Chapter 14. Rectilinear Figures [Quadrilaterals: Parallelogram, Rectangle, Rhombus, Square and Trapezium]

Exercise 14(A)

Solution 1:

The sum of the interior angle=4 times the sum of the exterior angles.

Therefore the sum of the interior angles = $4 \times 360^\circ = 1440^\circ$.

Now we have

$$\begin{aligned}(2n-4)\times 90^0 &= 1440^0\\ 2n-4 &= 16\\ 2n &= 20\\ n &= 10\end{aligned}$$

Thus the number of sides in the polygon is 10.

Solution 2:

Let the angles of the pentagon are 4x, 8x, 6x, 4x and 5x.

Thus we can write

$$\begin{aligned}4x+8x+6x+4x+5x &= 540^0\\ 27x &= 540^0\\ x &= 20^0\end{aligned}$$

Hence the angles of the pentagon are:

$4\times20^\circ = 80^\circ, 8\times20^\circ = 160^\circ, 6\times20^\circ = 120^\circ, 4\times20^\circ = 80^\circ, 5\times20^\circ = 100^\circ$

Solution 3:

Let the measure of each equal angles are x.

Then we can write

$$\begin{aligned}140^0+5x &= (2\times6-4)\times 90^0\\ 140^0+5x &= 720^0\\ 5x &= 580^0\\ x &= 116^0\end{aligned}$$

Therefore the measure of each equal angles are 116°

Solution 4:

Let the number of sides of the polygon is *n* and there are k angles with measure 195°.

Therefore we can write:

$$\begin{aligned}5\times 90^0+k\times195^0 &= (2n-4)90^0\\ 180^0 n-195^0 k &= 450^0-360^0\\ 180^0 n-195^0 k &= 90^0\\ 12n-13k &= 6\end{aligned}$$

In this linear equation n and k must be integer. Therefore to satisfy this equation the minimum value of

k must be 6 to get n as integer.
Hence the number of sides are: 5 + 6 = 11.

Solution 5:

Let the measure of each equal angles are x.

Then we can write:

$$\begin{aligned} 3\times 132^0 + 4x &= (2\times 7-4)90^0 \\ 4x &= 900^0 - 396 \\ 4x &= 504 \\ x &= 126^0 \end{aligned}$$

Thus the measure of each equal angles are 126°.

Solution 6:

Let the measure of each equal sides of the polygon is x.

Then we can write:

$$\begin{aligned} 142^0 + 176^0 + 6x &= (2\times 8-4)90^0 \\ 6x &= 1080^0 - 318^0 \\ 6x &= 762^0 \\ x &= 127^0 \end{aligned}$$

Thus the measure of each equal angles are 127°.

Solution 7:

Let the measure of the angles are 3x, 4x and 5x.

Thus

$$\begin{aligned} \angle A + \angle B + \angle C + \angle D + \angle E &= 540^0 \\ 3x + (\angle B + \angle C) + 4x + 5x &= 540^0 \\ 12x + 180^0 &= 540^0 \\ 12x &= 360^0 \\ x &= 30^0 \end{aligned}$$

Thus the measure of angle E will be $4\times 30^\circ = 120^\circ$

Solution 8:

(i)

Let each angle of measure x degree.

Therefore measure of each angle will be:

$$x = 180^0 - 2\times15^0 = 150^0$$

(ii)

Let each angle of measure x degree.

Therefore measure of each exterior angle will be:

$$\begin{aligned} x &= 180^0 - 150^0 \\ &= 30^0 \end{aligned}$$

(iii)

Let the number of each sides is n.

Now we can write

$$\begin{aligned} n\cdot150^0 &= (2n-4)\times90^0 \\ 180^0 n - 150^0 n &= 360^0 \\ 30^0 n &= 360^0 \\ n &= 12 \end{aligned}$$

Thus the number of sides are 12.

Solution 9:

Let measure of each interior and exterior angles are 3k and 2k.

Let number of sides of the polygon is n.

Now we can write:

$$\begin{aligned} n\cdot3k &= (2n-4)\times90^0 \\ 3nk &= (2n-4)90^0 \qquad \ldots(1) \end{aligned}$$

Again

$$\begin{aligned} n\cdot2k &= 360^0 \\ nk &= 180^0 \end{aligned}$$

From (1)

$$\begin{aligned} 3\cdot180^0 &= (2n-4)90^0 \\ 3 &= n-2 \\ n &= 5 \end{aligned}$$

Thus the number of sides of the polygon is 5.

Solution 10:

For (n-1) sided regular polygon:

Let measure of each angle is x.

Therefore

$$(n-1)x = (2(n-1)-4)90^0$$
$$x = \frac{n-3}{n-1}180^0$$

For (n+1) sided regular polygon:

Let measure of each angle is y.

Therefore

$$(n+2)y = (2(n+2)-4)90^0$$
$$y = \frac{n}{n+2}180^0$$

Now we have

$$y - x = 6^0$$
$$\frac{n}{n+2}180^0 - \frac{n-3}{n-1}180^0 = 6^0$$
$$\frac{n}{n+2} - \frac{n-3}{n-1} = \frac{1}{30}$$
$$30n(n-1) - 30(n-3)(n+2) = (n+2)(n-1)$$
$$-30n + 30n + 180 = n^2 + n - 2$$
$$n^2 + n - 182 = 0$$
$$(n-13)(n+14) = 0$$
$$n = 13, -14$$

Thus the value of n is 13.

Solution 11:

(i)

Let the measure of each exterior angle is x and the number of sides is n.

Therefore we can write:

$$n = \frac{360^0}{x}$$

Now we have

$$x + x + 90^0 = 180^0$$
$$2x = 90^0$$
$$x = 45^0$$

(ii)

Thus the number of sides in the polygon is:

$$n = \frac{360^0}{45^0}$$
$$= 8$$

Exercise 14(B)

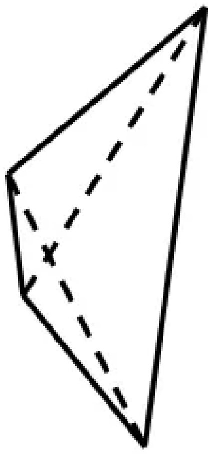
Solution 1:

(i)True.

This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii)False

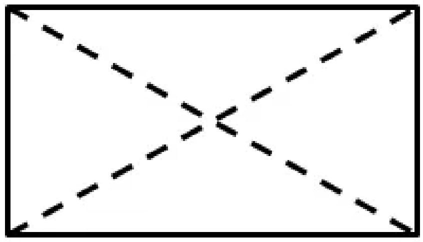
This is not true for any random quadrilateral. Observe the quadrilateral shown below.



Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

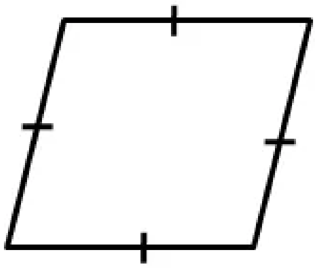
(iii)False

Consider a rectangle as shown below.

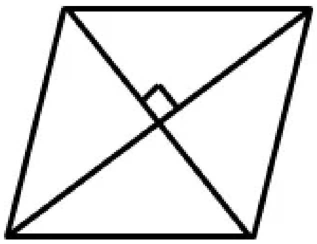


It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

(iv)True
Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.
(v)False
This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.
(vi)True



A parallelogram is a quadrilateral with opposite sides parallel and equal.
Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.
(vii)False
This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be a rhombus, except those parallelograms that have all equal sides.
(viii)False
This is a property of a rhombus. The diagonals of a rhombus need not be equal.
(ix)True
A parallelogram is a quadrilateral with opposite sides parallel and equal.
A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.
If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.
(x)False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

Solution 2:

From the given figure we conclude that

$$\angle A + \angle D = 180° \left[\text{since consecutive angles are supplementary}\right]$$

$$\frac{\angle A}{2} + \frac{\angle D}{2} = 90°$$

Again from the ΔADM

$$\frac{\angle A}{2} + \frac{\angle D}{2} + \angle M = 180°$$

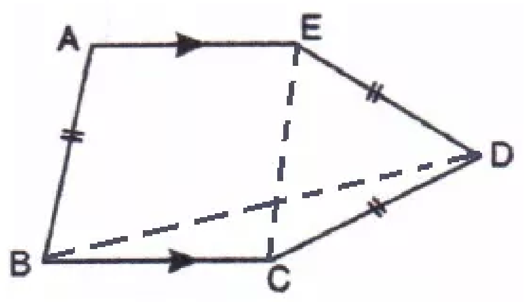
$$\Rightarrow 90° + \angle M = 180° \qquad \left[\sin ce \frac{\angle A}{2} + \frac{\angle D}{2} = 90°\right]$$

$$\Rightarrow \angle M = 90°$$

Hence $\angle AMD = 90°$

Solution 3:

In the given figure



Given that $AE = BC$

We have to find $\angle AEC$ $\angle BCD$

Let us join EC and BD.

In the quadrilateral AECB

$AE = BC$ and $AB = EC$

also $AE \parallel BC$

$\Rightarrow AB \parallel EC$

So quadrilateral is a parallelogram.

In parallelogram consecutive angles are supplementary

$\Rightarrow \angle A + \angle B = 180^\circ$

$\Rightarrow 102^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 78^\circ$

In parallelogram opposite angles are equal

$\Rightarrow \angle A = \angle BEC$ and $\angle B = \angle AEC$

$\Rightarrow \angle BEC = 102^\circ$ and $\angle AEC = 78^\circ$

Now consider ΔECD

$EC = ED = CD$ [Since $AB = EC$]

Therefore ΔECD is an equilateral triangle.

$\Rightarrow \angle ECD = 60^\circ$

$\angle BCD = \angle BEC + \angle ECD$

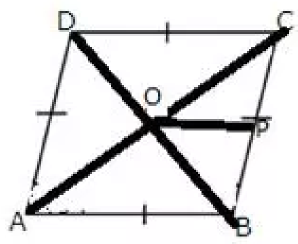
$\Rightarrow \angle BCD = 102^\circ + 60^\circ$

$\Rightarrow \angle BCD = 162^\circ$

Therefore $\angle AEC = 78^\circ$ and $\angle BCD = 162^\circ$

Solution 4:

Given ABCD is a square and diagonals meet at O.P is a point on BC such that OB=BP



In the

ΔBOC and ΔDOC

$\Rightarrow BD=BD$ [common side]

$\Rightarrow BO=CO$

$POD=OC$ [since diagonals cuts at O]

$\Delta BOC \cong \Delta DOC$ [by SSS]

Therefore

$\angle BOC = 90°$

NOW

$\angle POC = 22.5$

$\angle BOP = 67.5$ [since $\angle BOC = 67.5° + 22.5°$]

Again

ΔBDC

$\angle BDC = 45°$ [since $\angle B = 45°, \angle C = 90°$]

Therefore

$\angle BDC = 2\angle POC$

AGAIN

$\angle BOP = 67.5°$

$\Rightarrow \angle BOP = 2\angle POC$

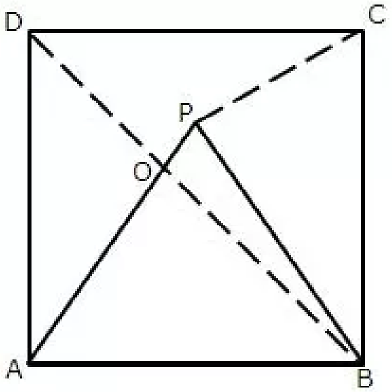
Hence proved that

i) $\angle PC = \left(22\frac{1}{2}^{\circ}\right)$

(ii) $\angle BDC = 2\angle POC$

(iii) $\angle BOP = 3\angle CPO$

Solution 5:



In the given figure ΔAPB is an equilateral triangle

Therefore all its angles are $60°$

Again in the

ΔADB

$\angle ABD = 45°$

$\angle AOB = 180° - 60° - 45°$

$= 75°$

Again

$\triangle BPC$

$\Rightarrow \angle BPC = 75°$ [Since BP = CB]

Now

$\angle C = \angle BCP + \angle PCD$

$\Rightarrow \angle PCD = 90° - 75°$

$\Rightarrow \angle PCD = 15°$

Therefore

$\angle APC = 60° + 75°$

$\Rightarrow \angle APC = 135°$

$\Rightarrow Reflex\ \angle APD = 360° - 135° = 225°$

(i) $\angle AOB = 75°$

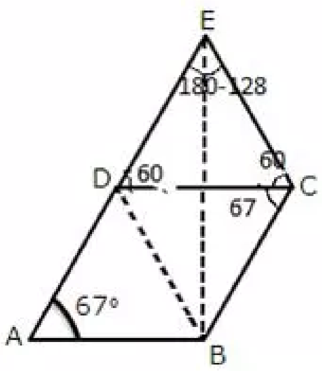
(ii) $\angle BPC = 75°$

(iii) $\angle PCD = 15°$

(iv) Reflex $\angle APD = 225°$

Solution 6:

Given that the figure ABCD is a rhombus with angle A = 67^{o}



In the rhombus We have

$\angle A = 67^{\circ} = \angle C$ [Opposite angles]

$\angle A + \angle D = 180^{\circ}$[Consecutive angles are supplementary]

$\Rightarrow \angle D = 113^{\circ}$

$\Rightarrow \angle ABC = 113^{\circ}$

Consider $\triangle DBC$,

$DC = CB$ [Sides of rhombous]

So $\triangle DBC$ is an isoscales triangle

$\Rightarrow \angle CDB = \angle CBD$

Also,

$\angle CDB + \angle CDB + \angle BCD = 180^{\circ}$

$\Rightarrow 2\angle CBD = 113^{\circ}$

$\Rightarrow \angle CDB = \angle CBD = 56.5^{\circ}$............$(i)$

Consider $\triangle DCE$,

$EC = CB$

So $\triangle DCE$ is an isoscales triangle

$\Rightarrow \angle CBE = \angle CEB$

Also,

$\angle CBE + \angle CEB + \angle BCE = 180^{\circ}$

$\Rightarrow 2\angle CBE = 53^{\circ}$

$\Rightarrow \angle CDE = 26.5^{\circ}$

From (i)

$\angle CBD = 56.5^{\circ}$

$\Rightarrow \angle CBE + \angle DBE = 56.5^{\circ}$

$\Rightarrow 26.5^{\circ} + \angle DBE = 56.5^{\circ}$

$\Rightarrow \angle DBE = 30.5^{\circ}$

Solution 7:

(i)ABCD is a parallelogram

Therefore

AD=BC
AB=DC

Thus

$$4y = 3x - 3 \ [\text{since AD=BC}]$$
$$\Rightarrow 3x - 4y = 3 \ (i)$$
$$6y + 2 = 4x \ \ [\text{since AB=DC}]$$
$$4x - 6y = 2 \ (ii)$$

Solving equations (i) and (ii) we have

x=5

y=3

(ii)

In the figure ABCD is a parallelogram

$$\angle A = \angle C$$
$$\angle B = \angle D \ [\text{since opposite angles are equal}]$$

Therefore

$$7y = 6y + 3y - 8^{\circ} \qquad (i) \quad [Since \ \angle A = \angle C]$$
$$4x + 20^{\circ} = 0 \qquad (ii)$$

Solving (i), (ii) we have

$$X = 12^{\circ}$$
$$Y = 16^{\circ}$$

Solution 8:

Given that the angles of a quadrilateral are in the ratio $3:4:5:6$ Let the angles be $3x, 4x, 5x, 6x$

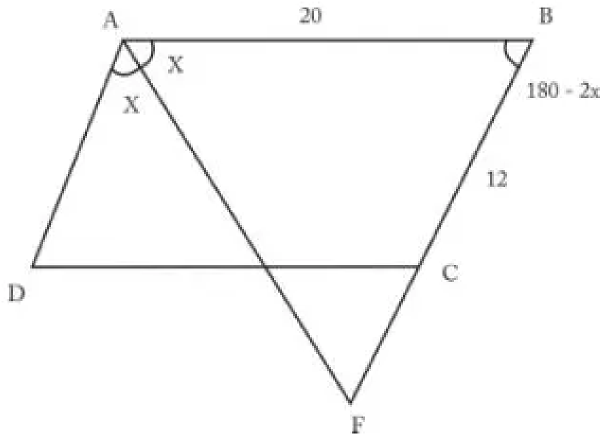
$$3x + 4x + 5x + 6x = 360^{\circ}$$
$$\Rightarrow x = \frac{360^{\circ}}{18}$$
$$\Rightarrow x = 20^{\circ}$$

Therefore the angles are

$$3 \times 20 = 60^{\circ},$$
$$4 \times 20 = 80^{\circ},$$
$$5 \times 20 = 100^{\circ},$$
$$6 \times 20 = 120^{\circ}$$

Since all the angles are of different degrees thus forms a trapezium

Solution 9:



Given AB = 20 cm and AD = 12 cm.

From the above figure, it's evident that ABF is an isosceles triangle with angle BAF = angle BFA = x

So AB = BF = 20

BF = 20

BC + CF = 20

CF = 20 - 12 = 8 cm

Solution 10:
We know that AQCP is a quadrilateral. So sum of all angles must be 360.
∴ x + y + 90 + 90 = 360
x + y = 180
Given x:y = 2:1
So substitute x = 2y
3y = 180
y = 60
x = 120
We know that angle C = angle A = x = 120
Angle D = Angle B = 180 – x = 180 – 120 = 60
Hence, angles of parallelogram are 120, 60, 120 and 60.

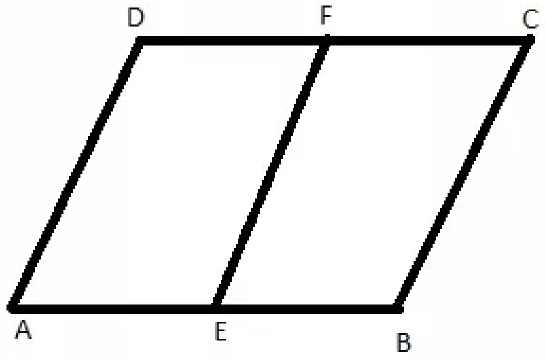
Exercise 14(C)

Solution 1:

Let us draw a parallelogram $ABCD$ Where F is the midpoint

Of side DC of parallelogram $ABCD$

To prove: $AEFD$ is a parallelogram



Proof:

Therefore $ABCD$

$$AB \parallel DC$$
$$BC \parallel AD$$
$$AB = DC$$
$$\frac{1}{2}AB = \frac{1}{2}DC$$
$$AE = DF$$

Also AD|| EF

therefore AEFC is a parallelogram.

Solution 2:

GIVEN: $ABCD$ is a parallelogram where the diagonal BD bisects

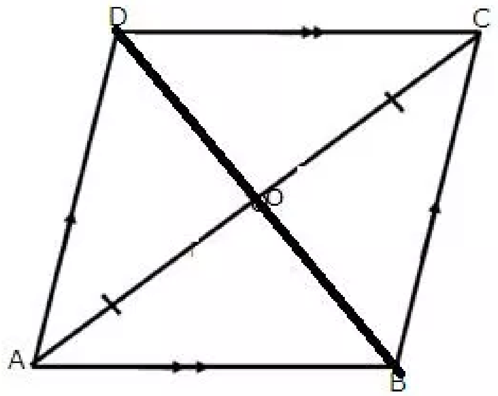
parallelogram $ABCD$ at angle B and D

TO PROVE: $ABCD$ is a rhombus

Proof : Let us draw a parallelogram $ABCD$ where the diagonal BD bisects the parallelogram at angle B and D

Consruction :Let us join AC as a diagonal of the parallelogram

$ABCD$



Since $ABCD$ is a parallelogram

Therefore

$$AB = DC$$
$$AD = BC$$

Diagonal BD bisects angle B and D

So $\angle COD = \angle DOA$

Again AC also bisects at A and C

Therefore $\angle AOB = \angle BOC$

Thus $ABCD$ is a rhombus.

Solution 3:

Given $ABCD$ is a parallelogram and $AE=EF=FC$.

We have to prove at first that $DEBF$ is a parallelogram.

Proof :From ΔADE and ΔBCF

$AE=FC$
$AD=BC$
$\angle D=\angle B$

$\Delta ADE \cong \Delta BCF$ [SAS]

Therefore $DE=FB$

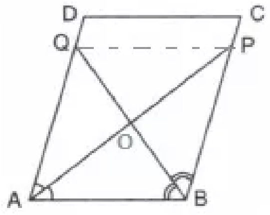
$DC=EF$ [since $AE+EF+FC=AC$ and $AE=EF=FC$]

Therefore $DEBF$ is a parallelogram.

So $DE \parallel FB$

Hence proved

Solution 4:



Let us join PQ.

Consider the $\triangle AOQ$ and $\triangle BOP$
$\angle AOQ=\angle BOP$ [opposite angles]
$\angle OAQ=\angle BPO$ [alternate angles]
$\Rightarrow \triangle AOQ \cong \triangle BOP$ [AA test]

$Hence\ AQ=BP$

Consider the $\triangle QOP$ and $\triangle AOB$
$\angle AOB=\angle QOP$ [opposite angles]
$\angle OAB=\angle APQ$ [alternate angles]
$\Rightarrow \triangle QOP \cong \triangle AOB$ [AA test]

$Hence\ PQ=AB=CD$

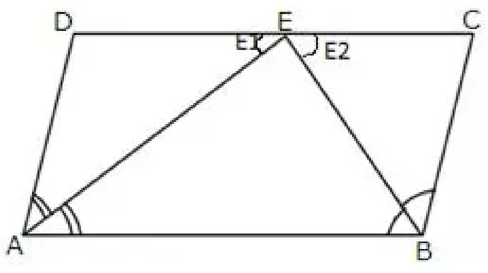
Consider the quadrilateral $QPCD$
$DQ=CP\ and\ DQ \parallel CP$ [Since $AD=BC\ and\ AD \parallel BC$]
Also $QP=DC\ and\ AB \parallel QP \parallel DC$

$Hence\ quadrilateral\ QPCD$ is a parallelogram.

Solution 5:

Given $ABCD$ is a parallelogram

To prove: $AB = 2BC$



Proof: $ABCD$ is a parallelogram

$\angle A + \angle D = \angle B + \angle C = 180°$

From the ΔAEB we have

$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \angle E = 180°$

$\Rightarrow \angle A - \frac{\angle A}{2} + \angle D + \angle E1 = 180°$ [*taking* E1 as new angle]

$\Rightarrow \angle A + \angle D + \angle E1 = 180° + \frac{\angle A}{2}$

$\Rightarrow \angle E1 = \frac{\angle A}{2}$ [*Since* $\angle A + \angle D = 180°$]

Again,

similarly,

$\angle E2 = \frac{\angle B}{2}$

NOW

AB=DE+EC

=AD+BC

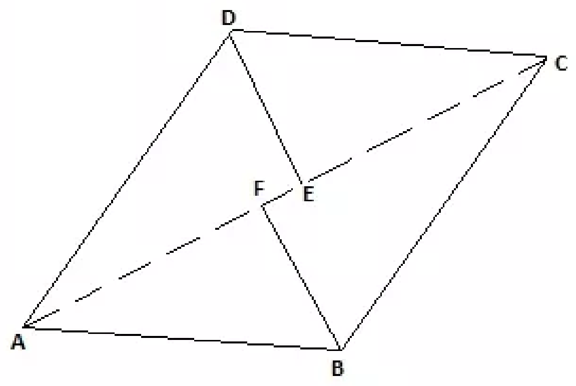
=2BC [since AD=BC]

Hence proved

Solution 6:

Given ABCD is a parallelogram. The bisectors of $\angle ADC$ and $\angle BCD$ meet at E. The bisectors of $\angle ABC$ and $\angle BCD$ meet at F

From the parallelogram $ABCD$ we have



$\angle ADC + \angle BCD = 180°$ [sum of adjacent angles of a parallelogram]

$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$

$\Rightarrow \angle EDC + \angle ECD = 90°$

In triangle ECD sum of angles $= 180°$

$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$

$\Rightarrow \angle CED = 90°$

Similarly taking triangle BCF it can be prove that $\angle BFC = 90°$

Now since

$\angle BFC = \angle CED = 90^{\circ}$

Therefore the lines DE and BF are parallel

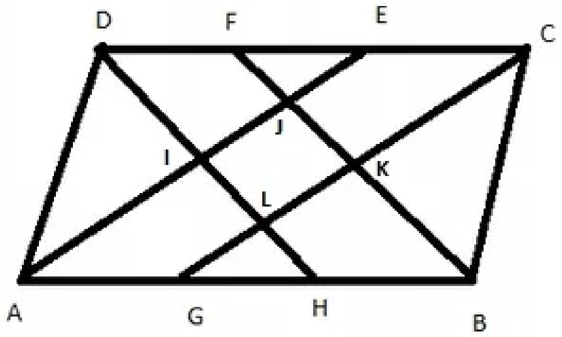
Hence proved

Solution 7:

Given: $ABCD$ is a parallelogram

AE bisects $\angle BAD$
BF bisects $\angle ABC$
CG bisects $\angle BCD$
DH bisecsts $\angle ADC$

TO PROVE: $LKJI$ is a rectangle



Proof :

$\angle BAD + \angle ABC = 180°$ [adjacent angles of a parallelogram are supplementary]

$\angle BAJ = \frac{1}{2}\angle BAD$ [AE bisects ÐBAD]

$\angle ABJ = \frac{1}{2}\angle ABC$ [DH bisect ÐABC]

$\angle BAJ + \angle ABJ = 90°$ [halves of supplementary angles are complementary]

ΔABJ is a right triangle because its acute interior angles are complementary.

Similarly

$\angle DLC = 90°$
$\angle AID = 90°$
Then $\angle JIL = 90°$ because $\angle AID$ and $\angle JIL$ are vertical angles

since 3 angles of quadrilateral $LKJI$ are right angles,si is the 4^{th} one and so $LKJI$ is a rectangle ,since its interior angles are all right angles

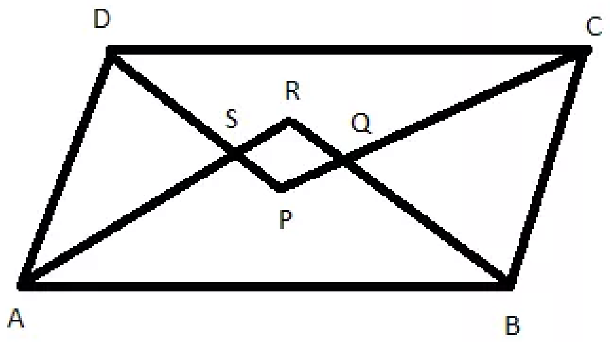
Hence proved

Solution 8:

Given: A parallelogram $ABCD$ in which AR, BR, CP, DP

Are the bisects of $\angle A, \angle B, \angle C, \angle D$ respectively forming quadrilaterals $PQRS$.

To prove: $PQRS$ is a rectangle



Proof :

$\angle DCB + \angle ABC = 180°$ [co-interior angles of parallelogram are supplementary]

$\Rightarrow \frac{1}{2}\angle DCB + \frac{1}{2}\angle ABC = 90°$

$\Rightarrow \angle 1 + \angle 2 = 90°$

Also in

$\Delta CQB, \angle 1 + \angle 2 + \angle CQB = 180°$

From the above equation we get

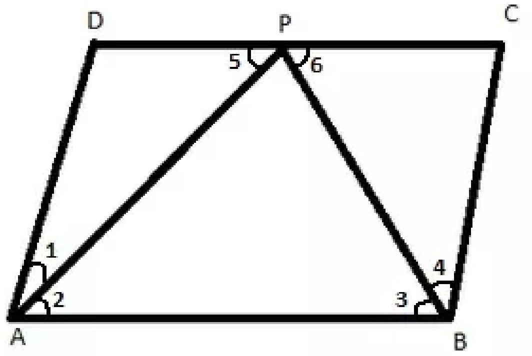
$\angle CQB = 180° - 90° = 90°$

$\angle RQP = 90°$ [$\angle CQB = \angle RQP$, *vertically* opposite angles]

$\angle QRP = \angle RSP = \angle SPQ = 90°$

Hence PQRS is a rectangle

Solution 9:



(i)Let $\mathrm{AD} = x$

$\mathrm{AB}=2\mathrm{AD} = 2x$

Also AP is the bisector $\angle A$

$\angle 1 = \angle 2$

Now,

$\angle 2 = \angle 5$ [alternate angles]

Therefore $\angle 1 = \angle 5$

Now

$\mathrm{AP}=\mathrm{DP} = x$ [sides opposite to equal angles are also equal]

Therefore

$AB=CD$ [opposite sides of parallelogram are equal]

$CD = 2x$

$\Rightarrow DP+PC=2x$

$\Rightarrow x+PC=2x$

$\Rightarrow PC = x$

Also , $BC=x$

In ΔBPC

$\Rightarrow \angle 6=\angle 4$ [*angles* opposite to equal sides are equal]

$\Rightarrow \angle 6 = \angle 3$

Therefore $\angle 3 = \angle 4$

Hence BP bisect $\angle B$

(ii)

Opposite angles are supplementary

Therefore

$$\angle 1+\angle 2+\angle 3+\angle 4 = 180°$$

$$\Rightarrow 2\angle 2+2\angle 3 = 180° \begin{bmatrix} \angle 1 = \angle 2 \\ \angle 3 = \angle 4 \end{bmatrix}$$

$$\Rightarrow \angle 2+\angle 3 = 90°$$

ΔAPB

$$\angle 2+\angle 3\angle APB = 180°$$

$$\Rightarrow \angle APB = 180° - 90° \text{ [by angle sum property]}$$

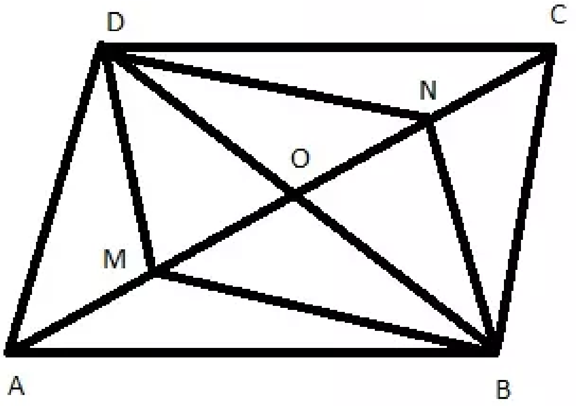
$$\Rightarrow \angle APB = 90°$$

Hence proved

Solution 10:

Points M and N are taken on the diagonal AC of a parallelogram $ABCD$ such that $AM=CN$.

Prove that $BMDN$ is a parallelogram



CONSTRUCTION: Join B to D to meet AC in O.

PROOF: We know that the diagonals of parallelogram bisect each other.

Now, AC and BD bisect each other at O.

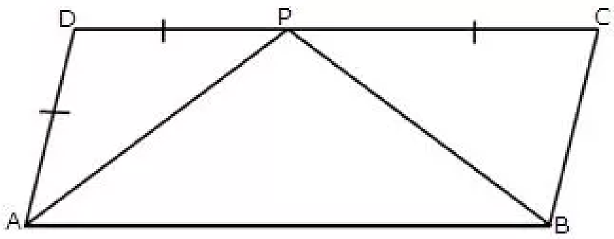
$$OC=OA$$
$$AM=CN$$
$$\Rightarrow OA-AM=OC-CN$$
$$\Rightarrow OM=ON$$

Thus in a quadrilateral $BMDN$, diagonal BD and MN are such that $OM=ON$ and $OD=OB$

Therefore the diagonals AC and PQ bisect each other.

Hence $BMDN$ is a parallelogram

Solution 11:



Consider ΔADP and ΔBCP

$AD = BC$ [since ABCD is a parallelogram]
$DC = AB$ [since ABCD is a parallelogram]
$\angle A = \angle C$ [opposite angles]
$\Delta ADP \cong \Delta BCP$ [SAS]

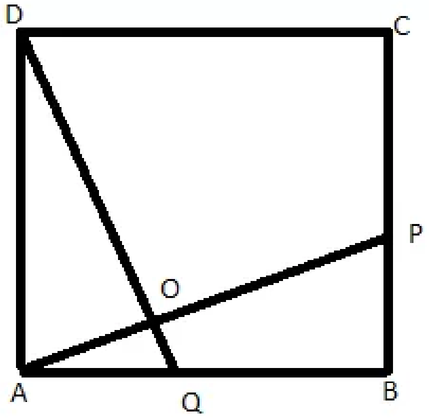
Therefore $AP = BP$

AP bisects $\angle A$
BP bisects $\angle B$

In ΔAPB
$AP = PB$
$\angle APB = \angle DAP + \angle BCP$

Hence proved

Solution 12:



ABCD is a square and $AP = PQ$

$$\text{Consider } \triangle DAQ \text{ and } \triangle ABP$$
$$\angle DAQ = \angle ABP = 90^{\circ}$$
$$DQ = AP$$
$$AD = AB$$
$$\triangle DAQ \cong \triangle ABP$$
$$\Rightarrow \angle PAB = \angle QDA$$

$$\text{Now,}$$
$$\angle PAB + \angle APB = 90^{\circ}$$
$$\text{also } \angle QDA + \angle APB = 90^{\circ} \quad [\angle PAB = \angle QDA]$$

$$\text{Consider } \triangle AOQ \text{ By ASP}$$
$$\angle QDA + \angle APB + \angle AOD = 180^{\circ}$$
$$\Rightarrow 90^{\circ} + \angle AOD = 180^{\circ}$$
$$\Rightarrow \angle AOD = 90^{\circ}$$

Hence AP and DQ are perpendicular.

Solution 13:

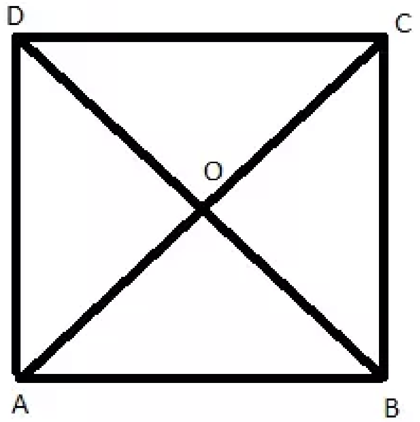
Given: $ABCD$ is quadrilateral,

$AB = AD$
$CB = CD$

To prove: (i) AC bisects angle BAD.

(ii) AC is perpendicular bisector of BD.

Proof :



In ΔABC and ΔADC

$AB = AD$ [given]

$CB = CD$ [given]

$AC = AC$ [common side]

$\Delta ABC \cong \Delta ADC$ [SSS]

Therefore AC bisects $\angle BAD$

$OD = OB$

$OA = OA$ [diagonals bisect each other at O]

Thus AC is perpendicular bisector of BD

Hence proved

Solution 14:

Given $ABCD$ is a trapezium, $AB \parallel DC$ and $AD=BC$

To prove(i) $\angle DAB = \angle CBA$

(ii) $\angle ADC = \angle BCD$

(iii) AC = BD

(iv) OA = OB and OC = OD

Proof :(i) Since $AD \parallel CE$ and transversal AE cuts them at A and E respectively.

Therefore, $\angle A + \angle B = 180°$

Since $AB \parallel CD$ and $AD \parallel BC$

Therefore $ABCD$ is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since ABCD is a parallelogram]

Therefore $\angle DAB = \angle CBA$

$\angle ADC = \angle BCD$

In ΔABC and ΔBAD ,we have

$BC=AD$ [given]

$AB=BA$ [common]

$\angle A = \angle B$ [proved]

$\Delta ABC \cong \Delta BAD$ [SAS]

Since $\Delta ABC \cong \Delta BAD$

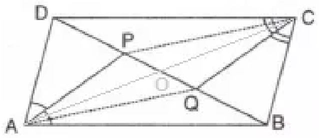
Therefore $AC=BD$ [corresponding parts of congruent triangles are equal]

Again $OA=OB$

$OC=OD$ [since diagonals bisect each other at O]

Hence proved

Solution 15:



Join AC to meet BD in O

We know that the diagonals of a parallelogram bisect each other. Therefore AC and BD bisect each other at O.

Therefore

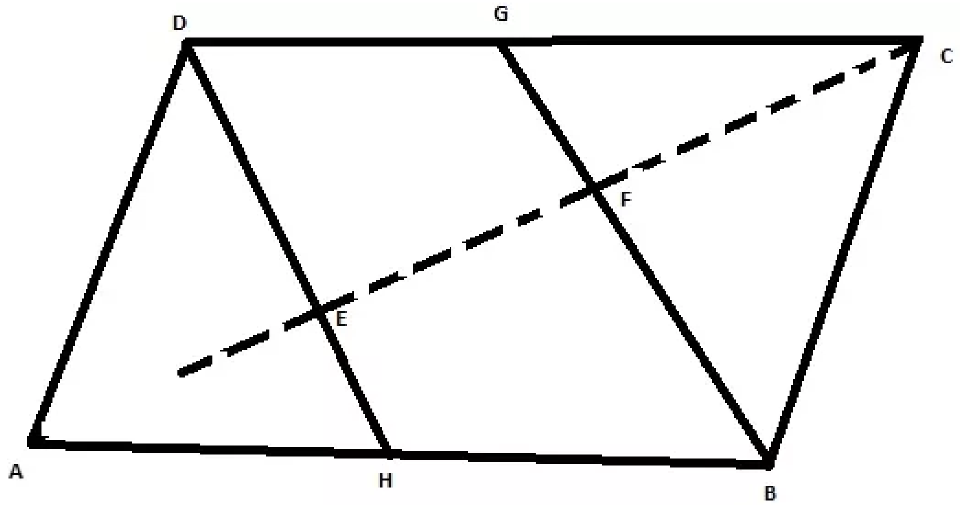
OB=OD
But
BQ=DP

OB-PQ=OD-DP
$\Rightarrow$ OQ=OP

Thus in a quadrilateral APCQ diagonals AC and PQ such that OQ=OP and OA=OC. Since diagonals AC and PQ bisect each other.

Hence APCQ is a parallelogram

Solution 16:



ABCD is a parallelogram ,the bisectors of $\angle ADC$ and $\angle BCD$ meet at a point E and the bisectors of $\angle BCD$ AND $\angle ABC$ meet at F.

We have to prove that the $\angle CED = 90°$ and $\angle CFG = 90°$

Proof :In the parallelogram ABCD

$$\angle ADC + \angle BCD = 180° \text{ [sum of adjacent angles of a parallelogram]}$$
$$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$$
$$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$$
$$\Rightarrow \angle CED = 90°$$

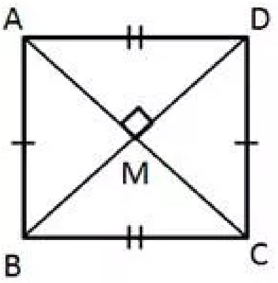
Similarly taking triangle BCF it can be proved that $\angle BFC = 90°$

Also $\angle BFC + \angle CFG = 180°$ [adjacent angles on a line]
$\Rightarrow \angle CFG = 90°$

Now since $\angle CFG = \angle CED = 90°$ [it means that the lines DE and BG are parallel]

Hence proved

Solution 17:



To prove : ABCD is a square,
that is, to prove that sides of the quadrilateral are equal
and each angle of the quadrilateral is 90°.
ABCD is a rectangle,
$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90°$ and diagonals bisect each other
that is, MD = BM...(i)
Consider ΔAMD and ΔAMB,
MD = BM (from (i))
$\angle AMD = \angle AMB = 90°$ (given)
AM = AM (common side)
$\Delta AMD \cong \Delta AMB$ (SAS congruence criterion)
$\Rightarrow$ AD = AB (cpctc)
Since ABCD is a rectangle, AD = BC and AB = CD
Thus, AB = BC = CD = AD and $\angle A = \angle B = \angle C = \angle D = 90°$
$\Rightarrow$ ABCD is a square.

Solution 18:

ABCD is a parallelogram
$\Rightarrow$ opposite angles of a parallelogram are congruent
$\Rightarrow \angle DAB = \angle BCD$ and $\angle ABC = \angle ADC = 120°$
In ABCD,
$\angle DAB + \angle BCD + \angle ABC + \angle ADC = 360°$
.......(sum of the measures of angles of a quadrilateral)
$\Rightarrow \angle BCD + \angle BCD + 120° + 120° = 360°$
$\Rightarrow 2\angle BCD = 360° - 240°$
$\Rightarrow 2\angle BCD = 120°$
$\Rightarrow \angle BCD = 60°$
PQRS is a parallelogram
$\Rightarrow \angle PQR = \angle PSR = 70°$
In ΔCMS,
$\angle CMS + \angle CSM + \angle MCS = 180°$ ....(angle sum property)
$\Rightarrow x + 70° + 60° = 180°$
$\Rightarrow x = 50°$

Solution 19:

ABCD is a rhombus $\Rightarrow AD = CD$ and $\angle ADC = \angle ABC = 56^\circ$

DCFE is a square $\Rightarrow ED = CD$ and $\angle FED = \angle EDC = \angle DCF = \angle CFE = 90^\circ$

$\Rightarrow AD = CD = ED$

In ΔADE,

$AD = ED \Rightarrow \angle DAE = \angle AED \ldots(i)$

$\angle DAE + \angle AED + \angle ADE = 180^\circ$

$\Rightarrow 2\angle DAE + 146^\circ = 180^\circ \quad \ldots$(Since $\angle ADE = \angle EDC + \angle ADC = 90^\circ + 56^\circ = 146^\circ$)

$\Rightarrow 2\angle DAE = 34^\circ$

$\Rightarrow \angle DAE = 17^\circ$

$\Rightarrow \angle DEA = 17^\circ \ldots(ii)$

In ABCD,

$\angle ABC + \angle BCD + \angle ADC + \angle DAB = 360^\circ$

$\Rightarrow 56^\circ + 56^\circ + 2\angle DAB = 360^\circ$ ($\because$ opposite angles of a rhombus are equal)

$\Rightarrow 2\angle DAB = 248^\circ$

$\Rightarrow \angle DAB = 124^\circ$

We know that diagonals of a rhombus, bisect its angles.

$\Rightarrow \angle DAC = \frac{124^\circ}{2} = 62^\circ$

$\Rightarrow \angle EAC = \angle DAC - \angle DAE = 62^\circ - 17^\circ = 45^\circ$

Now, $\angle FEA = \angle FED - \angle DEA$

$= 90^\circ - 17^\circ \quad \ldots$(from (ii) and each angle of a square is 90°)

$= 73^\circ$

We know that diagonals of a square bisect its angles.

$\Rightarrow \angle CED = \frac{90^\circ}{2} = 45^\circ$

So, $\angle AEC = \angle CED - \angle DEA$

$= 45^\circ - 17^\circ$

$= 28^\circ$

Hence, $\angle DAE = 17^\circ$, $\angle FEA = 73^\circ$, $\angle EAC = 45^\circ$ and $\angle AEC = 28^\circ$.