\boxed{x = 15}$$

Substitute x value in ①

$$15 + y = 20$$

$$y = 20 - 15$$

$$\boxed{y = 5}$$

8.

In $ABCD$ parallelogram.

$$\angle A + \angle B = 180^\circ \quad (\because \text{Adjacent angles are supplementary})$$

$$120^\circ + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 120^\circ$$

$$\angle B = 60^\circ$$

In $PQRS$ parallelogram

$$\angle P = \angle R \quad (\because \text{opposite angles are equal})$$

$$\angle P = 50^\circ$$

In $\triangle PBX$

$$\angle P + \angle B + x = 180^\circ \quad (\because \text{sum of angles in triangle})$$

$$50^\circ + 60^\circ + x = 180^\circ$$

$$110^\circ + x = 180^\circ$$

$$x = 180^\circ - 110^\circ$$

$$\boxed{x = 70^\circ}$$

9.

(i) $\angle CAD = ?$

$$\angle CBD = \angle ADB \quad (\because AD \parallel BC)$$

$$\angle ADB = 46^\circ$$

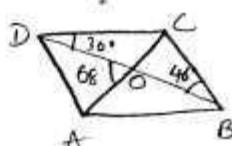
In $\triangle ADO$

$$\angle CAD + \angle ADB + 68^\circ = 180^\circ$$

$$\angle CAD + 46^\circ + 68^\circ = 180^\circ$$

$$\angle CAD + 114^\circ = 180^\circ$$

$$\boxed{\angle CAD = 66^\circ}$$



$\therefore \angle ACD = ?$

$$\angle DOA + \angle DOC = 180^\circ \quad (\because \text{straight line})$$

$$68 + \angle DOC = 180^\circ$$

$$\angle DOC = 180 - 68$$

$$\angle DOC = 112^\circ$$

In $\triangle DOC$

$$\angle CDO + \angle DOC + \angle ACD = 180^\circ$$

$$30 + 112 + \angle ACD = 180$$

$$\angle ACD + 142 = 180$$

$$\angle ACD = 38^\circ$$

$$\therefore \angle ADC = \angle ADO + \angle BDC$$

$$= 46 + 30$$

$$\angle ADC = 76^\circ$$

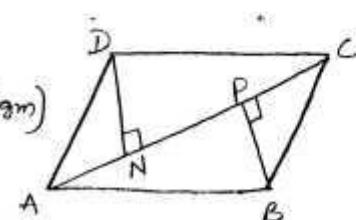
Q.

(i)

$$\overline{AD} = \overline{BC} \quad (\because \text{Sides are } \parallel)$$

$$\angle AND = \angle CPB = 90^\circ$$

$$\angle DAN = \angle BCP \quad (\because BC \parallel AD)$$



From $\triangle SAA$ Congruence

$$\text{And } \triangle AND \cong \triangle BPC$$

(ii) As $\triangle ABC \cong \triangle PQR$

$$\therefore \overline{AN} = \overline{CP}$$

ii. In parallelogram ABCR

$$\overline{AB} = \overline{RC} \quad (\because \text{opposite sides}) \rightarrow ①$$

In parallelogram ABCP

$$\overline{AB} = \overline{CP} \rightarrow ② \quad (\because \text{opposite sides})$$

$$① + ②$$

$$\overline{AB} + \overline{AB} = \overline{RC} + \overline{CP}$$

$$2\overline{AB} = \overline{RP} \rightarrow ③$$

By $2\overline{AC} = \overline{PV} \rightarrow ④$

$$2\overline{BC} = \overline{QR} \rightarrow ⑤$$

$$③ + ④ + ⑤$$

$$2(\overline{AB} + \overline{AC} + \overline{BC}) = \overline{PV} + \overline{QR} + \overline{RP}$$



1.

- i. Square, Rhombus
 ii. Square, Rectangle

2.

- i) I. Opposite sides are equal and opposite sides are parallel
 II. Adjacent angles are cong . Supplementary
 iii) I. It has four sides
 II. Sum of all interior angles is 360°
 iv) I. All the sides are equal
 II. All interior angles are 90°
 III. Diagonals cut perpendicularly.
 v) I. Opposite sides are equal
 II. All the interior angles are 90°

3.

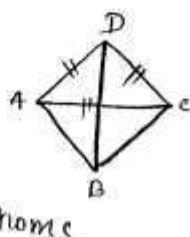
- i) parallelogram, square, rectangle, rhombus
 ii) square, rectangle, rhombus
 iii) square, rectangle

4. Given

Side of Rhombus = one

diagonal

of Rhombus



$$AD = DC = AC$$

$\therefore \triangle ADC$ is an equilateral triangle.

$$\therefore \angle ADC = \angle DAC = \angle DCA = 60^\circ$$

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$\triangle ACB$ is also an equilateral triangle

$$\therefore \angle CAB = \angle ABC = \angle BCA = 60^\circ$$

$$\angle ADB = \angle DAC + \angle CAB = 60^\circ + 60^\circ = 120^\circ$$

$$\angle DCB = \angle DCA + \angle ACB = 60^\circ + 60^\circ = 120^\circ$$

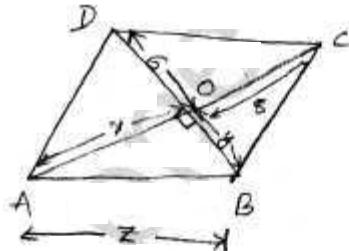
\therefore Angles of rhombus $60^\circ, 120^\circ, 60^\circ, 120^\circ$

5. $ABCD$ is a rhombus

diagonals of rhombus bisects each other

$$\therefore x = 8$$

$$y = 6.$$



Diagonals of rhombus cuts orthogonally

\therefore In $\triangle AOB$

$$\overline{OA}^2 + \overline{OB}^2 = \overline{AB}^2 \quad (\because \text{Pythagoras Theorem})$$

$$x^2 + y^2 = z^2$$

$$8^2 + 6^2 = z^2$$

$$z^2 = 64 + 36$$

$$z^2 = 100$$

$$\boxed{z = 10}$$

6. Given

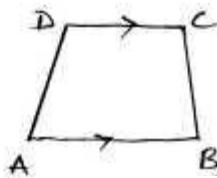
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ABCD is a trapezium

$$\angle A : \angle D = 5:7$$

$$\angle B = (3x+11)^\circ$$

$$\angle C = (5x-31)^\circ$$



From the property of trapezium

$$\angle A + \angle D = 180^\circ \quad \text{and} \quad \angle B + \angle C = 180^\circ$$

$$\text{let } \angle A, \angle D = 5y, 7y \quad 3x+11 + 5x-31 = 180^\circ$$

$$\therefore 5y + 7y = 180^\circ$$

$$8y - 20 = 180$$

$$12y = 180^\circ$$

$$8y = 180 + 20$$

$$y = 15^\circ$$

$$8y = 200$$

$$\angle A = 5y = 5 \times 15 = 75^\circ$$

$$8y = \frac{200}{8}$$

$$\angle D = 7y = 7 \times 15 = 105^\circ$$

$$\cancel{8y = 200}$$

$$\angle B = 3x+11.$$

$$= 3 \times 25 + 11$$

$$= 75 + 11$$

$$\angle B = 86^\circ$$

$$\angle C = 5x-31$$

$$= 5 \times 25 - 31$$

$$= 125 - 31$$

$$\angle C = 94^\circ$$

$$\therefore \angle A = 75^\circ, \angle B = 86^\circ, \angle C = 94^\circ, \angle D = 105^\circ$$

7.

$$\angle CEB : \angle ECB = 3:2$$

$$\angle CBE = 90^\circ \text{ (Angle in rectangle)}$$

\therefore In $\triangle ECB$

$$\text{Let } \angle CEB = 3x, \angle ECB = 2x.$$

$$\angle CEB + \angle ECB + \angle CBE = 180^\circ$$

$$3x + 2x + 90 = 180$$

$$5x + 90 = 180$$

$$5x = 180 - 90$$

$$5x = 90$$

$$x = \frac{90}{5}$$

$$x = 18^\circ$$

i) $\angle CEB = 3x = 3 \times 18 = 54^\circ$

ii) At 'C'

$$\angle CEB + \angle DCE = \angle DCB$$

$$54^\circ + \angle DCE = 90^\circ \quad (\text{Angle in rectangle})$$

$$\angle DCE = 90 - 54$$

$$\angle DCE = 36^\circ$$

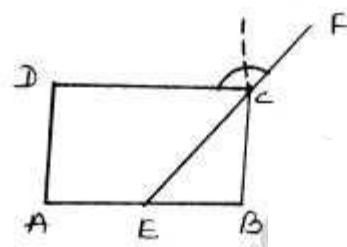
$$\angle DCE + \angle DCF = 180^\circ \quad (\text{Forms straight-line})$$

$$36^\circ + \angle DCF = 180^\circ$$

$$\angle DCF = 180 - 36$$

$$\angle DCF = 144^\circ$$

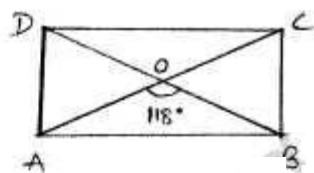
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8. Given ABCD is a rectangle

$$\overline{AO} = \overline{OB} (\because \text{intersect at } O)$$

$$\therefore \angle OAB = \angle OBA = x.$$



i) In $\triangle AOB$

$$\angle AOB + \angle ABO + \angle OAB = 180^\circ$$

$$118 + x + x = 180^\circ$$

$$2x = 180 - 118$$

$$2x = 62$$

$$x = \frac{62}{2}$$

$$x = 31^\circ$$

$$\therefore \angle ABO = 31^\circ$$

ii)

$$\angle AOB + \angle AOD = 180^\circ (\because \text{forms straight line})$$

$$118 + \angle AOD = 180^\circ$$

$$\angle AOD = 180 - 118$$

$$\angle AOD = 62^\circ$$

$$\overline{OD} = \overline{OA} (\because \text{diagonals bisect each other})$$

~~∴~~

$$\angle DAO = \angle ADO = y$$

In $\triangle AOD$

$$\angle DAO + \angle ADO + \angle AOD = 180^\circ$$

$$y + y + 62 = 180$$

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$$2y + 62 = 180$$

$$2y = 180 - 62$$

$$2y = 118$$

$$y = \frac{118}{2}$$

$$y = 59^\circ$$

$$\therefore \angle ADO = 59^\circ$$

iii) Similarly by taking $\triangle BOC$

We can prove $\angle OCB = 59^\circ$

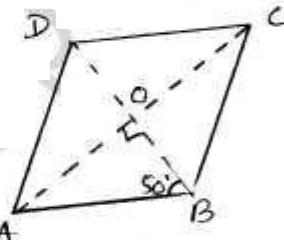
9. Given ABCD is a rhombus

$$\angle ABD = 50^\circ$$

In $\triangle AOB$

$$\angle AOB = 90^\circ (\because \text{In rhombus,}$$

diagonals cut perpendicularly)



$$\therefore \angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$90 + \angle OAB + 50 = 180$$

$$\angle OAB + 40 = 180$$

$$\angle OAB = 180 - 140$$

$$\boxed{\angle OAB = 40^\circ}$$

$$\therefore \angle CAB = \angle OAB = 40^\circ$$

$\therefore \angle BCD = ?$

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$$AB \parallel DC$$

$$\therefore \angle CAB = \angle ACD = 40^\circ$$

$\angle BCD = 2 \times \angle ACD$ (\because diagonal in rhombus, bisects
the angle)

$$\angle BCD = 2 \times 40^\circ$$

$$\angle BCD = 80^\circ$$

$\therefore \angle ADC$

$\angle ADC = \angle ABC$ (\because opposite angles are equal
in rhombus)

$$\angle ADC = 2 \times \angle ABD$$

$$\angle ADC = 2 \times 50^\circ$$

$$\angle ADC = 100^\circ$$

10. In a trapezium

$$\angle C + \angle B = 180^\circ$$

$$112^\circ + \angle B = 180^\circ$$

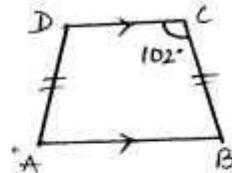
$$\angle B = 180 - 112$$

$$\angle B = 78^\circ$$

Given $\overline{AD} = \overline{CB}$

$$\therefore \angle C = \angle D = 102^\circ$$

$$\therefore \angle A = \angle B = 78^\circ$$



\therefore Angles in trapezium $78^\circ, 78^\circ, 102^\circ, 102^\circ$

11. Given $PQRS$ is a kite

$$\therefore \angle Q = \angle S = 120^\circ$$

$$x = 120^\circ$$

In a quadrilateral

$$\angle P + \angle Q + \angle R + \angle S = 360^\circ$$

$$y + 120 + 50 + 120 = 360$$

$$y + 290 = 360$$

$$y = 360 - 290$$

$$y = 70^\circ$$

