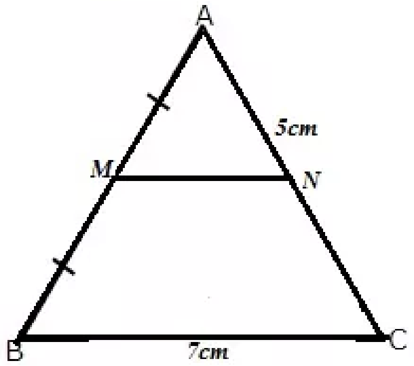
Chapter 12. Mid-point and Its Converse [Including Intercept Theorem]

Exercise 12(A)

Solution 1:

The triangle is shown below,



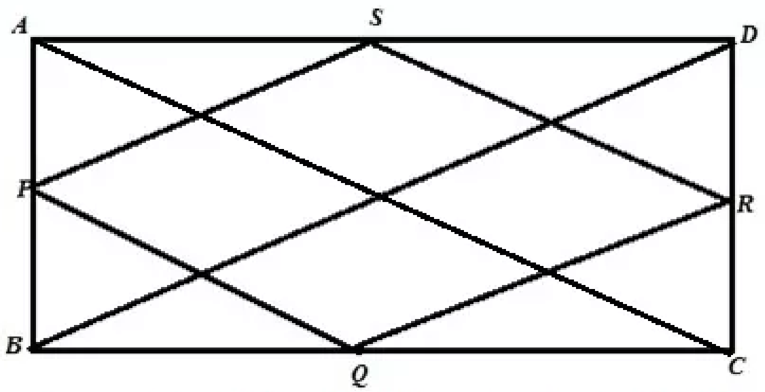
Since M is the midpoint of AB and MN||BC hence N is the midpoint of AC.Therefore

$$MN = \frac{1}{2}BC = \frac{1}{2}\times 7 = 3.5cm$$

And $AN = \frac{1}{2}AC = \frac{1}{2}\times 5 = 2.5cm$

Solution 2:

The figure is shown below,



Let ABCD be a rectangle where P,Q,R,S are the midpoint of AB,BC, CD, DA.We need to show that PQRS is a rhombus
For help we draw two diagonal BD and AC as shown in figure
Where BD=AC(Since diagonal of rectangle are equal)
Proof:
From ΔABD and ΔBCD

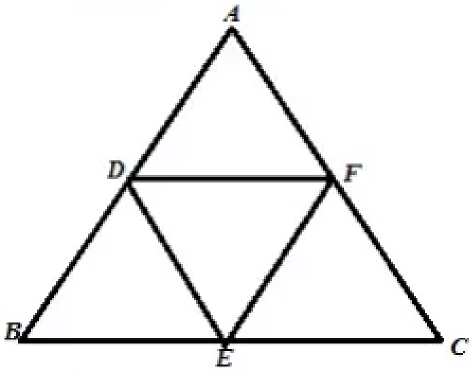
$$PS = \frac{1}{2}BD = QR \text{ and } PS \| BD \| QR$$

$$2PS = 2QR = BD \text{ and } PS \| QR \qquad \text{------ (1)}$$

Similarly 2PQ=2SR=AC and PQ||SR----- (2)
From (1) and (2) we get
PQ=QR=RS=PS
Therefore PQRS is a rhombus.
Hence proved

Solution 3:

The figure is shown below



Given that ABC is an isosceles triangle where AB=AC.
Since D,E,F are midpoint of AB,BC,CA therefore
2DE=AC and 2EF=AB this means DE=EF
Therefore DEF is an isosceles triangle an DE=EF.
Hence proved

Solution 4:

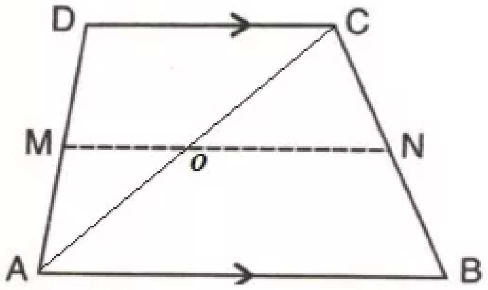
Here from triangle ABD P is the midpoint of AD and PR||AB, therefore Q is the midpoint of BD
Similarly R is the midpoint of BC as PR||CD||AB
From triangle ABD 2PQ=AB ....(1)
From triangle BCD 2QR=CD .....(2)
Now (1)+(2)=>
2(PQ+QR)=AB+CD

$$PR=\frac{1}{2}(AB+CD)$$

Hence proved

Solution 5:

Let we draw a diagonal AC as shown in the figure below,



(i)Given that AB=11cm,CD=8cm

From triangle ABC

$$ON = \frac{1}{2}AB = \frac{1}{2}\times 11 = 5.5cm$$

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2}\times 8 = 4cm$$

Hence MN=OM+ON=(4+5.5)=9.5cm

(ii)Given that CD=20cm,MN=27cm

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2}\times 20 = 10cm$$

Therefore ON=27-10=17cm

From triangle ABC

$$AB = 2ON = 2\times 17 = 34cm$$

(iii)Given that AB=23cm,MN=15cm

From triangle ABC

$$ON = \frac{1}{2}AB = \frac{1}{2}\times 23 = 11.5cm$$

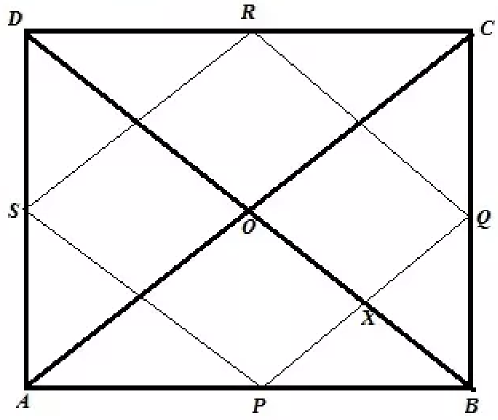
Therefore OM=15-11.5=3.5cm

From triangle ACD

$$CD = 2OM = 2\times 3.5 = 7cm$$

Solution 6:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA. Diagonal AC and BD intersects at right angle at point O. We need to show that PQRS is a rectangle

Proof:

From ΔABC and ΔADC

2PQ=AC and PQ||AC .....(1)

2RS=AC and RS||AC .....(2)

From (1) and (2) we get,

PQ=RS and PQ||RS

Similarly we can show that PS=RQ and PS||RQ

Therefore PQRS is a parallelogram.

Now PQ||AC, therefore $\angle AOD = \angle PXO = 90°$ $\quad$ $[Corresponding\ angle]$

Again BD||RQ, therefore $\angle PXO = \angle RQX = 90°$ $\quad$ $[Corresponding\ angle]$

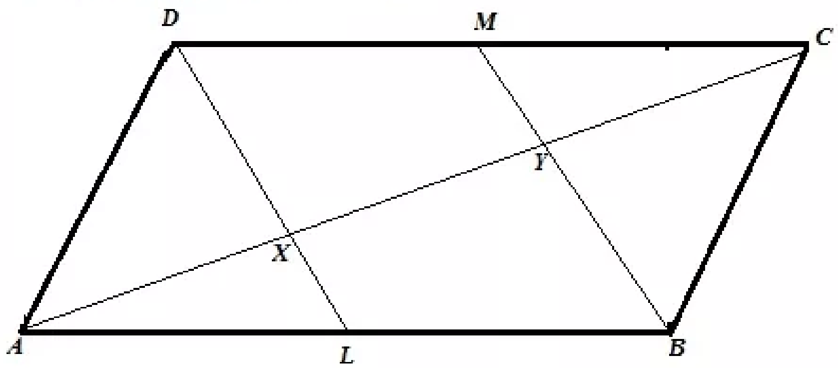
Similarly $\angle QRS = \angle RSP = \angle SPQ = 90°$

Therefore PQRS is a rectangle.

Hence proved

Solution 7:

The required figure is shown below



From figure,

BL=DM and BL||DM and BLMD is a parallelogram, therefore BM||DL

From triangle ABY

L is the midpoint of AB and XL||BY, therefore x is the midpoint of AY.ie AX=XY .....(1)

Similarly for triangle CDX

CY=XY .....(2)

From (1) and (2)

AX=XY=CY and AC=AX+XY+CY

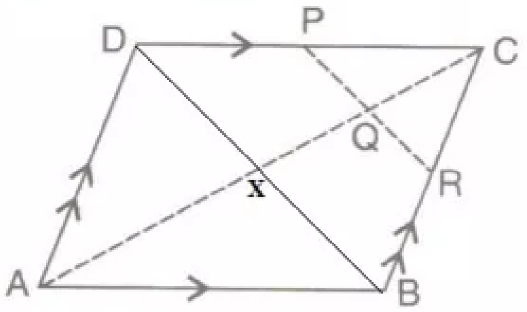
Hence proved

Solution 8:

Given that AD=BC .....(1)
From the figure,
For triangle ADC and triangle ABD
2GH=AD and 2EF=AD, therefore 2GH=2EF=AD .....(2)
For triangle BCD and triangle ABC
2GF=BC and 2EH=BC, therefore 2GF=2EH=BC .....(3)
From (1),(2),(3) we get,
2GH=2EF=2GF=2EH
GH=EF=GF=EH
Therefore EFGH is a rhombus.
Hence proved

Solution 9:

For help we draw the diagonal BD as shown below



The diagonal AC and BD cuts at point X.
We know that the diagonal of a parallelogram intersects equally each other. Therefore AX=CX and BX=DX
Given,

$$CQ = \frac{1}{4}AC$$

$$CQ = \frac{1}{4} \times 2CX$$

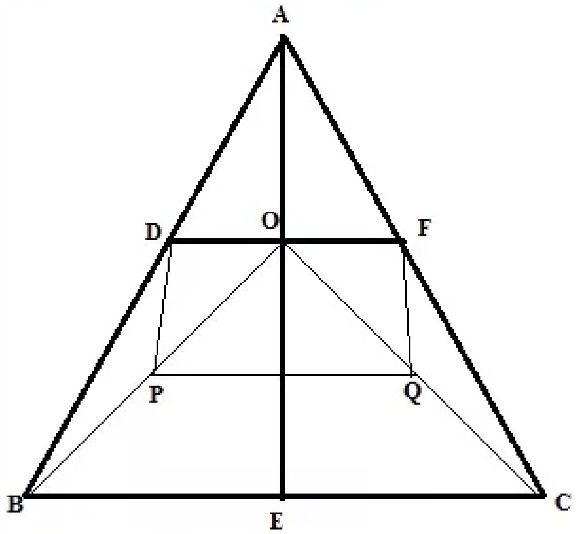
$$CQ = \frac{1}{2}CX$$

Therefore Q is the midpoint of CX.
(i)For triangle CDX PQ||DX or PR||BD

Since for triangle CBX
Q is the midpoint of CX and QR||BX. Therefore R is the midpoint of BC
(ii)For triangle BCD
As P and R are the midpoint of CD and BC ,therefore $PR = \frac{1}{2}DB$

Solution 10:

The required figure is shown below



For triangle ABC and OBC

2DE=BC and 2PQ=BC, therefore DE=PQ .....(1)

For triangle ABO and ACO

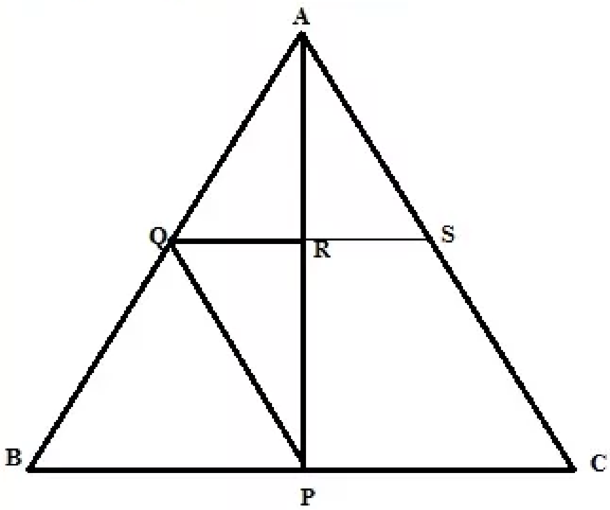
2PD=AO and 2FQ=AO, therefore PD=FQ .....(2)

From (1),(2) we get that PQFD is a parallelogram.

Hence proved

Solution 11:

The required figure is shown below



From the figure it is seen that P is the midpoint of BC and PQ||AC and QR||BC

Therefore Q is the midpoint of AB and R is the midpoint of AP

(i)Therefore AP=2AR

(ii)Here we increase QR so that it cuts AC at S as shown in the figure.

(iii)From triangle PQR and triangle ARS

$\angle PQR = \angle ARS \quad (Opposite\ angle)$

$PR = AR$

$PQ = AS \quad \left[PQ = AS = \frac{1}{2}AC\right]$

$\Delta PQR \cong \Delta ARS \quad \text{(SAS Postulate)}$

Therefore QR=RS

Now

$BC = 2QS$

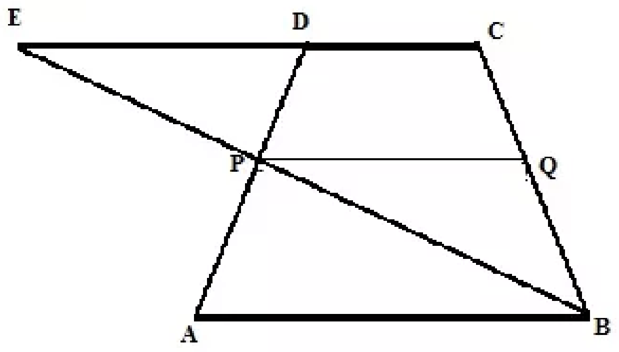
$BC = 2\times 2QR$

$BC = 4QR$

Hence proved

Solution 12:

The required figure is shown below



(i)

$From\ \Delta PED\ and\ \Delta ABP$

$PD = AP$ [P is the midpoint of AD]

$\angle DPE = \angle APB$ [Opposite angle]

$\angle PED = \angle PBA$ [$AB || CE$]

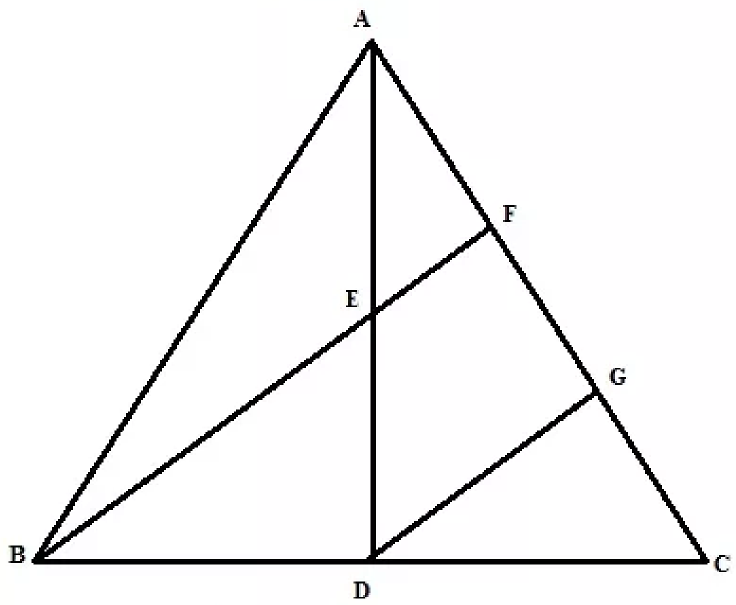
$\therefore \Delta PED \cong \Delta ABP$ [ASA postulate]

$\therefore EP = BP$

(ii)For tiangle ECB PQ||CE
Again CE||AB
Therefore PQ||AB
Hence proved

Solution 13:

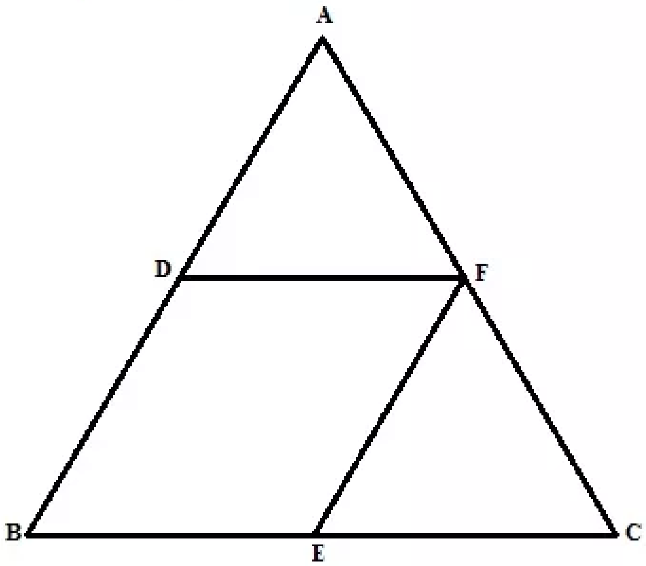
The required figure is shown below



For help we draw a line DG||BF
Now from triangle ADG, DG||BF and E is the midpoint of AD
Therefore F is the midpoint of AG,ie AF=GF .....(1)
From triangle BCF, DG||BF and D is the midpoint of BC
Therefore G is the midpoint of CF,ie GF=CF ...(2)
AC=AF+GF+CF
AC=3AF(From (1) and (2))
Hence proved

Solution 14:

The required figure is shown below



(i)Since F is the midpoint and EF||AB.
Therefore E is the midpoint of BC

So $BE = \frac{1}{2}BC$ and $EF = \frac{1}{2}AB$ .....(1)

Since D and F are the midpoint of AB and AC
Therefore DE||BC

SO $DF = \frac{1}{2}BC$ and $DB = \frac{1}{2}AB$ .....(2)

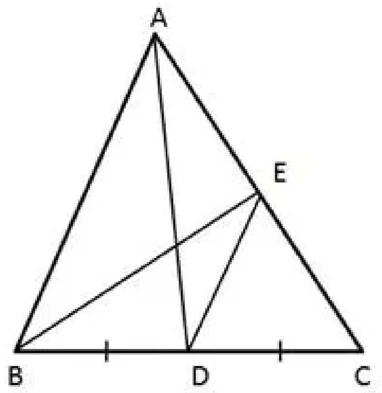
From (1),(2) we get
BE=DF and BD=EF
Hence BDEF is a parallelogram.
(ii)Since

$AB = 2EF$

$= 2\times 4.8$

$= 9.6cm$

Solution 15:



In ΔABC,
AD is the median of BC
$\Rightarrow$ D is the mid - point of BC.
Given that DE P BA
By the Converse of the Mid - point theorem,
$\Rightarrow$ DE bisects AC
$\Rightarrow$ E is the mid - point of AC
$\Rightarrow$ BE is the median of AC,
that is BE is also a median.

Solution 16:

Construction : Draw $DY \parallel BQ$

In ΔBCQ and ΔDCY,

$\angle BCQ = \angle DCY$ (Common)

$\angle BQC = \angle DYC$ (Corresponding angles)

So, $\Delta BCQ \sim \Delta DCY$ (AA Similarity criterion)

$\Rightarrow \frac{BQ}{DY} = \frac{BC}{DC} = \frac{CQ}{CY}$ (Corresponding sides are proportional)

$\Rightarrow \frac{BQ}{DY} = \frac{2CD}{CD}$ (D is the mid-point of BC)

$\Rightarrow \frac{BQ}{DY} = 2 \ldots (i)$

Similarly, $\Delta AEQ \sim \Delta ADY$

$\Rightarrow \frac{EQ}{DY} = \frac{AE}{ED} = \frac{1}{2}$ (E is the mid-point of AD)

that is $\frac{EQ}{DY} = \frac{1}{2} \ldots\ldots (ii)$

Dividing (i) by (ii), we get

$\Rightarrow \frac{BQ}{EQ} = 4$

$\Rightarrow BE + EQ = 4EQ$

$\Rightarrow BE = 3EQ$

$\Rightarrow \frac{BE}{EQ} = \frac{3}{1}$

Solution 17:

In ΔEDF,

M is the mid-point of AB and N is the mid-point of DE.

$\Rightarrow MN = \frac{1}{2}EF$ (Mid-point theorem)

$\Rightarrow EF = 2MN \ldots\ldots (i)$

In ΔABC,

M is the mid-point of AB and N is the mid-point of BC.

$\Rightarrow MN = \frac{1}{2}AC$ (Mid-point theorem)

$\Rightarrow AC = 2MN \ldots\ldots (ii)$

From (i) and (ii), we get

$\Rightarrow EF = AC$

Exercise 12(B)

Solution 1:

According to equal intercept theorem since CD=DE

Therefore AB=BC and EF=GF

(i)BC=AB=7.2cm

(ii)GE=EF+GF=2EF= 2×4 =8cm

Since B,D,F are the midpoint and AE||BF||CG

Therefore AE=2BD and CG=2DF

(iii)AE=2BD= 2×4.1 =8.2

(iv) $DF = \frac{1}{2}CG = \frac{1}{2} \times 11 = 5.5cm$

Solution 2:

Given that AD=AP=PB as 2AD=AB and p is the midpoint of AB

(i)From triangle DPR, A and Q are the midpoint of DP and DR.

Therefore AQ||PR

Since PR||BS ,hence AQ||BS

(ii)From triangle ABC, P is the midpoint and PR||BS

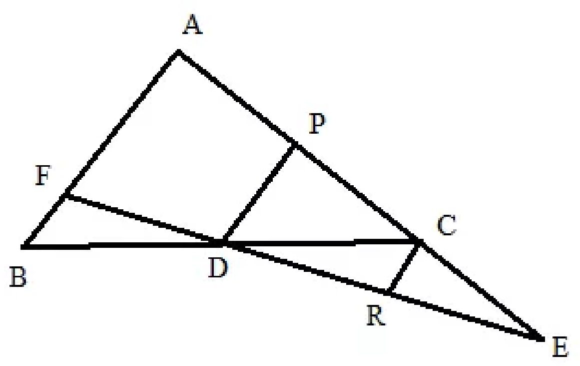
Therefore R is the midpoint of BC

$$
\begin{aligned}
&\text{From } \Delta BRS \text{ and } \Delta QRC\\
&\angle BRS = \angle QRC\\
&BR = RC\\
&\angle RBS = \angle RCQ\\
&\therefore\ \Delta BRS \cong \Delta QRC\\
&\therefore\ QR = RS
\end{aligned}
$$

DS=DQ+QR+RS=QR+QR+RS=3RS

Solution 3:

Consider the figure:



Here *D* is the midpoint of *BC* and *DP* is parallel to *AB*, therefore *P* is the midpoint of *AC* and $PD = \frac{1}{2}AB$

(i)

Again from the triangle *AEF* we have *AE* ||*PD*||*CR* and $AP = \frac{1}{3}AE$

Therefore $DF = \frac{1}{3}EF$ or we can say that 3DF = EF .

Hence it is shown.

(ii)

From the triangle *PED* we have *PD*||*CR* and *C* is the midpoint of *PE* therefore $CR = \frac{1}{2}PD$

Now

$PD = \frac{1}{2}AB$

$\frac{1}{2}PD = \frac{1}{4}AB$

$CR = \frac{1}{4}AB$

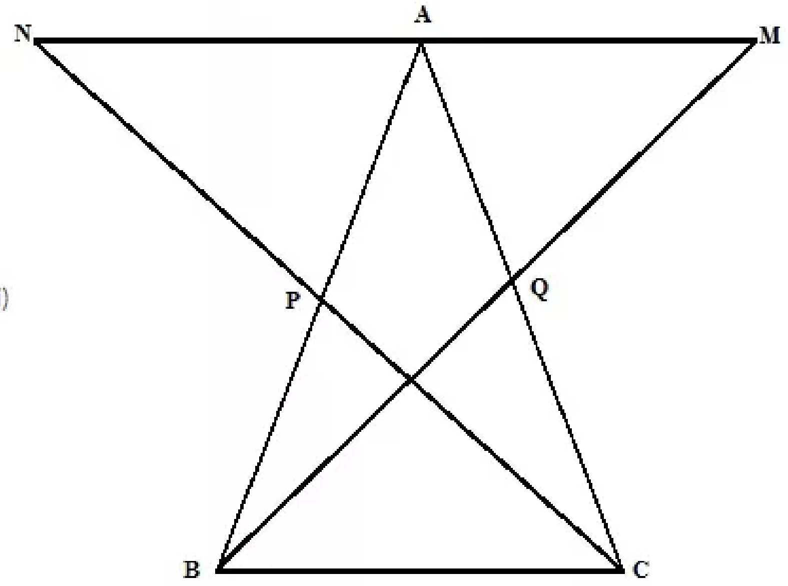
$4CR = AB$

Hence it is shown.

Solution 4:

The figure is shown below

(i)



From triangle BPC and triangle APN

$\angle BPC = \angle APN$ [Opposite angle]

$BP = AP$

$PC = PN$

$\therefore \Delta BPC \cong \Delta APN$ [SAS postulate]

$\therefore \angle PBC = \angle PAN$ ..... (1)

And BC=AN ......(3)

Similarly $\angle QCB = \angle QAN$ .....(2)

And BC=AM ......(4)

Now

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\angle PAN + \angle QAM + \angle BAC = 180°$ [(1),(2) we get]

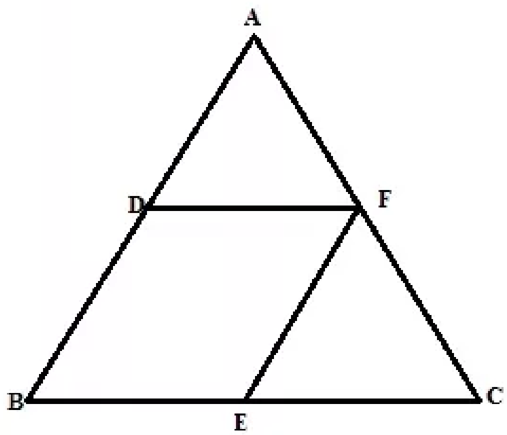
Therefore M,A,N are collinear

(ii) From (3) and (4) MA=NA

Hence A is the midpoint of MN

Solution 5:

The figure is shown below



From the figure EF||AB and E is the midpoint of BC.

Therefore F is the midpoint of AC.

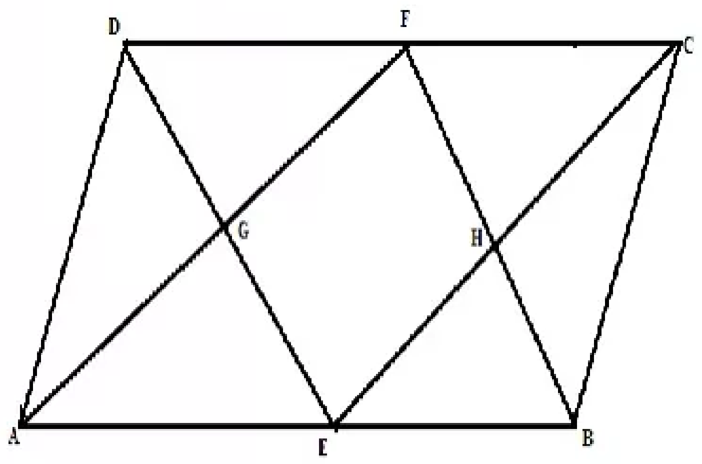
Here EF||BD, EF=BD as D is the midpoint of AB

BE||DF, BE=DF as E is the midpoint of BC.

Therefore BEFD is a parallelogram.

Solution 6:

The figure is shown below



(i)

$From\ \Delta HEB\ and\ \Delta FHC$

$BE = FC$

$\angle EHB = \angle FHC \quad [\text{Opposite angle}]$

$\angle HBE = \angle HFC$

$\therefore \Delta HEB \cong \Delta FHC$

$\therefore EH = CH, BH = FH$

(ii)

Similarly AG=GF and EG=DG .....(1)

For triangle ECD, F and H are the midpoint of CD and EC.

Therefore HF||DE and $HF = \frac{1}{2} DE$ .....(2)

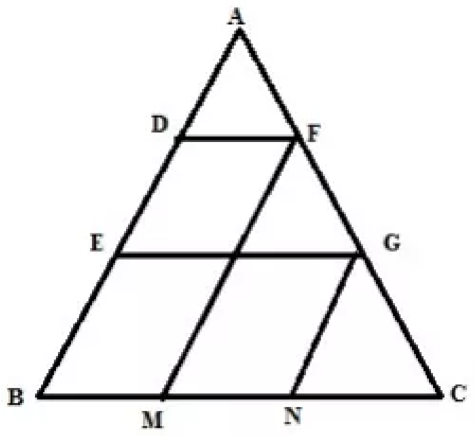
(1),(2) we get, HF=EG and HF||EG

Similarly we can show that EH=GF and EH||GF

Therefore GEHF is a parallelogram.

Solution 7:

The figure is shown below



For triangle AEG

D is the midpoint of AE and DF||EG||BC

Therefore F is the midpoint of AG.

AF=GF .....(1)

Again DF||EG||BC DE=BE, therefore GF=GC .....(2)

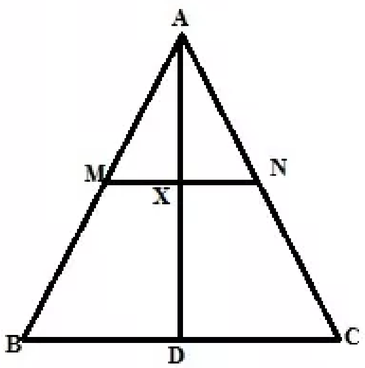
(1),(2) we get AF=GF=GC.

Similarly Since GN||FM||AB and AF=GF ,therefore BM=MN=NC

Hence proved

Solution 8:

The figure is shown below



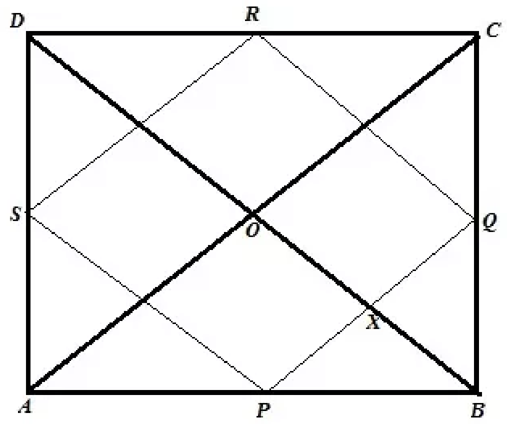
Since M and N are the midpoint of AB and AC, MN||BC

According to intercept theorem Since MN||BC and AM=BM,

Therefore AX=DX. Hence proved

Solution 9:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA.PQRS is a rectangle. Diagonal AC and BD intersect at point O. We need to show that AC and BD intersect at right angle.

Proof:

PQ||AC, therefore $\angle AOD = \angle PXO$ $\quad [Corresponding\ angle]$ ....(1)

||Again BD||RQ, therefore $\angle PXO = \angle RQX = 90^\circ [\text{Corresponding angle and angle of rectangle}]$ ...(2)

From (1) and (2) we get ,

$\angle AOD = 90^\circ$

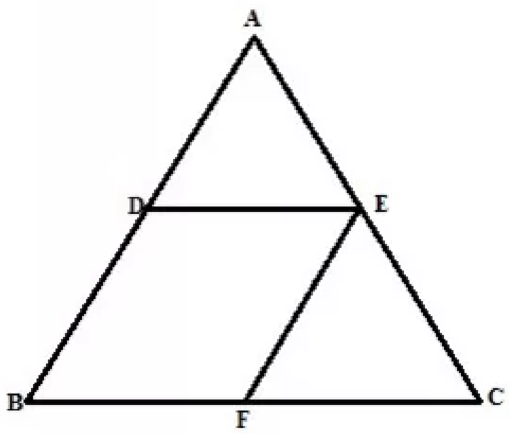
Similarly $\angle AOB = \angle BOC = \angle DOC = 90^\circ$

Therefore diagonals AC and BD intersect at right angle

Hence proved

Solution 10:

The figure is shown below



From figure since E is the midpoint of AC and EF||AB

Therefore F is the midpoint of BC and 2DE=BC or DE=BF

Again D and E are midpoint ,therefore DE||BF and EF=BD

Hence BDEF is a parallelogram.

Now

$$BD = EF = \frac{1}{2}AB = \frac{1}{2} \times 16 = 8cm$$

$$BF = DE = \frac{1}{2}BC = \frac{1}{2} \times 18 = 9cm$$

Therefore perimeter of BDEF=2(BF+EF)= $2(9+8) = 34cm$

Solution 11:

Given AD and CE are medians and DF || CE.
We know that from the midpoint theorem, if two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.
Consider triangle BEC. Given DF || CE and D is midpoint of BC.
So F must be the midpoint of BE.

So $FB = \frac{1}{2}BE$ but $BE = \frac{1}{2}AB$

Substitute value of BE in first equation, we get

$FB = \frac{1}{4}AB$

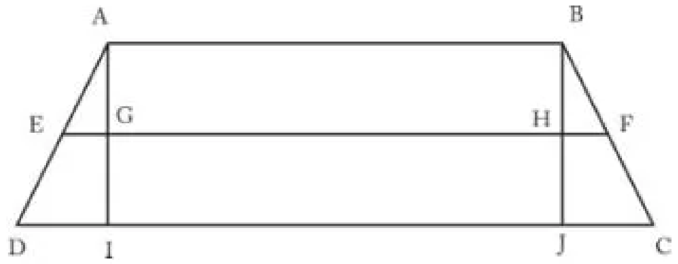
Hence Prove

Solution 12:

Given ABCD is parallelogram, so AD = BC, AB = CD.
Consider triangle APB, given EC is parallel to AP and E is midpoint of side AB. So by midpoint theorem, C has to be the midpoint of BP.
So BP = 2BC, but BC = AD as ABCD is a parallelogram.
Hence BP = 2AD
Consider triangle APB, AB || OC as ABCD is a parallelogram. So by midpoint theorem, O has to be the midpoint of AP.
Hence Proved

Solution 13:

Consider trapezium ABCD.
Given E and F are midpoints on sides AD and BC, respectively.



We know that AB = GH = IJ

From midpoint theorem, $EG = \frac{1}{2}DI$, $HF = \frac{1}{2}JC$

Consider LHS,
AB + CD = AB + CJ + JI + ID = AB + 2HF + AB + 2EG
So AB + CD = 2(AB + HF + EG) = 2(EG + GH + HF) = 2EF
AB + CD = 2EF
Hence Proved

Solution 14:

Given Δ ABC
AD is the median. So D is the midpoint of side BC.
Given DE || AB. By the midpoint theorem, E has to be midpoint of AC.
So line joining the vertex and midpoint of the opposite side is always known as median. So BE is also median of Δ ABC.