Exponents (Including Laws of Exponents)

EXERCISE 5 (A)

Question 1.
Find the value of:
(i) 6^2
(ii) 7^3
(iii) 4^4
(iv) 5^5
(v) 8^3
(vi) 7^5

Solution:
(i) $6^2 = 6 \times 6 = 36$
(ii) $7^3 = 7 \times 7 \times 7 = 343$
(iii) $4^4 = 4 \times 4 \times 4 \times 4 = 256$
(iv) $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$
(v) $8^3 = 8 \times 8 \times 8 = 512$
(vi) $7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16807$

Question 2.
Evaluate:
(i) $2^3 \times 4^2$
(ii) $2^3 \times 5^2$
(iii) $3^3 \times 5^2$
(iv) $2^2 \times 3^3$
(v) $3^2 \times 5^2$
(vi) $5^3 \times 2^4$
(vii) $3^2 \times 4^2$
(ix) $(5 \times 4)^2$

Solution:
(i) $2^3 \times 4^2$
$= 2 \times 2 \times 2 \times 4 \times 4$
$= 8 \times 16$
$= 128$
(ii) $2^3 \times 5^2$
$= 2 \times 2 \times 2 \times 5 \times 5$
$= 8 \times 25$
$= 200$
(iii) $3^3 \times 5^2$
$=3 \times 3 \times 3 \times 5 \times 5$
$= 27 \times 25$

= 675
(iv) $2^2 \times 3^3$
= 2 x 2 x 3 x 3 x 3
= 4 x 27
= 108
(v) $3^2 \times 5^3$
=3 x3 x 5 x 5 x 5
= 9 x 125
= 1125
(vi) $5^3 \times 2^4$
= 5 x 5 x 5 x 2 x 2 x 2 x 2
= 125 x 16
= 2000
(vii) $3^2 \times 4^2$
=3 x 3 x 4 x 4
= 9 x 16
=144
(viii) $(4 \times 3)^3$
=4 x 4 x 4 x 3 x 3 x 3
= 64 x 27
= 1728
(ix) $(5 \times 4)^2$
=5 x 5 x 4 x 4
= 25 x 16
= 400

Question 3.
Evaluate:

(i) $\left(\frac{3}{4}\right)^4$ (ii) $\left(-\frac{5}{6}\right)^5$

(iii) $\left(\frac{-3}{-5}\right)^3$

Solution:

(i) $\left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$

$= \frac{3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4} = \frac{81}{256}$

(ii) $\left(-\frac{5}{6}\right)^5$

$= \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right)$

$= \frac{(-5) \times (-5) \times (-5) \times (-5) \times (-5)}{6 \times 6 \times 6 \times 6 \times 6}$

$= -\frac{3125}{776}$

(iii) $\left(\frac{-3}{-5}\right)^3 = \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right) \times \left(\frac{-3}{-5}\right)$

$= \frac{(-3) \times (-3) \times (-3)}{(-5) \times (-5) \times (-5)}$

$= \frac{27}{125}$

Question 4.

Evaluate :

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$ (ii) $\left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$

Solution:

(*i*) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$

$= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$

$= \frac{8}{27} \times \frac{9}{16} = \frac{1}{6}$

(*ii*) $\left(-\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$

$= \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$

$\times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$

$= \frac{-27}{64} \times \frac{16}{81} = -\frac{1}{2}$

(*iii*) $\left(\frac{3}{5}\right)^2 \times \left(-\frac{2}{3}\right)^3$

$= \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$

$= \frac{9}{25} \times \left(\frac{-8}{27}\right)$

$= -\frac{8}{75}$

Question 5.
Which is greater :
(i) 2^3 or 3^2
(ii) 2^5 or 5^2
(iii) 4^3 or 3^4
(iv) 5^4 or 4^5

Solution:
(i) 2^3 or 3^3
Since, $2^3 = 2 \times 2 \times 2 = 8$
and, $3^2 = 3 \times 3 = 9$
$\because$9 is greater than 8 $\Rightarrow 3^2 > 2^3$
(ii) 2^5 or 5^2
Since, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
and, $5^2 = 5 \times 5 = 25$
$\because$32 is greater than 25 $\Rightarrow 2^{35} > 5^{32}$
(iii) 4^3 or 3^4
Since, $4^3 = 4 \times 4 \times 4 = 64$
and, $3^4 = 3 \times 3 \times 3 \times 3 = 81$
$\because$ 81 is greater than 64 $\Rightarrow 3^4 > 4^3$
(iv) 5^4 or 4^5
Since, $5^4 = 5 \times 5 \times 5 \times 5 = 625$
and, $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$
$\because$ 1024 is greater than 625 $\Rightarrow 4^5 > 5^4$

Question 6.
Express each of the following in exponential form :
(i) 512
(ii) 1250
(iii) 1458
(iv) 3600
(v) 1350
(vi) 1176

Solution:

(i) 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

(ii) 1250

2	1250
5	625
5	125
5	25
5	5
	1

$= 2 \times 5 \times 5 \times 5 \times 5 = 2 \times 5^4$

(iii) 1458

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^6$

(iv) 3600

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 2^4 \times 3^2 \times 5^2$

(v) 1350

2	1350
3	675
3	225
3	75
5	25
5	5
	1

$= 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 2 \times 3^3 \times 5^2$

(vi) 1176

2	1176
2	588
2	294
3	147
7	49
7	7
	1

$= 2 \times 2 \times 2 \times 3 \times 7 \times 7$

$= 2^3 \times 3 \times 7^2$

Question 7.
If a = 2 and b = 3, find the value of:
(i) $(a + b)^2$
(ii) $(b – a)^3$
(iii) (a x b)a (iv) (a x b)b

Solution:
(i) $(a + b)^2$
$= (2 + 3)^2 = (5)^2 = 5 \times 5 = 25$

(ii) $(b – a)^2$
$= (3 – 2)^2 = (1)^3$
$= 1 \times 1 \times 1 = 1$

(iii) $(a \times b)^a$
$= (2 \times 3)^2 – (6)^2$
$= 6 \times 6 = 36$

(iv) $(a \times b)^b$
$= (2 \times 3)^3 = (6)^3 = 6 \times 6 \times 6 = 216$

Question 8.
Express:
(i) 1024 as a power of 2.
(ii) 343 as a power of 7.
(iii) 729 as a power of 3.
Solution:
(i) 1024 as a power of 2.

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^{10}$

(ii) 343 as a power of 7.

$= 7 \times 7 \times 7 = 7^3$

7	343
7	49
7	7
	1

(iii) 729 as a power of 3.

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

3	729
3	243
3	81
3	27
3	9
3	3
	1

Question 9.

If 27 x 32 = 3^x x 2^y; find the values of x and y.

$27 \times 32 = 3^x \times 2^y$

$27 = 3^x$

3	27
3	9
3	3
	1

Solution:

$27 = 3 \times 3 \times 3$

$= 3^3 = 3^x$

$\therefore\ x = 3^x$

Also, $32 = 2^y$

2	32
2	16
2	8
2	4
2	2
	1

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5 = 2^y$

$\therefore\ y = 5$

Question 10.

If 64 x 625 = 2^a x 5^b; find :

(i) the values of a and b.

(ii) 2^b x 5^a

Solution:

(i) the values of a and b.

(i) $64 \times 625 = 2^a \times 5^b$

$64 = 2^a$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6$

$\therefore\ a = 6$

Also, $625 = 5^b$

5	625
5	125
5	25
5	5
	1

$625 = 5 \times 5 \times 5 \times 5$

$625 = 5^4$

$\therefore\ b = 4$

(ii) $2^b \times 5^a$

$$(ii)\ 2^b \times 5^a$$
$$= 2^4 \times 5^6$$
$$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$= 16 \times 15625 = 250000$$

EXERCISE 5 (B)

Question 1.
Fill in the blanks:
In $5^2 = 25$, base = …… and index = …….
If index = 3x and base = 2y, the number = ……

Solution:
(i) In $5^2 = 25$, base = 5 and index = 2
(ii) If index = 3x and base = 2y, the number = $2y^{3x}$

Question 2.
Evaluate:
(i) $2^8 \div 2^3$
(ii) $2^3 \div 2^8$
(iii) $(2^6)^0$
(iv) $(3^o)^6$
(v) $8^3 \times 8^{-5} \times 8^4$
(vi) $5^4 \times 5^3 + 5^5$
(vii) $5^4 \div 5^3 \times 5^5$
(viii) $4^4 \div 4^3 \times 4^0$
(ix) $(3^5 \times 4^7 \times 5^8)^0$

Solution:

$$= \frac{4^4}{4^3} = 4^{4-3} = 4^1 = 4$$

$$(ix)\ (3^5 \times 4^7 \times 5^8)^0 = 3^{5 \times 0} \times 4^{7 \times 0} \times 5^{8 \times 0}$$
$$= 3^0\, 4^0\, 5^0 = 1 \times 1 \times 1 = 1$$

(i) $2^8 \div 2^3 = \frac{2^8}{2^3} = 2^{8-3} = 2^5$

(ii) $2^3 \div 2^8 = \frac{2^3}{2^8} = 2^{3-8} = 2^{-5} = \frac{1}{2^5}$

(iii) $(2^6)^0 = 2^{6 \times 0} = 2^0 = 1$

(iv) $(3^0)^6 = 3^{0 \times 6} = 3^0 = 1$

(v) $8^3 \times 8^{-5} \times 8^4 = 8^{3+4-5} = 8^{7-5} = 8^2$

(vi) $5^4 \times 5^3 \div 5^5 = \frac{5^4 \times 5^3}{5^5}$

$= 5^{4+3-5} = 5^{7-5} = 5^2$

(vii) $5^4 \div 5^3 \times 5^5 = \frac{5^4}{5^3} \times 5^5 = 5^{4-3+5} = 5^6$

(viii) $4^4 \div 4^3 \times 4^0 = \frac{4^4}{4^3 4^0} = \frac{4^4}{4^3 \times 1}$

Question 3.

Simplify, giving Solutions with positive index:

(i) $2b^6 \cdot b^3 \cdot 5b^4$ (ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

(iii) $(-a^5)\ (a^2)$ (iv) $(-y^2)\ (-y^3)$

(v) $(-3)^2\ (3)^3$ (vi) $(-4x)\ (-5x^2)$

(vii) $(5a^2b)\ (2ab^2)\ (a^3b)$

(viii) $x^{2a+7} \cdot x^{2a-8}$ (ix) $3^y \cdot 3^2 \cdot 3^{-4}$

(x) $2^{4a} \cdot 3^{3a} \cdot 2^{-a}$ (xi) $4x^2y^2 \div 9x^3y^3$

(xii) $(10^2)^3\ (x^8)^{12}$ (xiii) $(a^{10})^{10}\ (1^6)^{10}$

(xiv) $(n^2)^2\ (-n^2)^2$ (xv) $-(3ab)^2\ (-5a^2bc^4)^2$

(xvi) $(-2)^2 \times (0)^3 \times (3)^3$

(xvii) $(2a^3)^4\ (4a^2)^2$

(xviii) $(4x^2y^3)^3 \div (3x^2y^3)^3$

(xix) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$

(xx) $\left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$

(xxi) $\frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7}$

(xxii) $\frac{(7p^2q^9r^5)^2 (4pqr)^3}{(14p^6q^{10}r^4)^2}$

Solution:

(i) $2b^6 \cdot b^3 \cdot 5b^4$

$= 2 \times 5 \times b^{6+3+4} = 10b^{13}$

(ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

$= 6 \times 9 \times x^{2+5+3}\, y^{3+1+4} = 54x^{10}y^8$

(iii) $(-a^5)\ (a^2)$

$= (-1 \times a)^5 \times a^2$

$= (-1)^5 \times a^{5+2}$

$= -1 \times a^7 = -a^7$

(iv) $(-y^2)\ (-y^3)$

$= (-1 \times y)^2 \cdot (-1 \times y)^3$

$= (-1)^2 \cdot y^2 \cdot (-1)^3 \times y^3$

$= 1^{2+3} \cdot y^{2+3}$

$= 1^5 y^5 = y^5$

(*v*) $(-3)^2\ (3)^3$

$= (-1 \times 3)^2 \cdot (3)^3$

$= (-1)^2 \times 3^2 \cdot 3^3$

$= -1^2 \cdot 3^{2+3} = 1 \cdot 3^5 = 3^5$

(*vi*) $(-4x)\ (-5x^2)$

$= (-1 \times 4 \times x) \cdot (-1 \times 5 \times x^2)^1$

$= -1 \times 4 \times x \cdot -1 \times 5 \times x^2$

$= -1 \times -1 \times 4 \times 5 \times x^{1+2}$

$= -1^{1+1} \cdot 4^1 \cdot 5^1\ x^3 = 20x^3$

(*vii*) $(5a^2b)\ (2ab^2)\ (a^3b)$

$= 5 \cdot 2 \cdot a^{2+1+3} b^{1+2+1} = 10a^6b^4$

(*viii*) $x^{2a+7} \cdot x^{2a-8}$

$= x^{2a+7+2a-8} = x^{4a-1}$

(*ix*) $3^y \cdot 3^2 \cdot 3^{-4}$

$$= 3^y \cdot \frac{3^2}{3^4} = 3^y = \frac{3 \times 3}{3 \times 3 \times 3 \times 3}$$

$$= 3^y \times \frac{1}{3^2} = 3^{y-2}$$

(*x*) $2^{4a} \cdot 2^{3a} \cdot 2^{-a}$

$= 2^{4a+3a-a} = 2^{7a-a} = 2^{6a}$

(*xi*) $4x^2y^2 \div 9x^3y^3$

$$= \frac{4x^2y^2}{9x^3y^3} = \frac{4x^{2-3}y^{2-3}}{9} = \frac{4x^{-1}y^{-1}}{9}$$

$$= \frac{4}{9xy}$$ (Since index should by positive)

(*xii*) $(10^2)^3\ (x^8)^{12}$

$= 10^{2 \times 3}\ x^{8 \times 12} = 10^6 x^{96}$

(*xiii*) $(a^{10})^{10}\ (1^6)^{10}$

$= a^{10 \times 10}\ 1^{6 \times 10} = a^{100}\ 1^{60} = a^{100}$

(*xiv*) $(n^2)^2\ (-n^2)^2$

$= n^{2 \times 2}\ (-n)^{2 \times 3} = n^4 \times (-n)^6$

$= -n^4 - 1^6\ n^6$

$= -n^{4+6} = -n^{10}$

(*xv*) $-(3ab)^2\ (-5a^2bc^4)^2$

$= -(3^2a^2b^2) \times (-1)^2 \times 5^2a^{2 \times 2}b^2c^{4 \times 2}$

$= -(3^2a^2b^2)\ (5^2a^4b^2c^8)$

$= -3^2 5^2 a^{2+4} b^{2+2} c^8$

$= -225a^6b^4c^8$

(*xvi*) $(-2)^2 \times (0)^3 \times (3)^3$

$= 4 \times 0 \times 27 = 0$

(*xvii*) $(2a^3)^4\ (4a^2)^2$

$= (2a^3)^4\ (2^2a^2)^2$

$= 2^4a^{3 \times 4} \cdot 2^{2 \times 2}\ a^{2 \times 2}$

$= 2^4a^{12} \cdot 2^4a^4$

$= 2^{4+4}a^{12+4}$

$= 2^8a^{16}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times a^{16}$

$= 256a^{16}$

(*xviii*) $(4x^2y^3)^3 \div (3x^2y^3)^3$

$$= \frac{4^3x^{2\times3}y^{3\times3}}{3^3x^{2\times3}y^{3\times3}} = \frac{4^3x^6y^9}{3^3x^6y^9} = \frac{4^3}{3^3} = \frac{64}{27}$$

(*xix*) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$

$$= \frac{1^3}{2^3 \times x^3} \times 6^2 \times x^2$$

$$= \frac{1^3 \times 6^2}{2^3 \times x^{3-2}} = \frac{6^2}{2^3x} = \frac{6 \times 6}{2 \times 2 \times 2 \times x} = \frac{9}{2x}$$

(xx) $\left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$

$$= \left(\frac{1}{4ab^2c}\right)^2 \times \left(\frac{2a^2bc^2}{3}\right)^4$$

$$= \frac{1^2}{4^2 a^2 b^{2\times2} \cdot c^2} \times \frac{2^4 a^{2\times4} \cdot b^4 \cdot c^{2\times4}}{3^4}$$

$$= \frac{1^2}{3^4} \times a^{8-2} b^{4-4} c^{8-2} \qquad (\because 2^4 = 4^2)$$

$$= \frac{1}{3\times3\times3\times3} a^6 b^0 c^6$$

$$= \frac{1}{81} a^6 c^6 \qquad (\because b^0 = 1)$$

(xxi) $\frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7} = \frac{5^3 x^{7\times3} \cdot 10^2 \cdot x^{2\times2}}{2^7 \cdot x^{6\times7}}$

$$= 5^3 \cdot 10^2 \cdot 2^{-7} x^{21+4-42}$$

$$= \frac{5^3 \times 10^2}{2^7 x^{17}} = \frac{5\times5\times5\times2\times5\times2\times5}{2\times2\times2\times2\times2\times2\times2 \cdot x^{17}}$$

$$= \frac{5^5}{2^5 x^{17}} = \frac{3125}{32x^{17}}$$

(xxii) $\frac{(7p^2q^9r^5)^2 (4pqr)^3}{(14p^6q^{10}r^4)^2}$

$$= \frac{(7^2 p^{2\times2} q^{9\times2} r^{5\times2})(4^3 p^3 q^3 r^3)}{14^2 p^{6\times2} q^{10\times2} r^{4\times2}}$$

$$= \frac{7\times7 p^4 q^{18} r^{10} \cdot 4\times4\times4 p^3 q^3 r^3}{2\times7\times2\times7\times p^{12} q^{20} r^8}$$

$$= p^{4-12+3} q^{18-20+3} r^{10-8+3}\ 4 \times 4$$

$$= 16p^{-5}qr^5$$

$$= \frac{16qr^5}{p^5}$$

Question 4.

Simplify and express the Solution in the positive exponent form :

(i) $\dfrac{(-3)^3 \times 2^6}{6 \times 2^3}$ (ii) $\dfrac{(2^3)^5 \times 5^4}{4^3 \times 5^2}$

(iii) $\dfrac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5}$ (iv) $-\dfrac{128}{2187}$

(v) $\dfrac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8}$

(vi) $(a^3 b^{-5})^{-2}$

Solution:

(i) $\dfrac{(-3)^3 \times 2^6}{6 \times 2^3} = \dfrac{(-3)^3 \times 2^6}{2 \times 3 \times 2^3} = \dfrac{(-3)^3 \times 2^6}{3 \times 2^{3+1}}$

$= -(3)^{3-1}\, 2^{6-4} = -(3)^2 2^2 = -3^2 2^2$

(ii) $\dfrac{(2^3)^5 \times 5^4}{4^3 \times 5^2} = \dfrac{2^{3\times5} \times 5^4}{2^3 \times 2^2 \times 5^2}$

$= \dfrac{2^{15} \times 5^4}{2^6 \times 5^2} = 2^{15-6} \times 5^{4-2}$

$= 2^9 \times 5^2$

(iii) $\dfrac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5} = \dfrac{6 \times 6 \times (-6)^2 \times 3^6}{3^3 \times 4^3 \times 3^5}$

$= \dfrac{(6)^2(-6)^2 \times 3^{6-3-5}}{4^3} = \dfrac{(6)^2(-6)^2 3^{-2}}{4^3}$

$= \dfrac{6^2(-6)^2}{3^2 \times 4^3} = \dfrac{6 \times 6 \times -6 \times -6}{3 \times 3 \times 4 \times 4 \times 4}$

$= \dfrac{9}{4} = \left(\dfrac{3}{2}\right)^2$

(iv) $-\dfrac{128}{2187}$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$= -\frac{2\times2\times2\times2\times2\times2\times2}{3\times3\times3\times3\times3\times3\times3} = -\frac{2^7}{3^7}$$

(v) $\dfrac{a^{-7}\times b^{-7}\times c^5\times d^4}{a^3\times b^{-5}\times c^{-3}\times d^8}$

$= a^{-7-3}\times b^{-7+5}\times c^{5-(-3)}\times d^{4-8}$

$= a^{-10}\times b^{-2}\times c^8\times d^{-4}$

$= \dfrac{c^8}{a^{10}\times b^2\times d^4}$

(vi) $(a^3b^{-5})^{-2} = a^{3\times-2}\,b^{-5\times-2}$

$= a^{-6}\,b^{10} = \dfrac{b^{10}}{a^6}$

Question 5.

Evaluate

(i) $6^{-2}\div(4^{-2}\times3^{-2})$

(ii) $\left[\left(\frac{5}{6}\right)^2\times\frac{9}{4}\right]\div\left[\left(-\frac{3}{2}^2\right)\times\frac{125}{216}\right]$

(iii) $5^3\times3^2+(17)^0\times7^3$

(iv) $2^5\times15^0+(-3)^3-\left(\frac{2}{7}\right)^{-2}$

(v) $(2^2)^0+2^{-4}\div2^{-6}+\left(\frac{1}{2}\right)^{-3}$

(vi) $5^n\times25^{n-1}\div(5^{n-1}\times25^{n-1})$

Solution:

(*i*) $6^{-2} \div (4^{-2} \times 3^{-2})$

$$= \left(\frac{1}{6}\right)^2 \div \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{36} \div \frac{1}{16} \times \frac{1}{9}$$

$$= \frac{1}{36} \div \frac{1}{144}$$

$$\begin{array}{r} 36\overline{)144}(4 \\ \underline{144} \\ \times \end{array}$$

$$= \frac{1}{36} \times \frac{144}{1} = 4$$

(*ii*) $\left[\left(\frac{5}{6}\right)^2 \times \frac{9}{4}\right] \div \left[\left(-\frac{3}{2}^2\right) \times \frac{125}{216}\right]$

$$= \left[\left(\frac{5\times5}{6\times6}\right)\times\frac{9}{4}\right] \div \left[\left(\frac{-3\times-3}{2\times2}\right)\times\frac{125}{216}\right]$$

$$= \left[\frac{25}{36}\times\frac{9}{4}\right] \div \left[\frac{9}{4}\times\frac{125}{216}\right]$$

$$= \left[\frac{25}{4}\times\frac{1}{4}\right] \div \left[\frac{1}{4}\times\frac{125}{24}\right]$$

$$= \left[\frac{25}{16}\right] \div \left[\frac{125}{96}\right]$$

$$= \frac{25}{16} \times \frac{96}{125}$$

$$= \frac{1}{1} \times \frac{6}{5} = 1\frac{1}{5}$$

(*iii*) $5^3 \times 3^2 + (17)^0 \times 7^3$

$= 5 \times 5 \times 5 \times 3 \times 3 + (17)^0 \times 7 \times 7 \times 7$

$(\because a^0 = 1)$

$= 125 \times 9 + 1 \times 343$

$= 1125 + 343 = 1468$

(*iv*) $2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7}\right)^{-2}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 1 + (-3) \times (-3) \times (-3) - \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$

$= 32 \times 1 - 27 - \frac{49}{4}$ $(\because a^0 = 1)$

$= \frac{32\times4}{1\times4} - \frac{27\times4}{1\times4} - \frac{49}{4\times1}$ $(\because \text{LCM} = 4)$

$$= \frac{128 - 108 - 49}{4} = \frac{-29}{4} = -7\frac{1}{4}$$

(*v*) $(2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2}\right)^{-3}$

$$(4)^0 + \left(\frac{1}{2}\right)^4 \div \left(\frac{1}{2}\right)^6 + \left(\frac{2}{1}\right)^3 \quad (\because a^0 = 1)$$

$$1 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \div$$

$$\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{2}{1} \times \frac{2}{1} \times \frac{2}{1}\right)$$

$$1 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\right) + 8$$

$$= 1 + 4 + 8 = 13$$

(*vi*) $5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$

$$= 5^n \times 25^{n-1} \times \frac{1}{5^{n-1} \times 25^{n-1}}$$

$$= 5^n \times \frac{1}{5^{n-1}} = 5^{n-n+1} = 5^1$$

Question 6.

If $m^2 = -2$ and $n = 2$; find the values of:

(i) $m + r^2 - 2mn$

(ii) $m^n + n^m$

(iii) $6m^{-3} + 4n^2$

(iv) $2n^3 - 3m$

Solution:

(i) $m^2 + n^2 - 2mn$

$m = -2, n = 2$

$= (-2)^2 + (2)^2 - 2(-2)(2)$

$= 4 + 4 - (-8)$

2	16
2	8
2	4
2	2
	1

$= 8 + 8 = 16 = (2^4)$

(ii) $m^n + n^m$

$m = -2, n = 2$

$= (-2)^2 + (2)^{-2}$

$= 4 + \frac{1}{2} \times \frac{1}{2}$

$= \frac{4 \times 4}{1 \times 4} + \frac{1}{4}$

$= \frac{16+1}{4} = \frac{17}{4} = 4\frac{1}{4}$

(iii) $6m^{-3} + 4n^2$

$m = -2, n = 2$

$= 6(-2)^{-3} + 4(2)^2$

$= 6 \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} + 4 \times 2 \times 2$

$= \frac{-3}{4} + 16$

$= \frac{-3 + 16 \times 4}{4} = \frac{-3+64}{4} = \frac{61}{4} = 15\frac{1}{4}$

(iv) $2n^3 - 3m$

$m = -2, n = 2$

$= 2(2)^3 - 3(-2)$

$= 2 \times (2 \times 2 \times 2) - 3 \times (-2)$

$= 16 - 3 \times (-2)$

$= 16 + 6 = 22$