

Algebraic Expressions and Identities

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1. Exercise 10.1

i) Given expression $12x^2yz - 4xy^2$

Terms	Numerical coefficient	Literal coefficient
$12x^2yz$	12	x^2yz
$-4xy^2$	-4	xy^2

ii) Given expression $8 + mn + nl - lm$

Terms	Numerical coefficient	Literal coefficient
8	8	-
mn	1	mn
nl	1	nl
$-lm$	-1	lm

iii) Given expression $\frac{x^2}{3} + \frac{4}{6} - xy^2$

Terms	Numerical co-efficient	Literal co-efficient
$\frac{x^2}{3}$	$\frac{1}{3}$	x^2
$\frac{4}{6}$	$\frac{1}{6}$	y
$-xy^2$	-1	xy^2

iv) Given expression $-4P + 2.3q + 1.7r$

Terms	Numerical co-efficient	Literal co-efficient
$-4P$	-4	P
$2.3q$	2.3	q
$1.7r$	1.7	r

Q.

- i) $5pxqr^2 \rightarrow$ Monomial
 ii) $3x^2y \div 2z \rightarrow$ Monomial
 iii) $-3 + 7x^2 \rightarrow$ Binomial
 iv) $\frac{5a^2 + 3b^2 + c}{2} \rightarrow$ Trinomial
 v) $7x^5 - \frac{3x}{y} \rightarrow$ Binomial
 vi) $5p \div 3q - 3pq^2 \rightarrow$ Binomial

3.

i) $\frac{2}{5}x^4 - \sqrt{3}x^3 + 5x - 1$

It is polynomial of degree 4

ii) $7x^3 - \frac{3}{x^2} + \sqrt{5}$

due to $-3x^{-2}$ term, It is not called as polynomial

iii) \therefore It is NOT Polynomial

iv) $4a^3b^2 - 3ab^4 + 5ab + \frac{2}{3}$

It is a polynomial of degree 5

v) $2x^2y - \frac{3}{xy} + 5y^3 + \sqrt{3}$

due to negative power in the $-3(xy)^{-1}$

\therefore It is NOT a Polynomial

4.

i) Arrange terms for column method

$$\begin{array}{r}
 ab - bc \\
 0 + bc - ca \\
 -ab \quad 0 \quad + ca \\
 \hline
 0 \quad 0 \quad + 0
 \end{array}$$

$$\therefore ab - bc + bc - ca + ca - ab = 0$$

ii) Arrange terms in columns

$$\begin{array}{r}
 5pq^2 + 4pq + 7 \\
 -2pq^2 + 9pq + 3 \\
 \hline
 3pq^2 + 13pq + 10
 \end{array}$$

iii) Arrange terms in columns

$$\begin{array}{r}
 l^2 + m^2 + n^2 + 0 + 0 + 0 \\
 0 + 0 + 0 + lm + mn + 0 \\
 0 + 0 + 0 + 0 + mn + nl \\
 0 + 0 + 0 + lm + 0 + nl \\
 \hline
 l^2 + m^2 + n^2 + 2lm + 2mn + 2nl
 \end{array}$$

iv) Arrange terms in columns

$$\begin{array}{r}
 4x^3 - 7x^2 + 0x + 9 \\
 \cancel{8} + 3x^2 + 5x + 4 \\
 7x^3 + 0 - 11x + 1 \\
 0 + 6x^2 - 13x + 0 \\
 \hline
 10x^3 + 2x^2 - 29x + 14
 \end{array}$$

5.

$$\text{i)} \quad 14a - 5ab + 7b - 5$$

$$\begin{array}{r} 8a + 3ab - 2b + 7 \\ (-) \qquad \qquad \qquad (+) \qquad - \\ \hline 6a - 8ab + 9b - 12 \end{array}$$

ii)

$$12xy - 3yz - 4zx + 5xyz$$

$$\begin{array}{r} 8xy + 4yz + 5zx + 0 \\ (-) \qquad \qquad \qquad (+) \qquad + \\ \hline 4xy - 7yz - 9zx + 5xyz \end{array}$$

iii)

$$4pq^2 + 3$$

$$4pq^2 + 3pq + 5pq^2$$

iv)

$$5p^2q^2 - 2pq^2 + 5pq - 11q^2 - 3p + 18$$

$$\begin{array}{r} 4p^2q^2 + 5pq^2 - 3pq + 7q^2 - 8p - 10 \\ (-) \qquad (-) \qquad (+) \qquad (+) \qquad (+) \qquad + \\ \hline p^2q^2 - 7pq^2 + 8pq - 18q^2 + 5p + 28 \end{array}$$

6. Horizontal method

$$3x^2 + 5xy + 7y^2 + 3 \rightarrow ①$$

$$18 - 3p - 11q + 8pq + 2pq^2$$

$$2x^2 - 4xy - 3y^2 + 7 \rightarrow ②$$

$$9x^2 - 8xy + 11y^2 \rightarrow ③$$

$$\textcircled{1} + \textcircled{2} - \textcircled{3}$$

$$\textcircled{3} - [\textcircled{1} + \textcircled{2}]$$

$$9x^2 - 8xy + 11y^2 - [3x^2 + 5xy + 7y^2 + 3 + 2x^2 - 4xy - 3y^2 + 7]$$

$$9x^2 - 8xy + 11y^2 - [5x^2 + xy + 4y^2 + 10]$$

$$4x^2 - 9xy.$$

$$9x^2 - 8xy + 11y^2 - 5x^2 - xy - 4y^2 - 10$$

$$\underline{4x^2 - 9xy + 7y^2 - 10}$$

7.

$$\text{det} \quad 3a^2 + 5ab +$$

$$\text{det} \quad 3a^2 - 5ab - 2b^2 - 3 \rightarrow ①$$

$$5a^2 - 7ab - 3b^2 + 3a \rightarrow ②$$

$$\text{do } ② - ①$$

$$5a^2 - 7ab - 3b^2 + 3a$$

$$\underline{\begin{array}{cccccc} 3a^2 & - 5ab & - 2b^2 & + 0 & - 3 \\ (-) & (+) & (+) & (-) & (+) \end{array}}$$

$$2a^2 - 2ab - b^2 + 3a + 3$$

8.

$$\text{Perimeter of triangle } P = 7P^2 - 5P + 11 \rightarrow ①$$

$$\text{Sides } S_1 = P^2 + 2P - 1 \rightarrow ②$$

$$S_2 = 3P^2 - 6P + 3 \rightarrow ③$$

$$S_3 = ?$$

$$P = S_1 + S_2 + S_3$$

$$S_3 = P - (S_1 + S_2)$$

$$= 7P^2 - 5P + 11 - [P^2 + 2P - 1 + 3P^2 - 6P + 3]$$

$$= 7P^2 - 5P + 11 - [4P^2 - 4P + 2]$$

$$= 3P^2 - 9P$$

Hence,

$$= 7P^2 - 5P + 11 - 4P^2 + 4P - 2$$

Third Side of

Triangle

$$\underline{3P^2 - P + 9}$$

Exercise 10.2

1.

$$\Rightarrow 4x^3 \text{ and } -3xy$$

$$4x^3 \times -3xy$$

$$(4x^3) \times (-3xy)$$

$$-12x^4y$$

$$\therefore 2xyz \text{ and } 0$$

$$(2xyz) \times 0$$

0

$$\therefore \left\{ -\frac{2}{3}pq^r, \frac{3}{4}pqr \text{ and } 5pqr \right.$$

$$\left(-\frac{2}{3}pq^r \right) \times \left(\frac{3}{4}pqr \right) (5pqr)$$

$$\left(-\frac{2}{3} \times \frac{3}{4} \times 5 \right) \times (pqr) \times (pqr^2) \times (pqr^2)$$

$$-\frac{5}{2} p^4 q^4 r$$

$$\therefore -7ab, -3a^3 \text{ and } -\frac{2}{7}ab^2$$

$$(-7ab) \times (-3a^3) \times \left(-\frac{2}{7}ab^2 \right)$$

$$\left(-7 \times -3 \times -\frac{2}{7} \right) \times ab \times a^3 \times ab^2$$

$$\underline{\underline{-6a^5 b^3}}$$

$$\Rightarrow -\frac{1}{2}x^7, -\frac{3}{5}xy, \frac{2}{3}yz \text{ and } \frac{5}{7}xyz$$

$$\left(-\frac{1}{2}x^7\right) \times \left(-\frac{3}{5}xy\right) \times \left(\frac{2}{3}yz\right) \times \left(\frac{5}{7}xyz\right)$$

$$\left(-\frac{1}{2} \times -\frac{3}{5} \times \frac{2}{3} \times \frac{5}{7}\right) \times x^7 \times xy \times yz \times xyz$$

$$\frac{1}{7}x^4y^3z^2$$

Q. $\Rightarrow (3x - 5y + 7z) \times (-3xyz)$

$$(3x \times -3xyz) + (-5y \times -3xyz) + (7z \times -3xyz)$$

$$\underline{-9x^2yz + 15xy^2z + 21xyz^2}$$

Q. ii) $2P^2 (2P^2 - 3Pq + 5q^2 + 5) \times (-2Pq)$

$$(2P^2 \times -2Pq) + (-3Pq \times -2Pq) + (5q^2 \times -2Pq) +$$

$$(5 \times -2Pq)$$

$$\underline{-4P^3q + 6P^2q^2 - 10Pq^3 - 10Pq^2}$$

iii) $\left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right) \times (35ab)$

$$\left(\frac{2}{3}a^2b \times 35ab\right) + \left(-\frac{4}{5}ab^2 \times 35ab\right) + \left(\frac{2}{7}ab \times 35ab\right) + (3 \times 35ab)$$

$$\underline{\frac{70}{3}a^3b - 28a^2b^3 + 10a^2b^2 + 105ab}$$

$$\text{iv) } (4x^2 - 10xy + 7y^2 - 8x + 4y + 3) \times (3xy)$$

$$(4x^2 \times 3xy) + (-10xy \times 3xy) + (7y^2 \times 3xy) + (-8x \times 3xy) \\ + (4y \times 3xy) + (3 \times 3xy)$$

$$12x^3y - 30x^2y^2 + 21xy^3 - 24x^2y + 12xy^2 + 9xy$$

3.

i) Given length (l) = pqr

breadth (b) = pqr^2

Rectangle - Area = $l \times b$

$$= (pqr) \times (pqr^2)$$

Area = $p^3q^3r^3$

ii) Given

length (l) = $5xy$

breadth (b) = $7xy^2$

Rectangle - Area = $l \times b$

$$= (5xy) \times (7xy^2)$$

Area = $35x^2y^3$

4. i) Given length (l) = $5ab$

breadth (b) = $3a^2b$

height (h) = $7a^4b^2$

Volume of rectangular box = $l \times b \times h$

$$= 5ab \times 3a^2b \times 7a^4b^2$$

$$= (3 \times 5 \times 7) \times ab \times a^2b \times a^4b^2$$

Volume = $\underline{\underline{105a^7b^4}}$

ii) Given
 length (l) = $2pq$
 breadth (b) = $4q^2$
 height (h) = $8rp$

$$\begin{aligned}\text{Volume of rectangular box} &= l \times b \times h \\ &= (2pq) \times (4q^2) \times (8rp) \\ &= (2 \times 4 \times 8) pqr \times q^2 \times rp\end{aligned}$$

$64 p^r q^3 r = 64 P^r q^3 r$

5.

$\therefore x^r(3 - 2x + x^r)$

$$3x^r - 2x^{r+1} + x^{2r}$$

For $x=1$

$$3 \times 1^r - 2 \times 1^{r+1} + 1^r$$

$$3 \times 1 - 2 \times 1 + 1$$

$$3 - 2 + 1$$

$$\underline{\underline{=}} \quad 2$$

for $x=1$

For $x=-1$

$$3x^r - 2x^{r+1} + x^{2r}$$

$$3 \times (-1)^r + (-2)(-1)^{r+1} + (-1)^{2r}$$

$$3 \times 1 + (-2 \times -1) + 1$$

$$3 + 2 + 1$$

6 for $x=-1$

$$\text{For } x = \frac{2}{3}$$

$$3x^7 - 2x^3 + x^4$$

$$3 \times \left(\frac{2}{3}\right)^7 - 2 \times \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4$$

$$\frac{3 \times 4}{9} - 2 \times \frac{8}{27} + \frac{16}{81}$$

$$\frac{12}{9} - \frac{16}{27} + \frac{16}{81}$$

$$\frac{108 - 48 + 16}{81}$$

$$\frac{76}{81} \text{ for } x = \frac{2}{3}$$

$$\text{For } x = -\frac{1}{2}$$

$$3x^7 - 2x^3 + x^4$$

$$3 \times \left(-\frac{1}{2}\right)^7 - 2 \times \left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^4$$

$$\frac{3 \times 1}{4} - 2 \times \left(-\frac{1}{8}\right) + \frac{1}{16}$$

$$\frac{3}{4} + \frac{2}{8} + \frac{1}{16}$$

$$\frac{12 + 4 + 1}{16}$$

$$\frac{17}{16} \text{ for } x = -\frac{1}{2}$$

$$\Rightarrow 5xy(3x+4y-7) - 3y(xy-x^2+9) - 8$$

$$(5x^2y + 20xy^2) - 5xy^2 + (-3xy^2) + (-3y^2x - 27y) - 8$$

$$+ (-3y \times 9) - 8$$

$$15x^2y + 20xy^2 - 35xy^2 - 3xy^2 + 3x^2y - 27y - 8$$

$$\underline{18x^2y + 17xy^2 - 62xy - 8}$$

For $x=2, y=-1$

$$18(2^2)(-1) + (17 \times 2 \times (-1)^2) - (62 \times 2 \times -1) - 8$$

$$-72 + 34 + 124 - 8$$

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6.

$$\Rightarrow \text{First expression} = 4p(2-p^2) = 8p - 4p^3$$

$$\text{Second expression} = 8p^3 - 3p$$

$$\text{Required sum} = (8p - 4p^3) + (8p^3 - 3p)$$

$$= 4p^3 + 5p$$

$$\Rightarrow \text{First expression} = 7xy(8x+2y-3) = 56x^2y + 14xy^2 - 21xy$$

$$\text{Second expression} = 3y(4x^2y - 5xy + 8xy^2)$$

$$= 12x^2y^2 - 15xy^2 + 24xy^3$$

$$\text{Required sum} = (56x^2y + 14xy^2 - 21xy) + (12x^2y^2 - 15xy^2 + 24xy^3)$$

ii) First expression = $7xy(8x+2y-3) = 56x^2y + 14xy^2 - 21xy$
 Second expression = $4x^2y^2(3y - 7x + 8) = 12x^2y^3 - 28x^3y^2 + 32x^2y^2$
 Required sum = $(56x^2y + 14xy^2 - 21xy) + (12x^2y^3 - 28x^3y^2 + 32x^2y^2)$
 $= 12x^2y^3 - 28x^3y^2 + 56x^2y + 46xy^2 - 21xy$

7.

iii) First expression = $6x(x-y+z) - 3y(x+y-z)$
 $= 6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz$
 Second expression = $2z(-x+y+z) = 6x^2 + 6xz - 6xy + 3yz - 3y^2 \rightarrow ①$
 $= -2xz - 2yz + 2z^2 \rightarrow ②$
 $② - ① = (-2xz - 2yz + 2z^2) - (6x^2 + 6xz - 6xy + 3yz - 3y^2)$
 $= -2xz - 2yz + 2z^2 - 6x^2 - 6xz + 6xy - 3yz + 3y^2$
 $= -6x^2 + 3y^2 + 2z^2 + 6xy - 5yz - 8xz$

iv)

First expression = $7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$
 $= 7x^3y - 14x^2y^2 + 21xy^3 - 8x^3y + 32x^2y^2$
 $= -7x^3y - 8x^2y^2 + 21xy^3 - 56x^2y^2$
 First expression = $-x^3y - 8x^2y - 70x^2y^2 + 32x^2y + 21xy^3 \rightarrow ③$
 $+ 21xy^3 \rightarrow ④$

$$\text{Second expression} = 3y(4x^2y - 5xy + 8xy^2)$$

$$= 12x^2y^2 - 15xy^2 + 24xy^3 \rightarrow ②$$

$$\begin{aligned}
 ② - ① &= 12x^2y^2 - 15xy^2 + 24xy^3 - (-x^3y + 21xy^3 - 70x^2y^2 \\
 &= 12x^2y^2 - 15xy^2 + 24xy^3 + x^3y + 32x^2y^2 \\
 &\quad - 21xy^3 + 70x^2y^2 \\
 &\quad - 82x^2y^2 + x^3y + 3xy^3 - 15xy^2 - 32x^2y \\
 &= x^3y + 3xy^2 + 82x^2y^2 - 15xy^2 - 32x^2y
 \end{aligned}$$

Exercise 10.3

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1.

i) $(5x-2)(3x+4)$

$$5x(3x+4) - 2(3x+4)$$

$$15x^2 + 20x - 6x - 8$$

$$15x^2 + 14x - 8$$

ii) $(ax+b)(cx+d)$

$$ax(cx+d) + b(cx+d)$$

$$acx^2 + adx + bcx + bd$$

$$acx^2 + (ad+bc)x + bd$$

iii) $(4p-7)(2-3p)$

$$4p(2-3p) - 7(2-3p)$$

$$8p - 12p^2 - 14 + 21p$$

$$-12p^2 + 29p - 14$$

iv) $(2x^2+3)(3x-5)$

$$2x^2(3x-5) + 3(3x-5)$$

$$6x^3 - 10x^2 + 9x - 15$$

$$6x^3 - 10x^2 + 9x - 15$$

$$\nabla \rightarrow (1.5a - 2.5b)(1.5a + 2.5b)$$

$$1.5a(1.5a + 2.5b) - 2.5b(1.5a + 2.5b)$$

$$(1.5 \times 1.5)a^2 + (1.5 \times 2.5)ab - (2.5 \times 1.5)ab - (2.5 \times 2.5)b^2$$

$$2.25a^2 + 3.75ab - 3.75ab - 6.25b^2$$

$$2.25a^2 + 0 - 6.25b^2$$

$$2.25a^2 - 6.25b^2$$

—————

$$\nabla \rightarrow \left(\frac{3}{7}p^2 + 4q^2\right)\left(7\left(p^2 - \frac{3}{4}q^2\right)\right)$$

$$\frac{3}{7}p^2 \times \left(7p^2 - \frac{21}{4}q^2\right) + 4q^2 \left(7p^2 - \frac{21}{4}q^2\right)$$

$$3p^4 - \frac{9}{4}q^2 p^2 + 28pq^2 - 21q^4$$

$$3p^4 + \frac{103}{4}pq^2 - 21q^4$$

2.

$$(x-2y+3)(x+2y)$$

$$x-2y+3$$

$$x+2y$$

—————
2xy - 4y² + 6y

$$x^2 - 2xy + 0 + 0 + 3x$$

—————
x² + 0 - 4y² + 6y + 3x

$$x^2 + 3x + 6y - 4y^2$$

—————

$$\text{ii)} (3-5x+2x^2)(4x-5)$$

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$$\begin{array}{r} 3-5x+2x^2 \\ \underline{-} 4x-5 \\ 2x^2-5x+3 \\ \underline{-} 4x-5 \\ 8x^3-20x^2+12x \\ \underline{-} 10x^2-25x-15 \\ 8x^3-30x^2-13x-15 \end{array}$$

$$3. \text{ ii)} (3x^2-2x-1)(2x^2+x-5)$$

$$\begin{array}{r} 3x^2-2x-1 \\ 2x^2+x-5 \\ \hline 6x^4-4x^3-2x^2 \\ + 3x^3-2x^2-x \\ \hline -15x^2+10x+5 \\ \hline 6x^4-x^3-19x^2+9x+5 \end{array}$$

$$\text{i)} (2-3y-5y^2)(2y-1+3y^2)$$

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$$\begin{array}{r} -5y^2 - 3y + 2 \\ 3y^2 + 2y - 1 \\ \hline -15y^4 - 9y^3 + 6y^2 \\ -10y^3 - 6y^2 + 4y \\ + 5y^2 + 3y - 2 \\ \hline -15y^4 - 19y^3 + 5y^2 + 7y - 2 \end{array}$$

4.

$$\text{i)} (x^2+3)(x-3) + 9$$

$$x^2(x-3) + 3(x-3) + 9$$

$$x^3 - 3x^2 + 3x - 9 + 9$$

$$x^3 - 3x^2 + 3x$$

$$\text{ii)} (x+3)(x-3)(x+4)(x-4)$$

$$[x(x-3)+3(x-3)][x(x-4)+4(x-4)]$$

$$[x^2 - 3x + 3x - 9][x^2 - 4x + 4x - 16]$$

$$[x^2 - 9][x^2 - 16]$$

$$x^2(x^2 - 16) - 9(x^2 - 16)$$

$$x^4 - 16x^2 - 9x^2 + 144$$

$$x^4 - 25x^2 + 144$$

$$\begin{aligned}
 \text{iii)} & (x+5)(x+6)(x+7) \\
 & [(x+5)(x+6)](x+7) \\
 & [x(x+6) + 5(x+6)](x+7) \\
 & (x^2 + 6x + 5x + 30)(x+7) \\
 & (x^2 + 11x + 30)x + (x^2 + 11x + 30)7 \\
 & (x^3 + 11x^2 + 30x + 7x^2 + 77x + 210) \\
 & \underline{x^3 + 18x^2 + 107x + 210}
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} & (p+q+r)(2p-q+r) - 4qr \\
 & p(2p-q+r) + q(2p-q+r) - 2r(2p-q+r) - 4qr \\
 & 2p^2 - pq + pr + 2pq - qr + qr - 4pr + 2qr - 2r^2 - 4qr
 \end{aligned}$$

$$\begin{aligned}
 \text{v)} & 2p^2 - qr^2 - 2r^2 + pq - qr - 3pr \\
 & (p+q)(r+s) + (p-q)(r-s) - 2(pr+qs) \\
 & p(r+s) + q(r+s) + p(r-s) - q(r-s) - 2pr - 2qs \\
 & pr + ps + qr + qs + pr - ps - qr + qs - 2pr - 2qs \\
 & 2pr - 2pr + ps - ps + qr - qr + 2qs - 2qs \\
 & 0 + 0 + 0 + 0 = 0
 \end{aligned}$$

$$vi) (x+y+z)(x-y+z) + (x+y-z)(-x+y+z) - 4zx$$

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$$x(x-y+z) + y(x-y+z) + z(x-y+z) + x(-x+y+z)$$

$$+ y(-x+y+z) - z(-x+y+z) - 4zx$$

$$x^2 - xy + xz + xy - y^2 + yz + xz - yz + z^2 - x^2 + xy + xz$$

$$\leftarrow -xy + y^2 + yz + xz - yz - z^2 - 4zx$$

$$x^2 - x^2 + 2xy - 2xy + 4yz - 4xz - y^2 + y^2 + 2yz - 2yz$$

$$+ z^2 - z^2$$

$$0 + 0 + 0 + 0 + 0 + 0 = 0$$

5.

$$\text{Sides of rectangle } S_1 = 5x^2 + 25xy + 4y^2$$

$$S_2 = 2x^2 - 2xy + 3y^2$$

$$\text{Area of rectangle} \approx S_1 \times S_2$$

$$A = (5x^2 + 25xy + 4y^2)(2x^2 - 2xy + 3y^2)$$

$$A = \underline{\underline{10x^4 + 40x^3y - 27x^2y^2 + 67xy^3 + 12y^4}}$$

$$5x^2 + 25xy + 4y^2$$

$$2x^2 - 2xy + 3y^2$$

$$10x^4 + 50x^3y + 8x^2y^2$$

$$- 10x^3y - 50x^2y^2 - 8xy^3$$

$$+ 152y^4 + 75xy^3 + 12y^4$$

$$\underline{\underline{10x^4 + 40x^3y - 27x^2y^2 + 67xy^3 + 12y^4}}$$

Exercise 10.4

1.

i)

$$-39pq^7r^5 \div -24p^3q^3r$$

$$= \frac{-39pq^7r^5}{-24p^3q^3r}$$

$$= \frac{13r^4}{8pq^4}$$

ii)

$$\begin{aligned} \frac{-3}{4}a^7b^3 \div \frac{6}{7}a^3b^2 &= \frac{-\frac{3}{4}a^7b^3}{\frac{6}{7}a^3b^2} \\ &= -\frac{3}{4} \times \frac{7}{6} \cdot \frac{b}{a} \\ &= -\frac{7b}{8a} \end{aligned}$$

2. i)

$$\begin{array}{r} 3x^3 - \frac{8}{3}x^2 - 4 \\ \hline 3x \Big) 9x^4 - 8x^3 - 12x + 3 \\ \underline{-9x^4} \\ 0 - 8x^3 \\ \underline{(+)} - 8x^3 \\ 0 - 12x \\ \underline{\underline{(+)} - 12x} \\ 0 + 3 \end{array}$$

Quotient = $3x^3 - \frac{8}{3}x^2 - 4$; Remainder = 3

ii)

$$\begin{array}{r}
 -7q^2 + 16pq \\
 \hline
 -2pq \sqrt{14p^2q^3 - 32p^3q^2 + 15pq^4 - 22p + 18q} \\
 \underline{-14p^2q^3} \\
 0 - 32p^3q^2 + 15pq^4 - 22p + 18q \\
 \underline{-32p^3q^2} \\
 0 + 15pq^4 - 22p + 18q
 \end{array}$$

Quotient = $-7q^2 + 16pq$, Remainder = $+15pq^4 - 22p + 18q$

iii) $6x^4$

$$\begin{array}{r}
 3x + 5 \\
 \hline
 2x+1 \sqrt{6x^4 + 13x^2 - 15} \\
 \underline{-6x^4 - 3x^3} \\
 + 10x^3 + 5 \\
 \underline{-10x^3 - 5} \\
 0
 \end{array}$$

Quotient = $3x + 5$, Remainder = 0

iv)

$$\begin{array}{r}
 y^2 - y - 1 \\
 \hline
 1+y \sqrt{y^3 + 1} \\
 \underline{-y^3 - 0 - y^2} \\
 -y^2 + 1 \\
 \underline{-y^2 - 0 - y} \\
 -y - 1 \\
 \underline{-1 - y} \\
 -1 - y
 \end{array}$$

2

Quotient = $y^2 - y - 1$: Remainder = 2

iii)

$$\begin{array}{r} -2x+3 \\ x+1 \sqrt{-2x^2+x+5} \\ \underline{-2x^2-2x} \\ (+) \quad (+) \\ \hline 3x+5 \\ +3x+3 \\ \hline \underline{2} \end{array}$$

Quotient = $-2x+3$, Remainder = 2

iv)

$$\begin{array}{r} x^2-4x+4 \\ x-2 \sqrt{x^3-6x^2+12x-8} \\ \underline{x^3-2x^2} \\ (-) \quad (+) \\ \hline -4x^2+12x-8 \\ -4x^2+8x \\ \hline (+) \quad (-) \\ 4x-8 \\ 4x-8 \\ \hline 0 \end{array}$$

Quotient = x^2-4x+4 , Remainder = 0

4. i)

$$\begin{array}{r} 2x^2+5x+3 \\ 3x-7 \sqrt{6x^3+x^2-26x-25} \\ \underline{6x^3-14x^2} \\ (-) \quad (+) \\ \hline 15x^2-26x-25 \\ 15x^2-35x \\ \hline (-) \quad (+) \\ 9x-25 \\ 9x-21 \\ \hline -4 \end{array}$$

Quotient = $2x^2+5x+3$, Remainder = -4

ii)

$$\begin{array}{r} m^2 - 5m - 5 \\ m-1 \sqrt{m^3 - 6m^2 + 7} \\ \underline{-m^3 + m^2} \\ -5m^2 + 7 \\ \underline{-5m^2 + 5m + 5m} \\ \underline{\underline{7 - 5m}} \\ \underline{\underline{5 - 5m}} \end{array}$$

$$\text{Quotient} = m^2 - 5m - 5, \text{ Remainder} = 2$$

5.

i)

$$\begin{array}{r} a+1 \\ a^3+a^2+a+1 \sqrt{a^3+2a^2+2a+1} \\ \underline{-a^3-a^2-a} \\ a^2+a+1 \\ \underline{-a^2-a-1} \\ \underline{\underline{0}} \end{array}$$

$$\text{Quotient} = a+1, \text{ Remainder} = 0$$

ii)

$$\begin{array}{r} 4x-3 \\ 3x^2-2x+5 \sqrt{12x^3 - 17x^2 + 26x - 18} \\ \underline{-12x^3 + 8x^2 - 20x} \\ -9x^2 + 6x - 18 \\ \underline{-9x^2 + 6x - 15} \\ \underline{\underline{-3}} \end{array}$$

$$\text{Quotient} = 4x-3, \text{ Remainder} = -3$$

6. Given Area of rectangle = $8x^2 - 45y^2 + 18xy$

$$\text{one side } S_1 = 4x + 15y$$

$$\text{other side } S_2 = ?$$

$$A = S_1 \times S_2$$

$$S_2 = A \div S_1$$

$$\begin{array}{r}
 & 2x - 3y \\
 4x + 15y & \sqrt{8x^2 - 45y^2 + 18xy} \\
 & \underline{- (8x^2 + 15xy)} \\
 & -45y^2 + 12xy \\
 & \underline{- (-45y^2 - 12xy)} \\
 & 0
 \end{array}$$

$$\text{Quotient} = 2x - 3y, \text{ Remainder} = 0$$

\therefore length of other side of rectangle

$$S_2 = 2x - 3y$$

$$1. \text{ i)} (3x+5)(3x+5)$$

$$(3x+5)^2 = (3x)^2 + 2(3x) \cdot 5 + 5^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$$

$$= 9x^2 + 30x + 25$$

$$\text{ii)} (9y-5)(9y-5)$$

$$(9y-5)^2 = (9y)^2 - 2(9y)5 + (-5)^2 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$

$$= 81y^2 - 90y + 25$$

$$\text{iii)} (4x+11y)(4x-11y)$$

$$(4x)^2 - (11y)^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$$

$$16x^2 - 121y^2$$

$$\text{iv)} \left(\frac{3}{2}m + \frac{2}{3}n\right) \left(\frac{3}{2}m - \frac{2}{3}n\right)$$

$$\left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$$

$$\frac{9}{4}m^2 - \frac{4}{9}n^2$$

$$\text{v)} \left(\frac{2}{a} + \frac{5}{b}\right) \left(\frac{2}{a} + \frac{5}{b}\right)$$

$$\left(\frac{2}{a} + \frac{5}{b}\right)^2 = \left(\frac{2}{a}\right)^2 + 2 \cdot \frac{2}{a} \cdot \frac{5}{b} + \left(\frac{5}{b}\right)^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$$

$$= \frac{4}{a^2} + \frac{20}{ab} + \frac{25}{b^2}$$

$$\text{vii)} \quad \left(\frac{p^2}{2} + \frac{2}{q^2}\right) \quad \left(\frac{p^2}{2} - \frac{2}{q^2}\right)$$

$$\left(\frac{p^2}{2}\right)^2 - \left(\frac{2}{q^2}\right)^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$$

$$\frac{p^4}{4} - \frac{4}{q^4}$$

2.

$$\text{i). } 81^2 = (80+1)^2$$

$$= 80^2 + 2 \cdot 80 \cdot 1 + 1^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$$

$$= 6400 + 160 + 1$$

$$81^2 = 6561$$

$$\text{ii). } 97^2 = (100-3)^2$$

$$= (100)^2 - 2 \cdot 100 \cdot 3 + 3^2 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$

$$= 10000 - 600 + 9$$

$$= 9409$$

$$\text{iii)} \quad 105^2 = (100+5)^2$$

$$= 100^2 + 2 \cdot 100 \cdot 5 + 5^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$$

$$= 10000 + 1000 + 25$$

$$= 11025$$

$$\text{iv)} \quad 997^2 = (1000-3)^2$$

$$= (1000)^2 - 2 \cdot 1000 \cdot 3 + 3^2 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$

$$= 1000000 - 6000 + 9$$

$$997^2 = 994009$$

$$\text{v)} \quad 6.1^2 = (6+0.1)^2$$

$$= 6^2 + 2 \cdot 6(0.1) + 0.1^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$$
$$= 36 + 1.2 + 0.01$$

$$6.1^2 = 37.21$$

$$\text{vi)} \quad 496 \times 504 = (500-4)(500+4)$$

$$= 500^2 - 4^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$$
$$= 250000 - 16$$

$$496 \times 504 = 249984$$

$$\text{vii)} \quad 20.5 \times 19.5 = (20+0.5)(20-0.5)$$

$$= 20^2 - 0.5^2 \quad (\because (a+b)(a-b) = a^2 - b^2)$$
$$= 400 - 0.25$$

$$20.5 \times 19.5 = 399.75$$

$$\text{viii)} \quad 9.6^2 = (10-0.4)^2$$

$$= 10^2 - 2 \cdot 10(0.4) + (0.4)^2 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$
$$= 100 - 8 + 0.16$$

$$9.6^2 = 92.16$$

5.

$$\text{i)} (pq + 5r)^2 = (pq)^2 + 2 \cdot pq \cdot 5r + (5r)^2 \quad (\because (a+b)^2 = a^2 + b^2 + 2ab)$$

$$= p^2q^2 + 10pqr + 25r^2$$

$$\text{ii)} \left(\frac{5}{2}a - \frac{3}{5}b\right)^2 = \left(\frac{5}{2}a\right)^2 - 2 \cdot \frac{5}{2}a \cdot \frac{3}{5}b + \left(\frac{3}{5}b\right)^2$$

$$(\because (a-b)^2 = a^2 - 2ab + b^2)$$

$$= \frac{25}{4}a^2 - 3ab + \frac{9}{25}b^2$$

$$\text{iii)} (\sqrt{2} \cdot a + \sqrt{3} \cdot b)^2 = (\sqrt{2} \cdot a)^2 + 2\sqrt{2} \cdot a \cdot \sqrt{3} \cdot b + (\sqrt{3} \cdot b)^2$$

$$(\because (a+b)^2 = a^2 + 2ab + b^2)$$

$$= 2a^2 + 2\sqrt{6}ab + 3b^2$$

$$\text{iv)} \left(\frac{2x}{3y} - \frac{3y}{2x}\right)^2 = \left(\frac{2x}{3y}\right)^2 - 2 \cdot \frac{2x}{3y} \cdot \frac{3y}{2x} + \left(\frac{3y}{2x}\right)^2$$

$$= \frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2} \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$

1

4.

$$\text{i. } (x+7)(x+3) = x^2 + (7+3)x + 7 \times 3 \quad (\because (x+a)(x+b) = x^2 + (a+b)x + ab)$$

$$= x^2 + 10x + 21$$

$$\text{ii. } (3x+4)(3x-5) = (3x)^2 + (4+(-5))(3x) + 4x - 5$$

$$(\because (x+a)(x+b) = x^2 + (a+b)x + ab)$$

$$= 9x^2 - 3x - 20$$

$$\text{iii. } (p^2+2q)(p^2-3q) = (p^2)^2 + (2q+(-3q))p^2 + 2q \cdot p^2 - 3q^2$$

$$(\because (x+a)(x+b) = x^2 + (a+b)x + ab)$$

$$= p^4 - 6p^2q^2 - 6q^2$$

$$= p^4 - p^2q^2 - 6q^2$$

$$\text{iv. } (abc+3)(abc-5) = (abc)^2 + (3+(-5)) \cdot abc + 3x-5$$

$$(\because (x+a)(x+b) = x^2 + (a+b)x + ab)$$

$$= (abc)^2 - 2abc - 15$$

5.

$$\text{i. } 203 \times 204 = (200+3)(200+4)$$

$$= (200)^2 + (3+4)200 + 3 \times 4 \quad (\because (x+a)(x+b) = x^2 + (a+b)x + ab)$$

$$= 40000 + 1400 + 12$$

$$= 41412$$

$$\text{ii. } 8.2 \times 8.7 = (8+0.2)(8+0.7)$$

$$= 8^2 + (0.2+0.7)8 + 0.2 \times 0.7 \quad (\because (x+a)(x+b) =$$

$$= 64 + 7.2 + 0.14 \quad x^2 + (a+b)x + ab)$$

$$= 71.34$$

$$\begin{aligned}
 \text{iii. } 107 \times 93 &= (100+7)(100-7) \\
 &= (100)^2 + (7+(-7)) \cdot 100 + 7 \times -7 \\
 &\quad (\because (a+b)(a+b) = a^2 + ab + b^2) \\
 &= 10000 + 0 \cdot 100 - 49 \\
 &= 9951
 \end{aligned}$$

6.

$$\text{i. } 53^2 - 47^2 = (53+47)(53-47) \quad (\because a^2 - b^2 = (a+b)(a-b))$$

$$\begin{aligned}
 &= (100)(6) \\
 &= 600
 \end{aligned}$$

$$\text{ii. } (2.05)^2 - (0.95)^2 = (2.05+0.95)(2.05-0.95)$$

$$\begin{aligned}
 &\approx 3 \times 0.1 \\
 &\approx 0.3
 \end{aligned}$$

$$\text{iii. } (14.3)^2 - (5.7)^2 = (14.3+5.7)(14.3-5.7)$$

$$\begin{aligned}
 &= (19)(9.6) \\
 &= 182.4
 \end{aligned}$$

7.

$$\text{i. } (2x+5y)^2 + (2x-5y)^2$$

$$(2x)^2 + (5y)^2 + 2 \cdot 2x \cdot 5y + (2x)^2 + (5y)^2 - 2 \cdot 2x \cdot 5y$$

$$\begin{aligned}
 &2(2x)^2 + 2(5y)^2 \quad (\because (a+b)^2 = a^2 + b^2 + 2ab) \\
 &2 \cdot [4x^2 + 25y^2] \quad (a-b)^2 = a^2 + b^2 - 2ab
 \end{aligned}$$

$$8x^2 + 50y^2$$

$$\text{ii)} \quad \left(\frac{7}{2}a - \frac{5}{2}b\right)^2 - \left(\frac{5}{2}a - \frac{7}{2}b\right)^2$$

III

$$\left(\frac{7}{2}a\right)^2 + \left(\frac{5}{2}b\right)^2 - 2 \cdot \frac{7}{2}a \cdot \frac{5}{2}b - \left[\left(\frac{5}{2}a\right)^2 + \left(\frac{7}{2}b\right)^2 - 2 \cdot \frac{5}{2}a \cdot \frac{7}{2}b\right]$$

$$\frac{49}{4}a^2 + \frac{25}{4}b^2 - 2 \cdot \frac{7}{2}a \cdot \frac{5}{2}b - \left[- \frac{25}{4}a^2 - \frac{49}{4}b^2 + 2 \cdot \frac{5}{2}a \cdot \frac{7}{2}b \right]$$

$$\left(\frac{49}{4} - \frac{25}{4}\right)a^2 + \left(\frac{25}{4} - \frac{49}{4}\right)b^2$$

$$\frac{24}{4} \cdot a^2 - \frac{24}{4} \cdot b^2$$

$$6(a^2 - b^2)$$

$$\text{iii)} \quad (p^2 - q^2)^2 + 2pq^2q$$

$$(p^2)^2 - 2 \cdot p^2 \cdot q^2 \cdot q + (q^2)^2 + 2pq^2q$$

$$p^4 - 2p^2q^2q + q^4 \cdot q^2 + 2pq^2q \quad (\because (a+b)^2 = a^2 + 2ab + b^2 \\ (a-b)^2 = a^2 - 2ab + b^2)$$

$$\underline{\underline{p^4 + q^4 \cdot q^2}}$$

8. LHS

$$\text{i). } (4x+7y)^2 - (4x-7y)^2$$

$$(4x)^2 + (7y)^2 + 2 \cdot 4x \cdot 7y - [(4x)^2 + (7y)^2 - 2 \cdot 4x \cdot 7y]$$

$$(4x)^2 + (7y)^2 + 2 \cdot 4x \cdot 7y - [4x]^2 - [7y]^2 + 2 \cdot 4x \cdot 7y \quad (\because (a-b)^2 = a^2 + b^2 - 2ab)$$

$$4 \cdot 4x \cdot 7y$$

$$\underline{\underline{112xy}} = \text{R.H.S}$$

$$\text{ii. } \frac{3}{7}P^r + \left(\frac{3}{7}P - \frac{7}{6}Q\right)^r + PQ$$

$$\left(\frac{3}{7}P\right)^r + \left(\frac{7}{6}Q\right)^r - 2 \cdot \frac{3}{7}P \cdot \frac{7}{6}Q + PQ$$

$$(\because (a-b)^r = a^r + b^r - 2ab)$$

$$\frac{9}{49}P^r + \frac{49}{36}Q^r - PQ + PQ$$

$$\frac{9}{49}P^r + \frac{49}{36}Q^r = R.H.S$$

$$\therefore L.H.S = R.H.S$$

$$\begin{aligned}\text{iii. } L.H.S &= (P-Q)(P+Q) + (Q-R)(Q+R) + (R-P)(R+P) \\ &= P(P+Q) - Q(P+Q) + Q(Q+R) - R(Q+R) \\ &= P^r - Q^r + Q^r - R^r + R^r - P^r \\ &(\because (a+b)(a-b) = a^r - b^r) \\ &= 0 = R.H.S\end{aligned}$$

$$\therefore L.H.S = R.H.S$$

9.

Given

$$\left(x + \frac{1}{x}\right) = 2$$

L

Squaring on both sides

i)

$$\left(x + \frac{1}{x}\right)^2 = 2^2$$

$$x^2 + 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 4 \quad (\because (a+b)^2 = a^2 + b^2 + 2ab)$$

$$x^2 + 2 + \frac{1}{x^2} = 4$$

$$x^2 + \frac{1}{x^2} = 4 - 2$$

$$x^2 + \frac{1}{x^2} = 2$$

ii)

Again squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 2^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 4$$

$$x^4 + 2 + \frac{1}{x^4} = 4$$

$$x^4 + \frac{1}{x^4} = 4 - 2$$

$$x^4 + \frac{1}{x^4} = 2$$

10.

∴

$$x - \frac{1}{x} = 7$$

VI

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = 7^2$$

$$x^2 - 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 49 \quad (\because (a-b)^2 = a^2 - 2ab + b^2)$$

$$x^2 - 2 + \frac{1}{x^2} = 49$$

$$x^2 + \frac{1}{x^2} = 49 + 2$$

$$\underline{x^2 + \frac{1}{x^2} = 51}$$

∴

$$x^2 + \frac{1}{x^2} = 51$$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 51^2$$

$$(x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 2601$$

$$x^4 + 2 + \frac{1}{x^4} = 2601$$

$$x^4 + \frac{1}{x^4} = 2601 - 2$$

$$\underline{\underline{x^4 + \frac{1}{x^4} = 2599}}$$

$$\text{ii. } x^2 + \frac{1}{x^2} = 23$$

$$\therefore x^2 + \frac{1}{x^2} = 23$$

Adding '2' on both sides

$$x^2 + \frac{1}{x^2} + 2 = 23 + 2$$

$$x^2 + \frac{1}{x^2}$$

$$(x^2 + \frac{1}{x^2})^2 + 2 \cdot x \cdot \frac{1}{x} = 25 \quad (\because a^2 + b^2 + 2ab = (a+b)^2)$$

$$(x^2 + \frac{1}{x^2})^2 = 25$$

$$\underline{x^2 + \frac{1}{x^2}} = 5$$

$$\text{ii. } x^2 + \frac{1}{x^2} = 23$$

Subtract '2' on both sides

$$x^2 + \frac{1}{x^2} - 2 = 23 - 2$$

$$(x^2 + \frac{1}{x^2})^2 - 2 \cdot x \cdot \frac{1}{x} = 21$$

$$(x^2 - \frac{1}{x^2})^2 = 21 \quad (\because a^2 + b^2 - 2ab = (a-b)^2)$$

$$x^2 - \frac{1}{x^2} = \sqrt{21}$$

$$\underline{x^2 - \frac{1}{x^2}} = 3\sqrt{3}$$

12. given $a+b=9$, $ab=10$

vi

Squaring on both sides

$$(a+b)^2 = 9^2$$

$$a^2 + b^2 + 2ab = 81$$

$$a^2 + b^2 + 2 \times 10 = 81 \quad (\because \text{given } ab=10)$$

$$a^2 + b^2 + 20 = 81$$

$$\underline{\underline{a^2 + b^2 = 61}}$$

13. given $a-b=6$, $a^2+b^2=42$

$$a-b=6$$

Squaring on both sides

$$(a-b)^2 = 6^2$$

$$a^2 + b^2 - 2ab = 36$$

$$42 - 2ab = 36 \quad (\because a^2 + b^2 = 42)$$

$$42 - 36 = 2ab$$

$$2ab = 6$$

$$\boxed{ab=3}$$

14.

Given $a^2+b^2=41$, $ab=4$

i) Consider

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a+b)^2 = 41 + 2 \times 4 = 41 + 8$$

$$(a+b)^2 = 49$$

ii) Consider

$$a+b = 7$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$= 41 - 2 \times 4 = 41 - 8 = 33$$

$$(a-b)^2 = 33$$

$$a-b = \sqrt{33}$$