

Factor Theorem

EXERCISE - 7.1

- Q1. Find the remainder (without division) on dividing $f(x)$ by $(x-2)$ where (i) $f(x) = 5x^2 - 7x + 4$ (ii) $f(x) = 2x^3 - 7x^2 + 3$.

Sol. Let $x-2=0 \Rightarrow x=2$

(i) Substituting the value of x in $f(x)$

$$f(x) = 5x^2 - 7x + 4$$

$$f(2) = 5(2)^2 - 7(2) + 4 = 20 - 14 + 4 = 10.$$

Hence remainder = 10.

(ii) $f(x) = 2x^3 - 7x^2 + 3$

$$f(2) = 2(2)^3 - 7(2)^2 + 3 = 16 - 28 + 3 = -9$$

Hence remainder = -9.

- Q2. Using remainder theorem, find the remainder on dividing $f(x)$ by $(x+3)$ where (i) $f(x) = 2x^2 - 5x + 1$
(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

Sol. Let $x+3=0 \Rightarrow x=-3$.

Substituting the value of x in $f(x)$.

(i) $f(x) = 2x^2 - 5x + 1$

$$f(-3) = 2(-3)^2 - 5(-3) + 1 = 18 + 15 + 1 = 34.$$

(ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

$$f(-3) = 3(-3)^3 + 7(-3)^2 - 5(-3) + 1$$

$$= -81 + 63 + 15 + 1$$

$$= -2.$$

Q3. Find the remainder (without division) on dividing $f(x)$ by $(2x+1)$ where (i) $f(x) = 4x^2 + 5x + 3$ (ii) $f(x) = 3x^3 - 7x^2 + 4x + 11$

Sol. Let $2x+1=0 \Rightarrow x = -\frac{1}{2}$

Sub. the value of x in $f(x)$.

$$(i) f(x) = 4x^2 + 5x + 3$$

$$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + 3 = 4\left(\frac{1}{4}\right) - \frac{5}{2} + 3 = 4 - \frac{5}{2} + 3 = \frac{3}{2}$$

$$\therefore \text{remainder} = \frac{3}{2}$$

$$(ii) f(x) = 3x^3 - 7x^2 + 4x + 11$$

$$f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 11 = -\frac{3}{8} - \frac{7}{4} + 9 \\ = \frac{-3 - 14 + 72}{8} = \frac{55}{8} = 6\frac{7}{8}$$

Q4. (i) find the remainder (without division) when $2x^3 - 3x^2 + 7x - 8$ is divided by $(x-1)$.

(ii) find the remainder (without division) on dividing $3x^2 + 5x - 9$ by $(3x+2)$.

Sol. (i) Let $x-1=0 \Rightarrow x = 1$

Sub. the value of x in $f(x)$

$$f(x) = 2x^3 - 3x^2 + 7x - 8$$

$$f(1) = 2(1)^3 - 3(1)^2 + 7(1) - 8 = 2 - 3 + 7 - 8 = -2.$$

(ii) Let $3x+2=0 \Rightarrow x = -\frac{2}{3}$

Sub. the value of x in $f(x)$.

$$f(x) = 3x^2 + 5x - 9$$

$$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 5\left(-\frac{2}{3}\right) - 9 = \frac{12}{9} - \frac{10}{3} - 9$$

$$= \frac{12 - 30 - 81}{9} = -\frac{99}{9} = -11$$

- Q5. When $kx^3 + 9x^2 + 4x - 10$ is divided by $(x+1)$, the remainder is 2. Find the value of k .

Sol. Let $x+1=0 \Rightarrow x = -1$.

Sub. the value of x in $f(x)$

$$f(x) = kx^3 + 9x^2 + 4x - 10$$

$$f(-1) = k(-1)^3 + 9(-1)^2 + 4(-1) - 10 = -k + 9 - 4 - 10 = -k - 5$$

$$\text{Remainder} = 2, \text{ then } -k - 5 = 2 \Rightarrow k = -7.$$

- Q6. Using remainder theorem, find the value of 'a' if the division of $x^3 + 5x^2 - ax + 6$ by $(x-1)$ leaves the remainder $2a$.

Sol. Let $x-1=0 \Rightarrow x = 1$

Sub. the value of x in $f(x)$:

$$f(x) = x^3 + 5x^2 - ax + 6$$

$$f(1) = (1)^3 + 5(1)^2 - a(1) + 6 = 12 - a.$$

$$\text{remainder} = 2a$$

$$12 - a = 2a \Rightarrow 3a = 12 \Rightarrow a = 4.$$

- Q7. (i) What number must be subtracted from $2x^2 - 5x$ so that the resulting polynomial leaves the remainder 2 when divided by $2x+1$?

- (ii) What number must be added to $2x^3 - 7x^2 + 2x$ so that the resulting polynomial leaves the remainder -2 when divided by $2x-3$?

- Sol. (i) Let 'a' be subtracted from $2x^2 - 5x$.

Dividing $2x^2 - 5x$ by $2x+1$,

$$\begin{array}{r} 2x+1) 2x^2 - 5x - a \\ \underline{- 2x^2 - x} \\ \hline - 6x - a \\ \underline{- 6x - 3} \\ \hline + + \\ - a + 3 \end{array}$$

Here remainder is $(3-a)$.

but we are given that remainder is 2.

$$3-a = 2 \Rightarrow a = 1$$

\therefore hence 1 is to be subtracted.

(ii) let 'a' be added to $2x^3 - 7x^2 + 2x$ dividing it by $x-3$, then

$$\begin{array}{r} 2x-3) 2x^3 - 7x^2 + 2x + a \\ \underline{- 2x^3 - 3x^2} \\ \hline + + \\ - 4x^2 + 2x \\ \underline{- 4x^2 + 6x} \\ \hline + + \\ - 4x + a \\ \underline{- 4x + 6} \\ \hline a - 6 \end{array}$$

But remainder is -2, then

$$a - 6 = -2 \Rightarrow a = 4$$

\therefore hence 4 is to be added.

- Q8. The polynomials $kx^3 + 3x^2 - 4$ and $2x^3 - 5x + 4k$ when divided by $(x+3)$ leave the same remainder. find the value of k .

Sol. let $x+3=0 \Rightarrow x = -3$.

Sub. the value of x in $f(x)$.

$$f(x) = kx^3 + 3x^2 - 4$$

$$f(-3) = k(-3)^3 + 3(-3)^2 - 4 = -27k + 27 - 4 \\ = -27k + 23 \quad \text{--- (i)}$$

and $f(x) = 2x^3 - 5x + 4k$

$$f(-3) = 2(-3)^3 - 5(-3) + 4k = -54 + 15 + 4k \\ = -39 + 4k \quad \text{--- (ii)}$$

Remainder is same in both cases,

from (i) & (ii),

$$-27k + 23 = -39 + 4k \\ \Rightarrow -27k + 4k = -39 - 23 \\ \Rightarrow -31k = -62 \Rightarrow k = 2$$

- Q9. By factor theorem, show that $(x+3)$ and $(2x-1)$ are factors of $2x^2 + 5x - 3$.

Sol. let $x+3=0 \Rightarrow x = -3$.

Sub. the value of x in $f(x)$

$$f(x) = 2x^2 + 5x - 3$$

$$f(-3) = 2(-3)^2 + 5(-3) - 3 = 18 - 15 - 3 = 0$$

$\therefore x+3$ is a factor of $f(x)$.

Again let $2x-1=0 \Rightarrow x = \frac{1}{2}$

Sub. the value of x in $f(x)$

$$f(x) = 2x^2 + 5x - 3$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 3 = \frac{1}{2} + \frac{5}{2} - 3 \\ = 3 - 3 = 0$$

$\therefore 2x-1$ is a factor of $f(x)$.

\therefore Hence proved.

Q10. Show that $(x-2)$ is a factor of $3x^2 - x - 10$. Hence factorise $3x^2 - x - 10$.

Sol. let $x-2=0 \Rightarrow x=2$.

Sub. the value of x in $f(x)$.

$$f(x) = 3x^2 - x - 10$$

$$f(2) = 3(2)^2 - 2 - 10 = 12 - 12 = 0$$

$\therefore (x-2)$ is a factor of $f(x)$.

Dividing $(3x^2 - x - 10)$ by $(x-2)$, we get

$$(x-2) \overline{)3x^2 - x - 10} (3x+5$$

$$\begin{array}{r} -3x^2 + 6x \\ \hline 5x - 10 \\ -5x + 10 \\ \hline (0) \end{array}$$

$$\therefore 3x^2 - x - 10 = (x-2)(3x+5)$$

Q11. Show that $(x-1)$ is a factor of $x^3 - 5x^2 - x + 5$. Hence factorise $x^3 - 5x^2 - x + 5$.

Sol. let $x-1=0 \Rightarrow x=1$

Sub. the value of x in $f(x)$,

$$f(x) = x^3 - 5x^2 - x + 5$$

$$f(1) = (1)^3 - 5(1)^2 - (1) + 5 = 1 - 5 - 1 + 5 = 0$$

$\therefore (x-1)$ is a factor of $x^3 - 5x^2 - x + 5$.

Now dividing $f(x)$ by $(x-1)$, we get

$$\begin{array}{r}
 (x-1) x^3 - 5x^2 - x + 5 (x^2 - 4x - 5) \\
 - \underline{x^3 - x^2} \\
 - 4x^2 - x \\
 - \underline{4x^2 + 4x} \\
 - 5x + 5 \\
 - 5x + 5 \\
 \hline (0)
 \end{array}$$

$$\begin{aligned}
 x^3 - 5x^2 - x + 5 &= (x-1)(x^2 - 4x - 5) = (x-1)(x^2 - 5x + x - 5) \\
 &= (x-1)[x(x-5) + 1(x-5)] \\
 &= (x-1)(x+1)(x-5)
 \end{aligned}$$

Q12. Show that $(x-3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence factorise $x^3 - 7x^2 + 15x - 9$.

Sol. Let $x-3=0 \Rightarrow x=3$.

Sub. the value of x in $f(x)$.

$$f(x) = x^3 - 7x^2 + 15x - 9$$

$$f(3) = (3)^3 - 7(3)^2 + 15(3) - 9 = 27 - 63 + 45 - 9 = 0$$

$\therefore (x-3)$ is a factor of $f(x)$.

Now dividing by $(x-3)$, we get

$$\begin{array}{r}
 (x-3) x^3 - 7x^2 + 15x - 9 (x^2 - 4x + 3) \\
 - \underline{x^3 - 3x^2} \\
 - 4x^2 + 15x \\
 - \underline{4x^2 + 12x} \\
 - 3x - 9 \\
 - \underline{3x - 9} \\
 (0)
 \end{array}$$

$$\therefore x^3 - 7x^2 + 15x - 9 = (x-3)(x^2 - 4x + 3)$$

$$\begin{aligned}
 &= (x-3)[x^2 - x - 3x + 3] \\
 &= (x-3)[x(x-1) - 3(x-1)] \\
 &= (x-3)(x-3)(x-1) \\
 &= (x-3)^2(x-1)
 \end{aligned}$$

Q13. Show that $(2x+1)$ is a factor of $4x^3 + 12x^2 + 11x + 3$. Hence factorise $4x^3 + 12x^2 + 11x + 3$.

Sol. Let $2x+1 = 0 \Rightarrow x = -\frac{1}{2}$

Sub. the value of x in $f(x)$.

$$f(x) = 4x^3 + 12x^2 + 11x + 3.$$

$$\begin{aligned}
 f\left(-\frac{1}{2}\right) &= 4\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 + 11\left(-\frac{1}{2}\right) + 3 \\
 &= -\frac{1}{2} + 3 - \frac{11}{2} + 3 = -6 + 6 = 0
 \end{aligned}$$

$\therefore (2x+1)$ is a factor of $4x^3 + 12x^2 + 11x + 3$.

Now dividing $f(x)$ by $(2x+1)$, we get

$$\begin{array}{r}
 2x+1) 4x^3 + 12x^2 + 11x + 3 \quad (2x^2 + 5x + 3 \\
 \underline{- 4x^3 - 2x^2} \\
 \hline
 10x^2 + 11x \\
 \underline{- 10x^2 - 5x} \\
 \hline
 6x + 3 \\
 \underline{- 6x - 3} \\
 \hline
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 4x^3 + 12x^2 + 11x + 3 &= (2x+1)(2x^2 + 5x + 3) \\
 &= (2x+1)(2x^2 + 2x + 3x + 3) \\
 &= (2x+1)[2x(x+1) + 3(x+1)] \\
 &= (x+1)(2x+1)(2x+3).
 \end{aligned}$$

Q14. Show that $(2x+7)$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence factorise the given expression completely, using the factor theorem.

Sol. Let $2x+7=0 \Rightarrow x = -\frac{7}{2}$

$$\begin{aligned} f\left(-\frac{7}{2}\right) &= 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14 \\ &= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14 \\ &= \frac{-343 + 245 + 154 - 56}{4} \\ &= 0 \end{aligned}$$

Hence, $(2x+7)$ is the factor of the given expression. On dividing $2x^3 + 5x^2 - 11x - 14 = 0$ by $2x+7$, we get

$$\begin{array}{r} 2x+7) 2x^3 + 5x^2 - 11x - 14 (x^2 - x - 2 \\ \underline{-} \quad \underline{2x^3 + 7x^2} \\ \underline{\quad \quad \quad - 2x^2 - 11x} \\ \underline{\quad \quad \quad + 2x^2 + 7x} \\ \underline{\quad \quad \quad - 4x - 14} \\ \underline{\quad \quad \quad + 4x + 14} \\ (0) \end{array}$$

$$\begin{aligned} \therefore 2x^3 + 5x^2 - 11x - 14 &= (2x+7)(x^2 - x - 2) \\ &= (2x+7)(x^2 - 2x + x - 2) \\ &= (2x+7)[x(x-2) + 1(x-2)] \\ &= (x+1)(x-2)(2x+7) \end{aligned}$$

Q15. Use factor theorem to factorise the following polynomials completely : (i) $x^3 + 2x^2 - 5x - 6$ (ii) $x^3 - 13x - 12$.

Sol. (i) $f(x) = x^3 + 2x^2 - 5x - 6$

let $x = -1$, then

$$\begin{aligned}f(-1) &= (-1)^3 + 2(-1)^2 - 5(-1) - 6 \\&= -1 + 2 + 5 - 6 = 0\end{aligned}$$

$\therefore (x+1)$ is a factor of $f(x)$.

Now dividing $f(x)$ by $x+1$, we get

$$\begin{array}{r}x+1 \Big| x^3 + 2x^2 - 5x - 6 \\ \underline{-x^3 - x^2} \\ x^2 - 5x \\ \underline{-x^2 - x} \\ -6x - 6 \\ \underline{-6x - 6} \\ 0\end{array}$$

$$\begin{aligned}\therefore f(x) &= (x+1)(x^2 + x - 6) = (x+1)(x^2 + 3x - 2x - 6) \\&= (x+1)[x(x+3) - 2(x+3)] \\&= (x+1)(x-2)(x+3)\end{aligned}$$

(ii) $x^3 - 13x - 12$

$$f(x) = x^3 - 13x - 12 \quad \text{--- (i)}$$

putting $x = -1$ in (i), we get

$$f(-1) = (-1)^3 - 13(-1) - 12 = -1 + 13 - 12 = 0$$

$\therefore (x+1)$ is a factor of $f(x)$.

On dividing $x^3 - 13x - 12$ by $(x+1)$, we get

$$(x+1) \quad x^3 - 13x - 12 \quad (x^2 - x - 12)$$

$$\begin{array}{r} \underline{-x^3 - x^2} \\ -x^2 - 13x \\ \underline{-x^2 - x} \\ -12x - 12 \\ \underline{-12x - 12} \\ (0) \end{array}$$

$$\begin{aligned} x^3 - 13x - 12 &= (x+1)(x^2 - x - 12) \\ &= (x+1)(x^2 - 4x + 3x - 12) \\ &= (x+1)[x(x-4) + 3(x-4)] \\ &= (x+1)(x+3)(x-4) \end{aligned}$$

- Q16. If $(2x+1)$ is a factor of $6x^3 + 5x^2 + ax - 2$, find the value of a .

Sol. let $2x+1 = 0 \Rightarrow x = -\frac{1}{2}$

Sub. the value of x in $f(x)$

$$\begin{aligned} \therefore f(x) &= 6x^3 + 5x^2 + ax - 2 \\ f\left(-\frac{1}{2}\right) &= 6\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) - 2 \\ &= -\frac{3}{4} + \frac{5}{4} - \frac{a}{2} - 2 \\ &= \frac{-3 + 5 - 2a - 8}{4} = \frac{-6 - 2a}{4} \end{aligned}$$

$\therefore 2x+1$ is a factor of $f(x)$

Remainder = 0

$$\begin{aligned} \therefore -\frac{6 - 2a}{4} &= 0 \Rightarrow -6 - 2a = 0 \Rightarrow 2a = -6 \\ &\Rightarrow a = -3. \end{aligned}$$

Q17. If $(3x-2)$ is a factor of $3x^3 - kx^2 + 21x - 10$, find the value of k .

Sol. Let $3x-2=0 \Rightarrow x = \frac{2}{3}$.

Sub. the value of x in $f(x)$,

$$f(x) = 3x^3 - kx^2 + 21x - 10$$

$$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10$$

$$= \frac{8}{9} - \frac{4k}{9} + 14 - 10$$

$$= \frac{8}{9} - \frac{4k}{9} + 4 = \frac{8-4k+36}{9} = \frac{44-4k}{9}$$

∴ Remainder is '0'

$$\therefore \frac{44-4k}{9} = 0 \Rightarrow 44-4k=0 \Rightarrow k=11.$$

Q18. What number must be added to $4x^3 - 8x^2 + 3x$ so that the resulting polynomial has a factor $2x+1$?

Sol. Let 'a' be added to $4x^3 - 8x^2 + 3x$, then

$$f(x) = 4x^3 - 8x^2 + 3x + a.$$

∴ $(2x+1)$ is a factor of $f(x)$, $f(x)=0$

$$\text{Now } 2x+1=0 \Rightarrow x = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + a$$

$$= -\frac{1}{2} - 2 - \frac{3}{2} + a = -4 + a$$

∴ $2x+1$ is a factor.

$$f\left(-\frac{1}{2}\right) = 0 \Rightarrow -4+a=0$$

$$\Rightarrow a=4$$

∴ Hence 4 is to be added.

Q19. If $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$, then find the value of p . Factorize the above expression completely.

Sol. Let $x-2=0 \Rightarrow x=2$

Now $f(x) = 2x^3 - x^2 + px - 2$

$$\begin{aligned} f(2) &= 2(2)^3 - (2)^2 + p(2) - 2 \\ &= 16 - 4 + 2p - 2 \\ &= 2p + 10 \end{aligned}$$

$\therefore f(2) = 0$, then $2p+10=0 \Rightarrow p=-5$

Now the polynomial will be $2x^3 - x^2 - 5x - 2$

$$\begin{aligned} 2x^3 - x^2 - 5x - 2 &= (x-2)(2x^2 + 3x + 1) \\ &= (x-2)(2x^2 + 2x + x + 1) \\ &= (x-2)[2x(x+1) + 1(x+1)] \\ &= (x+1)(x-2)(2x+1) \end{aligned}$$

$$\begin{array}{r} (x-2) \ 2x^3 - x^2 - 5x - 2 \quad (2x^2 + 3x + 1) \\ \underline{- 2x^3 - 4x^2} \\ \underline{\quad \quad \quad 3x^2 - 5x} \\ \underline{- 3x^2 - 6x} \\ \underline{\quad \quad \quad x - 2} \\ \underline{- \quad \quad \quad \quad \quad 0} \end{array}$$

Q20. Find the value of the constants a and b , if $(x-2)$ and $(x+3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$.

Sol. Let $x-2=0 \Rightarrow x=2$.

Sub. the value of x in $f(x)$

$$f(x) = x^3 + ax^2 + bx - 12$$

$$\begin{aligned}
 f(2) &= (2)^3 + a(2)^2 + b(2) - 12 \\
 &= 8 + 4a + 2b - 12 \\
 &= 4a + 2b - 4 \\
 \therefore x-2 \text{ is a factor} \\
 \therefore 4a + 2b - 4 &= 0 \Rightarrow 4a + 2b = 4 \\
 \Rightarrow 2a + b &= 2 \quad \text{--- (i)}
 \end{aligned}$$

Again let $x+3=0$, then $x=-3$

Sub the value of x in $f(x)$

$$\begin{aligned}
 f(-3) &= (-3)^3 + a(-3)^2 + b(-3) - 12 \\
 &= -27 + 9a - 3b - 12 \\
 &= +9a - 3b - 39 \\
 x+3 \text{ is a factor of } f(x) \\
 \therefore -39 + 9a - 3b &= 0 \Rightarrow 9a - 3b = 39 \\
 \Rightarrow 3a - b &= 13 \quad \text{--- (ii)}
 \end{aligned}$$

Adding (i) & (ii), $5a = 15 \Rightarrow a = 3$.

Sub the value of 'a' in (i)

$$2(3) + b = 2 \Rightarrow b = -4.$$

\therefore Hence $a = 3, b = -4$.

Q21. If $(x+2)$ and $(x-3)$ are factors of $x^3 + ax + b$. find the values of a and b . factorise the given expression.

Sol.

If $x+2=0 \Rightarrow x = -2$.

Sub the value of x in $f(x)$.

$$f(x) = x^3 + ax + b.$$

$$f(-2) = (-2)^3 + a(-2) + b \\ = -8 - 2a + b$$

$\therefore x+2$ is a factor

\therefore Remainder is zero.

$$-8 - 2a + b = 0 \Rightarrow 2a - b = -8 \quad \text{--- (i)}$$

$$\text{Again let } x-3=0 \Rightarrow x=3$$

Sub. the value of x in $f(x)$,

$$f(x) = x^3 + ax + b$$

$$f(3) = (3)^3 + a(3) + b = 27 + 3a + b$$

$(x-3)$ is a factor, remainder = 0

$$27 + 3a + b = 0 \Rightarrow 3a + b = -27 \quad \text{--- (ii)}$$

$$\text{Adding (i) \& (ii), } 5a = -35 \Rightarrow a = -7$$

$$\text{Sub. the value of 'a' in (i), } -7(-2) - b = -8$$

$$\Rightarrow 14 - b = -8 \Rightarrow b = -6$$

$$\therefore \text{Hence } a = -7, b = -6$$

$(x+2)$ and $(x-3)$ are the factors of $x^3 + ax + b$
 $\Rightarrow x^3 - 7x - 6$.

Now dividing $x^3 - 7x - 6$ by $(x+2)(x-3)$ or $x^2 - x - 6$, we get

$$\begin{array}{r} x^2 - x - 6) \overline{x^3 - 7x - 6} (x + 1 \\ \underline{- x^3 - x^2 - 6x} \\ \hline \underline{x^2 - x - 6} \\ \underline{- x^2 - x - 6} \\ \hline 0 \end{array}$$

$$\therefore x^3 - 7x - 6 = (x+1)(x+2)(x-3)$$

Q22. $(x-2)$ is a factor of the expression x^3+ax^2+bx+6 . When this expression is divided by $(x-3)$, it leaves the remainder 3. Find the values of a and b .

Sol. As it is given that $(x-2)$ is a factor of the expression x^3+ax^2+bx+6 — (i)

$$f(x) = x^3 + ax^2 + bx + 6$$

$$f(2) = (2)^3 + a(2)^2 + b(2) + 6 = 8a + 2b + 14$$

$\therefore (x-2)$ is a factor, remainder = 0

$$8a + 2b + 14 = 0 \Rightarrow 2a + b + 7 = 0$$

$$\Rightarrow 2a + b = -7 \quad \text{--- (ii)}$$

When exp (i) is divided by $(x-3)$, it leaves the remainder 3.

$$(x-3) \overline{)x^3 + ax^2 + bx + 6} \quad (x^2 + (3+a)x + (9+3a+b))$$

$$\begin{array}{r} x^3 - 3x^2 \\ \hline - + \end{array}$$

$$(3+a)x^2 + bx$$

$$\begin{array}{r} (3+a)x^2 - 3(3+a)x \\ \hline - + \end{array}$$

$$(9+3a+b)x + b$$

$$\begin{array}{r} (9+3a+b)x - 3(9+3a+b) \\ \hline - + \end{array}$$

$$33 + 9a + 3b.$$

$$\text{Remainder} = 33 + 9a + 3b = 3 \quad (\text{given})$$

$$\Rightarrow 9a + 3b = -30 \Rightarrow 3a + b = -10 \quad \text{--- (iii)}$$

Subtracting (iii) from (ii),

$$\begin{array}{r} 2a + b = -7 \\ - 3a + b = -10 \\ \hline - a = 3 \Rightarrow a = -3. \end{array}$$

put $a = -3$ in (iii) we get

$$b = -1$$

\therefore hence $a = -3, b = -1$.

- Q23. If $ax^3 + 3x^2 + bx - 3$ has a factor $(2x+3)$ and leaves remainder -3 when divided by $(x+2)$, find the values of a and b . With these values of a and b , factorize the given expression.

Sol.

$$\text{let } 2x+3=0 \text{ then } 2x = -3 \Rightarrow x = \frac{-3}{2}$$

Sub. the value of x in $f(x)$,

$$f(x) = ax^3 + 3x^2 + bx - 3$$

$$f\left(\frac{-3}{2}\right) = a\left(\frac{-3}{2}\right)^3 + 3\left(\frac{-3}{2}\right)^2 + b\left(\frac{-3}{2}\right) - 3$$

$$= -\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$$

$\therefore 2x+3$ is a factor of $f(x)$, remainder = 0

$$-\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3 = 0$$

$$\Rightarrow -27a + 54 - 12b - 24 = 0$$

$$\Rightarrow -27a - 12b = -30$$

$$\Rightarrow 9a + 4b = 10 \quad \text{--- (i)}$$

Again let $x+2=0 \Rightarrow x = -2$.

Sub. the value of x in $f(x)$,

$$f(x) = ax^3 + 3x^2 + bx - 3$$

$$f(-2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$$

$$= -8a + 12 - 2b - 3$$

$$= -8a - 2b + 9$$

\therefore remainder = -3

$$-8a - 2b + 9 = -3$$

$$\Rightarrow -8a - 2b = -12$$

$$\Rightarrow 4a+b = 6 \quad \text{---(ii)}$$

Multiply (ii) by 4 $\Rightarrow 16a+4b = 24$

$$(i) \Rightarrow \underline{\underline{-9a+4b=10}} \\ -7a = 14$$

$$\Rightarrow a = 2.$$

Sub. the value of 'a' in (i)

$$9(2) + 4b = 10 \Rightarrow 4b = -8 \Rightarrow b = -2.$$

$$\therefore \text{Hence } a = 2, b = -2$$

$$f(x) = ax^3 + 3x^2 + bx - 3 = 2x^3 + 3x^2 - 2x - 3$$

$\therefore 2x+3$ is a factor

\therefore Dividing $f(x)$ by $x+2$

$$\begin{array}{r} 2x+3) 2x^3 + 3x^2 - 2x - 3 (x^2 - 1 \\ \underline{-2x^3 - 3x^2} \\ \underline{\underline{-2x - 3}} \\ \underline{\underline{-2x - 3}} \\ \underline{\underline{0}} \end{array}$$

$$\therefore 2x^3 + 3x^2 - 2x - 3 = (2x+3)(x^2 - 1)$$

$$= (2x+3)(x+1)(x-1)$$

Q24. Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$.

If $x-2$ is a factor of $f(x)$ but leaves the remainder -15 when it divides $g(x)$, find the values of a and b with these values of a and b , factorize the expression $f(x) + g(x) + 4x^2 + 7x$.

sd.

$$\text{Given } f(x) = ax^2 + bx + 2, \quad g(x) = bx^2 + ax + 1$$

As $x-2$ is a factor of $f(x)$, $f(2) = 0$

$$\Rightarrow a(2)^2 + b(2) + 2 = 0$$

$$\Rightarrow 4a + 2b + 2 = 0$$

$$\Rightarrow 2a + b + 1 = 0 \quad \text{--- (1)}$$

when $g(x)$ divide by $(x-2)$, leaves remainder -15

$$g(2) = -15$$

$$\Rightarrow b(2)^2 + a(2) + 1 = -15$$

$$\Rightarrow 4b + 2a + 16 = 0$$

$$\Rightarrow 2b + a + 8 = 0 \quad \text{--- (2)}$$

from (1) & (2) -

$$① \times 2 \Rightarrow 4a + 2b + 2 = 0$$

$$② \times 1 \Rightarrow \underline{\underline{a + 2b + 8 = 0}}$$

$$3a - 6 = 0 \Rightarrow a = 2.$$

put the 'a' value in (1), $2(2) + b + 1 = 0$

$$\Rightarrow b = -5.$$

$$\text{Now, } f(x) = ax^2 + bx + 2 = 2x^2 - 5x + 2$$

$$g(x) = bx^2 + ax + 1 = -5x^2 + 2x + 1$$

$$\text{so, } f(x) + g(x) = 2x^2 - 5x + 2 - 5x^2 + 2x + 1 + 4x^2 + 7x$$

$$= x^2 + 4x + 3$$

$$= x^2 + 3x + x + 3$$

$$= x(x+3) + 1(x+3)$$

$$= (x+1)(x+3).$$