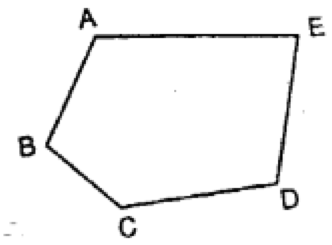
Polygons

IMPORTANT POINTS

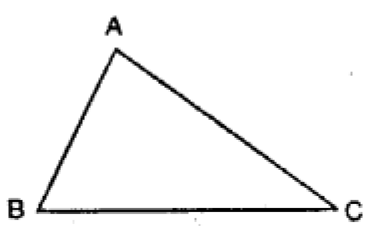
1. Polygon : A closed plane geometrical figure, bounded by at least three line segments, is called a polygon.
The adjoining figure is a polygon as it is :



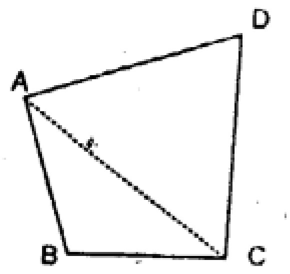
(i) Closed
(ii) bounded by five line segments AB, BC, CD, DE and AE.
Also, it is clear from the given polygon that:
(i) the line segments AB, BC, CD, DE and AE intersect at their end points.
(ii) two line segments, with a common vertex, are not collinear i.e. the angle at any vertex is not 180°.
A polygon is named according to the number of sides (line-segments) in it:

Note : No. of sides :	3	4	5	6
Name of polygon :	Triangle	Quadrilateral	Pentagon	Hexagon

2. Sum of Interior Angles of a Polygon
1. Triangle : Students already know that the sum of interior angles of a triangle is always 180°.
i.e. for Δ ABC, $\angle$B AC + $\angle$ABC + $\angle$ACB = 180°
$\Rightarrow$ ZA + ZB + ZC = 180°

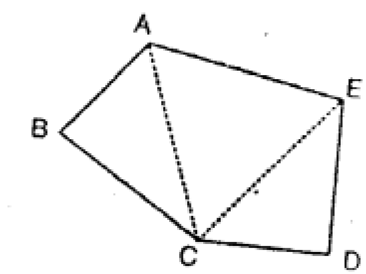


2. Quadrilateral : Consider a quadrilateral ABCD as shown alongside.
If diagonal AC of the quadrilaterals drawn, the quadrilateral will be divided into two triangles ABC and ADC.
Since, the sum of interior angles of a triangle is 180°.



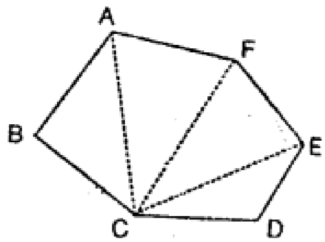
∴ In ∆ ABC, ∠ABC + ∠BAC +∠ACB = 180°
And, in ∆ ADC ∠DAC + ∠ADC + ∠ACD = 180°
Adding we get:
∠ABC + ∠BAC +∠ACB + ∠DAC + ∠ADC + ∠ACD = 180° +180°
⇒(∠BAC + ∠DAC) + ∠ABC + (∠ACB + ∠ACD) + ∠ADC = 360°
⇒∠BAD + ∠ABC + ∠BCD + ∠ADC = 360°
⇒ ∠A + ∠B + ∠C + ∠D = 360°
Alternative method : On drawing the diagonal AC, the given quadrilateral is divided into two triangles. And, we know the sum of the interior angles of a triangle is 180°.
∴ Sum of interior angles of the quadrilateral ABCD
= Sum of interior angles of ∆ ABC + sum of interior angles of ∆ ADC = 180°+ 180° = 360°

3. Pentagon : Consider a pentagon ABCDE as shown alongside.
On joining CA and CE, the given pentagon is divided into three triangles ABC, CDE and ACE.



Since, the sum of the interior angles of a triangle is 180°
Sum of the interior angles of the pentagon ABCDE = Sum of interior angles of (∆ ABC + ∆ CDE + ∆ACE)
= 180° + 180° + 180° = 540°

4. Hexagon :
It is clear from the given figure that the sum of the interior angles of the hexagon ABCDEF.



= Sum of inteior angles of
(ΔABC + Δ ACF + Δ FCE + Δ ECD)
= 180° + 180° + 180° + 180° = 720°

3. Using Formula : The sum of interior angles of a polygon can also be obtained by using the following formula:
Note : Sum of interior angles of a polygon = (n – 2) x 180°
where, n = number of sides of the polygon.

$\therefore$ (*i*) **For a trianlge :**

$n = 3$ (a triangle has 3 sides)

and, **sum of interior angles** $= (2n - 4) \times 90°$
$= (6 - 4) \times 90° = 180°$

(*ii*) **For a quadrilateral :**

$n = 4$

and, **sum of interior angles** $= (2n - 4) \times 90°$
$= (8 - 4) \times 90° = 360°$

(*iii*) **For a pentagon :**

$n = 5$

and, **sum of interior angles** $= (2n - 4) \times 90°$
$= (10 - 4) \times 90° = 6 \times 90° = 540°$

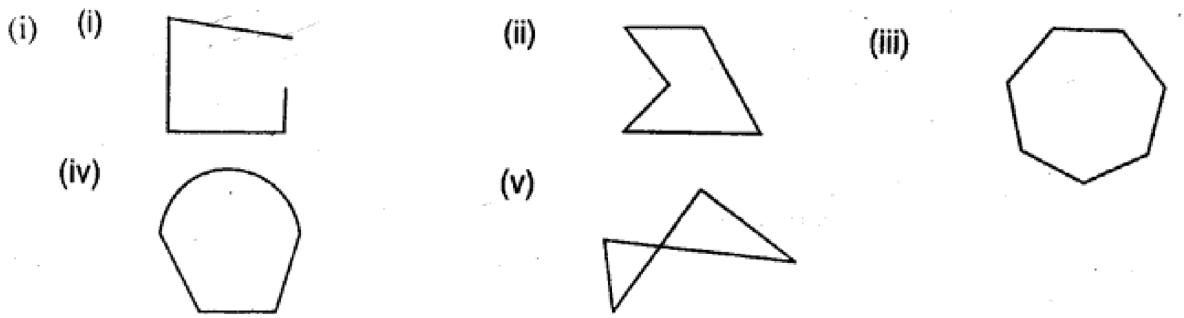
(*iv*) **For a hexagon :**

$n = 6$

and, **sum of interior angles** $= (2n - 4) \times 90°$
$= (12 - 4) \times 90° = 8 \times 90° = 720°$

EXERCISE 28 (A)

Question 1.
State, which of the following are polygons :



Solution:
Only figure (ii) and (iii) are polygons.

Question 2.
Find the sum of interior angles of a polygon with :
(i) 9 sides
(ii) 13 sides
(iii) 16 sides
Solution:
(i) 9 sides
No. of sides n = 9
∴Sum of interior angles of polygon = (2n – 4) x 90°
= (2 x 9 – 4) x 90°
= 14 x 90°= 1260°
(ii) 13 sides
No. of sides n = 13
∴ Sum of interior angles of polygon = (2n – 4) x 90° = (2 x 13 – 4) x 90° = 1980°
(iii) 16 sides
No. of sides n = 16
∴ Sum of interior angles of polygon = (2n – 4) x 90°
= (2 x 16 – 4) x 90°
= (32 – 4) x 90° = 28 x 90°
= 2520°

Question 3.
Find the number of sides of a polygon, if the sum of its interior angles is :
(i) 1440°
(ii) 1620°
Solution:

(i) $1440°$

Let no. of sides = n

$\therefore$ Sum of interior angles of polygon = $1440°$

$\therefore$ $(2n - 4) \times 90° = 1440°$

$$\Rightarrow 2n - 4 = \frac{1440°}{90°}$$

$$\Rightarrow 2(n - 2) = \frac{1440°}{90°}$$

$$\Rightarrow n - 2 = \frac{1440°}{2 \times 90°}$$

$\Rightarrow n - 2 = 8$

$\Rightarrow n = 8 + 2$

$\Rightarrow n = 10$

(ii) $1620°$

Let no. of sides = n

$\therefore$ Sum of interior angles of polygon = $1620°$

$\therefore$ $(2n - 4) \times 90° = 1620°$

$$\Rightarrow 2(n - 2) = \frac{1620°}{90°}$$

$$\Rightarrow n - 2 = \frac{1620°}{2 \times 90°}$$

$\Rightarrow n - 2 = 9$

$\Rightarrow n = 9 + 2 \quad \Rightarrow n = 11$

Question 4.

Is it possible to have a polygon, whose sum of interior angles is 1030°.

Solution:

Let no. of sides be = n

Sum of interior angles of polygon = $1030°$

$\therefore \quad (2n - 4) \times 90° = 1030°$

$$\Rightarrow \quad 2(n-2) = \frac{1030°}{90°}$$

$$\Rightarrow \quad (n-2) = \frac{1030°}{2 \times 90°}$$

$$\Rightarrow \quad (n-2) = \frac{103}{18}$$

$$\Rightarrow \quad n = \frac{103}{18} + 2$$

$$\Rightarrow \quad n = \frac{139}{18}$$

Which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is $1030°$.

Question 5.

(i) If all the angles of a hexagon arc equal, find the measure of each angle.

(ii) If all the angles of an octagon are equal, find the measure of each angle,

Solution:

(i) No. of sides of hexagon, $n = 6$

Let each angle be $= x°$

$\therefore$ Sum of angles $= 6x°$

$\therefore$ $(2n - 4) \times 90° =$ Sum of angles

$(2 \times 6 - 4) \times 90° = 6x°$

$(12 - 4) \times 90° = 6x°$

$\Rightarrow \dfrac{8 \times 90°}{6} = x°$

$\Rightarrow x = 120°$

$\therefore$ Each angle of hexagon $= 120°$

(ii) No. of sides of octagon $n = 8$

Let each angle be $= x°$

$\therefore$ Sum of angles $= 8x°$

$\therefore$ $(2n - 4) \times 90° =$ Sum of angles

$(2 \times 8 - 4) \times 90° = 8x°$

$12 \times 90° = 8x°$

$\Rightarrow x° = \dfrac{90° \times 12°}{8}$ $\Rightarrow x° = 135°$

$\therefore$ Each angle of octagon $= 135°$

Question 6.

One angle of a quadrilateral is 90° and all other angles are equal ; find each equal angle.

Solution:

Let the angles of a quadrilateral be $x°$, $x°$, $x°$, and $90°$

$\therefore$ Sum of interior angles of quadrilateral = 360°

$\Rightarrow x° + x° + x° + 90° = 360°$

$\Rightarrow 3x° = 360° - 90°$

$\Rightarrow x = \dfrac{270°}{3}$

$\Rightarrow x = 90°$

Question 7.

If angles of quadrilateral are in the ratio 4 : 5 : 3 : 6 ; find each angle of the quadrilateral.

Solution:

Let the angles of the quadrilateral be $4x$, $5x$, $3x$ and $6x$

$\therefore$ $4x + 5x + 3x + 6x = 360°$

$18x = 360°$

$$x = \frac{360°}{18} = 20°$$

$\therefore$ First angle $= 4x = 4 \times 20° = 180°$

Second angle $= 5x = 5 \times 20° = 100°$

Third angle $= 3x = 3 \times 20° = 60°$

Fourth angle $= 6x = 6 \times 20° = 120°$

Question 8.

If one angle of a pentagon is 120° and each of the remaining four angles is x°, find the magnitude of x.

Solution:

One angle of a pentagon = 120°
Let remaining four angles be x, x, x and x
Their sum = 4x + 120°
But sum of all the interior angles of a pentagon = (2n – 4) x 90°
= (2 x 5 – 4) x 90° = 540°
= 3 x 180° = 540°
∴ 4x+120o° = 540°
4x = 540° – 120°
4x = 420
x = $\frac{420}{4}$ ⇒ x = 105°
∴Equal angles are 105° (Each)

Question 9.

The angles of a pentagon are in the ratio 5 : 4 : 5 : 7 : 6 ; find each angle of the pentagon.

Solution:

Let the angles of the pentagon be $5x$, $4x$, $5x$, $7x$, $6x$

Their sum $= 5x + 4x + 5x + 7x + 6x = 27x$

Sum of interior angles of a polygon

$= (2n - 4) \times 90°$

$= (2 \times 5 - 4) \times 90° = 540°$

$\therefore\ 27x = 540 \Rightarrow \frac{540}{27} \Rightarrow x = 20°$

$\therefore$ Angles are $5 \times 20° = 100°$

$4 \times 20° = 80$

$5 \times 20° = 100°$

$7 \times 20° = 140°$

$6 \times 20° = 120°$

Question 10.

Two angles of a hexagon are 90° and 110°. If the remaining four angles arc equal, find each equal angle.

Solution:

Two angles of a hexagon are 90°, 110°

Let remaining four angles be x, x, x and x

Their sum $= 4x + 200°$

But sum of all the interior angles of a hexagon

$= (2n - 4) \times 90°$

$= (2 \times 6 - 4) \times 90° = 8 \times 90° = 720°$

$\therefore\ 4x + 200° = 720°$

$\Rightarrow\ 4x = 720° - 200° = 520°$

$\Rightarrow\ x = \frac{520°}{4} = 130°$

$\therefore$ Equal angles are 130° (each)

EXERCISE 28 (B)

Question 1.

Fill in the blanks :

In case of regular polygon, with

Number of sides	Each exterior angle	Each interior angle
(i) 6	....	....
(ii) 8	....	....
(iii)	36°	....
(iv)	20°	....
(v)	....	135°
(vi)	....	165°

Solution:

Number of sides	Each exterior angle	Each interior angle
(i) 6	**60°**	**120°**
(ii) **8**	**45°**	**135°**
(iii) **10**	36°	**144°**
(iv) **18**	20°	**160°**
(v) **8**	**45°**	135°
(vi) **24**	**15°**	165°

(i) Each exterior angle $= \frac{360°}{6} = 60°$

Each interior angle $= 180° - 60° = 120°$

(ii) Each exterior angle $= \frac{360°}{8} = 45°$

Each interior angle $= 180° - 45° = 135°$

(iii) Since each exterior angles $= 36°$

$\therefore$ Number of sides $= \frac{360°}{36°} = 10$

Also, interior angle $= 180° - 20° = 160°$

(iv) Since each exterior angles $= 20°$

$\therefore$ Number of sides $= \frac{360°}{20°} = 18$

Also, interior angle $180° - 20° = 160°$

(v) Since interior angle $= 135°$

$\therefore$ Exterior angle $= 180° - 135°$

$\therefore$ Number of sides $= \frac{360°}{45°} = 8$

(vi) Since interior angle $= 165°$

$\therefore$ Exterior angle $= 180° - 165° = 15°$

$\therefore$ Number of sides $= \frac{360°}{15°} = 24$

Question 2.

Find the number of sides in a regular polygon, if its each interior angle is :

(i) 160°

(ii) 150°

Solution:

(i) 160°

Let no. of sides of regular polygon be n

Each interior angle = 160°

$$\therefore \quad \frac{(2n-4)\times 90^\circ}{n} = 160^\circ$$

$180n - 360^\circ = 160n$

$180n - 160n = 360^\circ$

$$n = \frac{360^\circ}{20}$$

$n = 18$

(ii) 150°

Let no. of sides of regular polygon be n

Each interior angle = 150°

$$\therefore \quad \frac{(2n-4)\times 90^\circ}{n} = 150^\circ$$

$180n - 360^\circ = 150n$

$180n - 150n = 360^\circ$

$30n = 360^\circ$

$$n = \frac{360^\circ}{30}$$

$n = 12$.

Question 3.

Find number of sides in a regular polygon, if its each exterior angle is :

(i) 30°

(ii) 36°

Solution:

(i) 30°

Let number of sides = n

$$\therefore \quad \frac{360^\circ}{n} = 30^\circ$$

$$n = \frac{360^\circ}{30^\circ}$$

$$n = 12$$

(ii) 36°

Let number of sides = n

$$\therefore \quad \frac{360^\circ}{n} = 36^\circ$$

$$n = \frac{360^\circ}{36^\circ}$$

$$n = 10$$

Question 4.
Is it possible to have a regular polygon whose each interior angle is :
(i) 135°
(ii) 155°
Solution:

(i) $135°$

No. of sides $= n$

Each interior angle $= 135°$

$$\therefore \quad \frac{(2n-4)\times 90°}{n} = 135°$$

$180n - 360° = 135n$

$180n - 135n = 360°$

$$n = \frac{360°}{45°}$$

$n = 8$

Which is a whole number.

Hence, it is possible to have a regular polygon whose interior angle is $135°$.

(ii) $155°$

No. of sides $= n$

Each interior angle $= 155°$

$$\therefore \quad \frac{(2n-4)\times 90°}{n} = 155°$$

$180n - 360° = 155n$

$180n - 155n = 360°$

$25n = 360°$

$$n = \frac{360°}{25°}$$

$$n = \frac{72°}{5}$$

Which is not a whole number.

Hence, it is not possible to have a regular polygon having interior angle is of $138°$.

Question 5.

Is it possible to have a regular polygon whose each exterior angle is :

(i) 100°

(ii) 36°

Solution:

(i) 100°

Let no. of sides $= n$

Each exterior angle $= 100^\circ$

$$= \frac{360^\circ}{n} = 100^\circ$$

$$\therefore \quad n = \frac{360^\circ}{100^\circ}$$

$$n = \frac{18}{5}$$

Which is not a whole number.

Hence, it is not possible to have a regular polygon whose each exterior angle is 100°.

(ii) 36°

Let number of sides $= n$

Each exterior angle $= 36^\circ$

$$= \frac{360^\circ}{n} = 36^\circ$$

$$\therefore \quad n = \frac{360^\circ}{36^\circ}$$

$n = 10$

Which is a whole number.

Hence, it is possible to have a regular polygon whose each exterior angle is of 36°.

Question 6.

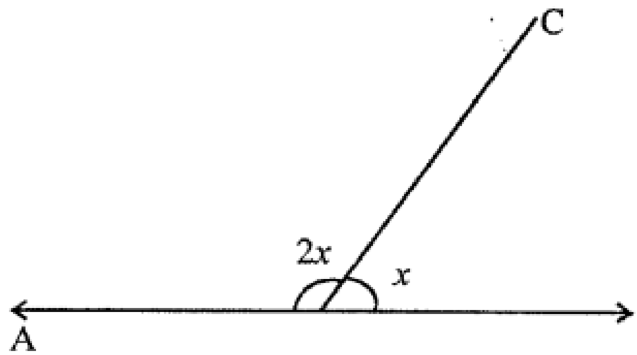
The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :
(i) each exterior angle of this polygon.
(ii) number of sides in the polygon.

Solution:

(i) Interior angle : exterior angle = 2 : 1

$\therefore$ Let interior angle = $2x°$

and exterior angle = $x°$



$\therefore \quad 2x° + x° = 180°$

$3x° = 180° \quad = x = \frac{180°}{3} = 60°$

(ii) $x = 60$

$\therefore$ Each exterior angle = 60°

$\therefore \quad \frac{360°}{n} = 60°$

$n = \frac{360°}{60°} = 6$ sides