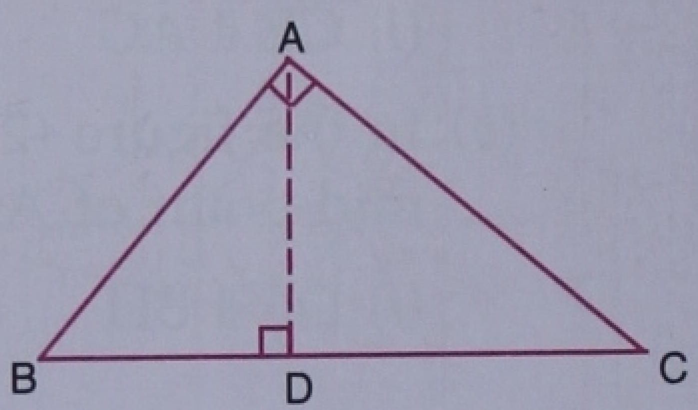
15

PYTHAGORAS THEOREM

15.1 PYTHAGORAS THEOREM

Theorem 11. *In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*



Given. ABC is a right angled triangle at A so that BC is its hypotenuse

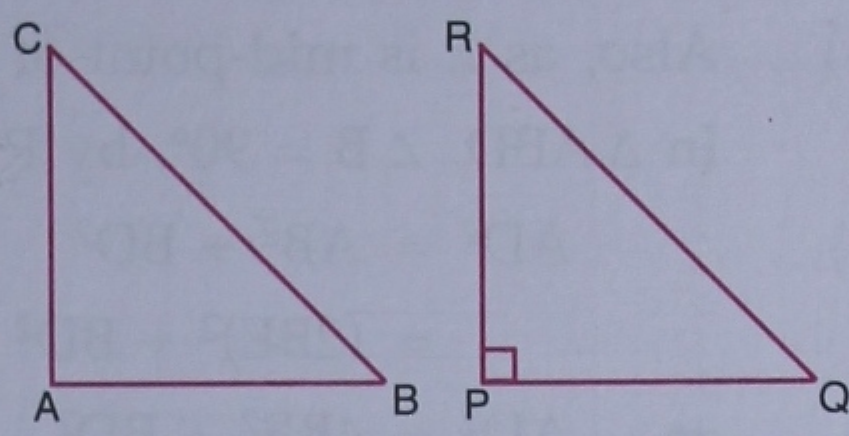
To prove. $BC^2 = AB^2 + AC^2$.

Construction. From A, draw AD perpendicular to BC.

Proof. *Statements*	*Reasons*
In Δs ABD and CBA	
1. $\angle ADB = \angle BAC$ (given) and $AD \perp BC$	1. Each angle = 90°, $\angle BAC = 90°$ (by construction).
2. $\angle ABD = \angle ABC$	2. Common.
3. $\Delta ABD \sim \Delta CBA$	3. A.A. axiom of similarity.
4. $\frac{AB}{BC} = \frac{BD}{AB}$ $\Rightarrow AB^2 = BD \times BC$	4. Corresponding sides of similar triangles are proportional.
5. $\Delta ACD \sim \Delta BCA$	5. As above.
6. $\frac{AC}{BC} = \frac{DC}{AC}$ $\Rightarrow AC^2 = DC \times BC$	6. Corresponding sides of similar triangles are proportional.
7. $AB^2 + AC^2 = BD \times BC + DC \times BC$ $= (BD + DC) \times BC$ $= BC \times BC$ $\Rightarrow AB^2 + AC^2 = BC^2$ **Q.E.D.**	7. Adding 5 and 8.

Theorem 12. *(Converse of Pythagoras Theorem)*

If in a triangle, the square of one side is equal to the sum of the squares of the other two sides, then the triangle is right angled at the angle contained by these two sides.



Given. In a triangle ABC,

$AB^2 + AC^2 = BC^2$.

To prove. $\angle A = 90°$.

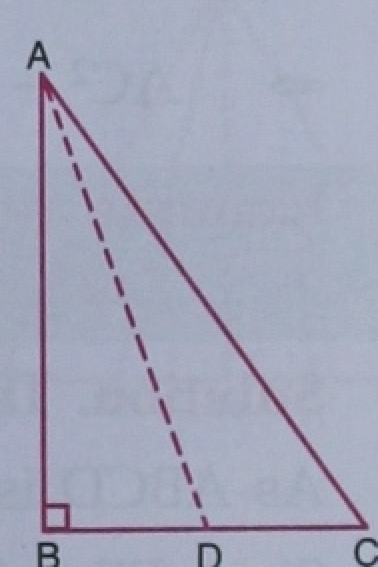
Construction. Construct a ΔPQR such that $\angle P = 90°$, PQ = AB and PR = AC.

Proof. *Statements*	*Reasons*
1. $QR^2 = PQ^2 + PR^2$	1. In ΔPQR, $\angle P = 90°$, by Pythagoras theorem $QR^2 = PQ^2 + PR^2$.
2. $QR^2 = AB^2 + AC^2$	2. PQ = AB and PR = AC (by construction).
3. $QR^2 = BC^2$	3. $AB^2 + AC^2 = BC^2$ (given).
$\Rightarrow QR = BC$	
In Δs ABC and PQR	
1′. AB = PQ	1′. By construction.
2′. AC = PR	2′. By construction.
3′. BC = QR	3′. From 3.
4′. $\Delta ABC \cong \Delta PQR$	4′. S.S.S. axiom of congruency.
5′. $\angle A = \angle P$	5′. c.p.c.t.
6′. $\angle A = 90°$	6′. $\angle P = 90°$ (by construction).
Q.E.D.	

15.2 APPLICATIONS OF PYTHAGORAS THEOREM

ILLUSTRATIVE EXAMPLES

Example 1. *In triangle ABC, $\angle B = 90°$ and D is the mid-point of BC. Prove that $AC^2 = AD^2 + 3CD^2$.*



Solution. In ΔABC, $\angle B = 90°$,

$\therefore \quad AC^2 = AB^2 + BC^2 \quad \ldots(i)$ (Pythagoras Th.)

In $\quad \Delta ABD$, $\angle B = 90°$,

$\therefore \quad AD^2 = AB^2 + BD^2$ (Pythagoras Th.)

$\Rightarrow \quad AB^2 = AD^2 - BD^2$

$\therefore \quad AC^2 = AD^2 - BD^2 + BC^2$ [using (*i*)]

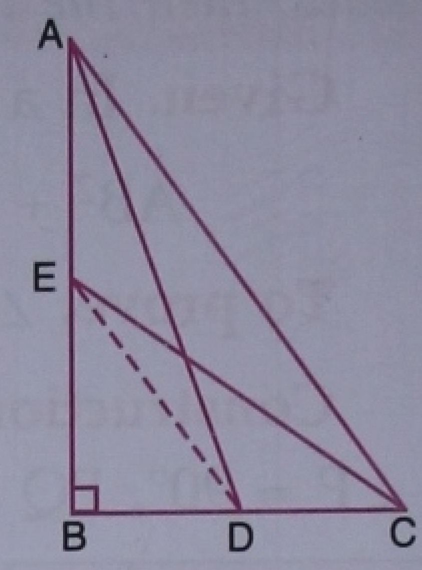
$\Rightarrow \quad AC^2 = AD^2 - CD^2 + (2CD)^2$

[$\because$ D is mid-point of BC, BD = CD and BC = 2CD]

$\Rightarrow \quad AC^2 = AD^2 - CD^2 + 4CD^2$

$\Rightarrow \quad AC^2 = AD^2 + 3CD^2$.

Example 2. *ABC is a right angled triangle at B. If D and E are mid-points of sides BC and AB respectively, prove that $AD^2 + CE^2 = 5\ DE^2$.*



Solution. As D is mid-point of BC, BC = 2 BD.

Also, as E is mid-point of AB, AB = 2BE.

In ΔABD, $\angle$B = 90°, by Pythgoras Th.,

$AD^2 = AB^2 + BD^2$

$= (2BE)^2 + BD^2$ $\quad$ ($\because$ AB = 2BE)

$\Rightarrow \quad AD^2 = 4BE^2 + BD^2$ $\quad$ ...(*i*)

In ΔEBC, $\angle$B = 90°, by Pythagoras Th.,

$CE^2 = BE^2 + BC^2$ $\quad$ ...(*ii*)

$= BE^2 + (2BD)^2$ $\quad$ ($\because$ BC = 2BD)

$\Rightarrow \quad CE^2 = BE^2 + 4BD^2$ $\quad$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$AD^2 + CE^2 = 5(BE^2 + BD^2)$ $\quad$ ...(*iii*)

In ΔEBD, $\angle$B = 90°, by Pythagoras Th.,

$DE^2 = BE^2 + BD^2$ $\quad$ ...(*iv*)

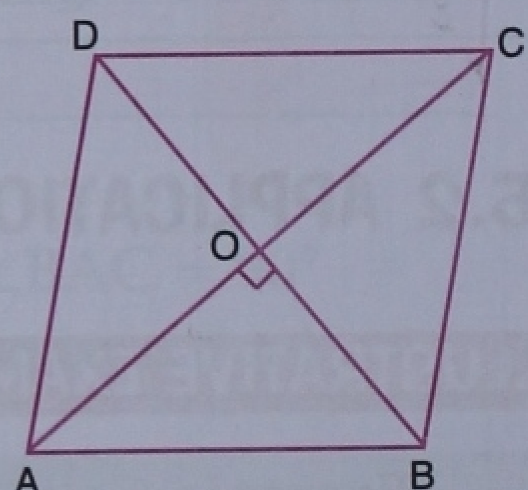
From (*iii*) and (*iv*), we get

$AD^2 + CE^2 = 5DE^2$, as required.

Example 3. *In a rhombus ABCD, prove that $AC^2 + BD^2 = 4\ AB^2$.*

Solution. Let the diagonals AC and BD of the rhombus intersect at O.

Since the diagonals of a rhombus are always at right angles, $\angle$AOB = 90°.



From ΔOAB, by Pythagoras Theorem, we get $AB^2 = OA^2 + OB^2$

$$\Rightarrow \quad AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$$

(Since the diagonals of a rhombus bisect each other)

$$\Rightarrow \quad AB^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2$$

$\Rightarrow \quad AC^2 + BD^2 = 4AB^2$, as required.

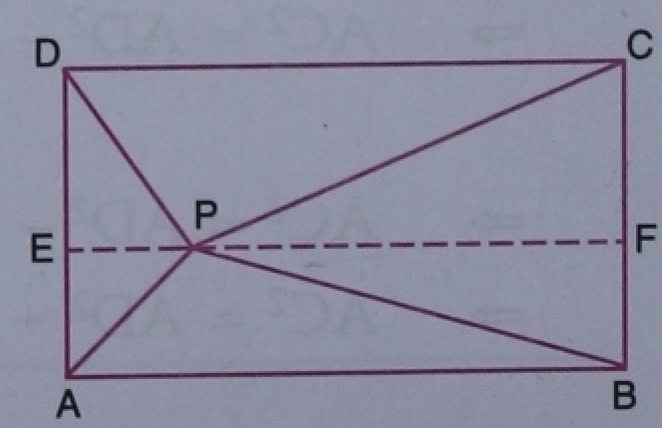
Example 4. *If P is any point inside a rectangle ABCD, prove that $AP^2 + CP^2 = BP^2 + DP^2$.*

Solution. Through P, draw a line EF parallel to AB.

As ABCD is a rectangle, AD $\perp$ AB.

Since EF ∥ AB and AD $\perp$ AB, therefore,

EF $\perp$ AD.



Similarly, EF $\perp$ BC.

In ΔAEP, $\angle$AEP = 90°,

$\therefore \quad AP^2 = AE^2 + EP^2$ $\quad$ ...(*i*)

In Δ CPF, $\angle$ CFP = 90°,

$\therefore \quad CP^2 = PF^2 + CF^2$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$AP^2 + CP^2 = AE^2 + EP^2 + PF^2 + CF^2$ ...(*iii*)

In Δ DEP, $\angle$DEP = 90°,

$\therefore \quad DP^2 = EP^2 + DE^2$ ...(*iv*)

In Δ BPF, $\angle$ BFP = 90°,

$\therefore \quad BP^2 = PF^2 + BF^2$ ...(*v*)

On adding, (*iv*) and (*v*), we get

$PB^2 + DP^2 = EP^2 + DE^2 + PF^2 + BF^2$ ...(*vi*)

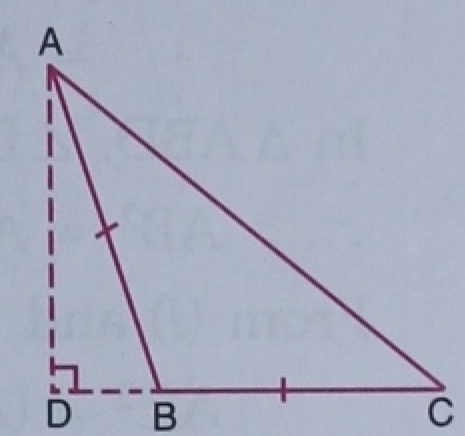
But in rectangle EABF, AE = BF and in rectange DEFC, DE = CF.

$\therefore$ From (*iii*) and (*vi*), we get

$AP^2 + CP^2 = BP^2 + DP^2$, as required.

Example 5. *In the adjoining figure, AB = BC and AD is perpendicular to CB produced. Prove that $AC^2 = 2BC.DC$.*

Solution. In Δ ADB, $\angle$ D = 90°,



$\therefore \quad AB^2 = AD^2 + DB^2$

$\Rightarrow \quad AD^2 = AB^2 - DB^2$ ...(*i*)

In Δ ADC, $\angle$ D = 90°,

$\therefore \quad AC^2 = AD^2 + DC^2$

$\Rightarrow \quad AC^2 = (AB^2 - DB^2) + (DB + BC)^2$ [using (*i*)]

$\Rightarrow \quad AC^2 = AB^2 - DB^2 + DB^2 + BC^2 + 2DB.BC$

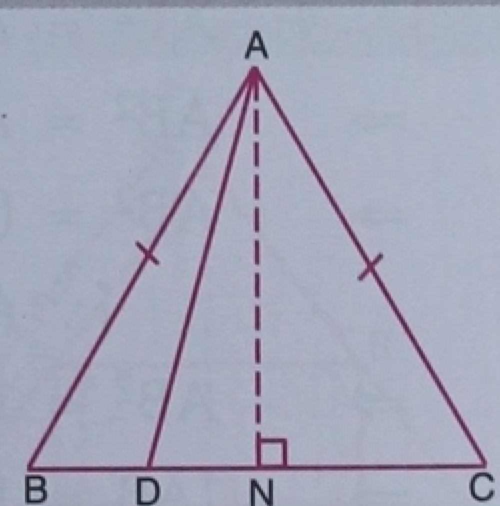
$= BC^2 + BC^2 + 2DB.BC$ [$\because$ AB = BC given]

$= 2BC^2 + 2DB.BC = 2BC\,(BC + DB)$

$\Rightarrow \quad AC^2 = 2BC.DC.$ (from figure)

Example 6. *ABC is a triangle in which AB = AC and D is a point on BC. Prove that $AB^2 - AD^2 = BD.DC$.*

Solution. Draw AN $\perp$ BC.



In Δs ABN and ANC

AB = AC (given)

$\angle$ ANB = 90° = $\angle$ ANC

and AN is common,

$\therefore \quad \Delta ABN \cong \Delta ANC$

$\Rightarrow \quad BN = NC.$

In Δ ABN, $\angle$ N = 90°,

$\therefore \quad AB^2 = AN^2 + BN^2$ ...(*i*)

In Δ ADN, $\angle$ N = 90°,

$\therefore \quad AD^2 = AN^2 + DN^2$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$AB^2 - AD^2 = BN^2 - DN^2$

$= (BN + DN)\ (BN - DN)$ $\qquad (\because\ BN = NC)$

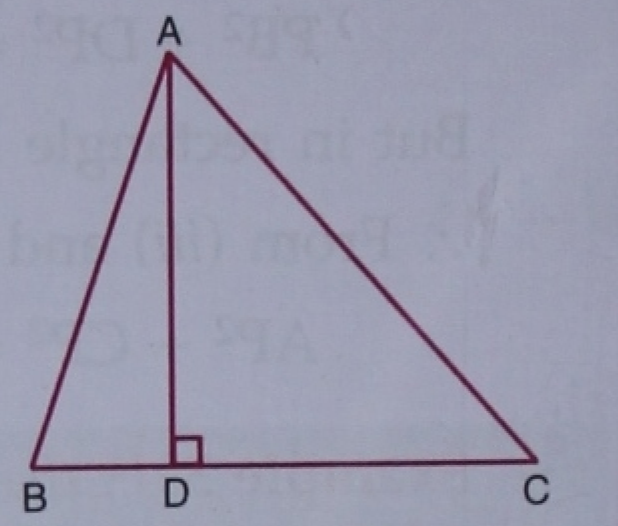
$= (NC + DN)\ .\ BD$

$= DC\ .\ BD$

Hence, $AB^2 - AD^2 = BD\ .\ DC$.

Example 7. *In the adjoining figure, AD ⊥ BC. If D divides BC in the ratio 1 : 3, prove that $2AC^2 = 2AB^2 + BC^2$.*

Solution. Given D divides BC in the ratio 1 : 3,



$\therefore\quad \frac{BD}{DC} = \frac{1}{3}$

$\Rightarrow\quad DC = 3BD$

$\therefore\quad BC = BD + DC = BD + 3BD = 4BD$

$\Rightarrow\quad BD = \frac{1}{4}\ BC.$

In Δ ADC, $\angle D = 90°$,

$\therefore\quad AC^2 = AD^2 + DC^2$

$= AD^2 + (3BD)^2 = AD^2 + 9BD^2$ $\qquad$ ...(*i*)

In Δ ABD, $\angle D = 90°$,

$\therefore\quad AB^2 = AD^2 + BD^2 \Rightarrow AD^2 = AB^2 - BD^2$ $\qquad$ ...(*ii*)

From (*i*) and (*ii*), we get

$AC^2 = (AB^2 - BD^2) + 9BD^2$

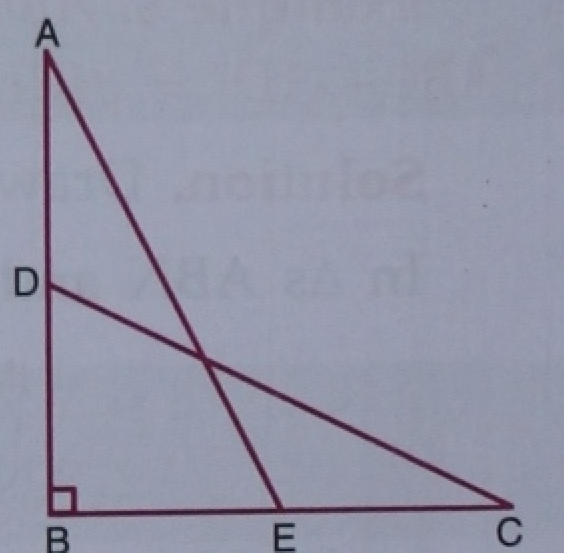
$= AB^2 + 8\ BD^2 = AB^2 + 8\ .\ \left(\frac{1}{4}BC\right)^2$

$= AB^2 + \frac{1}{2}BC^2$

$\Rightarrow\quad 2\ AC^2 = 2\ AB^2 + BC^2.$

Example 8. *In the adjoining figure, AE = DC = 13 cm, BE = 5 cm, ∠ABC = 90° and AD = EC = x cm. Calculate the length of AB and the value of x.*

Solution. In Δ ABE, $\angle B = 90°$,



$\therefore\quad AE^2 = AB^2 + BE^2$

$\Rightarrow\quad AB^2 = AE^2 - BE^2$

$\Rightarrow\quad AB^2 = (13)^2 - (5)^2$

$[\because\ AE = 13\text{ cm},\ BE = 5\text{ cm}]$

$\Rightarrow\quad AB^2 = 169 - 25 = 144$

$\Rightarrow\quad AB = 12\text{ cm}.$

From figure, $BD = AB - AD = (12 - x)$ cm $\qquad [\because\ AB = 12\text{ cm},\ AD = x\text{ cm}]$

and $\quad BC = BE + EC = (5 + x)$ cm.

In Δ BCD, $\angle B = 90°$,

$\therefore\quad CD^2 = BD^2 + BC^2$

$\Rightarrow\quad (13)^2 = (12 - x)^2 + (5 + x)^2$

$\Rightarrow\quad 169 = 144 + x^2 - 24x + 25 + x^2 + 10x$

$\Rightarrow \quad 169 = 169 + 2x^2 - 14x$

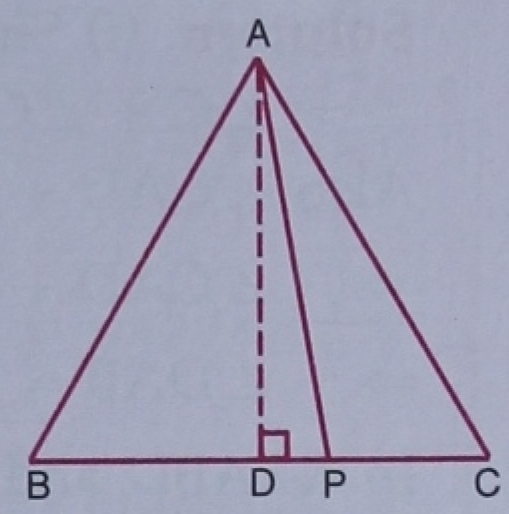
$\Rightarrow 2x^2 - 14x = 0 \Rightarrow 2x(x-7) = 0$

$\Rightarrow \quad x = 7$ cm. $\quad [\because x \neq 0]$

Example 9. *ABC is an equilateral triangle. P is a point on BC such that BP : PC = 2 : 1. Prove that $9AP^2 = 7AB^2$.*

Solution. Draw AD ⊥ BC.

As ABC is an equilateral triangle, D is mid point of BC.



Given BP : PC = 2 : 1.

Let $\quad PC = x$, then $BP = 2x$

$\therefore \quad BC = BP + PC = 2x + x = 3x$

$\Rightarrow \quad AB = 3x \quad \ldots(i) \; (\because AB = BC)$

As D is mid-point of BC,

$$BD = DC = \frac{1}{2}BC = \frac{1}{2}.3x = \frac{3}{2}x.$$

$$\therefore \quad DP = DC - PC = \frac{3}{2}x - x = \frac{1}{2}x.$$

In ΔABD, $\angle D = 90°$,

$\therefore \quad AB^2 = AD^2 + BD^2$

$$\Rightarrow \quad AD^2 = AB^2 - BD^2 = (3x)^2 - \left(\frac{3}{2}x\right)^2 = \frac{27}{4}x^2 \quad \ldots(ii)$$

In ΔADP, $\angle D = 90°$,

$\therefore \quad AP^2 = AD^2 + DP^2$

$$= \frac{27}{4}x^2 + \left(\frac{1}{2}x\right)^2 \quad \text{[using (ii)]}$$

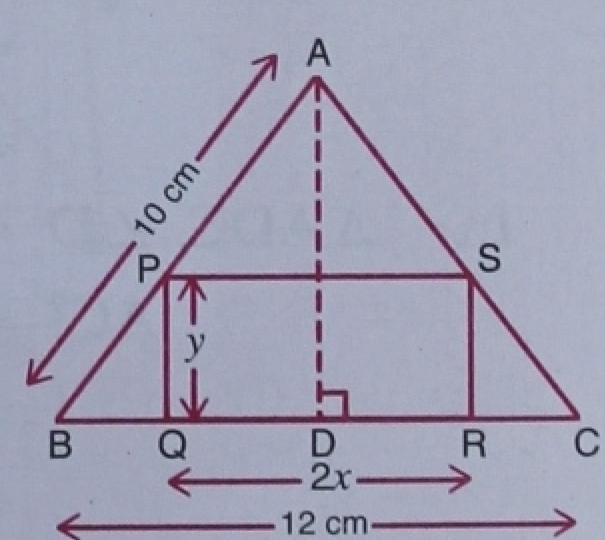
$$= 7x^2 = 7.\left(\frac{1}{3}AB\right)^2 \quad \text{[using (i)]}$$

$\Rightarrow \quad 9AP^2 = 7AB^2.$

Example 10. *ABC is an isosceles triangle. AB = AC = 10 cm, BC = 12 cm. PQRS is a rectangle drawn inside the isosceles triangle. Given PQ = SR = y cm and PS = QR = 2x cm. Prove that $x = 6 - \frac{3y}{4}$.*

Solution. In ΔABC, AB = AC.

Draw AD ⊥ BC, then, D is mid-point of BC.



But BC = 12 cm (given) ⇒ BD = 6 cm.

Also $\Delta PBQ \cong \Delta SRC \Rightarrow BQ = RC$

$\Rightarrow \quad QD = DR = x$ cm.

$\therefore \quad BQ = BD - QD = (6 - x)$ cm.

In ΔABD, $\angle D = 90°$,

$\therefore \quad AB^2 = AD^2 + BD^2 \Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow \quad AD^2 = (10)^2 - (6)^2 = 100 - 36 = 64$

$\Rightarrow \quad AD = 8$ cm.

In ΔABD, $PQ \parallel AD$,

$$\therefore \quad \frac{PQ}{AD} = \frac{BQ}{BD} \Rightarrow \frac{y}{8} = \frac{6-x}{6}$$

$$\Rightarrow \quad \frac{6y}{8} = 6 - x \Rightarrow x = 6 - \frac{3}{4}y, \text{ as required.}$$

Example 11. *In ΔABC, $\angle A = 90°$ and $AD \perp BC$.*

(i) Prove that ΔABD and ΔCAD are similar.

(ii) Calculate DC and AC given that AB = 5 cm and AD = 3 cm.

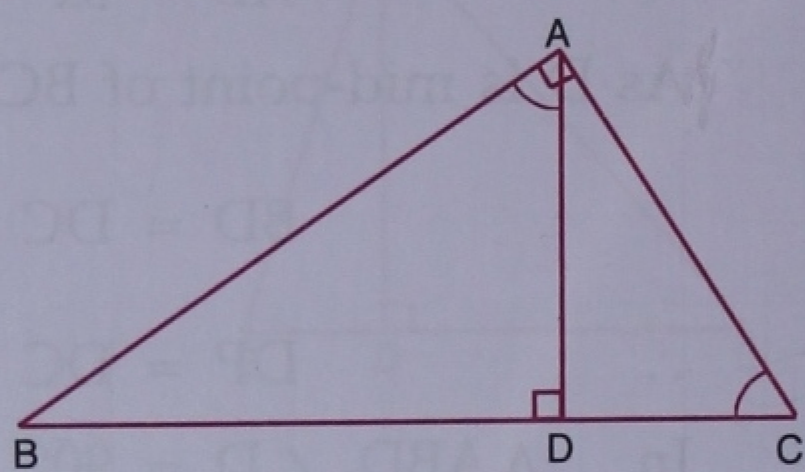
Solution. (*i*) Since $AD \perp BC$, $\angle ADB = 90° = \angle ADC$.

$\therefore \quad \angle C + \angle CAD = 90°$ [$\because$ In ΔACD, $\angle D = 90°$]

Also $\angle CAD + \angle DAB = \angle A = 90°$ (given)

$\Rightarrow \quad \angle CAD + \angle DAB = \angle C + \angle CAD$

$\Rightarrow \quad \angle DAB = \angle C.$



In Δs ABD and CAD,

$\angle ADB = \angle ADC$ (each = 90°)

and $\angle DAB = \angle C$ (proved above)

$\therefore \quad \Delta ABD \sim \Delta CAD$, as required.

(*ii*) In ΔABD, $\angle D = 90°$, by Pythagoras theorem, we get

$$AB^2 = BD^2 + AD^2 \Rightarrow 5^2 = BD^2 + 3^2$$

$$\Rightarrow \quad BD^2 = 5^2 - 3^2 = 25 - 9 = 16 \Rightarrow BD = 4 \text{ cm.}$$

Since $\Delta CAD \sim \Delta ABD$ [by part (*i*)]

$$\frac{DC}{AD} = \frac{AD}{BD} = \frac{AC}{AB} \Rightarrow \frac{DC}{3} = \frac{3}{4} = \frac{AC}{5}$$

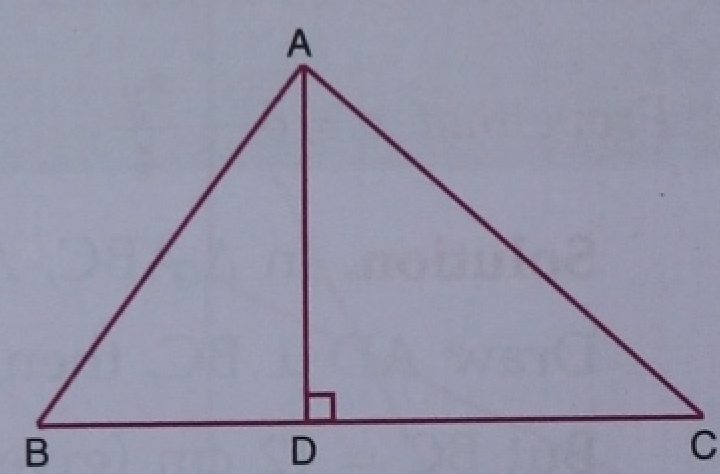
$$\Rightarrow \quad DC = \frac{9}{4} \text{ cm} = 2\frac{1}{4} \text{ cm and } AC = \frac{15}{4} \text{ cm} = 3\frac{3}{4} \text{ cm.}$$

Example 12. *In a Δ ABC, $AD \perp BC$. If $AD^2 = BD.DC$, prove that $\angle BAC = 90°$.*

Solution. Given, in Δ ABC, $AD \perp BC$ and $AD^2 = BD.DC$ ...(*i*)

In ΔABD, $\angle D = 90°$,

$$\begin{aligned} \therefore \quad AB^2 &= AD^2 + BD^2 \\ &= BD.DC + BD^2 \quad \text{[using (i)]} \\ &= BD.(DC + BD) \\ &= BD.BC \quad \text{...(ii)} \end{aligned}$$



In ΔADC, $\angle D = 90°$,

$$\begin{aligned} \therefore \quad AC^2 &= AD^2 + DC^2 \\ &= BD.DC + DC^2 \quad \text{[using (i)]} \\ &= DC.(BD + DC) \\ &= DC.BC \quad \text{...(iii)} \end{aligned}$$

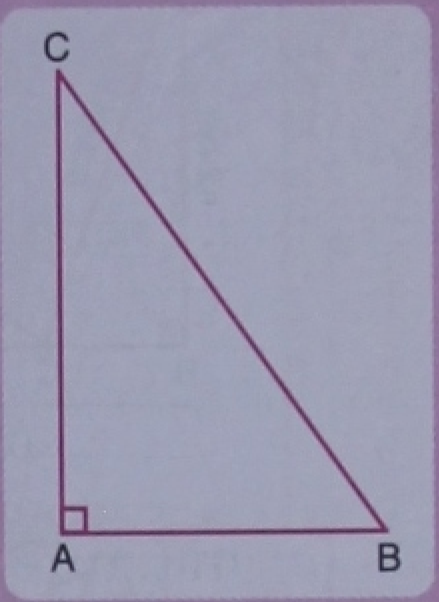
On adding (*ii*) and (*iii*), we get

$$\begin{aligned} AB^2 + AC^2 &= BD.BC + DC.BC \\ &= (BD + DC).BC = BC.BC = BC^2. \end{aligned}$$

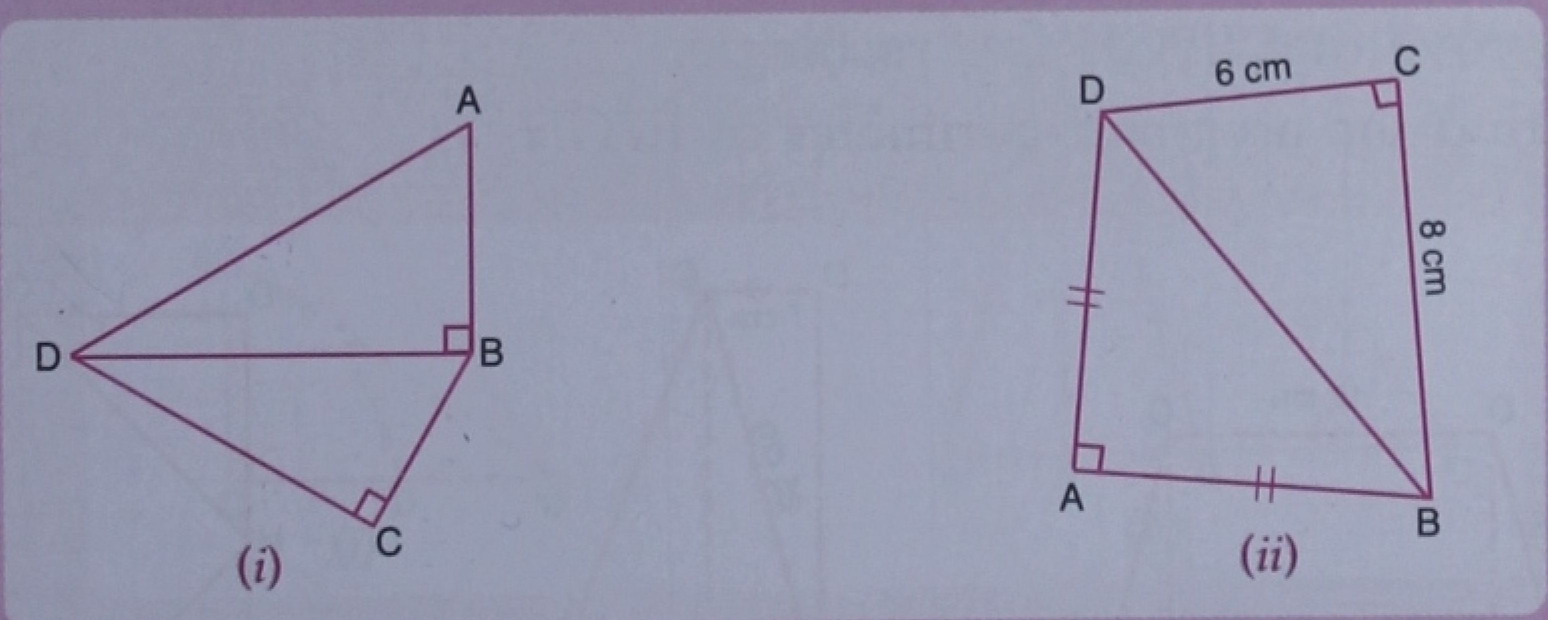
Hence, by converse of Pythagoras theorem, $\angle BAC = 90°$.

Exercise 15

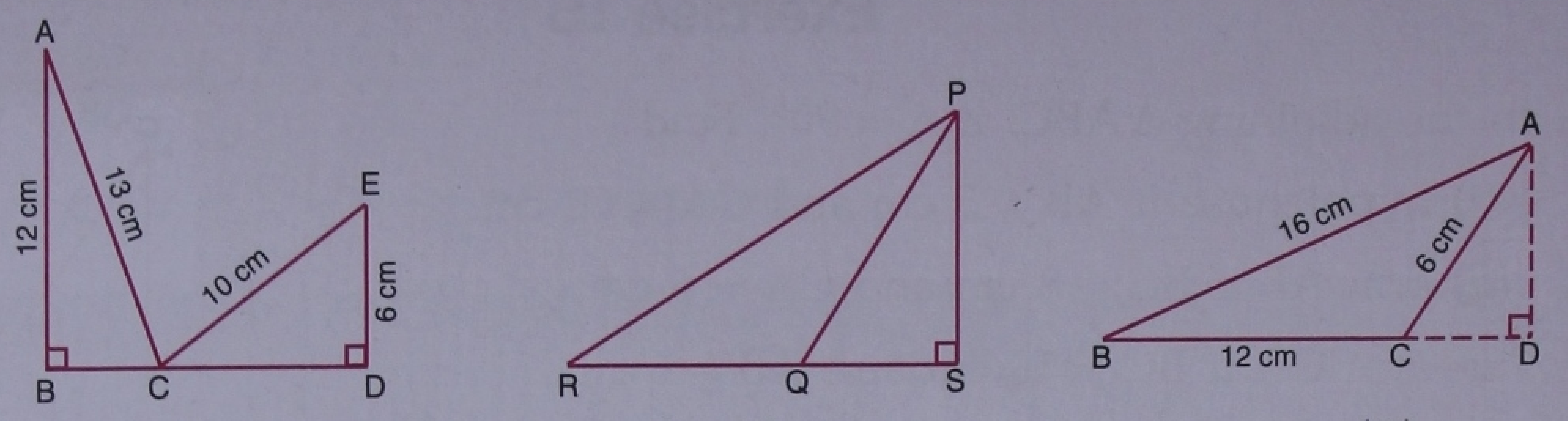
1. In the adjoining ΔABC, $\angle A = 90°$. Find
   (*i*) hypotenuse if AB = 5 cm and CA = 12 cm.
   (*ii*) arm AB if BC = 8 cm and CA = 6 cm.
   (*iii*) arm CA if BC = 10 cm and AB = 6 cm.



2. In a right-angled triangle, if hypotensue is 20 cm and the ratio of the other two sides is 4 : 3, find the sides.
3. If the sides of a triangle are in the ratio 3 : 4 : 5, prove that it is right-angled triangle.
4. The hypotenuse of a right triangle is 6 m more than twice the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle.
5. If ABC is an equilateral triangle with side 8 cm, find its height and calculate its area. Leave your answer in the surd form.
6. ABC is an isosceles triangle with AB = AC = 12 cm and BC = 8 cm. Find the altitude on BC and hence calculate its area.
7. Find the area and the perimeter of a square whose diagonal is 10 cm long.
8. (*a*) In figure (*i*) given below, ABCD is a quadrilateral in which AD = 13 cm, DC = 12 cm, BC = 3 cm, $\angle ABD = \angle BCD = 90°$. Calculate the length of AB.
   (*b*) In figure (*ii*) given below, ABCD is a quadrilateral in which AB = AD, $\angle A = 90° = \angle C$, BC = 8 cm and CD = 6 cm. Find AB and calculate the area of ΔABD.



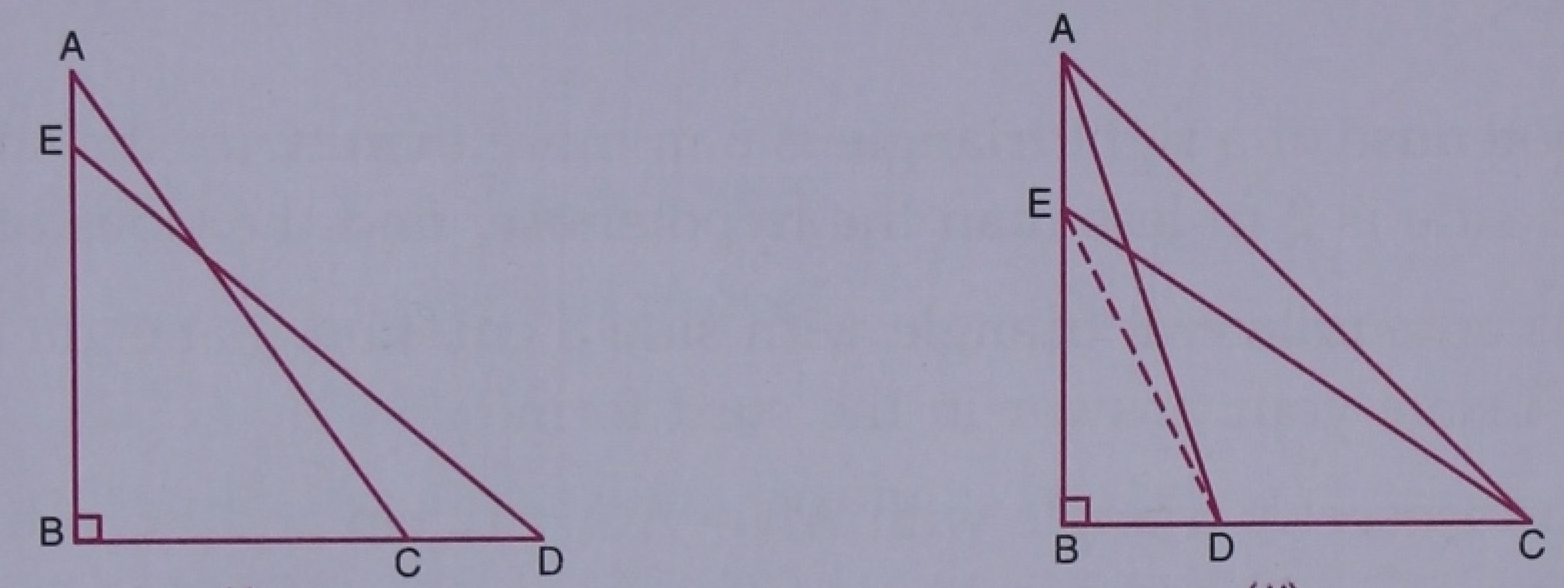
9. (*a*) In figure (*i*) given below, AB = 12 cm, AC = 13 cm, CE = 10 cm and DE = 6 cm. Calculate the length of BD.
   (*b*) In figure (*ii*) given below, $\angle PSR = 90°$, PQ = 10 cm, QS = 6 cm and RQ = 9 cm. Calculate the length of PR.
   (*c*) In figure (*iii*) given below, $\angle D = 90°$, AB = 16 cm, BC = 12 cm and CA = 6 cm. Find CD.



(i) (ii) (iii)

10. (*a*) In figure (*i*) given below, BC = 5 cm, ∠B = 90°, AB = 5AE, CD = 2AE and AC = ED. Calculate the lengths of EA, CD, AB and AC.

(*b*) In figure (*ii*) given below, ∠B = 90° and D, E are points on the sides BC, AB respectively of Δ ABC. Prove that $AD^2 + EC^2 = AC^2 + ED^2$.



(i) (ii)

11. In Δ ABC, AB = AC = *x*, BC = 10 cm and the area of Δ ABC is 60 cm². Find *x*.

12. In a rhombus, if diagonals are 30 cm and 40 cm, find its perimeter.

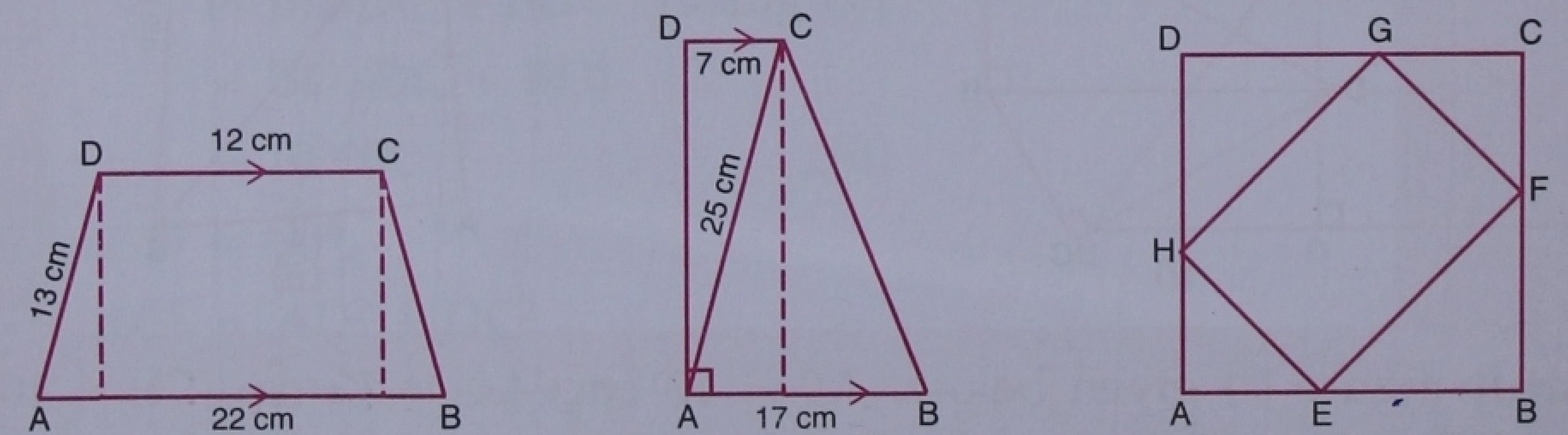
13. (*a*) In figure (*i*) given below, AB ‖ DC, BC = AD = 13 cm, AB = 22 cm and DC = 12 cm. Calculate the height of the trapezium ABCD.

(*b*) In figure (*ii*) given below, AB ‖ DC, ∠A = 90°, DC = 7 cm, AB = 17 cm and AC = 25 cm. Calculate BC.

(*c*) In figure (*iii*) given below, ABCD is a square of side 7 cm. If AE = FC = CG = HA = 3 cm,

(*i*) prove that EFGH is a rectangle.

(*ii*) find the area and perimeter of EFGH.



(i) (ii) (iii)

Hint

(*c*) (*i*) ∠AEH = 45°, ∠BEF = 45°.

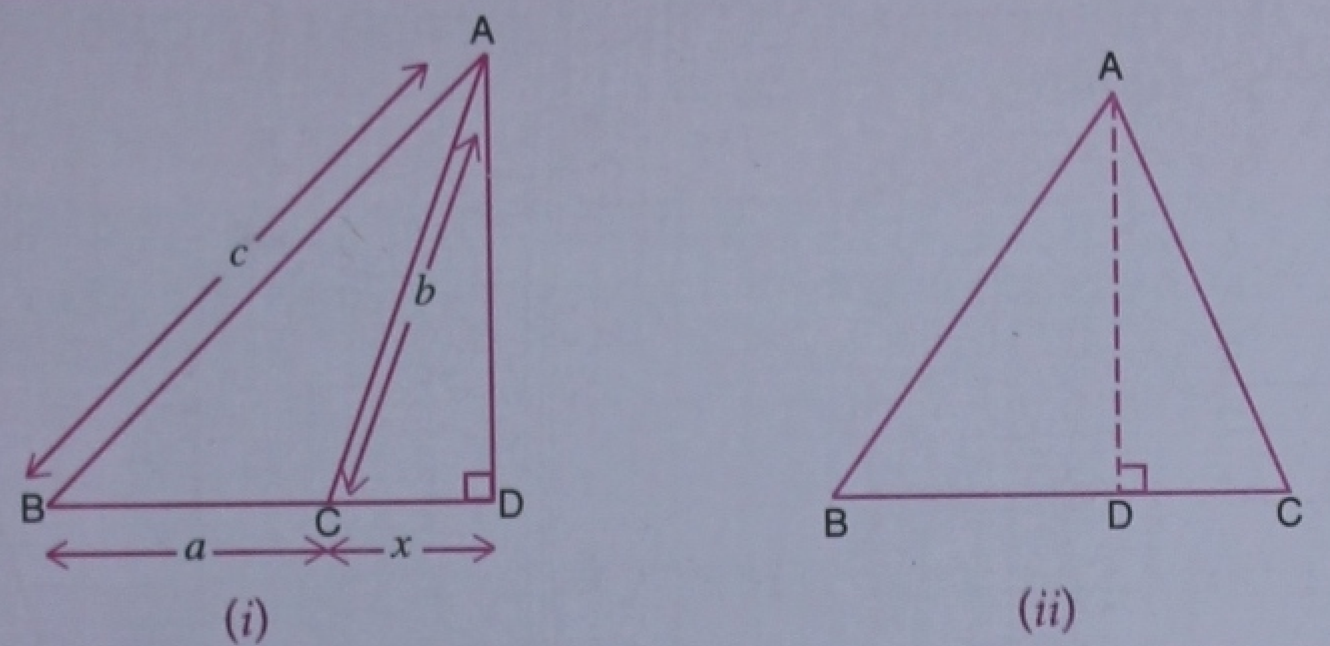
14. If in an isosceles triangle ABC, ∠A = 90°, prove that $BC^2 = 2AB^2$.

15. AD is perpendicular to the side BC of an equilateral ΔABC. Prove that $4AD^2 = 3AB^2$.

16. In ΔABC, $\angle B = 90°$ and M is a point on BC. Prove that $AM^2 + BC^2 = AC^2 + BM^2$.

17. (*a*) In figure (*i*) given below, AD is perpendicular to BC produced, prove that $c^2 = a^2 + b^2 + 2ax$.

(*b*) In the figure (*ii*) given below, $\angle C$ is acute and AD is perpendicular to BC. Prove that $AB^2 = BC^2 + CA^2 - 2BC.DC$.



(*i*) (*ii*)

18. (*a*) In figure (*i*) given below, D and E are mid points of the sides BC and CA respectively of a ΔABC, right angled at C. Prove that :

(*i*) $4AD^2 = 4AC^2 + BC^2$

(*ii*) $4BE^2 = 4BC^2 + AC^2$

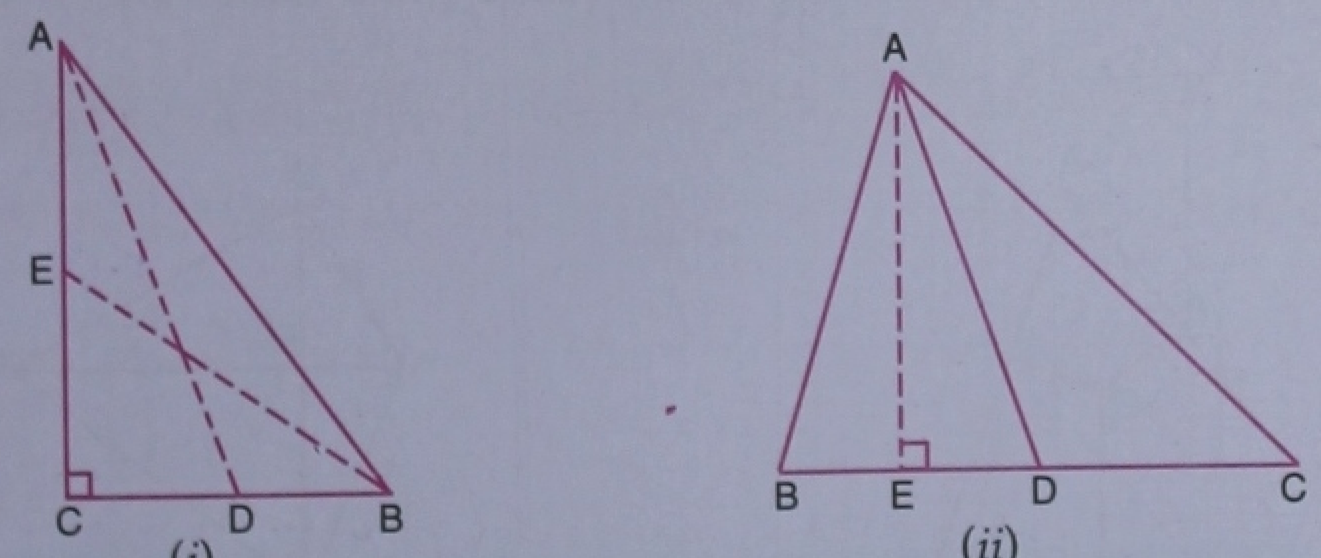
(*iii*) $4(AD^2 + BE^2) = 5AB^2$.

(*b*) In figure (*ii*) given below, AD is median of ΔABC and $AE \perp BC$. Prove that

(*i*) $AC^2 = AD^2 + BC.ED + \frac{1}{4}BC^2$

(*ii*) $AB^2 = AD^2 - BC.ED + \frac{1}{4}BC^2$

(*iii*) $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.



(*i*) (*ii*)

Hint

(*b*) (*i*) $AC^2 = AE^2 + EC^2 = AE^2 + (ED + DC)^2$

$= AE^2 + (ED + \frac{1}{2}BC)^2 = AE^2 + ED^2 + BC.ED + \frac{1}{4}BC^2$

$= (AE^2 + ED^2) + BC.ED + \frac{1}{4}BC^2$

$= AD^2 + BC.ED + \frac{1}{4}BC^2$.

19. If AD, BE and CF are medians of ΔABC, prove that
$3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$.

Hint

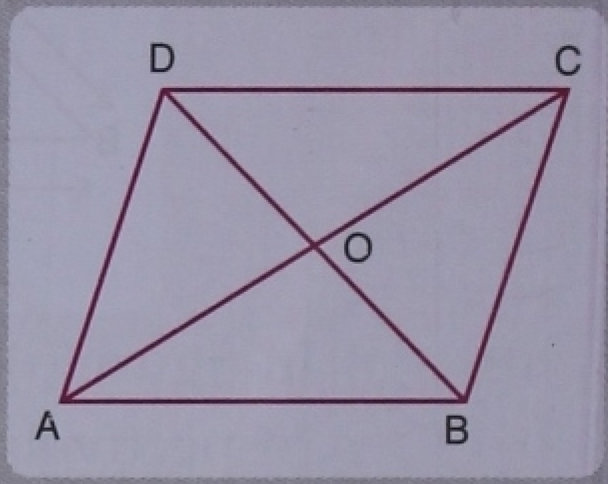
From part (*iii*) of problem 18(*b*), $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Similarly, $BC^2 + CA^2 = 2BE^2 + \frac{1}{2}CA^2$, $CA^2 + AB^2 = 2CF^2 + \frac{1}{2}AB^2$. Add all these.

20. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Hint

Let ABCD be a parallelogram and its diagonals AC and BD meet at O, then $AO = \frac{1}{2}AC$.



In ΔABD, AO is median.

From part (*iii*) of problem 18(*b*),

$$AB^2 + AD^2 = 2AO^2 + \frac{1}{2}BD^2$$

$$= 2 \cdot \left(\frac{1}{2}AC\right)^2 + \frac{1}{2}BD^2 = \frac{1}{2}(AC^2 + BD^2).$$

In ΔBCD, CO is median.

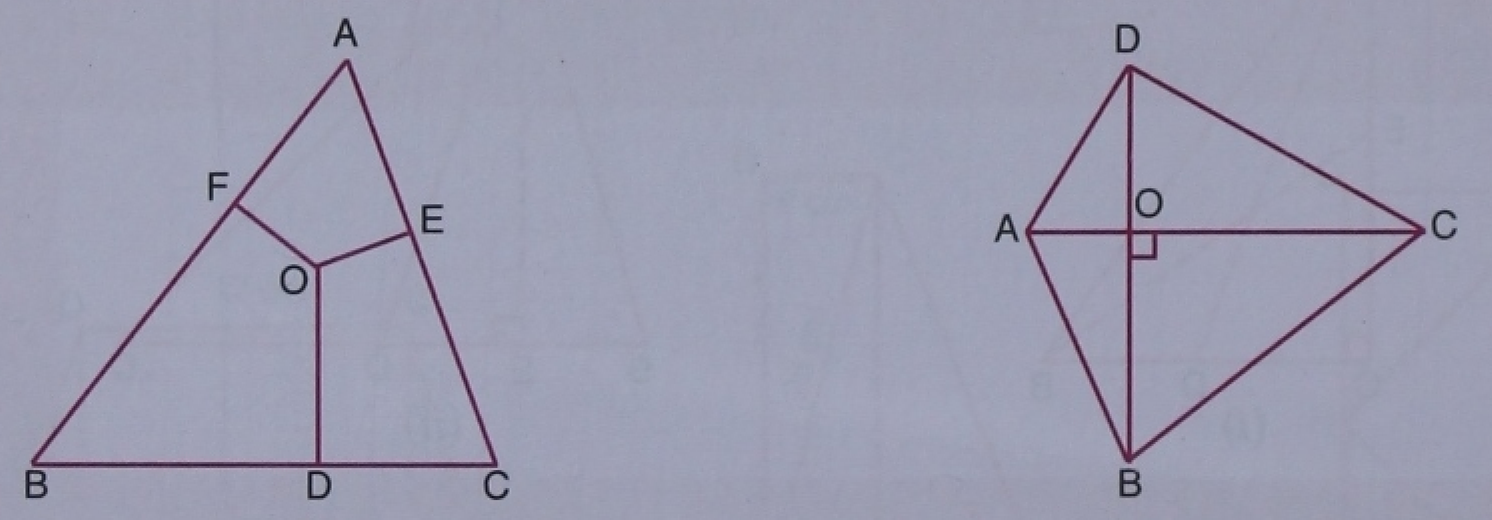
$\therefore\ BC^2 + CD^2 = \frac{1}{2}(AC^2 + BD^2)$.

21. (*a*) In figure (*i*) given below, $OD \perp BC$, $OE \perp CA$ and $OF \perp AB$. Prove that

(*i*) $OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OD^2 + OE^2 + OF^2$.

(*ii*) $AF^2 + BD^2 + CE^2 = FB^2 + DC^2 + EA^2$.

(*b*) In figure (*ii*) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $AB^2 + CD^2 = AD^2 + BC^2$.



(*i*) (*ii*)

22. In a quadrilateral ABCD, $\angle B = 90° = \angle D$. Prove that

$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$.

23. PQRS is a rhombus and the diagonals PR and SQ intersect at O. Prove that

$OP^2 + OR^2 = PS^2 + SR^2 - \frac{1}{2}SQ^2$.

24. In a rectangle ABCD, prove that

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2.$$

25. In a ΔABC, $\angle A = 90°$, CA = AB and D is a point on AB produced. Prove that

$$DC^2 - BD^2 = 2AB \times AD.$$

26. In an isosceles triangle ABC, AB = AC and D is a point on BC produced. Prove that

$$AD^2 = AC^2 + BD \times CD.$$

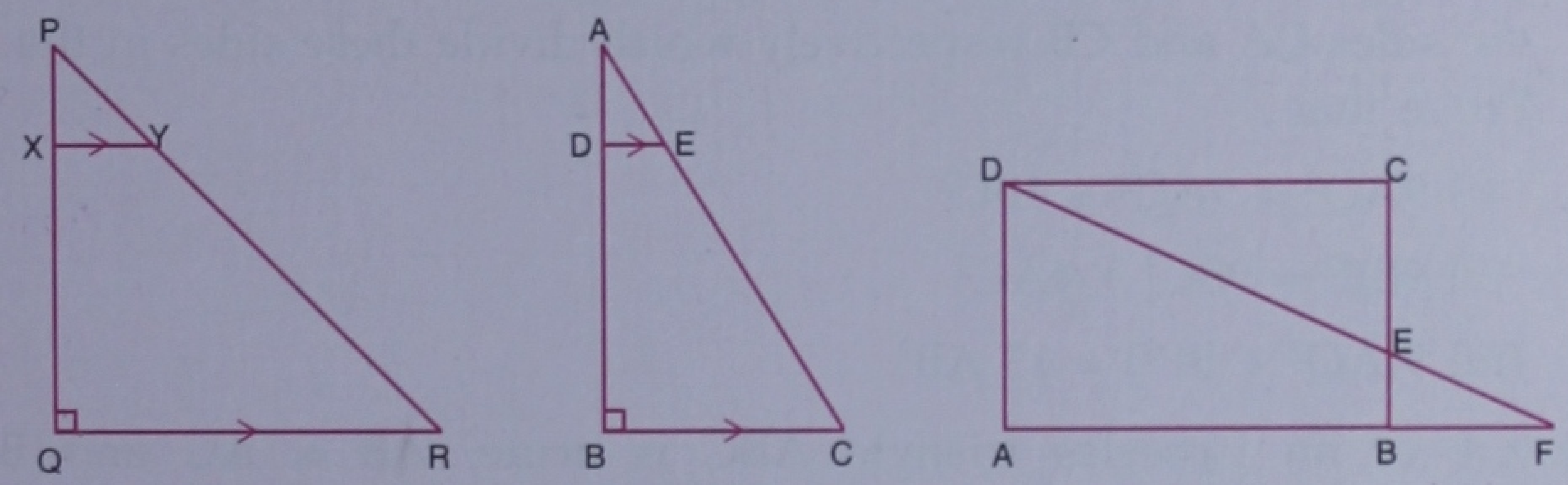
27. (*a*) In figure (*i*) given below, PQR is a right angled triangle, right angled at Q. XY is parallel to QR. PQ = 6 cm, PY = 4 cm and PX : QX = 1 : 2. Calculate the length of PR and QR.

(*b*) In figure (*ii*) given below, ABC is a right angled triangle, right angled at B. DE ∥ BC, AB = 12 cm, AE = 5 cm and AD : DB = 1 : 2. Calculate the perimeter of Δ ABC.

(*c*) In figure (*iii*) given below, ABCD is a rectangle, AB = 12 cm, BC = 8 cm and E is a point on BC such that CE = 5 cm. DE when produced meets AB produced at F.

(*i*) Calculate the length DE.

(*ii*) Prove that $\Delta DEC \sim \Delta FEB$ and hence compute EF and BF.



(*i*) (*ii*) (*iii*)

28. ABC is a right angled triangle, right angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB. Prove that

(*i*) $cp = ab$ (*ii*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Hint

Let CN ⊥ AB, then CN = p. $\Delta ACN \sim \Delta ABC$

$\Rightarrow \frac{CN}{BC} = \frac{AC}{AB} \Rightarrow \frac{p}{a} = \frac{b}{c}$. Also $c^2 = a^2 + b^2$.

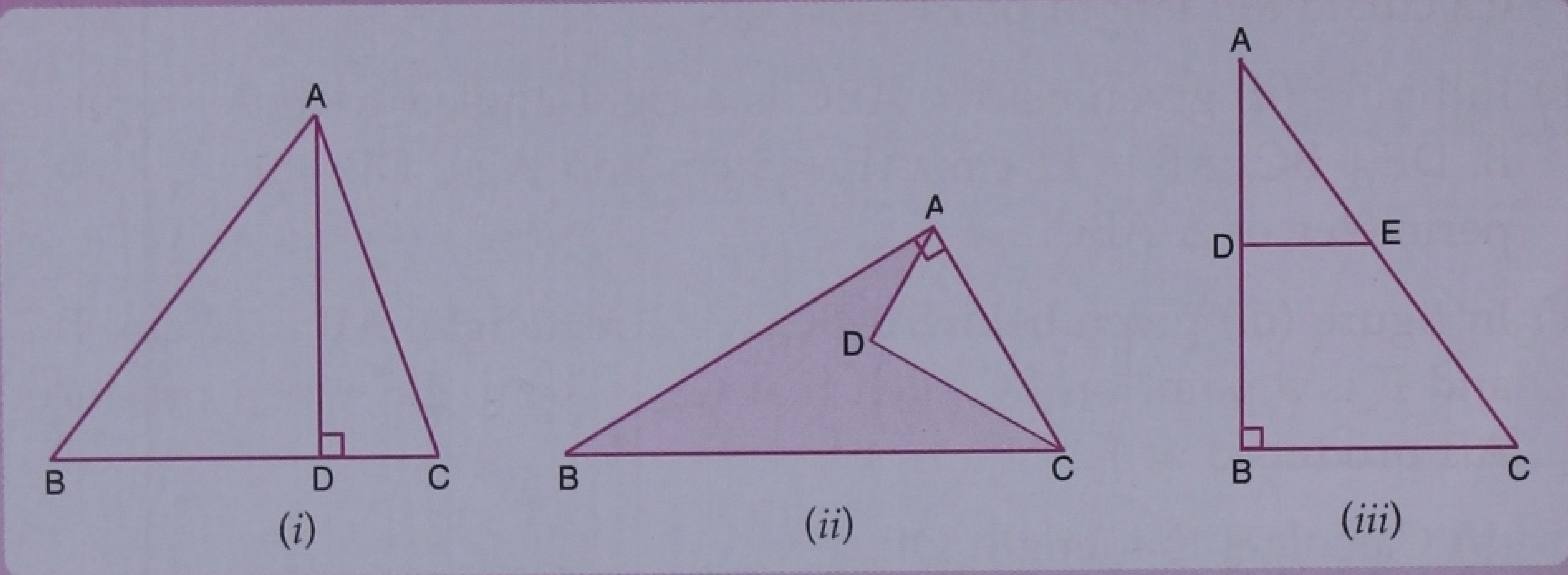
CHAPTER TEST

1. (*a*) In figure (*i*) given below, AD ⊥ BC, AB = 25 cm, AC = 17 cm and AD = 15 cm. Find the length of BC.

   (*b*) In figure (*ii*) given below, ∠BAC = 90°, ∠ADC = 90°, AD = 6 cm, CD = 8 cm and BC = 26 cm. Find

   (*i*) AC (*ii*) AB (*iii*) area of the shaded region.

   (*c*) In figure (*iii*) given below, triangle ABC is right angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are mid-points of the sides AB and AC respectively, calculate (*i*) the length of BC (*ii*) the area of Δ ADE.



2. If in Δ ABC, AB > AC and AD ⊥ BC, prove that $AB^2 - AC^2 = BD^2 - CD^2$.

3. In a right angled triangle ABC, right angled at C, P and Q are the points on the sides CA and CB respectively which divide these sides in the ratio 2 : 1. Prove that

   (*i*) $9AQ^2 = 9AC^2 + 4BC^2$

   (*ii*) $9BP^2 = 9BC^2 + 4AC^2$

   (*iii*) $9(AQ^2 + BP^2) = 13\ AB^2$.

4. ∠A of an isosceles triangle ABC is acute, AB = AC and BD ⊥ AC. Prove that

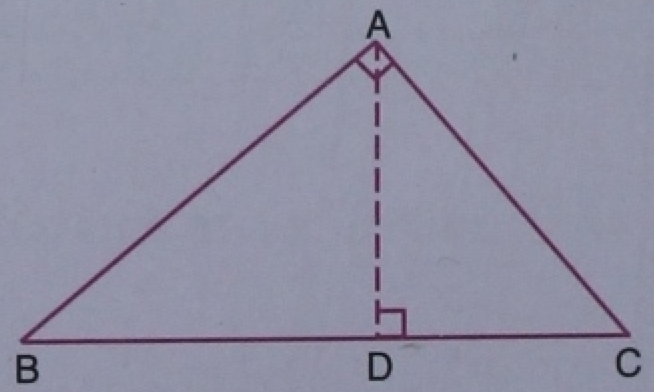
   $BC^2 = 2AC \times CD$.

Hint

$BC^2 = AB^2 + AC^2 - 2AC \times AD$, see Q. 17(*b*) excercise 15.

5. In a triangle ABC, AB = AC and D is a point on side AC such that $BC^2 = AC \times CD$. Prove that BD = BC.

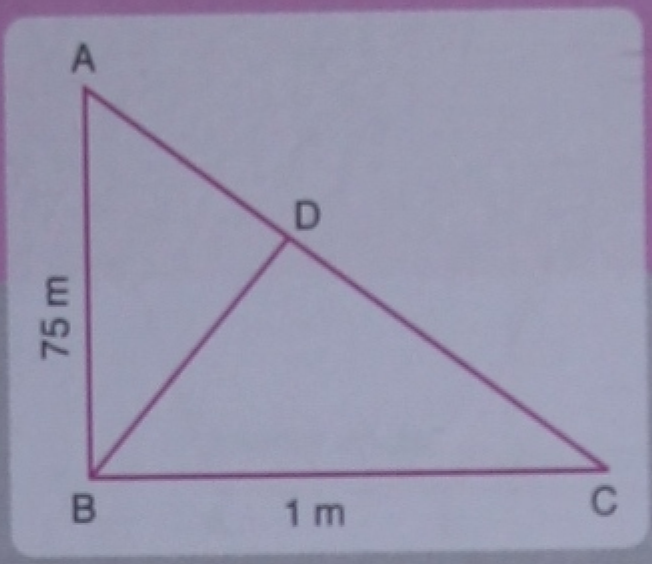
6. In the adjoining figure, ∠BAC = 90° and AD ⊥ BC. If AB = 10 cm and BD = 8 cm, find DC and AC.



Hint

Find AD. Δ ABD ~ Δ CAD.

7. In the adjoing figure, AB ⊥ BC and BD ⊥ AC. Find BD.



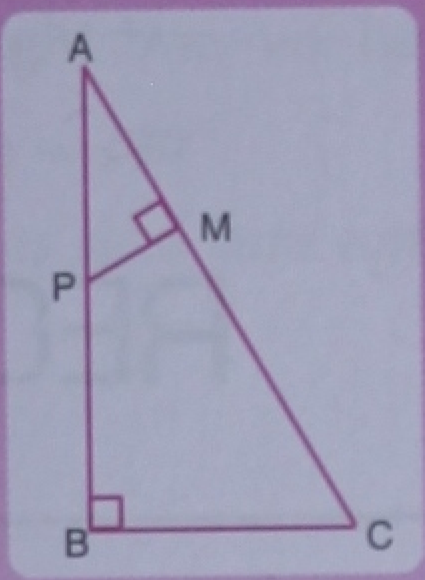
Hint

Find AC. Check Δ ABD ~ Δ ACB,

$$\Rightarrow \frac{BD}{BC} = \frac{AB}{AC}.$$

8. In the adjoining figure, ABC is a right angled triangle at B. P is a point on AB such that AP = 10 cm. From P, PM is drawn perpendicular to AC. If MP = 6 cm and BC = 18 cm, find

(*i*) AM (*ii*) BP (*iii*) MC.



9. P is a point in the interior of rectangle ABCD. If P is joined to each of the vertices of the rectangle and the lengths PA, PB and PC are 3 cm, 4 cm and 5 cm respectively, find the length of PD.

Hint

Use example 4.