

Section Formula

EXERCISE - 11.1

Q1. Find the distance between the following pair of points :

(i) $(-5, 3), (3, 1)$

Sol. Distance = $\sqrt{[3 - (-5)]^2 + (1 - 3)^2} = \sqrt{64 + 4} = \sqrt{68}$
 $= 2\sqrt{17}$.

(ii) $(4, 5), (-3, 2)$

Distance = $\sqrt{(-3 - 4)^2 + (2 - 5)^2} = \sqrt{49 + 9} = \sqrt{58}$

(iii) $(-1, -4), (3, 5)$

Distance = $\sqrt{[3 - (-1)]^2 + [5 - (-4)]^2} = \sqrt{16 + 81} = \sqrt{97}$

Q2. Calculate the distance between $A(7, 3)$ and B on the x -axis whose abscissa is 11.

Sol. $A(7, 3)$, $B(11, 0)$ as B lies on x -axis

$\therefore AB = \sqrt{(11 - 7)^2 + (0 - 3)^2} = \sqrt{25} = 5$

Q3. A is a point on the y -axis whose ordinate is 5 and B is the point $(-3, 1)$. Calculate the length of AB .

Sol. A lies on y -axis, $x = 0$ and $y = 5$

$A(0, 5)$, $B(-3, 1)$

$AB = \sqrt{(-3 - 0)^2 + (1 - 5)^2} = \sqrt{9 + 16} = 5$

Q4. Show that the point $(4,4)$ is equidistant from the points $A(1,0)$ and $B(-1,4)$

Sol. Distance between $A(1,0)$ & $(4,4) = \sqrt{(4-1)^2 + (4-0)^2} = \sqrt{25} = 5$

Distance between $B(-1,4)$ & $(4,4) = \sqrt{(4-(-1))^2 + (4-4)^2} = 5$

∴ Hence proved.

Q5. A is a point of y-axis whose ordinate is 4 and B is a point on x-axis whose abscissa is -3. Find the length of the line segment AB.

Sol. A lies on y-axis, $A(0,4)$

B lies on x-axis, $B(-3,0)$

$$AB = \sqrt{(-3-0)^2 + (0-4)^2} = \sqrt{25} = 5$$

Q6. The distance between $A(1,3)$ and $B(x,7)$ is 5. Calculate the possible values of x .

Sol.

$$\text{Distance} = 5$$

$$\Rightarrow \sqrt{(x-1)^2 + (7-3)^2} = 5 \Rightarrow x^2 - 2x + 1 + 16 = 25$$

$$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow x = 4, -2.$$

Q7. Point $A(5,-1)$ on reflection in x-axis is mapped as A' . Also A on reflection in y-axis is mapped as A'' . Write the coordinates of A' and A'' . Also calculate the distance AA'' .

Sol.

Coordinates of A' are $(5,1)$

Coordinates of A'' are $(-5,-1)$

Q9. B and C have coordinates (3, 2) and (6, 3). find:

(i) The image B' of B under reflection in the x-axis.

(ii) The image C' of C under reflection in the line BB'.

(iii) The length of B'C'.

sol. (i) The coordinates of B' reflected on x-axis will be (3, -2).

(ii) The coordinates of C', reflected on line BB' = (6, 3)

(iii) $B'C' = \sqrt{(6-3)^2 + (3-(-2))^2} = \sqrt{9+25} = \sqrt{34} = 5.83$.

Q10. what points on y-axis are at a distance of 10 units from the point (8, 8)?

sol. let the coordinates of points be (x, y) which are at a distance of 10 units from the point (8, 8).

$$\sqrt{(8-x)^2 + (8-y)^2} = 10$$

$$\Rightarrow 64 + x^2 - 16x + 64 + y^2 - 16y = 100$$

$$\Rightarrow x^2 + y^2 - 16x - 16y + 28 = 0$$

\therefore points are on y-axis, $x = 0$

Hence $y^2 - 16y + 28 = 0 \Rightarrow y^2 - 14y - 2y + 28 = 0$

$$\Rightarrow (y-14)(y-2) = 0$$

$$\Rightarrow y = 2, 14$$

\therefore points will be (0, 14) and (0, 2).

Q11. find the points which are at a distance of $\sqrt{10}$ from the point (4, 3) given that the ordinate of the points is twice the abscissa.

sol. let the abscissa of point = x

$$\text{ordinate} = 2x$$

$$\sqrt{(x-4)^2 + (2x-3)^2} = \sqrt{10}$$

$$\Rightarrow x^2 - 8x + 16 + 4x^2 + 9 - 12x = 10$$

$$\Rightarrow 5x^2 - 20x + 15 = 0 \Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow x^2 - 3x - x + 3 = 0 \Rightarrow (x-3)(x-1) = 0$$

$$\Rightarrow x = 1, 3$$

\therefore points will be $(1, 2)$ and $(3, 6)$.

Q12. $A(2, 2)$, $B(-2, 4)$ and $C(2, 6)$ are the vertices of a triangle ABC . prove that ABC is an isosceles triangle.

Sol.

$$AB = \sqrt{(-2-2)^2 + (4-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$BC = \sqrt{(2-(-2))^2 + (6-4)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$AC = \sqrt{(2-2)^2 + (6-2)^2} = \sqrt{16} = 4.$$

$$\therefore AB = BC = 2\sqrt{5}$$

$\therefore \triangle ABC$ is an isosceles triangle.

Q13. show that the points $(1, 1)$, $(-1, -1)$ and $(-\sqrt{3}, \sqrt{3})$ form an equilateral triangle?

Sol.

$$AB = \sqrt{(1-(-1))^2 + (1-(-1))^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{[-\sqrt{3}-(-1)]^2 + [\sqrt{3}-(-1)]^2} = \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$$

$$= \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}} = \sqrt{8} = 2\sqrt{2}$$

$$AC = \sqrt{(-\sqrt{3}-1)^2 + (\sqrt{3}-1)^2} = \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}}$$

$$= \sqrt{8} = 2\sqrt{2}$$

$$\therefore AB = BC = AC = 2\sqrt{2}$$

$\therefore \triangle ABC$ is an equilateral triangle.

Q14. Show that the points $(3, 3)$, $(9, 0)$ and $(12, 21)$ are the vertices of a right angled triangle.

Sol. let $A(3, 3)$, $B(9, 0)$, $C(12, 21)$

$$AB = \sqrt{(9-3)^2 + (0-3)^2} = \sqrt{45} \Rightarrow AB^2 = 45$$

$$BC^2 = (12-9)^2 + (21-0)^2 = 9 + 441 = 450$$

$$AC^2 = (12-3)^2 + (21-3)^2 = 81 + 324 = 405$$

$$\therefore AB^2 + AC^2 = 405 + 45 = 450 = BC^2$$

$\therefore \triangle ABC$ is a right angled triangle.

Q15. Show that the points $(0, -1)$, $(-2, 3)$, $(6, 7)$ and $(8, 3)$ are the vertices of a rectangle.

Sol. let $A(0, -1)$, $B(-2, 3)$, $C(6, 7)$ and $D(8, 3)$

$$AB = \sqrt{(-2-0)^2 + (3-(-1))^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$BC = \sqrt{(-2-6)^2 + (3-7)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$CD = \sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$AD = \sqrt{(8-0)^2 + (3-(-1))^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$\therefore AB = CD \text{ and } BC = AD$$

$\therefore ABCD$ is a rectangle.

Q16. Show that the points $(7, 3)$, $(3, 0)$, $(0, -4)$ and $(4, -1)$ are the vertices of a rhombus. Also find the area of the rhombus.

Sol. let $A(7, 3)$, $B(3, 0)$, $C(0, -4)$ and $D(4, -1)$

$$AB = \sqrt{(3-7)^2 + (0-3)^2} = \sqrt{16+9} = 5$$

$$BC = \sqrt{(0-3)^2 + (-4-0)^2} = \sqrt{9+16} = 5$$

$$CD = \sqrt{(4-0)^2 + (-1+4)^2} = \sqrt{16+9} = 5$$

$$AD = \sqrt{(-1-4)^2 + (3-(-1))^2} = \sqrt{9+16} = 5$$

$$\therefore AB = BC = CD = DA$$

\therefore ABCD is a rhombus.

$$AC = \sqrt{(0-7)^2 + (-4-3)^2} = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2}$$

$$BD = \sqrt{(4-3)^2 + (-1+0)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 7\sqrt{2} \times \sqrt{2} = 7 \text{ sq units.}$$

Q17. The points $A(0,3)$, $B(-2,a)$ and $C(-1,4)$ are the vertices of a right angled triangle at A , find the value of a .

sol. $AB^2 = (-2-0)^2 + (a-3)^2 = 4 + (a-3)^2$

$$AC^2 = (-1-0)^2 + (4-3)^2 = 1+1 = 2$$

$$BC^2 = (-1+2)^2 + (4-a)^2 = 1 + (4-a)^2$$

$$\therefore AB^2 + AC^2 = BC^2 \quad \left\{ \because \text{By Pythagoras theorem} \right\}$$

$$\Rightarrow 4 + (a-3)^2 + 2 = 1 + (4-a)^2$$

$$\Rightarrow 4 + a^2 - 6a + 9 + 2 = 1 + 16 + a^2 - 8a$$

$$\Rightarrow a^2 - 6a + 15 = a^2 - 8a + 17$$

$$\Rightarrow 2a = 2 \Rightarrow a = 1.$$

Q18. Show by distance formula that the points $(-1,-1)$, $(2,3)$ and $(8,11)$ are collinear.

sol. let $A(-1,-1)$, $B(2,3)$, $C(8,11)$

$$AB = \sqrt{(2-(-1))^2 + (3-(-1))^2} = \sqrt{9+16} = 5$$

$$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{36+64} = 10$$

$$AC = \sqrt{(8-(-1))^2 + (11-(-1))^2} = \sqrt{81+144} = 15$$

$$\therefore AB + BC = 5 + 10 = 15 = AC.$$

\therefore Hence A, B & C are collinear.

Q19. what point on the x-axis is equidistance from (7,6) and (-3,4)?

Sol. let P(x,0) be the equidistance from given points A(7,6) and B(-3,4), then

$$PA = \sqrt{(7-x)^2 + (6-0)^2} = \sqrt{(7-x)^2 + 36}$$

$$PB = \sqrt{(-3-x)^2 + (4-0)^2} = \sqrt{(-3-x)^2 + 16} = \sqrt{(3+x)^2 + 16}$$

$$\therefore PA = PB$$

$$\sqrt{(7-x)^2 + 36} = \sqrt{(3+x)^2 + 16}$$

Squaring on both sides,

$$\Rightarrow (7-x)^2 + 36 = (3+x)^2 + 16$$

$$\Rightarrow 49 + x^2 - 14x + 36 = 9 + x^2 + 6x + 16$$

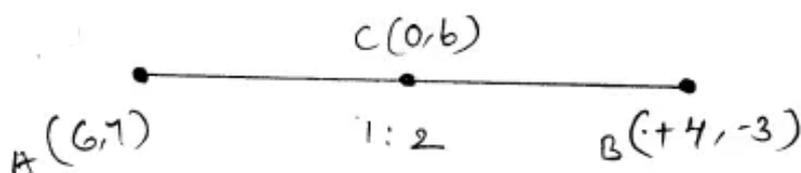
$$\Rightarrow x^2 - 14x + 85 = x^2 + 6x + 25$$

$$\Rightarrow -20x = -60 \Rightarrow x = 3.$$

$$\therefore P = (3,0).$$

Q20. find the points on the y-axis whose distances from the points (6,7) and (4,-3) are in the ratio 1:2

Sol. let the point on y-axis is (0,b).



$$AC = \sqrt{(0-b)^2 + (b-7)^2} = \sqrt{36 + (b-7)^2}$$

$$BC = \sqrt{(4-0)^2 + (-3-b)^2} = \sqrt{16 + (b+3)^2}$$

According to given, $\frac{AC}{BC} = \frac{1}{2}$

$$\Rightarrow 2AC = BC$$

$$\Rightarrow 2\sqrt{36 + (b-7)^2} = \sqrt{16 + (b+3)^2}$$

Squaring on both sides,

$$\Rightarrow 4(36 + b^2 + 49 - 14b) = 16 + b^2 + 9 + 6b$$

$$\Rightarrow 144 + 4b^2 + 196 - 56b = 16 + b^2 + 9 + 6b$$

$$\Rightarrow 3b^2 - 62b + 315 = 0$$

$$\Rightarrow 3b^2 - 27b - 35b + 315 = 0$$

$$\Rightarrow 3b(b-9) - 35(b-9) = 0 \Rightarrow (b-9)(3b-35) = 0$$

$$\Rightarrow b = 9, \frac{35}{3}$$

\therefore Hence the point is $(0, 9)$ or $(0, \frac{35}{3})$

Q21. find the abscissa of points whose ordinate is 4 and which are at a distance 5 units from $(5, 0)$.

Sol.

let abscissa of point = x

\therefore coordinates of point $(x, 4)$

Distance between $(5, 0)$ & $(x, 4) = 5$

$$\Rightarrow \sqrt{(x-5)^2 + (4-0)^2} = 5$$

$$\Rightarrow x^2 + 25 - 10x + 16 = 25$$

$$\Rightarrow x^2 - 10x + 16 = 0$$

$$\Rightarrow x^2 - 8x - 2x + 16 = 0$$

$$\Rightarrow (x-8)(x-2) \Rightarrow x = 2 \text{ or } 8$$

Q22. find the value of x such that $AB = BC$ where A, B and C are the points $(6, -1), (1, 3), (x, 8)$ respectively.

Sol. $A(6, -1), B(1, 3), C(x, 8)$

$$AB = \sqrt{(6-1)^2 + (-1-3)^2} = \sqrt{25+16} = \sqrt{41}$$

$$BC = \sqrt{(x-1)^2 + (8-3)^2} = \sqrt{(x-1)^2 + 25}$$

$$\therefore AB = BC \quad \{ \text{given} \}$$

$$\sqrt{41} = \sqrt{(x-1)^2 + 25}$$

Squaring on both sides

$$41 = (x-1)^2 + 25 \Rightarrow (x-1)^2 = 16$$

$$\Rightarrow x^2 - 2x + 1 = 16 \Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow x^2 - 5x + 3x - 15 = 0 \Rightarrow (x-5)(x+3) = 0$$

$$\Rightarrow x = 5, -3$$

Q23. If A, B and P are the points $(-4, 3), (0, -2)$ and (α, β) respectively and P is equidistant from A and B , Show that $8\alpha - 10\beta + 21 = 0$

Sol. $A(-4, 3), B(0, -2)$ and $P(\alpha, \beta)$

$$PA = \sqrt{(\alpha - (-4))^2 + (\beta - 3)^2} = \sqrt{(\alpha + 4)^2 + (\beta - 3)^2}$$

$$PB = \sqrt{(\alpha - 0)^2 + (\beta - (-2))^2} = \sqrt{\alpha^2 + (\beta + 2)^2}$$

$$PA = PB$$

$$\sqrt{(\alpha + 4)^2 + (\beta - 3)^2} = \sqrt{\alpha^2 + (\beta + 2)^2}$$

Squaring on both sides

$$(\alpha + 4)^2 + (\beta - 3)^2 = \alpha^2 + (\beta + 2)^2$$

$$\Rightarrow \alpha^2 + 8\alpha + 16 + \beta^2 - 6\beta + 9 = \alpha^2 + \beta^2 + 4\beta + 4$$

$$\Rightarrow 8\alpha - 10\beta + 21 = 0$$

\therefore Hence proved.

Q24. The Centre of a Circle is $(2\alpha-1, 3\alpha+1)$ and it passes through the points $(-3, -1)$. find the values of α if a diameter of the circle is of length 20 units.

Sol. Centre $O(2\alpha-1, 3\alpha+1)$, $A(-3, -1)$

$$AO = \sqrt{(2\alpha-1+3)^2 + (3\alpha+1+1)^2} = \sqrt{(2\alpha+2)^2 + (3\alpha+2)^2}$$

$$\therefore \text{Diameter} = 20$$

$$2 \times AO = 20 \Rightarrow AO = 10$$

$$\Rightarrow \sqrt{(2\alpha+2)^2 + (3\alpha+2)^2} = 10$$

$$\Rightarrow 4\alpha^2 + 4 + 8\alpha + 9\alpha^2 + 4 + 12\alpha = 100$$

$$\Rightarrow 13\alpha^2 + 20\alpha - 92 = 0$$

$$\Rightarrow 13\alpha^2 + 46\alpha - 26\alpha - 92 = 0$$

$$\Rightarrow \alpha(13\alpha + 46) - 2(13\alpha + 46) = 0$$

$$\Rightarrow (13\alpha + 46)(\alpha - 2) = 0$$

$$\Rightarrow \alpha = 2, -46/13$$

Q25. find the centre of Circle passing through the points $(8, 12)$, $(11, 3)$, $(0, 14)$. Also find its radius.

Sol. let $O(x, y)$ be the centre of circle

$$\therefore AO = \sqrt{(x-8)^2 + (y-12)^2}$$

$$BO = \sqrt{(x-11)^2 + (y-3)^2}, \quad CO = \sqrt{(x-0)^2 + (y-14)^2}$$

$\therefore AO = BO = CO$ (radii of the same circle)

$$\sqrt{(x-8)^2 + (y-12)^2} = \sqrt{(x-11)^2 + (y-3)^2}$$

Squaring on both sides

$$\Rightarrow x^2 - 16x + 64 + y^2 + 144 - 24y = x^2 - 22x + 121 + y^2 - 6y + 9$$

$$\Rightarrow 6x - 18y + 78 = 0$$

$$\Rightarrow x - 3y = -13 \quad \text{--- (i)}$$

$$\text{Again } \sqrt{(x-11)^2 + (y-3)^2} = \sqrt{x^2 + (y-14)^2}$$

Squaring on both sides

$$\Rightarrow x^2 - 22x + 121 + y^2 - 6y + 9 = x^2 + y^2 - 28y + 196$$

$$\Rightarrow -22x + 22y = 66 \Rightarrow x - y = -3 \quad \text{--- (ii)}$$

$$\text{Subtracting (i) \& (ii) } \quad -2y = -10 \Rightarrow y = 5$$

Substituting the values of y in (ii)

$$x - 5 = -3 \Rightarrow x = 2$$

$$\therefore x = 2, \quad y = 5$$

$$OA = \sqrt{(x-8)^2 + (y-12)^2} = \sqrt{(2-8)^2 + (5-12)^2}$$

$$= \sqrt{36 + 49} = \sqrt{85} \text{ units.}$$

EXERCISE - 11.2

Q1. Find the coordinates of the midpoints of the line segments joining the following pairs of points:

(i) $(2, -3), (-6, 7)$

Sol. midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-6}{2}, \frac{-3+7}{2}\right)$
 $= (-2, 2)$

(ii) $(5, -11), (4, 3)$

Sol. midpoint = $\left(\frac{5+4}{2}, \frac{-11+3}{2}\right) = \left(\frac{9}{2}, -4\right)$

(iii) $(a+3, 5b), (2a-1, 3b+4)$

Sol. midpoint = $\left(\frac{a+3+2a-1}{2}, \frac{5b+3b+4}{2}\right) = \left(\frac{3a+2}{2}, (4b+2)\right)$

Q2. Given that the coordinates of points A $(-3, 2)$ and B $(9, 7)$ respectively, find: (i) The coordinates of midpoint of AB (ii) The distance between A and B.

Sol. $A(-3, 2) \text{ ; } B(9, 7)$

(i) midpoint of AB = $\left(\frac{-3+9}{2}, \frac{2+7}{2}\right) = \left(3, \frac{9}{2}\right)$

(ii) AB = $\sqrt{(9-(-3))^2 + (7-2)^2} = \sqrt{144+25} = \sqrt{169}$
 $= 13 \text{ units.}$

Q3. The coordinates of two points A and B are $(-3, 3)$ and $(12, -7)$ respectively. P is a point on the line segment AB such that $AP:PB = 2:3$

$$A(-3, 3), B(12, -7)$$

Let $P(x, y)$ be the point which divides AB in the ratio of $m_1:m_2$ i.e., $2:3$ then coordinates of P will be

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{(2)(12) + 3(-3)}{2+3} = \frac{15}{5} = 3$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{2(-7) + 3(3)}{2+3} = \frac{-5}{5} = -1$$

\therefore Coordinates of P are $(3, -1)$.

Q4. P divides the distance between $A(-2, 1)$ and $B(1, 4)$ in the ratio $2:1$, calculate the coordinates of the point P.

Sol. $A(-2, 1), B(1, 4) \quad m_1:m_2 = 2:1$

Let $P(x, y)$

Coordinates of P.

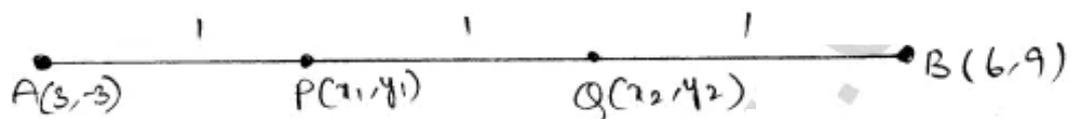
$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{2 \times 1 + 1 \times (-2)}{2+1} = \frac{0}{3} = 0$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{2 \times 4 + 1 \times 1}{2+1} = \frac{9}{3} = 3$$

$\therefore P(x, y) = (0, 3)$.

Q5. Find the coordinates of the points of trisection of the line segment joining the points $(3, -3)$, $(6, 9)$

Sol. let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the points which trisect the line segment joining the points $A(3, -3)$, $B(6, 9)$



$P(x_1, y_1)$ divides AB in the ratio of $1:2$

$$x_1 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 6 + 2 \times 3}{1 + 2} = \frac{12}{3} = 4$$

$$y_1 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 9 + 2 \times (-3)}{1 + 2} = \frac{3}{3} = 1$$

$$\therefore P = (4, 1)$$

Again $Q(x_2, y_2)$ divides the line segment AB in the ratio of $2:1$

$$x_2 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{2 \times 6 + 1 \times 3}{2 + 1} = \frac{15}{3} = 5$$

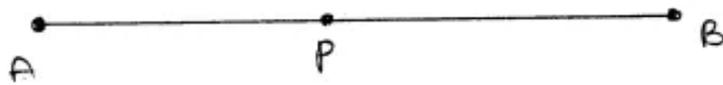
$$y_2 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{2 \times 9 + 1 \times (-3)}{2 + 1} = \frac{15}{3} = 5$$

\therefore coordinates of $Q = (5, 5)$

Q6. Find the coordinates of the point which $\frac{3}{4}$ th of the way from $A(3, 1)$ to $B(-2, 5)$

Sol. let $P = (x, y)$

$$\frac{AP}{AB} = \frac{3}{4}$$

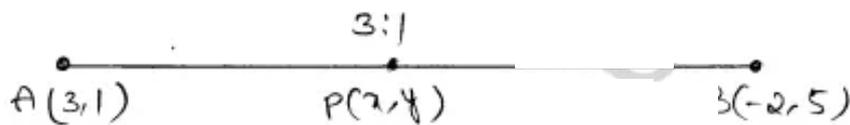


Applying section formula to the coordinate of $\frac{AP}{AB}$

$$\therefore \frac{AP}{PB} = \frac{AP}{AB - AP}$$

$$\Rightarrow \frac{PB}{AP} = \frac{AB - AP}{AP} = \frac{AB}{AP} - 1$$

$$\Rightarrow \frac{PB}{AP} = \frac{4}{3} - 1 = \frac{1}{3} \Rightarrow \frac{AP}{PB} = \frac{3}{1}$$



Now apply section formula.

$$x = \frac{3(-2) + 1(3)}{3+1} = \frac{-3}{4}, \quad y = \frac{3(5) + 1(1)}{3+1} = 4$$

$$\therefore P = (x, y) = \left(-\frac{3}{4}, 4\right)$$

Q7. Point $P(3, -5)$ is reflected to P' in the x -axis. Also P on reflection in the y -axis is mapped as P'' .

(i) find the coordinates of P' and P'' .

(ii) compute the distance $P'P''$.

(iii) find the middle point of the line segment $P'P''$.

(iv) on which coordinate axis does the middle point of the line segment $P'P''$ lie?

Sol. (i) coordinates of P' = image of $P(3, -5)$ reflected on x -axis
 $= (3, 5)$

Coordinates of P'' = image of $P(3, -5)$ reflected on
y-axis
= $(3, -5)$

(ii) length of $PP'' = \sqrt{(-3-3)^2 + (-5-5)^2} = \sqrt{36+100} = \sqrt{136}$
= $2\sqrt{34}$

(iii) let the coordinates of middle point M be (x, y)

$$\therefore x = \frac{x_1 + x_2}{2} = \frac{3-3}{2} = 0, \quad y = \frac{y_1 + y_2}{2} = \frac{-5+5}{2} = 0$$

\therefore middle point is $(0, 0)$

(iv) midpoint of PP'' be $N(x_1, y_1)$

$$x_1 = \frac{3-3}{2} = 0, \quad y_1 = \frac{-5-5}{2} = -5$$

Coordinates of middle point of PP'' are $(0, -5)$

As $x=0$, this point lies on y-axis.

Q8.

Use graph paper for this question. Take $1\text{cm} = 1\text{unit}$ on both axes. Plot the points $A(3, 0)$ and $B(0, 4)$

(i) Write down the coordinates of A_1 , the reflection of A in the y-axis.

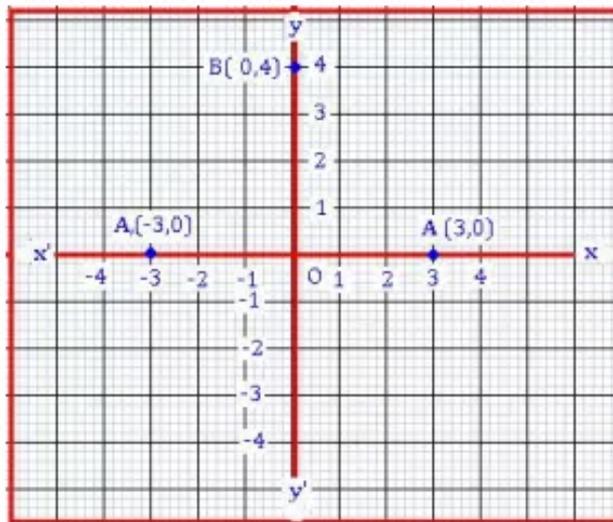
(ii) Write down the coordinates of B_1 , the reflection of B in the x-axis.

(iii) Assign the special name to quadrilateral ABA_1B_1 .

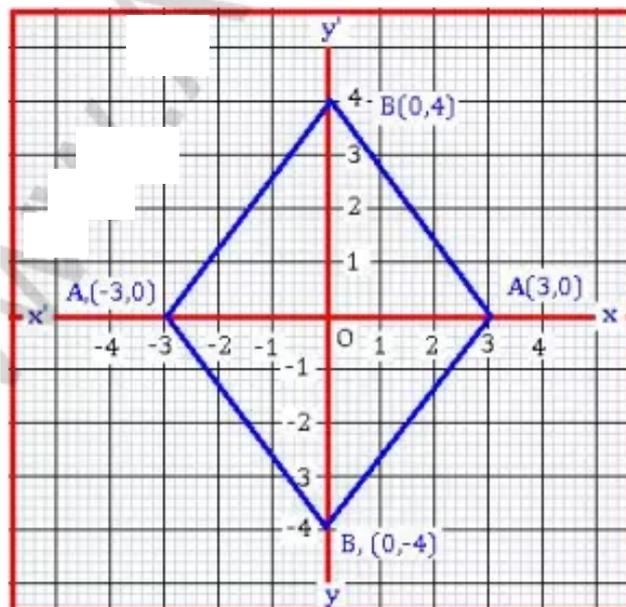
(iv) If C is the midpoint of AB , write down the coordinates of C_1 , the reflection of C in the origin.

(v) Assign the special name to quadrilateral ABC_1B_1 .

Sol. (i)



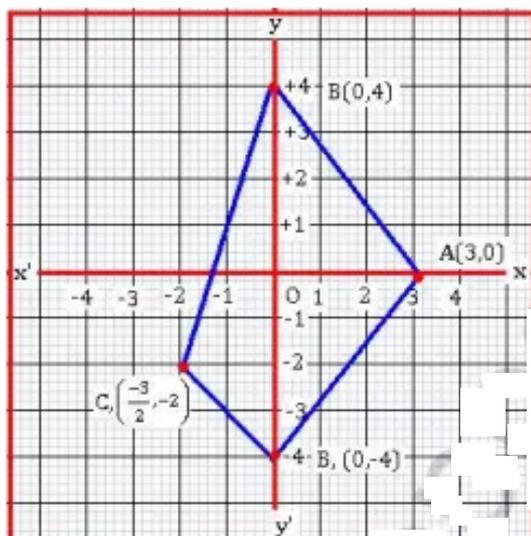
(ii)



As $AB = BA$, so the figure is rhombus.

(iv) As C is the midpoint of AB then coordinates of C are $\left(\frac{-3+0}{2}, \frac{0-4}{2}\right) = \left(-\frac{3}{2}, -2\right)$

(v)



The figure obtained after meeting $ABCD$ is trapezium.

Q9. The line segment joining $A(-3, 1)$ and $B(5, -4)$ is a diameter of a circle whose centre is C . Find the coordinates of the point C .

sol. C is the midpoint of AB .

$$\text{let } C = (x, y)$$

$$(x, y) = \left(\frac{-3+5}{2}, \frac{1-4}{2}\right) = \left(1, -\frac{3}{2}\right)$$

$$\therefore C = (x, y) = \left(1, -\frac{3}{2}\right)$$

Q10. The midpoint of the line segment joining $(2a, 4)$, $(-2, 3b)$ is $(1, 2a+1)$. Find the values of a & b .

sol. $1 = \frac{2a-2}{2}$ and $2a+1 = \frac{4+3b}{2}$

$$\Rightarrow a = 2 \quad \Rightarrow 4a+2 = 4+3b$$

$$\Rightarrow 4a - 3b = 2$$

$$\Rightarrow 4(2) - 3b = 2 \Rightarrow b = 3b \Rightarrow b = 2$$

$$\therefore a = 2, b = 2$$

Q11. The coordinates of the midpoint of the line segment PQ are (1, -2). The coordinates of P are (-3, 2). Find the coordinates of Q.

Sol. Let the coordinates of Q be (x, y), P(-3, 2)

Midpoint of PQ are (1, -2)

$$\text{then } 1 = \frac{-3+x}{2} \Rightarrow -3+x = 2 \Rightarrow x = 5$$

$$-2 = \frac{2+y}{2} \Rightarrow 2+y = -4 \Rightarrow y = -6$$

\therefore Hence coordinates of Q are (5, -6).

Q12. The centre O of a circle has the coordinates (4, 5) and one point on the circumference is (8, 10). Find the coordinates of the other end of the diameter of the circle through this point.

Sol. Centre O (4, 5), A(8, 10)

Let other end of diameter be (x, y) then

O is the midpoint of AB

$$4 = \frac{8+x}{2} \Rightarrow 8+x = 8 \Rightarrow x = 0$$

$$5 = \frac{10+y}{2} \Rightarrow 10+y = 10 \Rightarrow y = 0$$

\therefore Coordinates of other end are (0, 0).

Q13. find the reflection (image) of the point $(5, -3)$ in the point $(-1, 3)$.

Sol. let the coordinates of the image of the point $A(5, -3)$ be $A_1(x, y)$ in the point $(-1, 3)$ then the point $(-1, 3)$ will be the midpoint of AA_1 ,

$$-1 = \frac{5+x}{2} \Rightarrow 5+x = -2 \Rightarrow x = -7$$

$$3 = \frac{-3+y}{2} \Rightarrow -3+y = 6 \Rightarrow y = 9$$

\therefore coordinates of the image A will be $(-7, 9)$.

Q14. The line segment joining $A(-1, \frac{5}{3})$ and $B(a, 5)$ is divided in the ratio $1:3$ at P , the point where the line segment AB intersects y -axis. Calculate (i) The value of a (ii) The co-ordinates of P .

Sol. let $P(x, y)$ divides the line segment joining the points $A(-1, \frac{5}{3})$, $B(a, 5)$ in the ratio $1:3$

$$x = \frac{1 \times a + 3(-1)}{1+3} = \frac{a-3}{4}$$

$$y = \frac{1 \times 5 + 3(\frac{5}{3})}{1+3} = \frac{10}{4} = \frac{5}{2}$$

(i) AB intersects y -axis at P , $\frac{a-3}{4} = 0 \Rightarrow a = 3$.

(ii) coordinates of P are $(0, \frac{5}{2})$.

Q15. The point $P(-4, 1)$ divides the line segment joining the points $A(2, -2)$ and B in the ratio of $3:5$. find the point B .

Sol.

let the coordinates of B be (x, y) .

A (2, -2) and P(-4, 1) divides AB in the ratio of 3:5

$$-4 = \frac{3x + 5(2)}{3+5} \Rightarrow -32 = 3x + 10 \Rightarrow 3x = -42$$
$$\Rightarrow x = -14$$

$$1 = \frac{3y + 5(-2)}{3+5} \Rightarrow 3y - 10 = 8 \Rightarrow y = 6$$

\therefore Coordinates of B = (-14, 6)

Q16. (i) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6)?

Sol. (R) let the ratio be $m_1 : m_2$ that the point (5, 4) divides the line segment joining the points (2, 1), (7, 6).

$$5 = \frac{m_1 \times 7 + m_2 \times 2}{m_1 + m_2} \Rightarrow 7m_1 + 2m_2 = 5m_1 + 5m_2$$

$$\Rightarrow 2m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{2} \Rightarrow m_1 : m_2 = 3 : 2$$

(ii) In what ratio does the point (-4, b) divide the line segment joining the points P(2, -2), Q(-14, 6). Hence find the value of b.

Sol. let the ratio be $m_1 : m_2$, the point (-4, b) divides the line segment joining the points P(2, -2) and Q(-14, 6)

$$-4 = \frac{m_1(-14) + m_2 \times 2}{m_1 + m_2} \Rightarrow -14m_1 + 2m_2 = -4m_1 - 4m_2$$

$$\Rightarrow 10m_1 = 6m_2 \Rightarrow \frac{m_1}{m_2} = \frac{6}{10} = \frac{3}{5}$$

Again,

$$b = \frac{m_1 \times 6 + m_2(-2)}{m_1 + m_2} = \frac{6m_1 - 2m_2}{m_1 + m_2}$$

$$\Rightarrow b = \frac{6 \times 3 - 2 \times 5}{3 + 5} = \frac{8}{8} = 1$$

$$\therefore b = 1$$

Q17. The line segment joining $A(2, 3)$ and $B(6, -5)$ is intersected by x -axis at a point K . Write down the ordinate of the point K . Hence, find the ratio in which K divides AB .

Sol. let the coordinates of K be $(x, 0)$ as it intersects x -axis.

let point K divides the line segment joining the point $A(2, 3)$ and $B(6, -5)$ in the ratio $m_1 : m_2$.

$$0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{m_1(-5) + m_2 \times 3}{m_1 + m_2}$$

$$\Rightarrow -5m_1 + 3m_2 = 0 \Rightarrow 5m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{5}$$

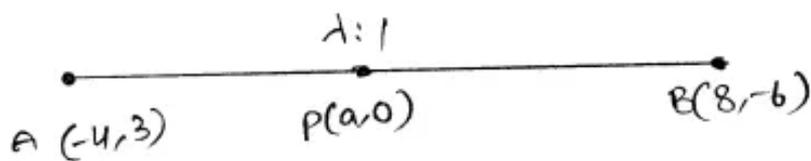
ordinate of point $K = 0$.

Q18. If $A = (-4, 3)$ and $B = (8, -6)$, (i) find the length of AB (ii) In what ratio is the line joining AB , divided by the x -axis?

Sol. $A = (-4, 3)$, $B(8, -6)$

$$(i) AB = \sqrt{(8 - (-4))^2 + (-6 - 3)^2} = \sqrt{144 + 81} = 15$$

(ii) let the ratio be $\lambda : 1$ and the point on the x -axis is $P(a, 0)$



By section formula, $0 = \frac{-6\lambda + 3}{\lambda + 1} \Rightarrow \lambda = -\frac{1}{2}$

Hence the required ratio is 1:2

Q19. Calculate the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.

Sol. Let the point P divides the line segment joining the points A(3, 4) and B(-2, 1) in the ratio of $m_1:m_2$ and let the coordinates of P be (0, y) as it intersects y-axis

$$0 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{m_1(-2) + m_2 \times 3}{m_1 + m_2}$$

$$\Rightarrow -2m_1 + 3m_2 = 0 \Rightarrow 2m_1 = 3m_2 \Rightarrow \frac{m_1}{m_2} = \frac{3}{2}$$

Q20. (i) Write down the coordinates of the point P that divides the line joining A(-4, 1) and B(17, 10) in the ratio 1:2

(ii) Calculate the distance OP where O is the origin.

(iii) In what ratio does the y-axis divide the line AB?

Sol. (i) Let coordinates of P be (x, y) which divides the line segment joining the points A(-4, 1) and B(17, 10) in the ratio of 1:2

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 17 + 2(-4)}{1 + 2} = \frac{9}{3} = 3$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times 1}{1 + 2} = \frac{12}{3} = 4$$

\therefore coordinates of P are (3, 4).

(ii) Distance of OP where O is origin i.e., $O = (0, 0)$

$$\text{Distance} = \sqrt{(3-0)^2 + (4-0)^2} = 5$$

(iii) let y-axis divides the AB in the ratio of $m_1 : m_2$ at P and let co-ordinates of P be (0, y)

$$0 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{m_1 \times 17 + m_2 \times (-4)}{m_1 + m_2}$$

$$\Rightarrow 17m_1 - 4m_2 = 0 \Rightarrow \frac{m_1}{m_2} = \frac{4}{17}$$

Q21. Calculate the length of the median through the vertex A of the triangle ABC with vertices $A(7, -3)$, $B(5, 3)$ and $C(3, -1)$

Sol. let $D(x, y)$ be the median of $\triangle ABC$ through A to BC.

\therefore D will be the midpoint of BC.

Coordinates of D will be,

$$x = \frac{5+3}{2} = 4, \quad y = \frac{3-1}{2} = 1$$

$$\therefore D = (4, 1)$$

$$\text{length of DA} = \sqrt{(7-4)^2 + (-3-1)^2}$$

$$= \sqrt{9+16}$$

$$= 5$$

Q22. prove by section formula that the points $(10, -6)$, $(2, -6)$, $(-4, -2)$ and $(4, -2)$ taken in this order, are the vertices of a parallelogram.

Sol. let $A(10, -6)$, $B(2, -6)$, $C(-4, -2)$, $D(4, -2)$

$$AB = \sqrt{(10-2)^2 + (-6+6)^2} = \sqrt{64} = 8$$

$$BC = \sqrt{(2+4)^2 + (-6+2)^2} = \sqrt{36+16} = \sqrt{52}$$

$$CD = \sqrt{(4+4)^2 + (-2+2)^2} = \sqrt{64} = 8$$

$$DA = \sqrt{(4-10)^2 + (-2+6)^2} = \sqrt{36+16} = \sqrt{52}$$

$$\therefore AB = CD \text{ and } BC = DA$$

∴ Hence proved

Q23. Three consecutive vertices of a parallelogram ABCD are $A(1, 2)$, $B(1, 0)$, $C(4, 0)$. find the fourth vertex D.

Sol. let O is the midpoint of AC the diagonal of ABCD

$$\therefore \text{Coordinates of O will be } \left(\frac{1+4}{2}, \frac{2+0}{2}\right) = \left(\frac{5}{2}, 1\right)$$

∴ O is also the midpoint of second diagonal BD and let co-ordinates of D be (x, y)

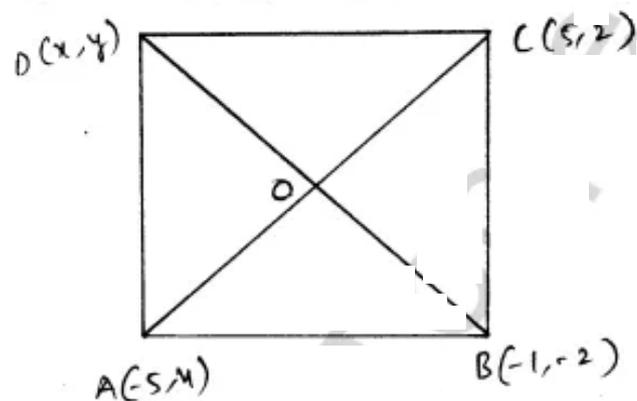
$$\frac{5}{2} = \frac{1+x}{2} \Rightarrow x = 4$$

$$1 = \frac{0+y}{2} \Rightarrow y = 2$$

∴ Coordinates of D are $(4, 2)$.

Q24. prove that the points $A(-5, 4)$, $B(-1, -2)$ and $C(5, 2)$ are the vertices of an isosceles right-angled triangle. find the co-ordinates of D so that $ABCD$ is a square.

Sol. Given points $A(-5, 4)$, $B(-1, -2)$, $C(5, 2)$
 If they are vertices of an isosceles right-angled triangle ABC then $AB = BC$.



$$AB = \sqrt{(-1 - (-5))^2 + (-2 - 4)^2} = \sqrt{16 + 36} = \sqrt{52}$$

$$BC = \sqrt{(5 - (-1))^2 + (2 - (-2))^2} = \sqrt{36 + 16} = \sqrt{52}$$

$$\therefore AB = BC$$

$\triangle ABC$ is an isosceles triangle.

$$AC = \sqrt{(5 - (-5))^2 + (4 - (-2))^2} = \sqrt{100 + 4} = \sqrt{104}$$

$$\text{Now } AC^2 = AB^2 + BC^2$$

$$(\sqrt{104})^2 = (\sqrt{52})^2 + (\sqrt{52})^2$$

$$\Rightarrow 104 = 104$$

$\therefore \triangle ABC$ is an isosceles right-angled triangle.

Let the coordinates of D be (x, y) and diagonals AC and BD of $ABCD$ is a square,

then diagonals bisect each other at O.
 If ABCD is a square, then diagonals bisect each other at O.

O is midpoint of AC

$$O = \left(\frac{5-5}{2}, \frac{2+4}{2} \right) = (0, 3)$$

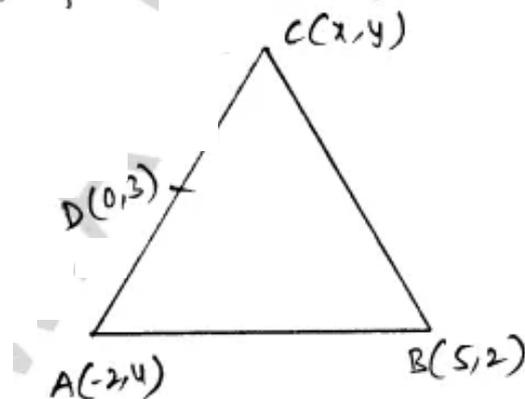
O is midpoint of BD also.

$$O = \left(\frac{-1+x}{2}, \frac{2+y}{2} \right) \Rightarrow x=1, y=8$$

$$\therefore D = (1, 8)$$

Q25. Find the third vertex of a triangle if its two vertices are $(-1, 4)$ and $(5, 2)$ and midpoint of one side is $(0, 3)$.

Sol. Let $D(0, 3)$ be the midpoint of AC and co-ordinates of C be (x, y) .



$$O = \left(\frac{-1+x}{2}, \frac{4+y}{2} \right) \Rightarrow x=1, y=2$$

\therefore Co-ordinates will be $(1, 2)$

If we take midpoint $D(0, 3)$ of BC, then

$$0 = \frac{5+x}{2} \Rightarrow x = -5, \quad 3 = \frac{x+y}{2} \Rightarrow y = 4$$

\therefore Co-ordinates of C will be $(-5, 4)$.

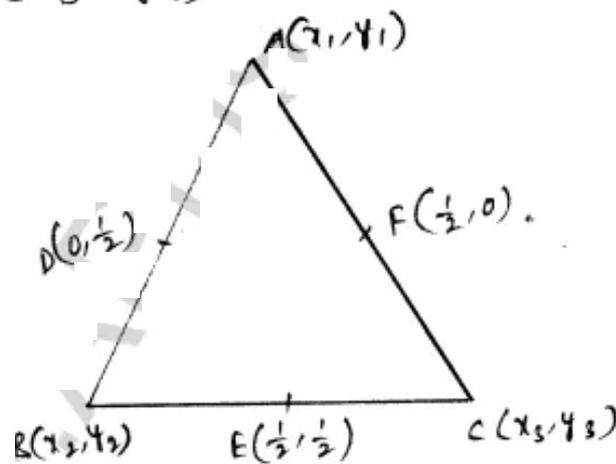
The third vertex will be $(1, 2)$ or $(-5, 4)$.

Q26. Find the co-ordinates of the vertices of the triangle the middle points of whose sides are $(0, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, 0)$.

Sol.

Let ABC be a triangle, in which $D(0, \frac{1}{2})$, $E(\frac{1}{2}, \frac{1}{2})$, $F(\frac{1}{2}, 0)$ are the midpoints of sides AB, BC and CA respectively.

Let co-ordinates of A be (x_1, y_1) , B (x_2, y_2) and C (x_3, y_3) .



$$0 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 0 \quad \text{--- (i)}$$

$$\frac{1}{2} = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 1 \quad \text{--- (ii)}$$

$$\text{Again } \frac{1}{2} = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 1 \quad \text{--- (iii)}$$

$$\text{and } \frac{1}{2} = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 1 \quad \text{--- (iv)}$$

$$\frac{1}{2} = \frac{\lambda_3 + \lambda_1}{2} \Rightarrow \lambda_3 + \lambda_1 = 1 \quad \text{--- (v)}$$

$$0 = \frac{\mu_3 + \mu_1}{2} \Rightarrow \mu_3 + \mu_1 = 0 \quad \text{--- (vi)}$$

Adding (i), (iii) & (v)

$$2(\lambda_1 + \lambda_2 + \lambda_3) = 0 + 1 + 1 = 2$$

$$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Now subtracting (iii), (v) and (i) respectively, we get

$$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 1$$

Again adding (ii), (iv) and (v)

$$2(\mu_1 + \mu_2 + \mu_3) = 1 + 1 + 0 = 2$$

$$\Rightarrow \mu_1 + \mu_2 + \mu_3 = 1$$

Now subtracting (iv), (v) and (ii) respectively, we get

$$\mu_1 = 0, \mu_2 = 1, \mu_3 = 0$$

$$\therefore A = (0, 0), B(0, 1), C(1, 0)$$

Q27. Show by section formula that the points $(3, -2)$, $(5, 2)$ and $(8, 8)$ are collinear.

sol.

Let the point $(5, 2)$ divides the line joining the points $(3, -2)$ and $(8, 8)$ in the ratio of $m_1 : m_2$

$$5 = \frac{m_1 \times 8 + m_2 \times 3}{m_1 + m_2} \Rightarrow 8m_1 + 3m_2 = 5m_1 + 5m_2$$

$$\Rightarrow 3m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3} \quad \text{--- (i)}$$

$$\text{Again, } 2 = \frac{8m_1 - 2m_2}{m_1 + m_2} \Rightarrow 8m_1 - 2m_2 = 2m_1 + 2m_2$$

$$\Rightarrow 6m_1 = 4m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3} \quad \text{--- (ii)}$$

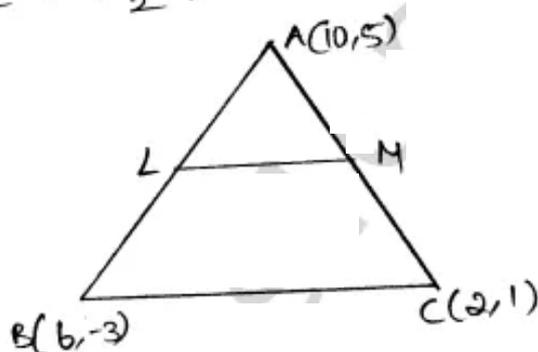
From (i) & (ii) it is clear that point $(5, 2)$ lies on the line joining the points $(3, -2)$ and $(8, 8)$.
 \therefore Hence proved.

Q28. $A(10, 5)$, $B(6, -3)$ and $C(2, 1)$ are the vertices of a triangle ABC . L is the midpoint of AB and M is the midpoint of AC . Write down the co-ordinates of L and M . Show that $LM = \frac{1}{2} BC$

Sol.

$$L = \left(\frac{10+6}{2}, \frac{5-3}{2} \right) = (8, 1)$$

$$M = \left(\frac{10+2}{2}, \frac{5+1}{2} \right) = (6, 3)$$



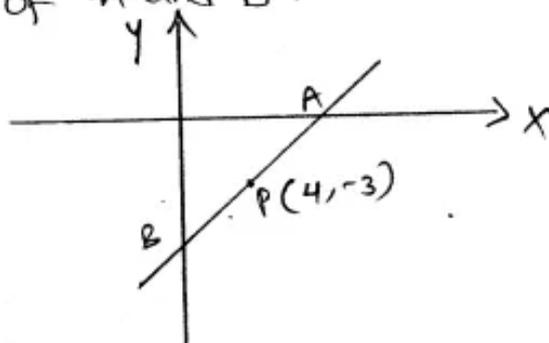
$$LM = \sqrt{(6-8)^2 + (3-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ --- (i)}$$

$$BC = \sqrt{(2-6)^2 + (1-(-3))^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ --- (ii)}$$

From (i) & (ii), $LM = \frac{1}{2} BC$

Q29. The midpoint of the line segment AB shown in the adjoining diagram is $(4, -3)$. Write down the co-ordinates of A and B .

Sol.



A lies on x-axis and B on y-axis
let co-ordinates of A be $(x, 0)$ and B be $(0, y)$

$P(4, -3)$ is the third point of AB

$$4 = \frac{x+0}{2} \Rightarrow x = 8, \quad -3 = \frac{0+y}{2} \Rightarrow y = -6$$

\therefore co-ordinates of A will be $(8, 0)$ and B $(0, -6)$

Q30. find the co-ordinates of the centroid of a triangle whose vertices are: $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$

Sol. The co-ordinates of centroid $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right)$
 $\Rightarrow G = \left(\frac{-1+1+5}{3}, \frac{3-1-1}{3} \right) = \left(\frac{5}{3}, 1 \right)$

Q31. two vertices of a triangle are $(3, -5)$ and $(-7, 4)$. find the third vertex, given that the centroid is $(2, -1)$.

Sol. let the co-ordinates of third vertex is (x, y)
and other two vertices are $(3, -5)$ and $(-7, 4)$
and centroid $= (2, -1)$

$$2 = \frac{3-7+x}{3} \Rightarrow \frac{x-4}{3} = 2 \Rightarrow x = 10$$

$$\Rightarrow -1 = \frac{-5+4+y}{3} \Rightarrow y-1 = -3 \Rightarrow y = -2$$

\therefore co-ordinates are $(10, -2)$.