Chapter 16. Area Theorems [Proof and Use]

Exercise 16(A)

Solution 1:

(i) ΔADE and parallelogram ABED are on the same base AB and between the same parallels DE//AB, so area of the triangle ΔADE is half the area of parallelogram ABED.

Area of ABED = 2 (Area of ADE) = 120 cm^2

(ii)Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between the same parallels

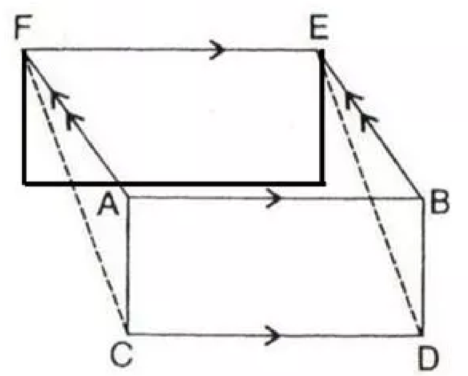
Area of ABCF = Area of ABED = 120 cm^2

(iii)We know that area of triangles on the same base and between same parallel lines are equal

Area of ABE=Area of ADE =60 cm^2

Solution 2:

After drawing the opposite sides of AB, we get



Since from the figure, we get CD//FE therefore FC must parallel to DE. Therefore it is proved that the quadrilateral CDEF is a parallelogram.
Area of parallelogram on same base and between same parallel lines is always equal and area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between same parallel lines.
So Area of CDEF= Area of ABDC + Area of ABEF
Hence Proved

Solution 3:

(i)

Since POS and parallelogram PMLS are on the same base PS and between the same parallels i.e. SP//LM.

As O is the center of LM and Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.

The area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels.

So 2(Area of PSO)=Area of PMLS

Hence Proved.

(ii)

Consider the expression $Area(\triangle POS) + Area(QOR)$;

LM is parallel to PS and PS is parallel to RQ, therefore, LM is

Since triangle POS lie on the base PS and in between the parallels PS and LM, we have, $Area(\triangle POS) = \frac{1}{2} Area(\square PSLM)$,

Since triangle QOR lie on the base QR and in between the parallels LM and RQ, we have,

$$Area(\triangle QOR) = \frac{1}{2} Area(\square LMQR)$$

$$\begin{aligned} Area(\triangle POS) + Area(\triangle QOR) &= \frac{1}{2} Area(\square PSLM) + \frac{1}{2} Area(\square LMQR) \\ &= \frac{1}{2}[Area(\square PSLM) + Area(\square LMQR)] \\ &= \frac{1}{2}[Area(\square PQRS)] \end{aligned}$$

(iii)

In a parallelogram, the diagonals bisect each other.

Therefore, OS = OQ

Consider the triangle PQS, since OS = OQ, OP is the median of the triangle PQS.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

$Area(\triangle POS) = Area(\triangle POQ) \ldots .(1)$

Similarly, since OR is the median of the triangle QRS, we have,

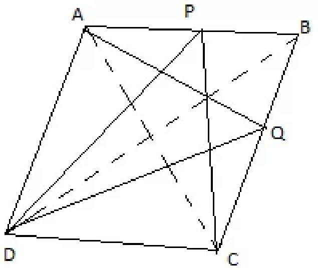
$Area(\triangle QOR) = Area(\triangle SOR) \ldots .(2)$

Adding equations (1) and (2), we have,

$Area(\triangle POS) + Area(\triangle QOR) = Area(\triangle POQ) + Area(\triangle SOR)$

Hence Proved.

Solution 4:



(i)

Given ABCD is a parallelogram. P and Q are any points on the sides AB and BC respectively, join diagonals AC and BD.
proof:
since triangles with same base and between same set of parallel lines have equal areas
area (CPD)=area(BCD)...... (1)
again, diagonals of the parallelogram bisects area in two equal parts
area (BCD)=(1/2) area of parallelogram ABCD...... (2)

from (1) and (2)
area(CPD)=1/2 area(ABCD)...... (3)
similarly area (AQD)=area(ABD)=1/2 area(ABCD)...... (4)
from (3) and (4)
area(CPD)=area(AQD),
hence proved.
(ii)
We know that area of triangles on the same base and between same parallel lines are equal
So Area of AQD= Area of ACD= Area of PDC = Area of BDC = Area of ABC=Area of APD + Area of BPC
Hence Proved

Solution 5:

(i)

Since triangle BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$So\ Area(\triangle BEC) = \frac{1}{2} \times Area(\square ABCD) = \frac{1}{2} \times 48 = 24\ cm^2$$

(ii)

$$\begin{aligned} Area(\square ANMD) &= Area(\square BNMC) \\ &= \frac{1}{2} Area(\square ABCD) \\ &= \frac{1}{2} \times 2 \times Area(\triangle BEC) \\ &= Area(\triangle BEC) \end{aligned}$$

Therefore, Parallelograms ANMD and NBCM have areas equal to triangle BEC

Solution 6:

Since $\triangle$ DCB and $\triangle$ DEB are on the same base DB and between the same parallels i.e. DB//CE, therefore we get

$$\begin{aligned} \text{Ar.}(\Delta DCB) &= \text{Ar.}(\Delta DEB) \\ \text{Ar.}(\Delta DCB + \Delta ADB) &= \text{Ar.}(\Delta DEB + \Delta ADB) \\ \text{Ar.}(ABCD) &= \text{Ar.}(\Delta ADE) \end{aligned}$$

Hence proved

Solution 7:

$\triangle$ APB and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.

$$\therefore \text{Ar.}(\Delta APB) = \frac{1}{2}\text{Ar.}(\text{parallelogram ABCD}) \quad ......(i)$$

$\triangle$ ADQ and parallelogram ABCD are on the same base AD and between the same parallel lines AD and BQ.

$$\therefore \text{Ar.}(\Delta ADQ) = \frac{1}{2}\text{Ar.}(\text{parallelogram ABCD}) \quad ......(ii)$$

Adding equation (i) and (ii), we get

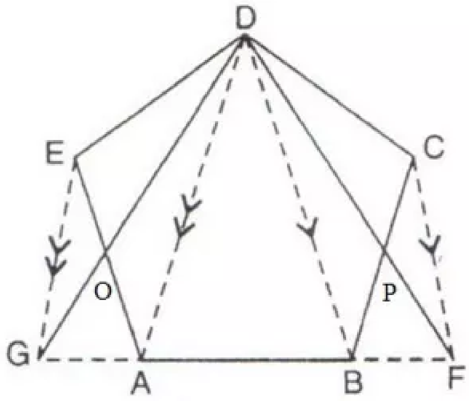
$$\begin{aligned} \therefore \text{Ar.}(\Delta APB) + \text{Ar.}(\Delta ADQ) &= \text{Ar.}(\text{parallelogram ABCD}) \\ \text{Ar.}(\text{quad.} ADQB) - \text{Ar.}(\Delta BPQ) &= \text{Ar.}(\text{parallelogram ABCD}) \\ \text{Ar.}(\text{quad.} ADQB) - \text{Ar.}(\Delta BPQ) &= \text{Ar.}(\text{quad.} ADQB) - \text{Ar.}(\Delta DCQ) \\ \text{Ar.}(\Delta BPQ) &= \text{Ar.}(\Delta DCQ) \end{aligned}$$

Subtracting Ar. $\triangle$ PCQ from both sides, we get

$$\begin{aligned} \text{Ar.}(\Delta BPQ) - \text{Ar.}(\Delta PCQ) &= \text{Ar.}(\Delta DCQ) - \text{Ar.}(\Delta PCQ) \\ \text{Ar.}(\Delta BCP) &= \text{Ar.}(\Delta DPQ) \end{aligned}$$

Hence proved.

Solution 8:



Since triangle EDG and EGA are on the same base EG and between the same parallel lines EG and DA, therefore

$Ar.(\Delta EDG) = Ar.(\Delta EGA)$

Subtracting ΔEOG from both sides, we have

$Ar.(\Delta EOD) = Ar.(\Delta GOA)$ (i)

Similarly

$Ar.(\Delta DPC) = Ar.(\Delta BPF)$ (ii)

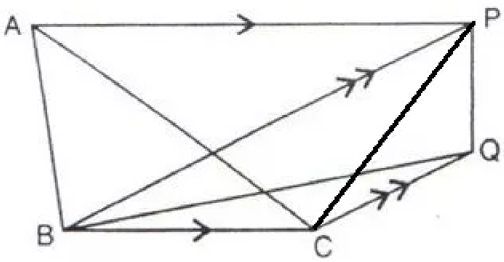
Now

$$\begin{aligned} Ar.(\Delta GDF) &= Ar.(\Delta GOA) + Ar.(\Delta BPF) + Ar.(\text{pen}.ABPDO) \\ &= Ar.(\Delta EOD) + Ar.(\Delta DPC) + Ar.(\text{pen}.ABPDO) \\ &= Ar.(\text{pen}.\text{ABCDE}) \end{aligned}$$

Hence proved

Solution 9:

Joining PC we get



Δ ABC and Δ BPC are on the same base BC and between the same parallel lines AP and BC.

$\therefore$ Ar.$(\Delta ABC) =$ Ar.(ΔBPC) ......(i)

Δ BPC and Δ BQP are on the same base BP and between the same parallel lines BP and CQ.

$\therefore$ Ar.$(\Delta BPC) =$ Ar.(ΔBQP) ......(ii)

From (i) and (ii), we get

$\therefore$ Ar.$(\Delta ABC) =$ Ar.(ΔBQP)

Hence proved.

Solution 10:

(i)

$$\angle EAC = \angle EAB + \angle BAC$$
$$\angle EAC = 90° + \angle BAC \qquad ......(i)$$
$$\angle BAF = \angle FAC + \angle BAC$$
$$\angle BAF = 90° + \angle BAC \qquad ......(ii)$$

From (i) and (ii), we get

$\angle EAC = \angle BAF$

In $\triangle$EAC and $\triangle$BAF, we have, EA=AB

$\angle EAC = \angle BAF$ and AC=AF

$\therefore$ $\triangle$EAC $\cong$ $\triangle$BAF (SAS axiom of congruency)

(ii)

Since $\triangle ABC$ is a right triangle, we have,

$$AC^2 = AB^2 + BC^2 \quad [\text{Using Pythagoras Theorem in } \triangle ABC]$$
$$\Rightarrow AB^2 = AC^2 - BC^2$$
$$\Rightarrow AB^2 = (AR + RC)^2 - (BR^2 + RC^2) \quad [\text{Since } AC = AR + RC \text{ and Using Pythagoras Theorem in } \triangle BRC]$$
$$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2) \quad [\text{Using the identity}]$$
$$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2) \quad [\text{Using Pythagoras Theorem in } \triangle ABR]$$
$$\Rightarrow 2AB^2 = 2AR^2 + 2AR \times RC$$
$$\Rightarrow AB^2 = AR(AR + RC)$$
$$\Rightarrow AB^2 = AR \times AC$$
$$\Rightarrow AB^2 = AR \times AF$$
$$\Rightarrow Area(\square ABDE) = Area(rectangle\ ARHF)$$

Solution 11:

(i)

In $\triangle$ABC, D is midpoint of AB and E is the midpoint of AC.

$$\frac{AD}{AB} = \frac{AE}{AC}$$

DE is parallel to BC.

$$\therefore Ar.(\Delta ADC) = Ar.(\Delta BDC) = \frac{1}{2} Ar.(\Delta ABC)$$

Again

$$\therefore Ar.(\Delta AEB) = Ar.(\Delta BEC) = \frac{1}{2} Ar.(\Delta ABC)$$

From the above two equations, we have

Area ($\triangle$ADC) = Area($\triangle$AEB).

Hence Proved

(ii)

We know that area of triangles on the same base and between same parallel lines are equal

Area(triangle DBC)= Area(triangle BCE)

Area(triangle DOB) + Area(triangle BOC) = Area(triangle BOC) + Area(triangle COE)

So Area(triangle DOB) = Area(triangle COE)

Solution 12:

(i)

Since $\triangle$EBC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$\therefore \text{Ar.}(\Delta EBC)=\frac{1}{2}\times\text{Ar.}(\text{parallelogram ABCD})$$

$$(\text{parallelogram ABCD})=2\times\text{Ar.}(\Delta EBC)$$

$$=2\times480\text{cm}^2$$

$$=960\text{cm}^2$$

(ii)

Parallelograms on same base and between same parallels are equal in area

Area of BCFE = Area of ABCD= 960 cm^2

(iii)

Area of triangle ACD=480 = (1/2) x 30 x Altitude

Altitude=32 cm

(iv)

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.

Therefore,

$$Area(\triangle ECF)=\frac{1}{2}Area(\square CBEF)$$

$$Similarly,\ Area(\triangle BCE)=\frac{1}{2}Area(\square CBEF)$$

$$\Rightarrow Area(\triangle ECF)=Area(\triangle BCE)=480\ cm^2$$

Solution 13:

Here AD=DB and EC=DB, therefore EC=AD

Again, $\angle EFC=\angle AFD$ (opposite angles)

Since ED and CB are parallel lines and AC cut this line, therefore

$\angle ECF=\angle FAD$

From the above conditions, we have

$\Delta EFC=\Delta AFD$

Adding quadrilateral CBDF in both sides, we have

Area of // gm BDEC= Area of $\triangle$ABC

Solution 14:

In Parallelogram PQRS, AC // PS // QR and PQ // DB // SR.

Similarly, AQRC and APSC are also parallelograms.

Since $\triangle$ABC and parallelogram AQRC are on the same base AC and between the same parallels, then

Ar.($\triangle$ABC)= $\frac{1}{2}$ Ar.(AQRC)......(i)

Similarly,

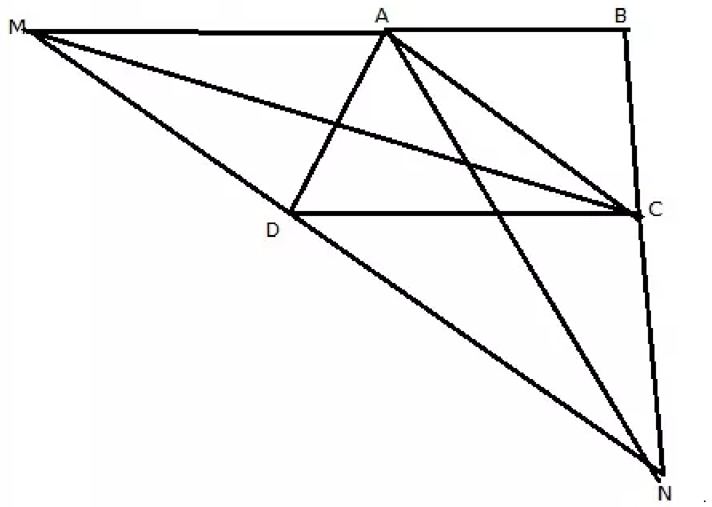
Ar.($\triangle$ADC)= $\frac{1}{2}$ Ar.(APSC).......(ii)

Adding (i) and (ii), we get

Area of quadrilateral PQRS = 2 × Area of quad. ABCD

Solution 15:

Given: ABCD is a trapezium



AB || CD, MN || AC

Join C and M

We know that area of triangles on the same base and between same parallel lines are equal.

So Area of Δ AMD = Area of Δ AMC

Similarly, consider AMNC quadrilateral where MN || AC.

Δ ACM and Δ ACN are on the same base and between the same parallel lines. So areas are equal.

So, Area of Δ ACM = Area of Δ CAN

From the above two equations, we can say

Area of Δ ADM = Area of Δ CAN

Hence Proved.

Solution 16:

We know that area of triangles on the same base and between same parallel lines are equal.
Consider ABED quadrilateral; AD||BE
With common base, BE and between AD and BE parallel lines, we have
Area of ΔABE = Area of ΔBDE
Similarly, in BEFC quadrilateral, BE||CF
With common base BC and between BE and CF parallel lines, we have
Area of ΔBEC = Area of ΔBEF
Adding both equations, we have
Area of ΔABE + Area of ΔBEC = Area of ΔBEF + Area of ΔBDE
=> Area of AEC = Area of DBF
Hence Proved

Solution 17:

Given: ABCD is a parallelogram.
We know that
Area of ΔABC = Area of ΔACD
Consider ΔABX,
Area of ΔABX = Area of ΔABC + Area of ΔACX
We also know that area of triangles on the same base and between same parallel lines are equal.
Area of ΔACX = Area of ΔCXD
From above equations, we can conclude that
Area of ΔABX = Area of ΔABC + Area of ΔACX = Area of ΔACD+ Area of ΔCXD = Area of ACXD Quadrilateral
Hence Proved

Solution 18:

Join B and R and P and R.
We know that the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram

are on the same base and between the parallels
Consider ABCD parallelogram:
Since the parallelogram ABCD and the triangle ABR lie on AB and between the parallels AB and DC, we have

$$Area(\square ABCD) = 2 \times Area(\triangle ABR) \text{....(1)}$$

We know that the area of triangles with same base and between the same parallel lines are equal.

Since the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,

$$Area(\triangle ABR) = Area(\triangle APR) \text{....(2)}$$

From equations (1) and (2), we have,

$$Area(\square ABCD) = 2 \times Area(\triangle APR) \text{....(3)}$$

Also, the triangle APR and the parallelogram ARQP
lie on the same base AR band between the parallels, AR and QP,

$$Area(\triangle APR) = \frac{1}{2} \times Area(\square ARQP) \text{....(4)}$$

Using (4) in equation (3), we have,

$$Area(\square ABCD) = 2 \times \frac{1}{2} \times Area(\square ARQP)$$

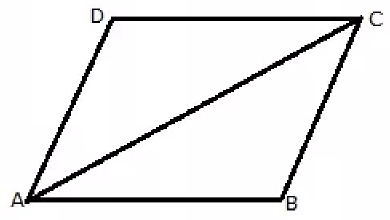
$$Area(\square ABCD) = Area(\square ARQP)$$

Hence proved.

Exercise 16(B)

Solution 1:

(i) Suppose ABCD is a parallelogram (given)

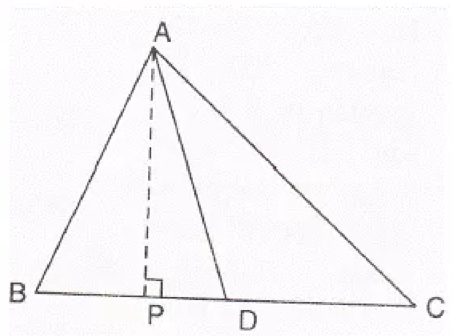


Consider the triangles ABC and ADC:
$AB = CD$ [ABCD is a parallelogram]
$AD = BC$ [ABCD is a parallelogram]
$AD = AD$ [common]
By Side – Side – Side criterion of congruence, we have,
$\triangle ABC \cong \triangle ADC$

Area of congruent triangles are equal.

Therefore, Area of ABC = Area of ADC

(ii) Consider the following figure:



Here $AP \perp BC$

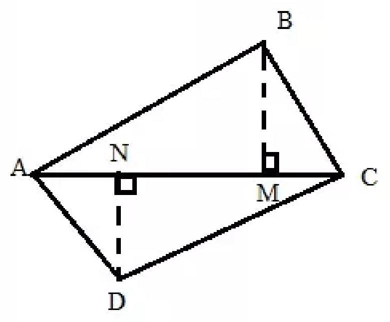
Since Ar.(ΔABD)= $\frac{1}{2}BD \times AP$

And, Ar.(ΔADC)= $\frac{1}{2}DC \times AP$

$$\therefore \frac{Area(\Delta ABD)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BD \times AP}{\frac{1}{2}DC \times AP} = \frac{BD}{DC},$$

hence proved

(iii) Consider the following figure:



Here

Ar.(ΔABC)= $\frac{1}{2}BM \times AC$

And, Ar.(ΔADC)= $\frac{1}{2}DN \times AC$

$$\therefore \frac{Area(\Delta ABC)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BM \times AC}{\frac{1}{2}DN \times AC} = \frac{BM}{DN},$$

hence proved

Solution 2:

AD is the median of Δ ABC. Therefore it will divide Δ ABC into two triangles of equal areas.

$\therefore$ Area(Δ ABD)= Area(Δ ACD) (i)

ED is the median of Δ EBC

$\therefore$ Area(Δ EBD)= Area(Δ ECD) (ii)

Subtracting equation (ii) from (i), we obtain

Area(Δ ABD)- Area(Δ EBD)= Area(Δ ACD)- Area(Δ ECD)

Area (Δ ABE) = Area (Δ ACE). Hence proved

Solution 3:

AD is the median of $\triangle$ ABC. Therefore it will divide $\triangle$ ABC into two triangles of equal areas.

$\therefore$ Area($\triangle$ ABD)= Area($\triangle$ ACD)

Area ($\triangle$ ABD)= $\frac{1}{2}$ Area($\triangle$ ABC) (i)

In $\triangle$ ABD, E is the mid-point of AD. Therefore BE is the median.

$\therefore$ Area($\triangle$ BED)= Area($\triangle$ ABE)

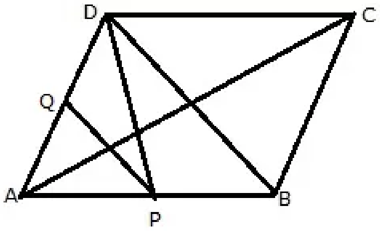
Area($\triangle$ BED)= $\frac{1}{2}$ Area($\triangle$ ABD)

Area($\triangle$ BED)= $\frac{1}{2} \times \frac{1}{2}$ Area($\triangle$ ABC)[from equation (i)]

Area($\triangle$ BED)= $\frac{1}{4}$ Area($\triangle$ ABC)

Solution 4:

We have to join PD and BD.



BD is the diagonal of the parallelogram ABCD. Therefore it divides the parallelogram into two equal parts.

$\therefore$ Area($\triangle$ ABD)= Area($\triangle$ DBC)

= $\frac{1}{2}$ Area (parallelogram ABCD) (i)

DP is the median of $\triangle$ ABD. Therefore it will divide $\triangle$ ABD into two triangles of equal areas.

$\therefore$ Area($\triangle$ APD)= Area($\triangle$ DPB)

= $\frac{1}{2}$ Area ($\triangle$ ABD)

= $\frac{1}{2} \times \frac{1}{2}$ Area(parallelogram ABCD)[from equation (i)]

= $\frac{1}{4}$ Area (parallelogram ABCD) (ii)

In $\triangle$ APD, Q is the mid-point of AD. Therefore PQ is the median.

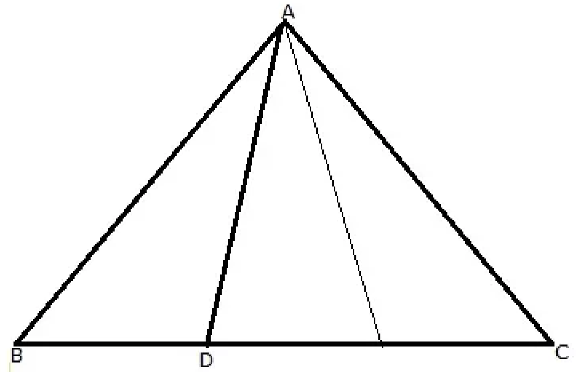
$\therefore$ Area($\triangle$ APQ)= Area($\triangle$ DPQ)

= $\frac{1}{2}$ Area($\triangle$ APD)

= $\frac{1}{2} \times \frac{1}{4}$ Area (parallelogram ABCD) [from equation (ii)]

Area ($\triangle$ APQ)= $\frac{1}{8}$ Area (parallelogram ABCD),hence proved

Solution 5:



In $\triangle$ ABC, $\because$ BD = $\frac{1}{2}$ DC $\Rightarrow \frac{BD}{DC} = \frac{1}{2}$

$\therefore$ Ar.($\triangle$ABD):Ar.($\triangle$ADC)=1:2

But Ar.($\triangle$ABD)+Ar.($\triangle$ADC)=Ar.($\triangle$ABC)

Ar.($\triangle$ABD)+2Ar.($\triangle$ABD)=Ar.($\triangle$ABC)

3 Ar.($\triangle$ABD)= Ar.($\triangle$ABC)

Ar.($\triangle$ABD)= $\frac{1}{3}$ Ar.($\triangle$ABC)

Solution 6:

Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases. So, we have

$$\frac{\text{Area of DPB}}{\text{Area of PCB}} = \frac{\text{DP}}{\text{PC}} = \frac{3}{2}$$

Given: Area of ΔDPB = 30 sq. cm

Let 'x' bet the area of the triangle PCB

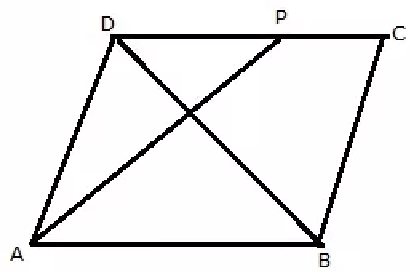
Therefore, we have,

$$\frac{30}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{30}{3} \times 2 = 20\ sq.\ cm.$$

So area of ΔPCB = 20 sq. cm

Consider the following figure.



From the diagram, it is clear that,

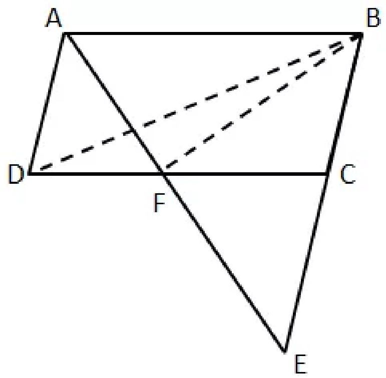
$$\begin{aligned} Area(\triangle CDB) &= Area(\triangle DPB) + Area(\triangle CPB) \\ &= 30 + 20 \\ &= 50\ sq.\ cm \end{aligned}$$

Diagonal of the parallelogram divides it into two triangles $\triangle ADB$ and $\triangle CDB$ of equal area.

Therefore,

$$Area(||gm\ ABCD) = 2 \times \triangle CDB$$
$$= 2 \times 50 = 100\ sq.\ cm$$

Solution 7:



$BC = CE$ (given)

Also, in parallelogram ABCD, $BC = AD$

$\Rightarrow AD = CE$

Now, in ΔADF and ΔECF, we have

$AD = CE$

$\angle ADF = \angle ECF$ (Alternate angles)

$\angle DAF = \angle CEF$ (Alternate angles)

$\therefore \Delta ADF \cong \Delta ECF$ (ASA Criterion)

$\Rightarrow Area(\Delta ADF) = Area(\Delta ECF)$ ....(1)

Also, in ΔFBE, FC is the median (Since $BC = CE$)

$\Rightarrow Area(\Delta BCF) = Area(\Delta ECF)$ ....(2)

From (1) and (2),

$Area(\Delta ADF) = Area(\Delta BCF)$ ....(3)

Again, ΔADF and ΔBDF are on the base DF and between parallels DF and AB.

$\Rightarrow Area(\Delta BDF) = Area(\Delta ADF)$ ....(4)

From (3) and (4),

$Area(\Delta BDF) = Area(\Delta BCF) = 30\ cm^2$

Þ$Area(\Delta BCD) = Area(\Delta BDF) + Area(\Delta BCF) = 30 + 30 = 60\ cm^2$

Hence, Area of parallelogram ABCD $= 2 \times Area(\Delta BCD) = 2 \times 60 = 120\ cm^2$

Solution 8:

In ΔABC,

R and Q are the mid - points of AC and BC respectively.

$\Rightarrow RQ \parallel AB$

that is $RQ \parallel PB$

So, area(ΔPBQ) = area(ΔAPR)...(i)..(Since AP = PB and triangles on the same base and between the same parallels are equal in area)

Since P and R are the mid - points of AB and AC respectively.

$\Rightarrow PR \parallel BC$

that is $PR \parallel BQ$

So, quadrilateral PMQR is a parallelogram.

Also, area(ΔPBQ) = area(ΔPQR)...(ii)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

from (i) and (ii),

area(ΔPQR) = area(ΔPBQ) = area(ΔAPR)...(iii)

Similarly, P and Q are the mid - points of AB and BC respectively.

$\Rightarrow PQ \parallel AC$

that is $PQ \parallel RC$

So, quadrilateral PQCR is a parallelogram.

Also, area(ΔRQC) = area(ΔPQR)...(iv)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

From (iii) and (iv),

area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area(ΔPBQ) = $\frac{1}{4}$area(ΔABC)...(v)

Also, since S is the mid - point of PQ,

BS is the median of ΔPBQ

So, area(ΔQSB) = $\frac{1}{2}$area(ΔPBQ)

from (v),

area(ΔQSB) = $\frac{1}{2} \times \frac{1}{4}$area($\Delta ABC$)

$\Rightarrow$ area(ΔABC) = 8 area(ΔQSB)

Exercise 16(C)

Solution 1:

(i)

Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases. So, we have:

$$\frac{\text{Area of } \Delta \text{ DOC}}{\text{Area of } \Delta \text{ BOC}} = \frac{\text{DO}}{\text{BO}} = 1 \text{ ----1}$$

Similarly

$$\frac{\text{Area of } \Delta \text{ DOA}}{\text{Area of } \Delta \text{ BOA}} = \frac{\text{DO}}{\text{BO}} = 1 \text{ ------2}$$

We know that area of triangles on the same base and between same parallel lines are equal.

Area of Δ ACD = Area of Δ BCD

Area of Δ AOD + Area of Δ DOC = Area of Δ DOC + Area of Δ BOC

=> Area of Δ AOD = Area of Δ BOC ------3

From 1, 2 and 3 we have

Area (Δ DOC) = Area (Δ AOB)

Hence Proved.

(ii)

Similarly, from 1, 2 and 3, we also have

Area of Δ DCB = Area of Δ DOC + Area of Δ BOC = Area of Δ AOB + Area of Δ BOC = Area of Δ ABC

So Area of Δ DCB = Area of Δ ABC

Hence Proved.

(iii)

We know that area of triangles on the same base and between same parallel lines are equal.

Given: triangles are equal in area on the common base, so it indicates AD|| BC.

So, ABCD is a parallelogram.

Hence Proved

Solution 2:

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.

So, we have

$$\frac{\text{Area of } \Delta \text{ APD}}{\text{Area of } \Delta \text{ BPD}} = \frac{AP}{BP} = \frac{1}{2}$$

Area of parallelogram ABCD = 324 sq.cm
Area of the triangles with the same base and between the same parallels are equal.

We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.

Therefore, we have,

$$\begin{aligned} Area(\triangle ABD) &= \frac{1}{2} \times Area(\|gm\ ABCD) \\ &= \frac{324}{2} \\ &= 162\ sq.\ cm \end{aligned}$$

From the diagram it is clear that,

$$Area(\triangle ABD) = Area(\triangle APD) + Area(\triangle BPD)$$
$$\Rightarrow 162 = Area(\triangle APD) + 2Area(\triangle APD)$$
$$\Rightarrow 162 = 3Area(\triangle APD)$$
$$\Rightarrow Area(\triangle APD) = \frac{162}{3}$$
$$\Rightarrow Area(\triangle APD) = 54\ sq.\ cm$$

(ii)

Consider the triangles $\triangle AOP$ and $\triangle COD$

$\angle AOP = \angle COD$ [vertically opposite angles]

$\angle CDO = \angle APD$ [AB and DC are parallel and DP is the transversal, alternate interior angles are equal]

Thus, by Angle – Angle similarity, $\triangle AOP \sim \triangle COD$.

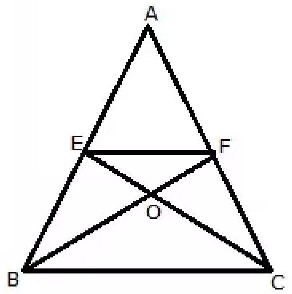
Hence the corresponding sides are proportional.

$$\begin{aligned} \frac{AP}{CD} = \frac{OP}{OD} &= \frac{AP}{AB} \\ &= \frac{AP}{AP+PB} \\ &= \frac{AP}{3AP} \\ &= \frac{1}{3} \end{aligned}$$

Solution 3:

E and F are the midpoints of the sides AB and AC.

Consider the following figure.



Therefore, by midpoint theorem, we have, EF || BC

Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC

$$\begin{aligned}&\text{Therefore, } Ar.(\triangle BEF) = Ar.(\triangle CEF)\\&\Rightarrow Ar.(\triangle BOE) + Ar.(\triangle EOF) = Ar.(\triangle EOF) + Ar.(\triangle COF)\\&\Rightarrow Ar.(\triangle BOE) = Ar.(\triangle COF)\end{aligned}$$

Now BF and CE are the medians of the triangle ABC

Medians of the triangle divides it into two equal areas of triangles.

Thus, we have, Ar. $\triangle$ ABF=Ar. $\triangle$ CBF

Subtracting Ar. $\triangle$ BOE on the both the sides, we have

Ar. $\triangle$ ABF - Ar. $\triangle$ BOE=Ar. $\triangle$ CBF - Ar. $\triangle$ BOE

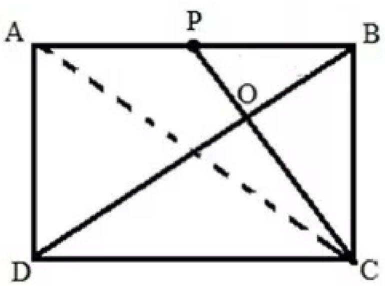
Since, Ar.($\triangle$ BOE)= Ar.($\triangle$ COF),

Ar. $\triangle$ ABF- Ar. $\triangle$ BOE=Ar. $\triangle$ CBF- Ar. $\triangle$ COF

Ar. (quad. AEOF)=Ar.($\triangle$ OBC), hence proved

Solution 4:

(i) Joining AC we have the following figure



$$\begin{aligned}&\text{Consider the triangles } \triangle POB \text{ and } \triangle COD\\&\angle POB = \angle DOC \quad [\text{vertically opposite angles}]\\&\angle OPB = \angle ODC \quad [AB \text{ and } DC \text{ are parallel, } CP \text{ and } BD \text{ are the}\\&\qquad\qquad\qquad \text{transversals, alternate interior angles are equal}]\\&\text{Therefore, by Angle} - \text{Angle similarity criterion of congruence,}\\&\triangle POB \sim \triangle COD\\&\text{Since } P \text{ is the midpoint } AP = BP, \text{ and } AB = CD, \text{ we have } CD = 2BP\\&\text{Therefore, we have,}\\&\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}\\&\Rightarrow OP:OC = 1:2\end{aligned}$$

(ii)

Since from part (i), we have

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2},$$

Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides.

Here, $\triangle DOC$ and $\triangle POB$ are similar triangles.

Thus, we have,

$$\frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{DC^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{(2PB)^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = \frac{4PB^2}{PB^2}$$

$$\Rightarrow \frac{Ar.(\triangle DOC)}{Ar.(\triangle POB)} = 4$$

$$\Rightarrow Ar.(\triangle DOC) = 4Ar.(\triangle POB)$$
$$= 4 \times 40$$
$$= 160\ cm^2$$

Now consider $Ar.(\triangle DBC) = Ar.(\triangle DOC) + Ar.(\triangle BOC)$
$$= 160 + 80$$
$$= 240\ cm^2$$

Two triangles are equal in area if they are on the equal bases and between the same parallels.

Therefore, $Ar.(\triangle DBC) = Ar.(\triangle ABC) = 240\ cm^2$

Median divides the triangle into areas of two equal triangles.

Thus, CP is the median of the triangle ABC.

Hence, $Ar.(\triangle ABC) = 2Ar.(\triangle PBC)$

$$\Rightarrow Ar.(\triangle PBC) = \frac{Ar.(\triangle ABC)}{2}$$

$$\Rightarrow Ar.(\triangle PBC) = 120\ cm^2$$

(iii)

From part(ii) we have,

$Ar.(\triangle ABC) = 2Ar.(PBC) = 240\ cm^2$

Area of a triangle is half the area of the Parallelogram if both are on equal bases and between the same parallels.

Thus, $Ar.(\triangle ABC) = \frac{1}{2}Ar.(||gm\ ABCD)$

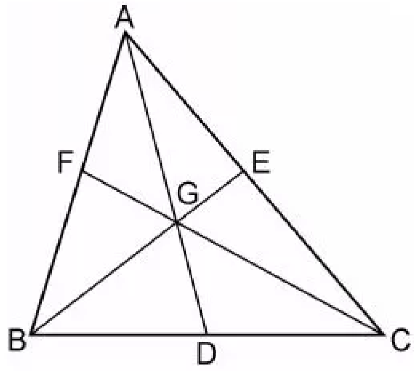
$$\Rightarrow Ar.(||gm\ ABCD) = 2\ Ar.(\triangle ABC)$$

$$\Rightarrow Ar.(||gm\ ABCD) = 2 \times 240$$

$$\Rightarrow Ar.(||gm\ ABCD) = 480\ cm^2$$

Solution 5:

(i) The figure is shown below



Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio $2:1$

That is $AG:GD = 2:1$

Since BG divides AD in the ratio $2:1$, *we have,*

$$\frac{Area(\triangle AGB)}{Area(\triangle BGD)} = \frac{2}{1}$$

$\Rightarrow Area(\triangle AGB) = 2Area(\triangle BGD)$

From the figure, it is clear that,

$Area(\triangle ABD) = Area(\triangle AGB) + Area(\triangle BGD)$

$\Rightarrow Area(\triangle ABD) = 2Area(\triangle BGD) + Area(\triangle BGD)$

$\Rightarrow Area(\triangle ABD) = 3Area(\triangle BGD)....(1)$

(ii)

Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio $2:1$

That is $AG:GD = 2:1$

Similarly CG divides AD in the ratio $2:1$, *we have,*

$$\frac{Area(\triangle AGC)}{Area(\triangle CGD)} = \frac{2}{1}$$

$\Rightarrow Area(\triangle AGC) = 2Area(\triangle CGD)$

From the figure, it is clear that,

$Area(\triangle ACD) = Area(\triangle AGC) + Area(\triangle CGD)$

$\Rightarrow Area(\triangle ACD) = 2Area(\triangle CGD) + Area(\triangle CGD)$

$\Rightarrow Area(\triangle ACD) = 3Area(\triangle CGD)....(2)$

(iii)

Adding equations (1) *and* (2), *we have,*

$Area(\triangle ABD) + Area(\triangle ACD) = 3Area(\triangle BGD) + 3Area(\triangle CGD)$

$\Rightarrow Area(\triangle ABC) = 3[Area(\triangle BGD) + Area(\triangle CGD)]$

$\Rightarrow Area(\triangle ABC) = 3[Area(\triangle BGC)]$

$$\Rightarrow \frac{Area(\triangle ABC)}{3} = [Area(\triangle BGC)]$$

$$\Rightarrow Area(\triangle BGC) = \frac{1}{3}Area(\triangle ABC)$$

Solution 6:

Consider that the sides be x cm, y cm and (37-x-y) cm. also, consider that the lengths of altitudes be 6a cm, 5a cm and 4a cm.

$\therefore$ Area of a triangle= $\frac{1}{2} \times \text{base} \times \text{altitude}$

$$\therefore \frac{1}{2} \times x \times 6a = \frac{1}{2} \times y \times 5a = \frac{1}{2} \times (37 - x - y) \times 4a$$

$$6x = 5y = 148 - 4x - 4y$$

$6x = 5y$ and $6x = 148 - 4x - 4y$

$6x - 5y = 0$ and $10x + 4y = 148$

Solving both the equations, we have

X=10 cm, y=12 cm and (37-x-y)cm=15 cm

Solution 7:

(i)

Consider the triangles $\triangle AFE$ *and* $\triangle DFC$.

$\angle AFE = \angle DEC$ [*Vertically opposite angles*]

$\angle FAE = \angle DCF$ [*AB and DC are parallel lines, AC is a transversal, alternate interior angles are equal*]

Thus, by Angle – Angle similarity, we have,

$\triangle AFE \sim \triangle DFC$

Therefore, we have,

$$\frac{DF}{FE} = \frac{DC}{AE} = \frac{CF}{AF} = \frac{2}{1}$$

$\Rightarrow DF : FE = 2 : 1$

(ii)

$Since\ from\ part(i)\ we\ have\ DF:FE=2:1,\ therefore,$
$Area(\triangle DCF)=4Area(\triangle AFE)...(1)$
$Also\ we\ know\ that,$
$Area(\triangle ADF)+Area(\triangle AFE)=Area(\triangle ADE)$
$\Rightarrow 60+Area(\triangle AFE)=Area(\triangle ADE) \quad [Area(\triangle ADF)=60\ cm^2]$
$\Rightarrow 2Area(\triangle ADE)=2[60+Area(\triangle AFE)]$
$Median\ divides\ the\ triangle\ into\ two\ equal\ areas\ of\ triangle.$
$Therefore,\ 2Area(\triangle ADE)=Area(\triangle ABD)$
$\Rightarrow Area(\triangle ABD)=2[60+Area(\triangle AFE)]$
$\Rightarrow Area(\triangle ABD)=120+2Area(\triangle AFE)...(2)$
$Triangles\ with\ equal\ bases\ and\ between\ the\ parallels\ are\ of$
$equal\ area.$
$Area(\triangle ABD)=Area(\triangle ACD)$
$Thus,\ Equation\ (2),\ becomes,$
$Area(\triangle ACD)=120+2Area(\triangle AFE)...(3)$
$From\ the\ figure,\ it\ is\ clear\ that,$
$Area(\triangle ACD)=Area(\triangle DCF)+Area(\triangle ADF)$
$\Rightarrow Area(\triangle ACD)=Area(\triangle DCF)+60$
$\Rightarrow Area(\triangle ACD)=4Area(\triangle AEF)+60...(4)$
$Equating\ equations\ (3)\ and\ (4),\ we\ have,$
$120+2Area(\triangle AFE)=4Area(\triangle AEF)+60$

$\Rightarrow 2Area(\triangle AFE)=60$
$$\Rightarrow Area(\triangle AFE)=\frac{60}{2}$$
$\Rightarrow Area(\triangle AFE)=30$
$\Rightarrow Arear(\triangle ADE)=Area(\triangle ADF)+Area(\triangle AFE)$
$\Rightarrow Arear(\triangle ADE)=60+30$
$\Rightarrow Arear(\triangle ADE)=90\ cm^2$

(iii)

$Median\ of\ a\ trianlge\ divides\ it\ intot\ two\ equal\ areas\ of\ triangle.$
$Arear(\triangle ADB)=2Arear(\triangle ADE)$
$\Rightarrow Arear(\triangle ADB)=2Arear(\triangle ADE)$
$\Rightarrow Arear(\triangle ADB)=2\times 90\ cm^2$
$\Rightarrow Arear(\triangle ADB)=180\ cm^2$

(iv)

Since DB divides the parallelogram ABCD into two equal triangle, therefore Area of $\triangle$ DBC= Area of $\triangle$ ADB

$=180\ cm^2$

Thus the area of the parallelogram ABCD = Area of $\triangle$ ADB+ Area

of $\triangle$ DBC

$=180\ cm^2+180\ cm^2$

$=360\ cm^2$

Solution 8:

Here BCED is a parallelogram, since BD = CE and BD || CE.

ar.(ΔDBC) = ar.(ΔEBC)...(Since they have the same base and are between the same parallels)

In ΔABC,

BE is the median,

So, ar.(ΔEBC) = $\frac{1}{2}$ar.(ΔABC)

Now, ar.(ΔABC) = ar.(ΔEBC) + ar.(ΔABE)

Also, ar.(ΔABC) = 2ar.(ΔEBC)

$\Rightarrow$ ar.(ΔABC) = 2ar.(ΔDBC)

Solution 9:

Given:

ΔCAD = 140 cm^2

ΔODC = 172 cm^2

AB || CD

As Triangle DBC and ΔCAD have same base CD and between the same parallel lines Hence,

Area of ΔDBC = Area of ΔCAD = 140 cm^2

Area of ΔOAC = Area of ΔCAD + Area of ΔODC = 140 cm^2 + 172 cm^2 = 312 cm^2

Area of ΔODB = Area of ΔDBC + Area of ΔODC = 140 cm^2 + 172 cm^2 = 312 cm^2