GEOMETRY

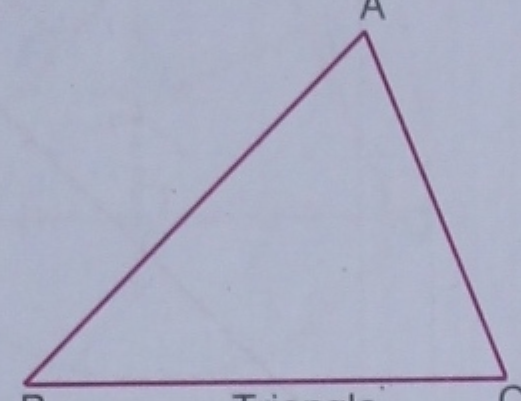
12

TRIANGLES

12.1 TRIANGLE

*A **triangle** is a closed figure bounded by three line segments.*

The adjoining figure shows a triangle ABC. The line segments AB, BC and CA are called its ***sides***. The angles CAB, ABC and BCA are called its ***interior angles*** or simply the ***angles***. The points A, B, and C are called its ***vertices.***

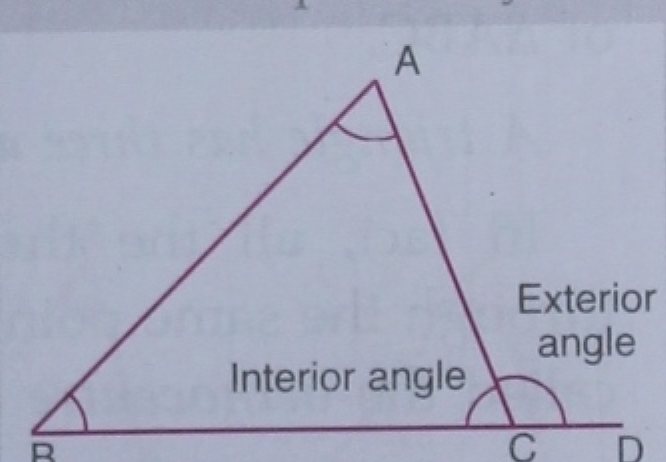


Triangle

Thus, a triangle has three sides and three angles, and all the six are called ***elements*** of the triangle ABC.

Remarks

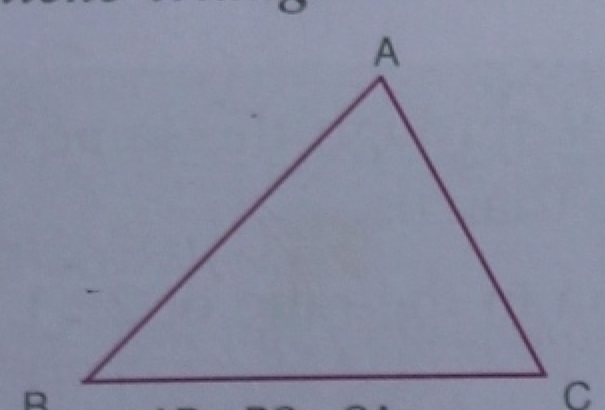
- Usually, triangle ABC is written as ΔABC.
- $\angle CAB$, $\angle ABC$ and $\angle BCA$ are written as $\angle A$, $\angle B$ and $\angle C$ respectively.
- If a side, say BC, is produced to a point D, then $\angle ACD$ is called **exterior angle** at C. The two interior angles of ΔABC that are opposite to $\angle ACD$ are called its ***opposite*** (or ***remote***) **interior angles.** Thus $\angle A$ and $\angle B$ are opposite interior angles of $\angle ACD$.



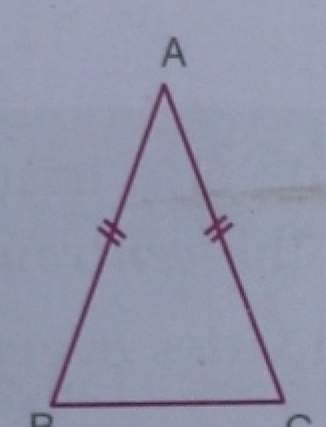
12.1.1 Types of triangles

1. Types of triangles on the basis of sides.

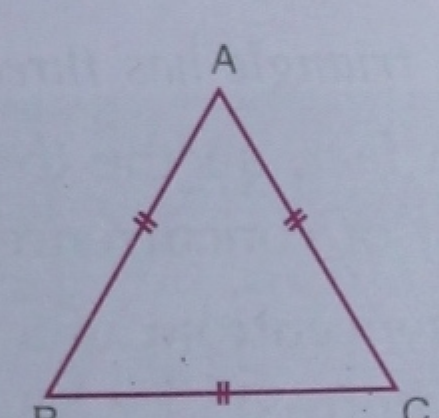
(*i*) **Scalene triangle.** If all the three sides of a triangle are unequal, it is called a ***scalene triangle***.



$AB \neq BC \neq CA$
Scalene triangle



$AB = AC$
Isosceles triangle



$AB = BC = CA$
Equilateral triangle

(*ii*) **Isosceles triangle.** If any two sides of a triangle are equal, it is called an ***isosceles triangle***.

(*iii*) **Equilateral triangle.** If all the three sides of a triangle are equal, it is called an ***equilateral triangle***.

Note

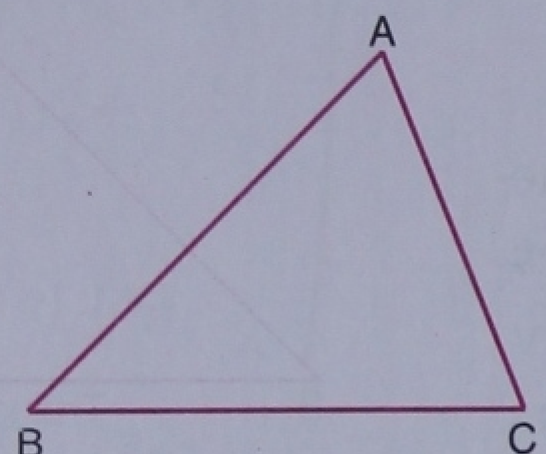
Every equilateral triangle is an isosceles triangle but the converse is not always true.

2. Types of triangles on the basis of angles.

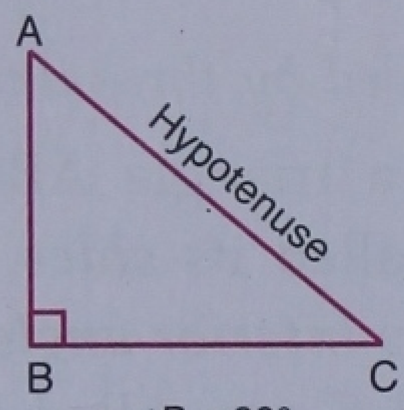
(*i*) **Acute-angled triangle.** If each angle of a triangle is an acute angle (less than 90°), it is called an ***acute-angled triangle***.

(*ii*) **Right-angled triangle.** If one angle of a triangle is a right angle (= 90°), it is called a ***right-angled triangle***. The side opposite to right angle is called ***hypotenuse***.

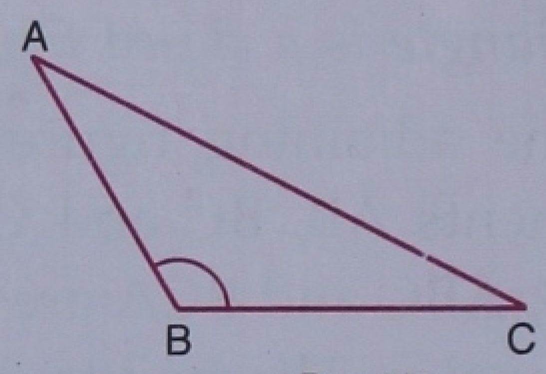
(*iii*) **Obtuse-angled triangle.** If one angle of a triangle is obtuse (greater than 90°), it is called an ***obtuse-angled triangle***.



Acute-angled triangle



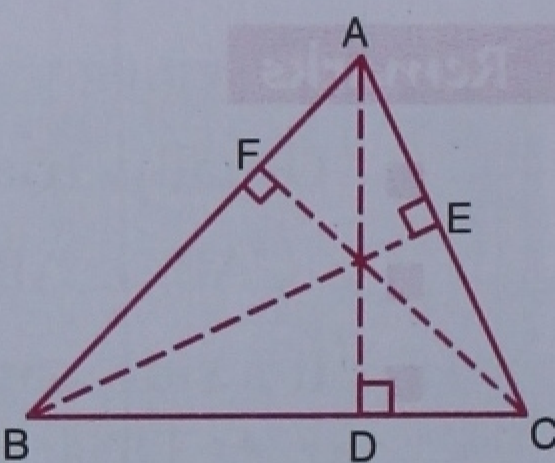
$\angle B = 90°$
Right-angled triangle



$\angle B > 90°$
Obtuse-angled triangle

12.1.2 Some terms connected with a triangle

Altitude. *Perpendicular from a vertex of a triangle to the opposite side is called an* ***altitude*** *of the triangle.*

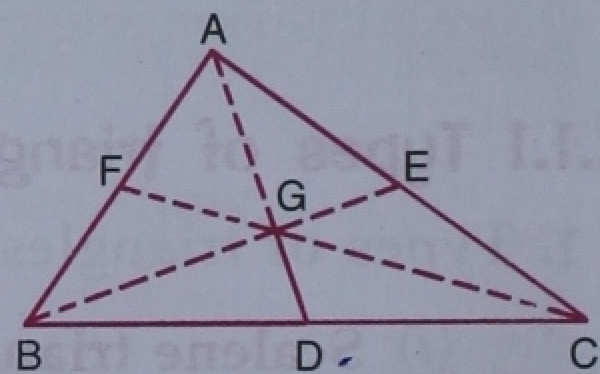


In the adjoining figure, AD ⊥ BC, so AD is an altitude of ΔABC.

A triangle has three altitudes.

In fact, all the three altitudes of a triangle pass through the same point and the point of concurrence is called the ***orthocentre*** of the triangle.

Median. The straight line joining a vertex of a triangle to the mid-point of the opposite side is called a ***median*** of the triangle.



In the adjoining figure, D is mid-point of BC, so AD is a median of ΔABC.

A triangle has three medians.

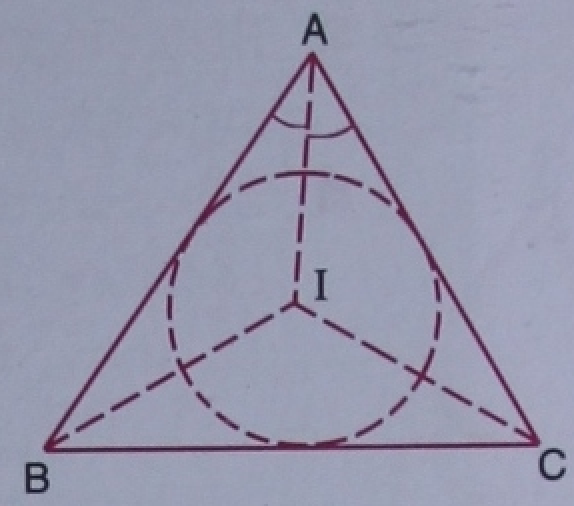
In fact, all the three medians of a triangle pass through the same point and the point of concurrence is called the ***centroid*** of the triangle.

The centroid of a triangle divides every median in the ratio of 2 : 1. Thus, if G is the centroid of ΔABC, then

AG : GD = 2 : 1, BG : GE = 2 : 1 and CG : GF = 2 : 1.

Incentre and incircle

Line bisecting an (interior) angle of a triangle is called the (internal) ***bisector*** of the angle of the triangle.



In the adjoining figure, ∠BAI = ∠IAC, so AI is the (internal) bisector of ∠A.

A triangle has three internal bisectors of its angles.

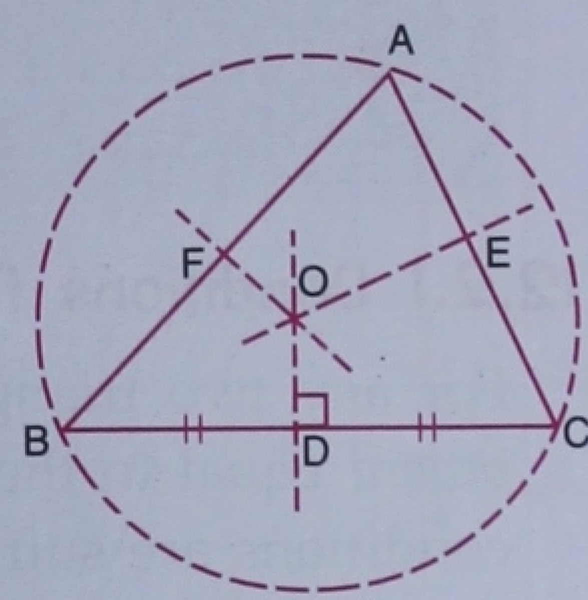
In fact, all the three (internal) bisectors of the angles of a triangle pass through the same point and the point of concurrence is called ***incentre*** of the triangle.

In the above figure, IA, IB and IC are the (internal) bisectors of ∠A, ∠B and ∠C respectively. So I is the incentre of ΔABC.

Moreover, incentre is the centre of a circle which touches all the sides of Δ ABC and this circle is called ***incircle*** of Δ ABC.

Circumcentre and circumcircle

Line bisecting a side of a triangle and perpendicular to it is called the right ***bisector*** of the side of the triangle.



In the adjoining figure, D is mid-point of BC and OD ⊥ BC, so OD is the right bisector of the side BC.

A triangle has three right bisectors of its sides.

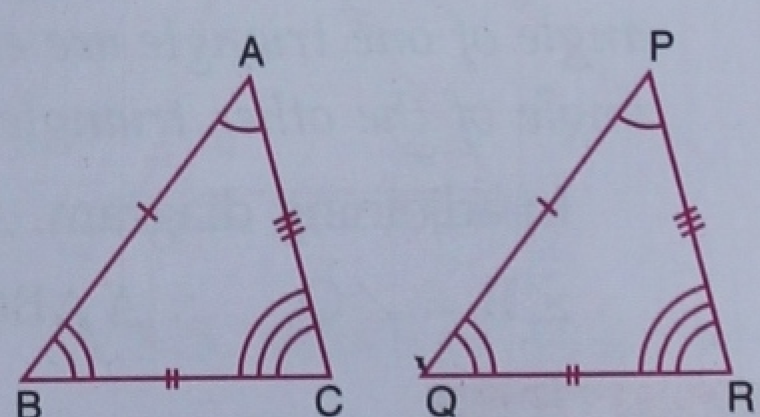
In fact, all the three right bisectors of the sides of a triangle pass through the same point and the point of concurrence is called the ***circumcentre*** of the triangle.

In the above diagram, OD, OE and OF are the right bisectors of the sides BC, CA and AB respectively of Δ ABC. So O is the circumcentre of Δ ABC.

Moreover, circumcentre is the centre of a circle which passes through the vertices of ΔABC and this circle is called ***circumcircle*** of ΔABC.

12.2 CONGRUENCY OF TRIANGLES

Congruent triangles. *Two triangles are called* ***congruent*** *if and only if they have exactly the same shape and same size.*



In this diagram, two triangles ABC and PQR are such that AB = PQ, BC = QR, CA = RP ; ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R *i.e.* they have exactly the same shape and same size, therefore, these triangles are congruent.

We use the symbol ≡ or ≅ (read as 'congruent to') to indicate congruency of triangles.

Remarks

- Congruent triangles are 'equal in all respects,' *i.e.* they are the exact ***duplicate*** of each other.
- If Δ ABC ≡ Δ PQR, then any one triangle can be ***superposed*** on the other to cover it exactly.

- In congruent triangles, the sides and the angles which coincide by superposition are called ***corresponding sides*** and ***corresponding angles***.
- The corresponding sides lie ***opposite to the equal angles*** and the corresponding angles lie ***opposite to the equal sides.***

In the above diagram, ∠A = ∠P, therefore, the corresponding sides BC and QR are equal. Similarly, ∠B = ∠Q, the corresponding sides AC and PR are equal; ∠C = ∠R, the corresponding sides AB and PQ are equal.

Also BC = QR, therefore, the corresponding angles A and P are equal, and so on.

- The order of the letters in the names of two triangles will indicate the correspondence between the vertices of the two triangles.

  Thus, ΔABC ≅ ΔPQR will mean that ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R and BC = QR, CA = RP, AB = PQ.

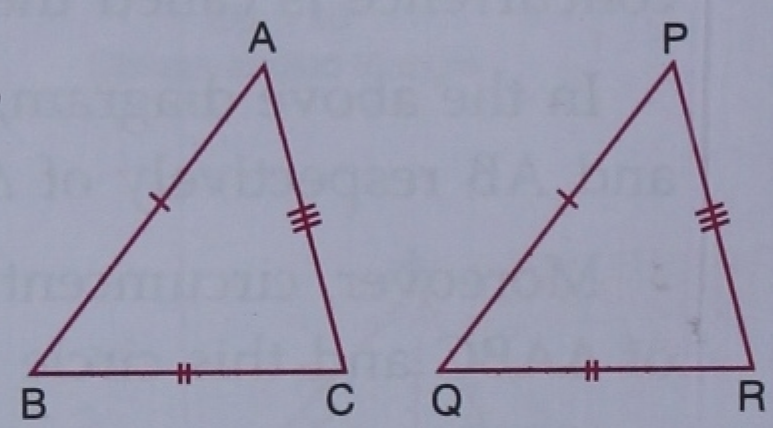
  Writing any other correspondence *i.e.* ΔABC ≅ ΔPRQ or Δ BCA ≅ Δ PQR etc. will be incorrect.
- The abbreviation 'c.p.c.t.' will be used for *corresponding parts of congruent triangles.*
- Two geometrical figures are called congruent if and only if they have exactly the same shape and same size.

12.2.1 Conditions for congruency of triangles

For any two triangles to be congruent, the *six elements* of one triangle *need not be proved* equal to the corresponding six elements of the other triangle. The following conditions are sufficient to ensure the congruency of two triangles :

1. S.S.S. (Side-Side-Side) axiom of congruency.

Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.



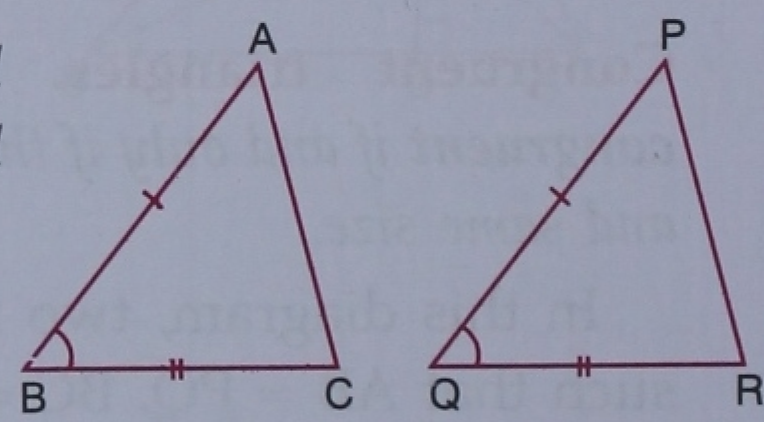
In adjoining diagram,

AB = PQ, BC = QR and CA = RP,

∴ Δ ABC ≅ Δ PQR.

2. S.A.S. (Side-Angle-Side) axiom of congruency.

Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle.



In adjoining diagram, AB = PQ, BC = QR and

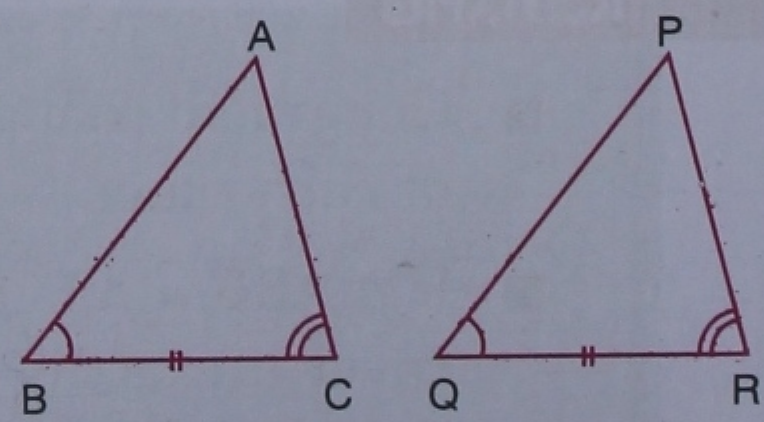
∠B = ∠Q, ∴ Δ ABC ≅ Δ PQR.

Note

The equality of the *'included angle'* is essential.

3. A.S.A. (Angle-Side-Angle) axiom of congruency.

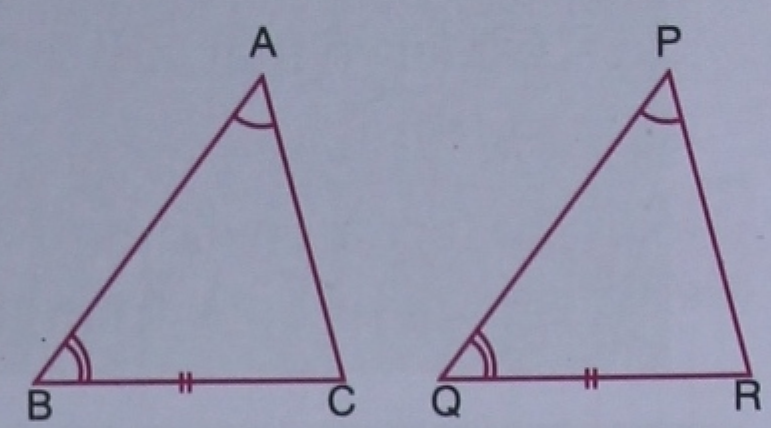
Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.



In adjoining diagram, ∠B = ∠Q, ∠C = ∠R and BC = QR, ∴ ΔABC ≅ Δ PQR.

Corollary. A.A.S. (Angle-Angle-Side) axiom of congruency.

Two triangles are congruent if any two angles and a (non-included) side of one triangle are equal to two angles and corresponding side of the other triangle.



In adjoining diagram,

$\angle A = \angle P$, $\angle B = \angle Q$ and

$BC = QR$, $\therefore\ \Delta ABC \cong \Delta PQR$.

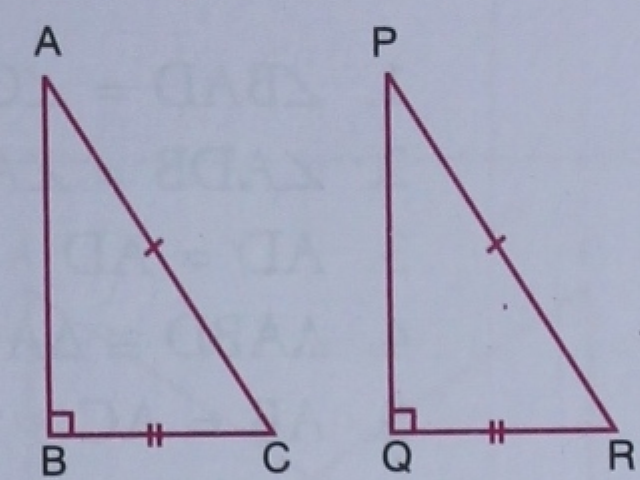
[This result follows immediately from the above axiom, for, if two angles of a triangle are equal to two angles of another triangle, then the third angles of both the triangles are also equal because the sum of three angles of a triangle is 180°].

Note

The equality of corresponding sides is essential.

4. R.H.S. (Right Angle-Hypotenuse-Side) axiom of congruency.

Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle.



In adjoining diagram,

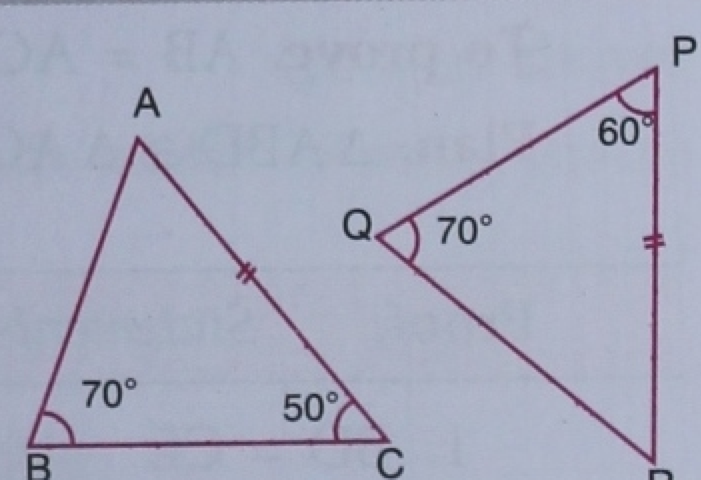
$\angle B$ = a right angle = $\angle Q$, $AC = PR$ and $BC = QR$,

$\therefore\ \Delta ABC \cong \Delta PQR$.

12.2.2 Applications of congruency of triangles

ILLUSTRATIVE EXAMPLES

Example 1. *State whether the adjoining triangles are congruent or not.*



Solution. Since the sum of angles of a triangle is 180°,

$\angle A = 180° - (70° + 50°) = 60°$.

In Δs ABC and PQR

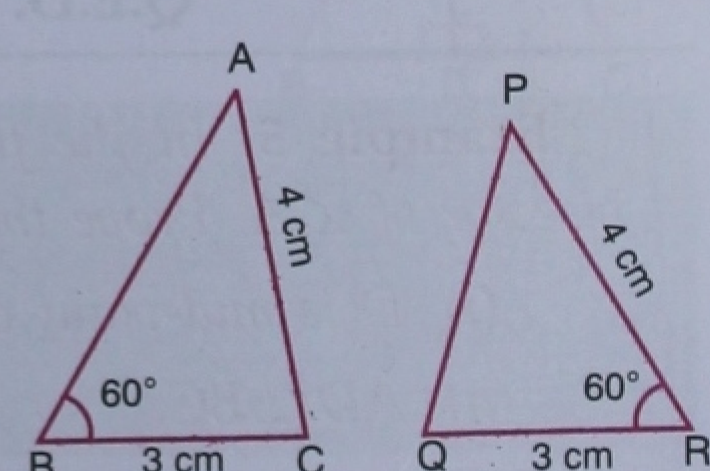
$\angle A = \angle P$ (each = 60°)

$\angle B = \angle Q$ (each = 70°, given)

and the corresponding sides $AC = PR$ (given)

$\therefore\ \Delta ABC \cong \Delta PQR$ (A.A.S. axiom of congruency)

Example 2. *State, giving reasons, whether the adjoining triangles are congruent or not.*



Solution. Here we find that

$BC = QR$ (each = 3 cm)

and $AC = PR$ (each = 4 cm)

but the included angles are not equal.

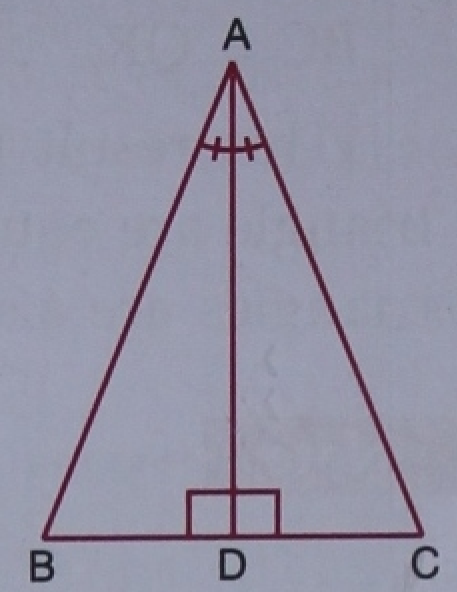
So the given triangles ABC and PQR are not congruent.

Note

Observe that ∠B = ∠R (each = 60°) but ∠B is not included between the given sides.

In fact, two sides of one triangle equal to two sides of the other triangle and one angle of both triangles equal may not ensure the congruency of two triangles.

Example 3. *In a triangle ABC, the bisector AD of ∠A is perpendicular to the side BC. Prove that ΔABC is isosceles.*



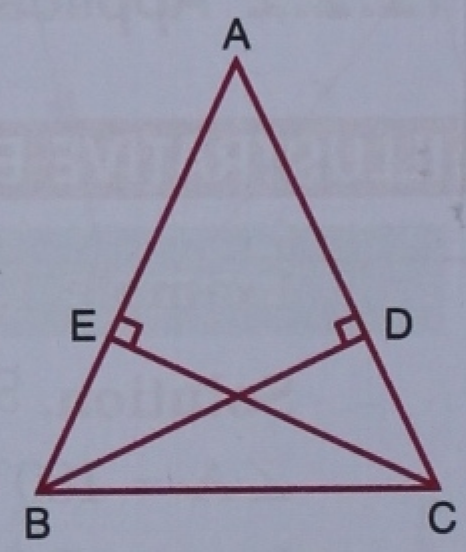
Given. A triangle ABC, AD is bisector of ∠A and AD ⊥ BC.

To prove. AB = AC.

Plan. Δ ABD ≅ Δ ACD.

Proof. *Statements*	*Reasons*
1. ∠BAD = ∠CAD	1. AD is bisector of A.
2. ∠ADB = ∠ADC	2. Each angle = 90°, ∵ AD ⊥ BC.
3. AD = AD	3. Common.
4. ΔABD ≅ ΔACD	4. A.S.A. axion of congruency.
5. AB = AC	5. 'c.p.c.t.'
Hence ABC is an isosceles triangle	
Q.E.D.	

Example 4. *If two altitudes of a triangle are equal, prove that it is an isosceles triangle.*



Given. A triangle ABC, BD ⊥ AC, CE ⊥ AB and BD = CE.

To prove. AB = AC.

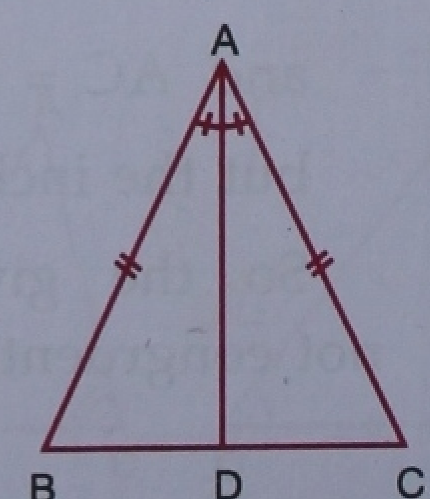
Plan. Δ ABD ≅ Δ ACE.

Proof. *Statements*	*Reasons*
1. BD = CE	1. Given.
2. ∠ ADB = ∠ AEC	2. Each is a right angle.
3. ∠BAD = ∠CAE	3. Common.
4. Δ ABD ≅ Δ ACE	4. A.A.S. (Axiom of congruency).
5. AB = AC.	5. 'c.p.c.t.'
Q.E.D.	

Example 5. *In the figure alongside, AB = AC and AD is bisector of ∠A. Prove that*

(i) D is mid-point of BC

(ii) AD ⊥ BC.



Given. Δ ABC, AB = AC and ∠BAD = ∠CAD.

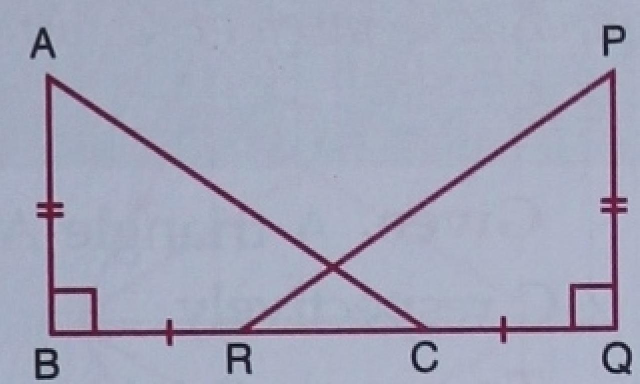
To prove. (*i*) BD = DC

(*ii*) ∠ ADB = 90°.

Plan. Δ ABD ≅ Δ ACD.

Proof. *Statements*	*Reasons*
1. AB = AC	1. Given.
2. ∠BAD = ∠CAD	2. Given.
3. AD = AD	3. Common.
4. Δ ABD ≅ Δ ACD	4. S.A.S. (Axiom of congruency)
5. (*i*) BD = DC	5. 'c.p.c.t.'
6. ∠ADB = ∠ADC	6. 'c.p.c.t.'
7. ∠ADB + ∠ADC = 180°	7. BDC is a st. line.
8. (*ii*) 2 ∠ADB = 180° ⇒ ∠ADB = 90°. Hence, (*i*) BD = DC and (*ii*) ∠ADB = 90°. **Q.E.D.**	8. From 6 and 7.

Example 6. *In the figure given alongside, AB = PQ, BR = QC, AB ⊥ BC and PQ ⊥ RQ. Prove that AC = PR.*



Solution. Given BR = QC

⇒ BR + RC = QC + RC (Adding RC to both sides)

⇒ BR + RC = RC + CQ

⇒ BC = QR.

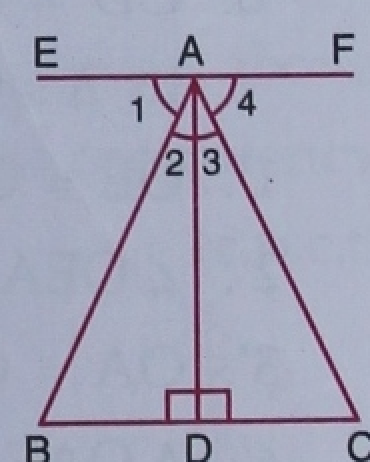
In Δs ABC and PQR, we have

1. ∠B = ∠Q (each angle = 90°, ∵ AB ⊥ BC, PQ ⊥ RQ)
2. AB = PQ (given)
3. BC = QR (proved above)

∴ Δ ABC ≅ Δ PQR (S.A.S. axiom of congruency)

⇒ AC = PR (c.p.c.t.)

Example 7. *In the figure alongside, AD ⊥ BC, AD ⊥ EF and ∠ 1 = ∠ 4. Prove that Δ ABD ≅ Δ ACD.*



Solution. Since AD ⊥ EF, ∠EAD = 90° = ∠DAF.

∴ ∠1 + ∠2 = 90° and ∠3 + ∠4 = 90°

but ∠1 = ∠4 (given)

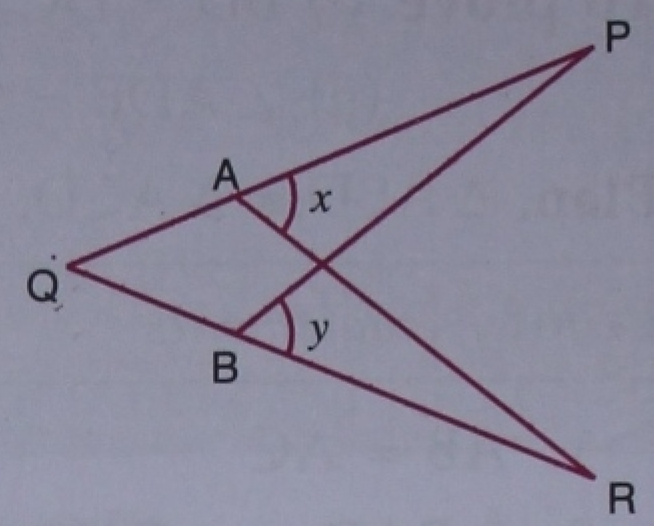
⇒ ∠2 = ∠3.

In Δs ABD and ACD, we have

1. ∠2 = ∠3 (Proved above)
2. ∠ADB = ∠ADC [each angle = 90°, ∵ AD ⊥ BC]
3. AD is common.

∴ Δ ABD ≅ Δ ACD (A.S.A. axiom of congruency)

Example 8. *In the adjoining figure, PQ = RQ and ∠x = ∠y. Prove that BP = AR.*



Solution. For ΔAQR, ∠x = ∠Q + ∠ARQ

(ext. ∠ = sum of two opp. int. ∠s)

For ΔBPQ, ∠y = ∠Q + ∠BPQ

(ext. ∠ = sum of two opp. int. ∠s)

But ∠x = ∠y (given)

⇒ ∠Q + ∠ARQ = ∠Q + ∠BPQ ⇒ ∠ARQ = ∠BPQ.

In ΔBPQ and ΔARQ,

∠ARQ = ∠BPQ (proved above)

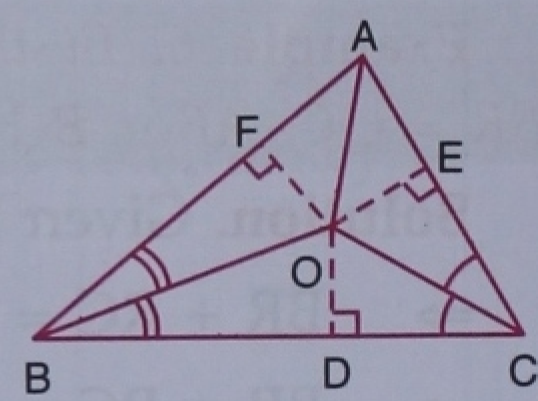
∠Q = ∠Q (common)

PQ = RQ (given)

∴ ΔBPQ ≅ ΔARQ (ASA axiom of congruency)

∴ BP = AR (c.p.c.t.)

Example 9. *In a triangle ABC, the internal bisectors of ∠B and ∠C meet at O. Prove that OA is also internal bisector of ∠A.*



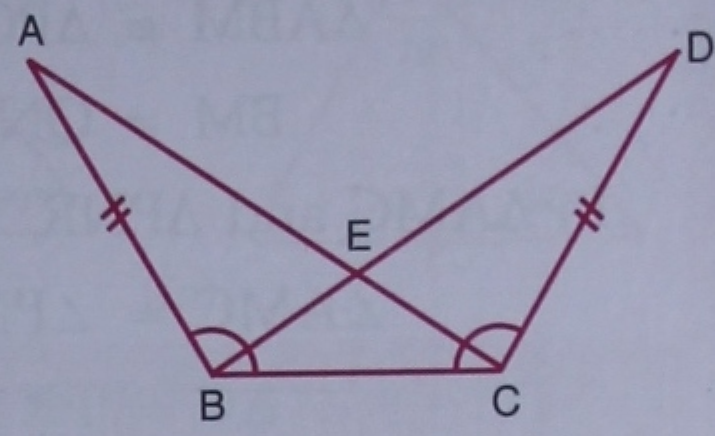
Given. A triangle ABC, OB and OC bisectors of ∠B and ∠C respectively.

To prove. OA bisects ∠A.

Construction. Draw OD ⊥ BC, OE ⊥ CA and OF ⊥ AB.

Proof. *Statements*	*Reasons*
In Δ s ODC and OEC	
1. ∠ODC = ∠OEC	1. Each being a right angle (by construction).
2. ∠OCD = ∠OCE	2. OC bisects ∠C.
3. OC = OC	3. Common.
4. ΔODC ≅ ΔOEC	4. A.A.S. (Axiom of congruency).
5. OD = OE	5. 'c.p.c.t.'
6. OD = OF	6. ΔOBD ≅ ΔOBF (similarly).
In Δ s OAE and OAF	
1′. OE = OF	1′. From 5 and 6.
2′. ∠OEA = ∠OFA	2′. Each being a rt. angle.
3′. OA = OA	3′. Common.
4′. ΔOAE ≅ ΔOAF	4′. R.H.S. (Axiom of congruency).
5′. ∠OAE = ∠OAF	5′. 'c.p.c.t.'
⇒ OA bisects ∠A.	**Q.E.D.**

Example 10. *In the adjoining figure, AB = CD and ∠ABC = ∠BCD. Prove that (i) AC = BD (ii) BE = CE.*



Solution. In Δs ABC and DCB,

AB = CD (given)

∠ABC = ∠BCD (given)

BC = BC (common)

∴ ΔABC ≅ ΔDCB (S.A.S. axiom of congruency)

(*i*) AC = BD (c.p.c.t.)

Also ∠BAC = ∠BDC (c.p.c.t.)

(*ii*) In Δs ABE and DEC,

∠BAE = ∠EDC (∵ ∠BAC = ∠BDC, proved above)

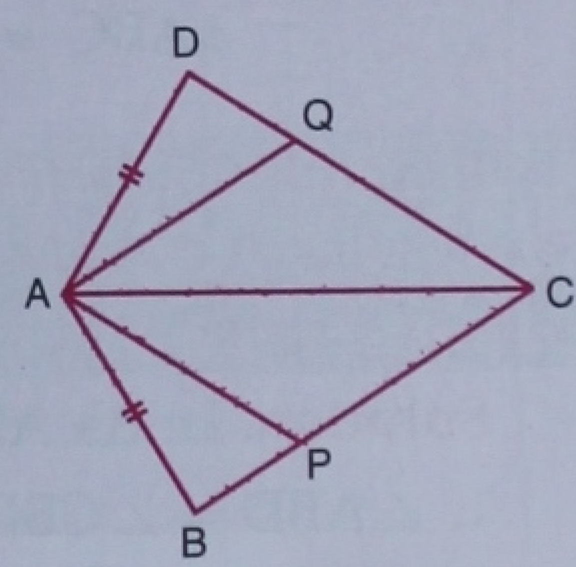
∠AEB = ∠CED (vert. opp. ∠s)

AB = CD (given)

∴ ΔABE ≅ ΔDEC (A.A.S.. axiom of congruency)

⇒ BE = CE (c.p.c.t.)

Example 11. *In the figure given alongside, AB = AD, ∠BAP = ∠QAD and ∠PAC = ∠CAQ. Prove that AP = AQ.*



Solution. Given ∠BAP = ∠QAD

and ∠PAC = ∠CAQ

⇒ ∠BAP + ∠PAC = ∠QAD + ∠CAQ

⇒ ∠BAC = ∠CAD

In Δs ABC and ADC, we have

1. AB = AD (given)
2. ∠BAC = ∠CAD (proved above)
3. AC = AC (common)

∴ ΔABC ≅ ΔADC (S.A.S. axiom of congruency)

⇒ ∠ABC = ∠ADC (c.p.c.t.)

In Δs ABP and ADQ, we have

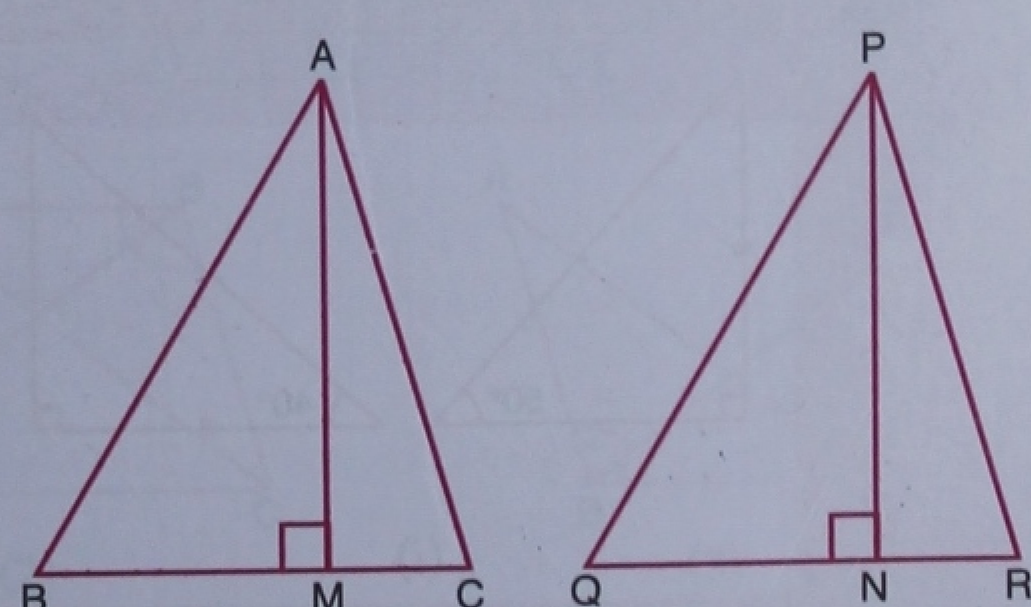
1. AB = AD (given)
2. ∠BAP = ∠DAD (given)
3. ∠ABC = ∠ADC (proved above)

∴ ΔABP ≅ ΔADQ (S.A.S. axiom of congruency)

⇒ AP = AQ (c.p.c.t.)

Example 12. *In the adjoining figure, two sides AB, AC and altitude AM of ΔABC are respectively equal to two sides PQ, PR and altitude PN of ΔPQR. Prove that*

ΔABC ≅ ΔPQR.



Solution. In ΔABM and ΔPQN,

But ∠AMB = ∠PNQ (each = 90°)

AB = PQ (given)

AM = PN (given)

∴ ΔABM ≅ ΔPQN (RHS axiom of congruency)

∴ BM = QN ...(*i*) (c.p.c.t.)

In ΔAMC and ΔPNR,

∠AMC = ∠PNR (each = 90°)

AC = PR (given)

AM = PN (given)

∴ ΔAMC ≅ ΔPNR (RHS axiom of congruency)

∴ MC = NR ...(*ii*) (c.p.c.t.)

From (*i*) and (*ii*), we get

BM + MC = QN + NR ⇒ BC = QR (from fig.)

In ΔABC and ΔPQR,

AB = PQ (given)

AC = PR (given)

BC = QR (proved above)

∴ ΔABC ≅ ΔPQR (SSS axiom of congruency)

Example 13. *In the Δ ABC given below, BD bisects ∠B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.*

Solution. In Δs ABD and CBD,

1. ∠ABD = ∠CBD [∵ BD bisects ∠B]
2. ∠ADB = ∠CDB [each being = 90°, ∵ BD ⊥ AC]
3. Side BD is common

∴ Δ ABD ≅ ΔCBD [A.S.A. axiom of congruency]

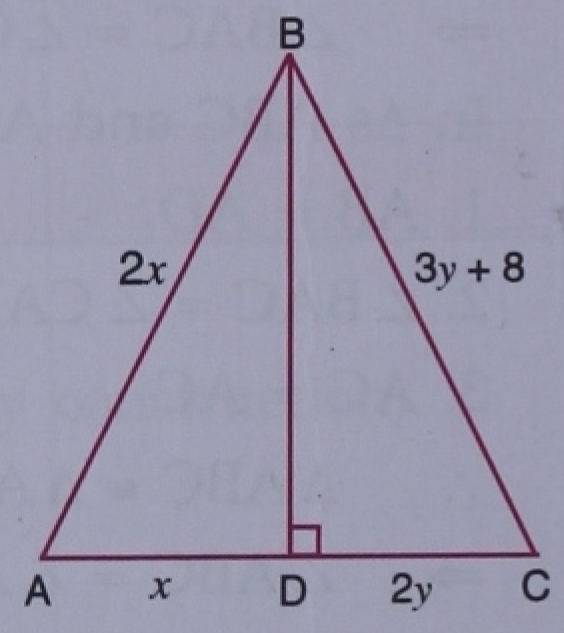
∴ AD = DC ⇒ $x = 2y$ ...(*i*)

and AB = BC ⇒ $2x = 3y + 8$ ...(*ii*)

Substituting the value of x from (*i*) in (*ii*), we get

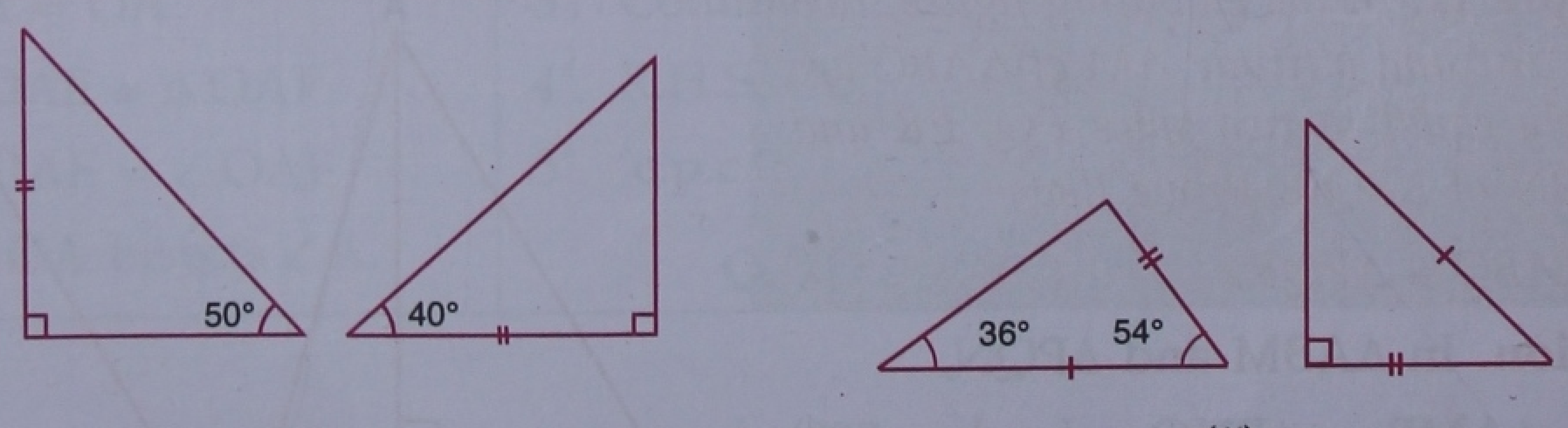
$4y = 3y + 8 \Rightarrow y = 8$, $\therefore x = 16$ [from (*i*)]

Hence $x = 16$, $y = 8$.

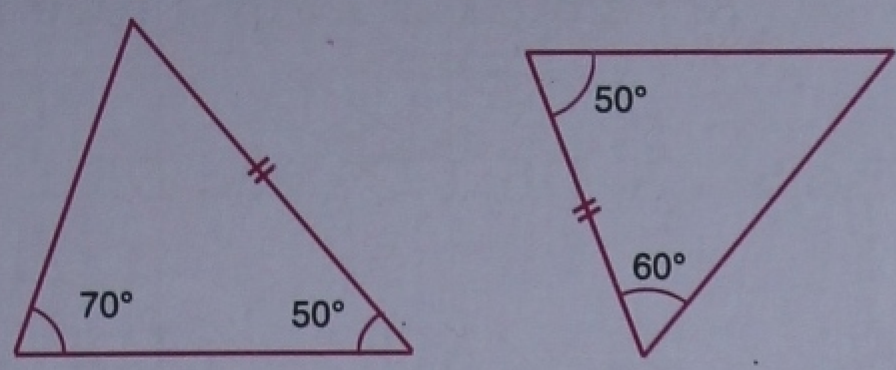


Exercise 12.1

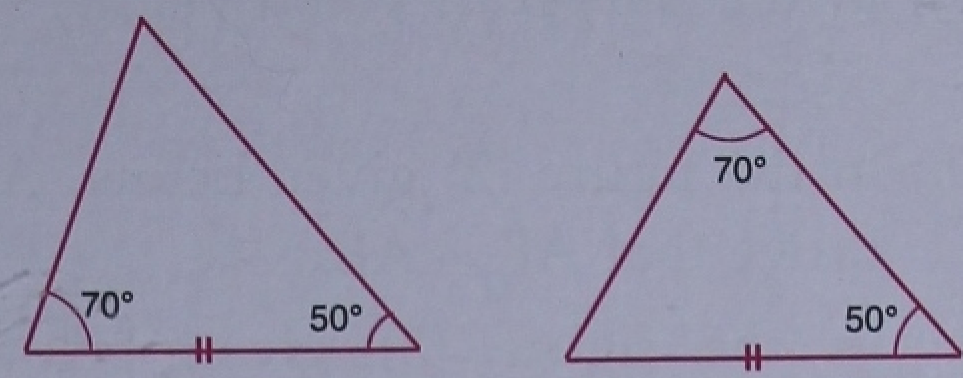
1. State, giving reasons, whether the following pairs of triangles are congruent or not.



(*i*) (*ii*)



(*iii*)



(*iv*)

2. Which of the following pairs of triangles are congruent ? Give reasons.

(*i*) Δ ABC : AB = 4 cm, BC = 5 cm, ∠ B = 70°
Δ PQR : QR = 4 cm, RP = 5 cm, ∠ R = 70°.

(*ii*) Δ ABC : AB = 4 cm, BC = 5 cm, ∠ B = 70°
Δ PQR : PQ = 4 cm, RP = 5 cm, ∠ R = 70°.

(*iii*) Δ ABC : BC = 6 cm, ∠ A = 90°, ∠ C = 50°
Δ PQR : QR = 6 cm, ∠ R = 50°, ∠ Q = 40°.

(*iv*) Δ ABC : AB = 5 cm, BC = 7 cm, CA = 9 cm
Δ PQR : PQ = 7 cm, QR = 5 cm, RP = 9 cm.

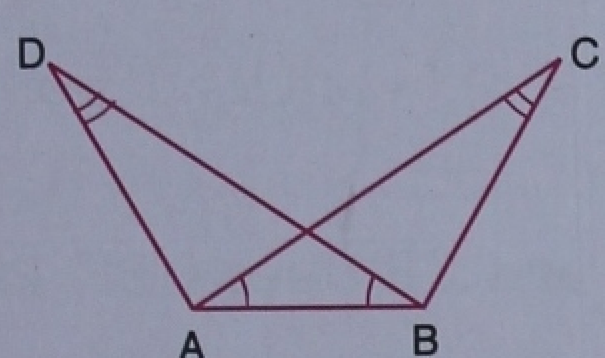
(*v*) Δ ABC : ∠ A = 90°, BC = 8 cm, AB = 5 cm
Δ PQR : ∠ P = 90°, QR = 8 cm, PR = 5 cm.

(*vi*) Δ ABC and Δ ADC in which AB = AD and BC = CD.

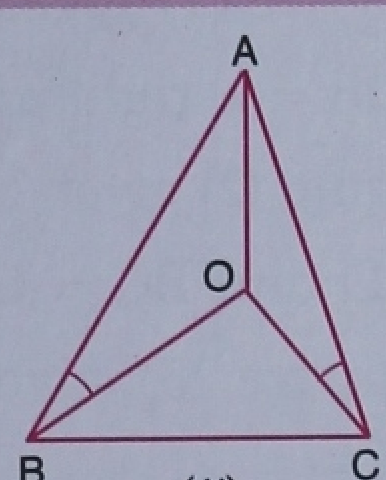
(*vii*) Δ ABC and Δ ABD in which AC = AD and ∠ ABC = ∠ ABD = 90°.

3. (*a*) In the figure (*i*) given below, ∠ C = ∠ D and ∠ CAB = ∠ ABD. Prove that AD = BC.

(*b*) In the figure (*ii*) given below, OA bisects ∠ A and ∠ OBA = ∠ OCA. Prove that Δ OAB ≅ Δ OAC.



(*i*)



(*ii*)

4. (*a*) In the figure (1) given below, AD = BC and BD = AC. Prove that

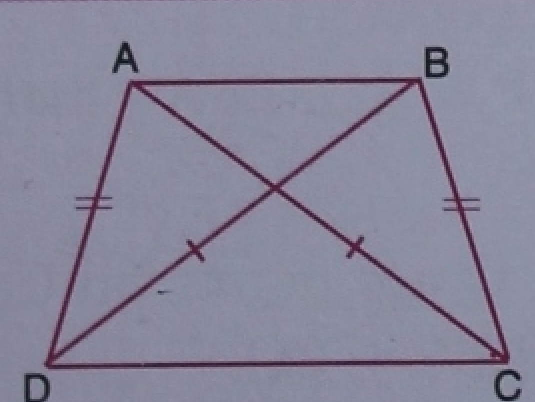
(*i*) ∠ ADB = ∠ BCA (*ii*) ∠ DAB = ∠ ABC.

(*b*) In the figure (2) given below, AD = BC and ∠ ABC = ∠ DAB. Prove that

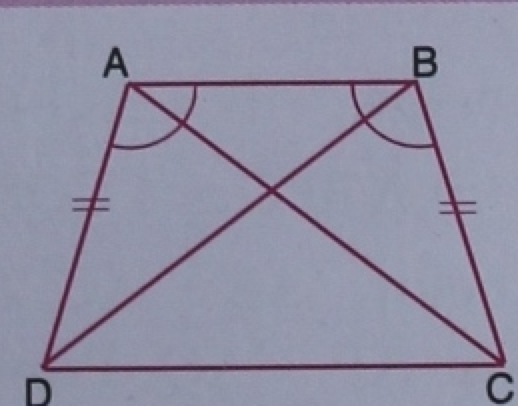
(*i*) AC = BD (*ii*) ∠ BAC = ∠ ABD.

(*c*) In the figure (3) given below,
AB = DC, ∠ACB = ∠CBD and ∠ABD = ∠ACD. Prove that

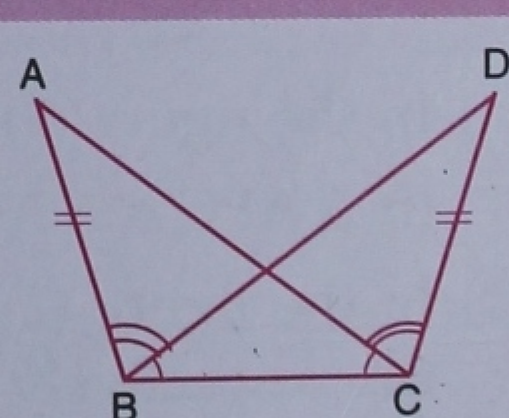
(*i*) AC = BD (*ii*) ∠CAB = ∠BDC.



(1)



(2)

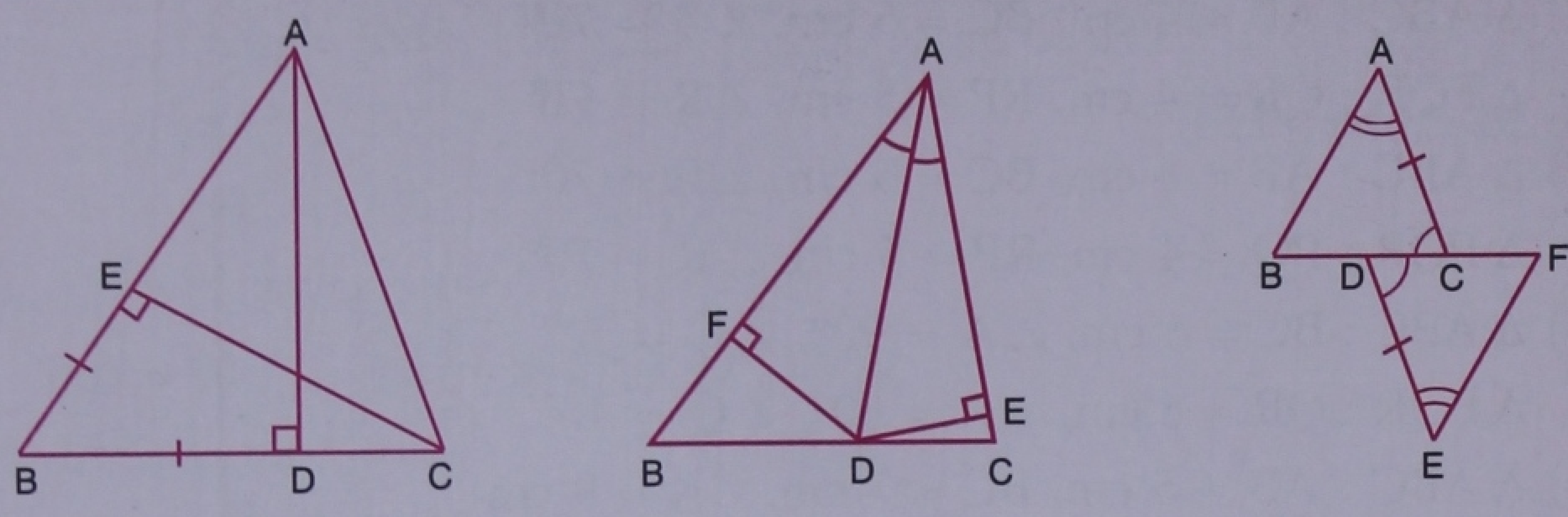


(3)

5. (*a*) In the figure (1) given below, AB = BC, AD ⊥ BC, CE ⊥ AB. Prove that AD = CE.

(*b*) In the figure (2) given below, AD bisects ∠A, DE ⊥ CA and DF ⊥ AB. Prove that AF = AE.

(*c*) In the figure (3) given below, AC = DE, ∠ACB = ∠EDF and ∠BAC = ∠DEF. Prove that AB = EF.

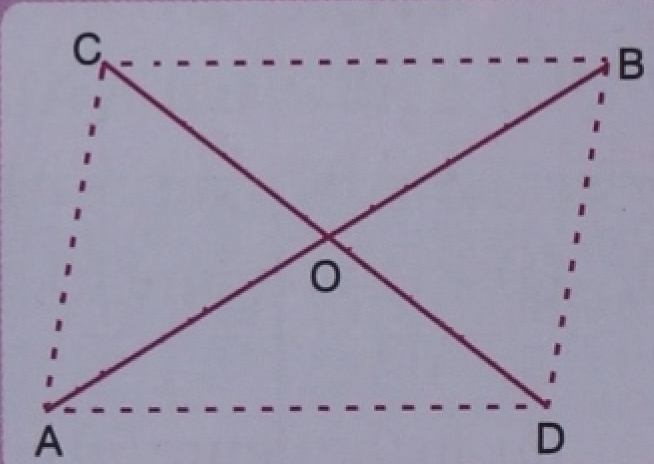


(1) (2) (3)

6. Two line segments AB and CD bisect each other at O. Prove that

(*i*) AC = BD (*ii*) ∠CAB = ∠ABD

(*iii*) AD ∥ CB (*iv*) AD = CB.

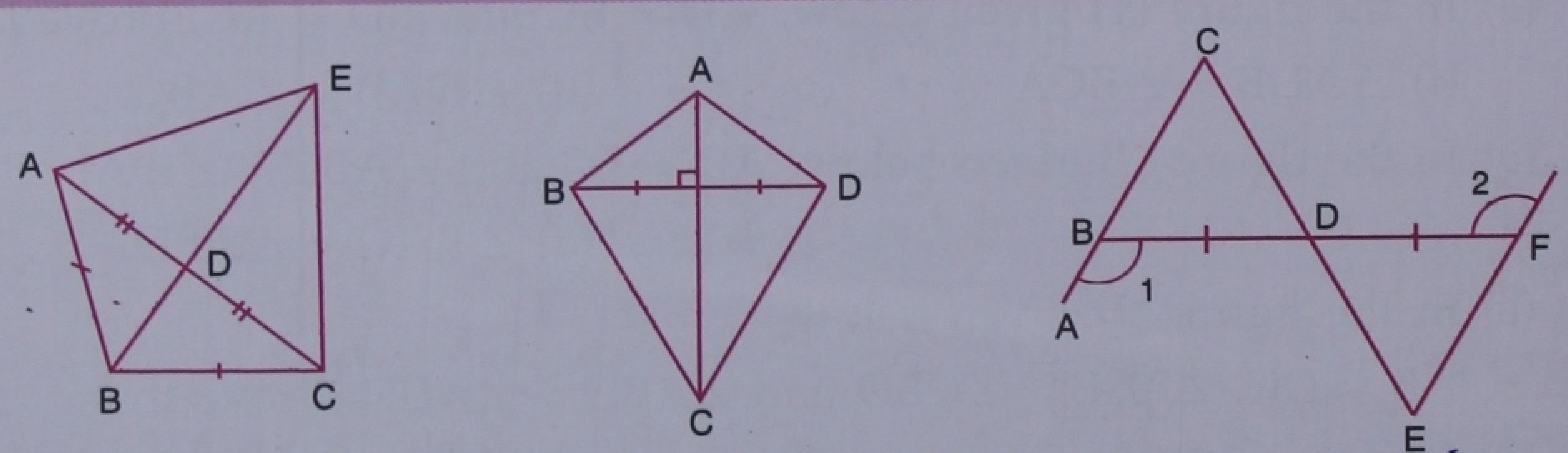


7. (*a*) In the figure (1) given below, AB = BC, AD = CD. Prove that

(*i*) ∠ADE = a right angle (*ii*) AE = EC.

(*b*) In the figure (2) given below, AC bisects BD at right angles. Prove that AB = AD and BC = CD.

(*c*) In the figure (3) given below, ∠1 = ∠2 and CE bisects BF. Prove that ∠C = ∠E.



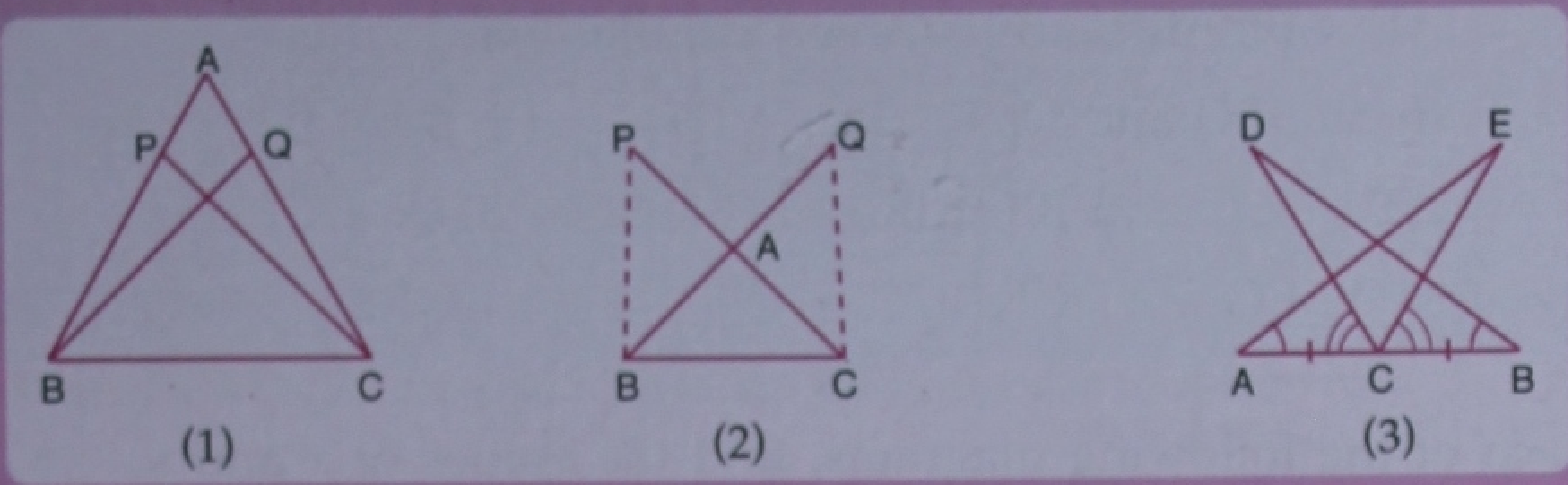
(1) (2) (3)

8. (*a*) In the figure (1) given below, AB = AC and AP = AQ. Prove that

(*i*) Δ APC ≅ Δ AQB (*ii*) Δ BPC ≅ Δ CQB.

(*b*) In the figure (2) given below, AB = AC, P and Q are points on CA and BA respectively such that AP = AQ. Prove that BP = CQ.

(*c*) In the figure (3) given below, AC = CB, ∠A = ∠B and ∠ECB = ∠ACD. Prove that (*i*) Δ BCD ≅ Δ ACE (*ii*) AE = BD.

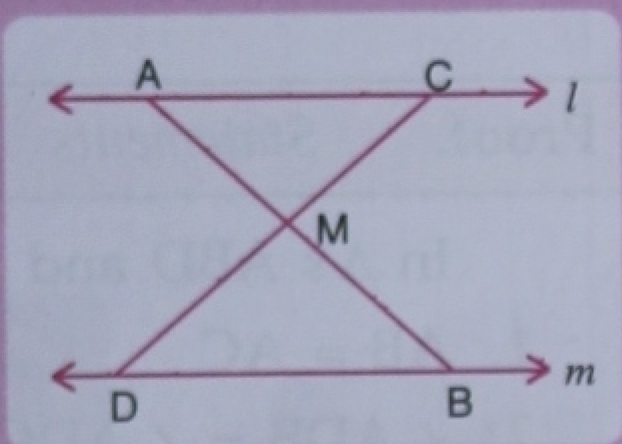


9. AD is median of Δ ABC and PM is median of Δ PQR. If AB = PQ, BC = QR and AD = PM, prove that Δ ABC ≅ Δ PQR.

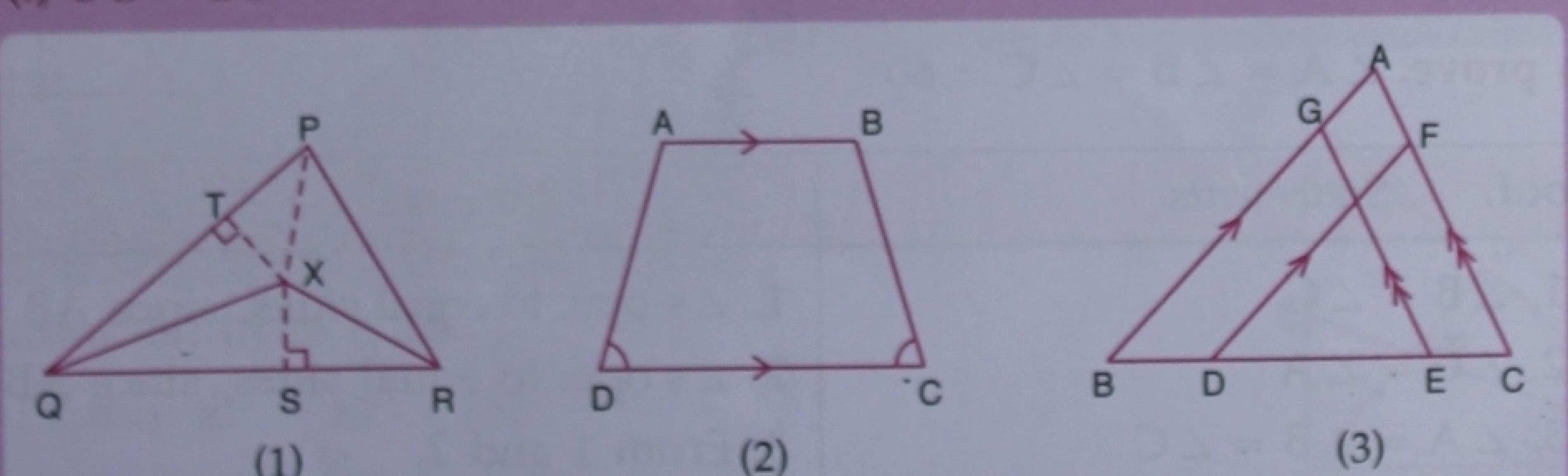
Hint

BC = QR ⇒ 2BD = 2 QM ⇒ BD = QM. Show that Δ ABD ≅ Δ PQM ⇒ ∠B = ∠D.

10. If all the altitudes of a triangle are equal, prove that it is an equilateral triangle.
11. Prove that the perpendiculars drawn from any point on the internal bisector of an angle, to the arms of the angle, are equal.
12. In a triangle ABC, AB = AC. Prove that the altitude AD is also a median.
13. AB is a line segment. AX and BY are equal line segments drawn on opposite sides of line segment AB such that AX ∥ BY. If AB and XY intersect each other at P, prove that
(*i*) Δ APX ≅ Δ BPY (*ii*) AB and XY bisect each other at P.
14. In the adjoining figure, $l \parallel m$ and M is mid-point of line segment AB, where A and B are any points on l and m respectively. Prove that M is also mid-point of any other line segment CD, where C and D are points on l and m respectively.



15. (*a*) In the figure (1) given below, QX, RX are bisectors of angles PQR and PRQ respectively of Δ PQR. If XS ⊥ QR and XT ⊥ PQ, prove that
(*i*) Δ XTQ ≅ Δ XSQ (*ii*) PX bisects the angle P.
(*b*) In the figure (2) given below, AB ∥ DC and ∠C = ∠D. Prove that
(*i*) AD = BC (*ii*) AC = BD.
(*c*) In the figure (3) given below, BA ∥ DF and CA ∥ EG and BD = EC. Prove that
(*i*) BG = DF (*ii*) EG = CF.

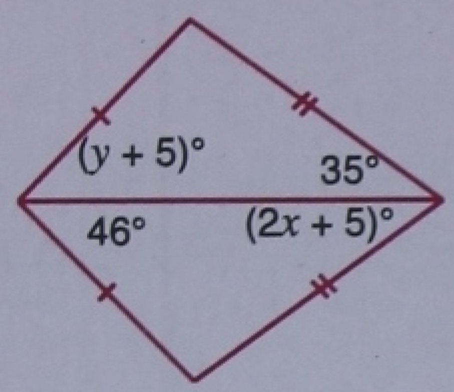


Hint

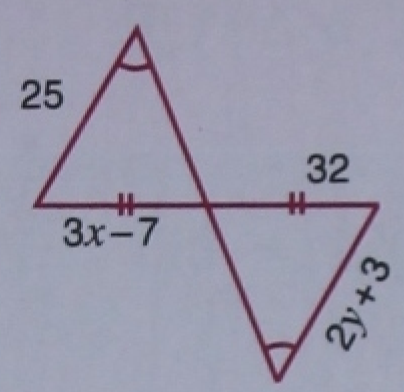
(*b*) Draw AE ⊥ CD, BF ⊥ CD.
AE = BF (distance between parallel lines).

(*c*) In Δs BEG and DCF, ∠B = ∠D (∵ BA ∥ DF, corres. ∠s equal),
∠E = ∠C and BE = BC − EC = BC − BD = DC
⇒ ΔBEG ≅ ΔDCF.

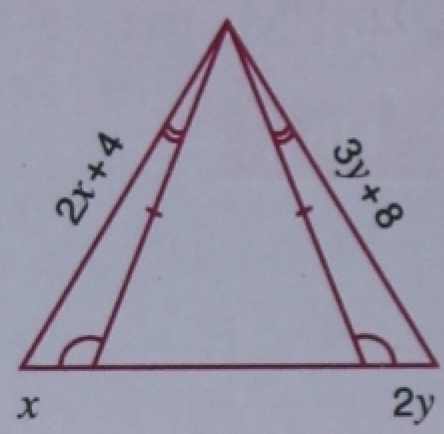
16. In each of the following diagrams, find the values of x and y.



(*i*)

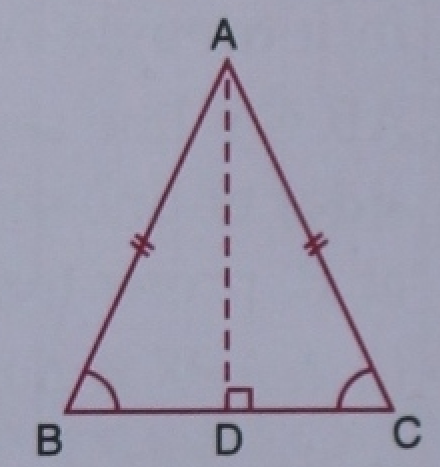


(*ii*)



(*iii*)

12.3 ISOSCELES TRIANGLES



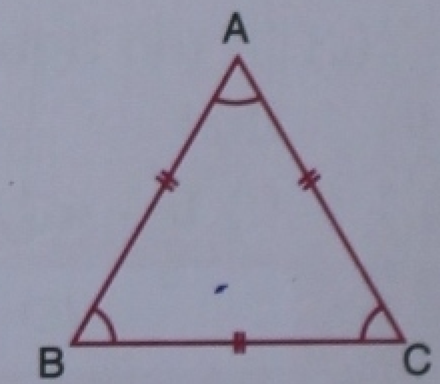
Theorem 1. *If two sides of a triangle are equal, then the angles opposite to them are also equal.*

Given. A triangle ABC, AB = AC.

To prove. ∠B = ∠C.

Construction. From A, draw AD ⊥ BC.

Proof. *Statements*	*Reasons*
In Δs ABD and ACD	
1. AB = AC	1. Given.
2. ∠ADB = ∠ADC	2. Each = 90°, since AD ⊥ BC.
3. AD = AD	3. Common.
4. Δ ABD ≅ Δ ACD	4. R.H.S. (Axiom of congruency).
5. ∠B = ∠C.	5. 'c.p.c.t.'
Q.E.D.	



Corollary. *In an equilateral triangle, each angle is 60°.*

Given. A triangle ABC,
AB = BC = CA.

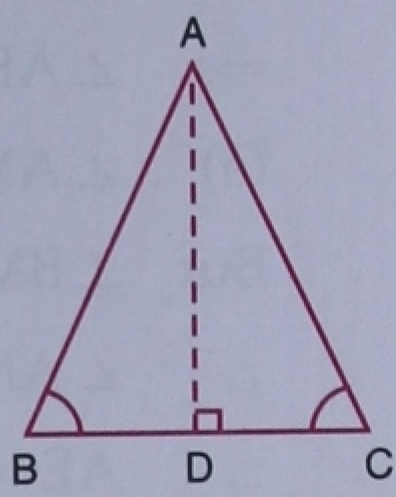
To prove. ∠A = ∠B = ∠C = 60°.

Proof. *Statements*	*Reasons*
1. ∠B = ∠C	1. ∠s opp. to equal sides, since AB = AC.
2. ∠C = ∠A	2. ∠s opp. to equal sides, since AB = BC.
3. ∠A = ∠B = ∠C	3. From 1 and 2.

4. $\angle A + \angle B + \angle C = 180°$	4. Angles of a triangle.
5. $3\angle A = 180°$	5. From 3 and 4.
$\Rightarrow \angle A = 60°$.	
Hence $\angle A = \angle B = \angle C = 60°$.	
Q.E.D.	

Theorem 2. (*Converse of theorem 1*)

If two angles of a triangle are equal, then the sides opposite to them are also equal.



Given. A triangle ABC, $\angle B = \angle C$.

To prove. AC = AB.

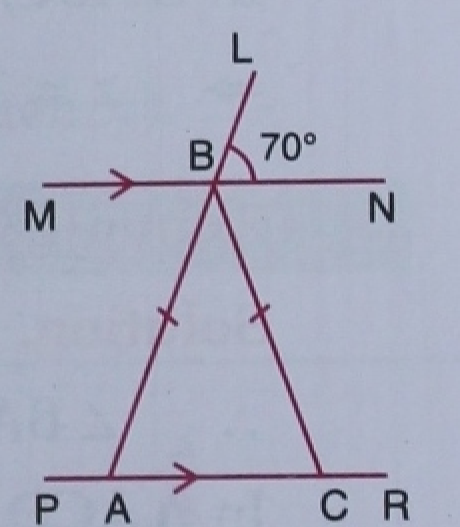
Construction. From A, draw $AD \perp BC$.

Proof. *Statements*	*Reasons*
In Δs ABD and ACD	
1. $\angle B = \angle C$	1. Given.
2. $\angle ADB = \angle ADC$	2. Each = 90°, since $AD \perp BC$.
3. AD = AD	3. Common.
4. $\Delta ABD \cong \Delta ACD$	4. A.A.S. (Axiom of congruency).
5. AC = AB.	5. 'c.p.c.t.'
Q.E.D.	

Corollary. *Every equiangular triangle is equilateral.*

ILLUSTRATIVE EXAMPLES

Example 1. *In the adjoining figure, MN is parallel to PR, $\angle LBN = 70°$ and AB = BC. Find the value of $\angle ABC$.*



Solution. $\angle BAC = \angle LBN = 70°$ ...(*i*)

[$\because$ MN ∥ PR, corres. $\angle$s are equal]

$\therefore \angle ACB = \angle BAC = 70°$ ...(*ii*)

[$\because$ AB = BC, angles opp. to equal sides are equal]

In ΔABC, $\angle ABC + \angle BAC + \angle ACB = 180°$

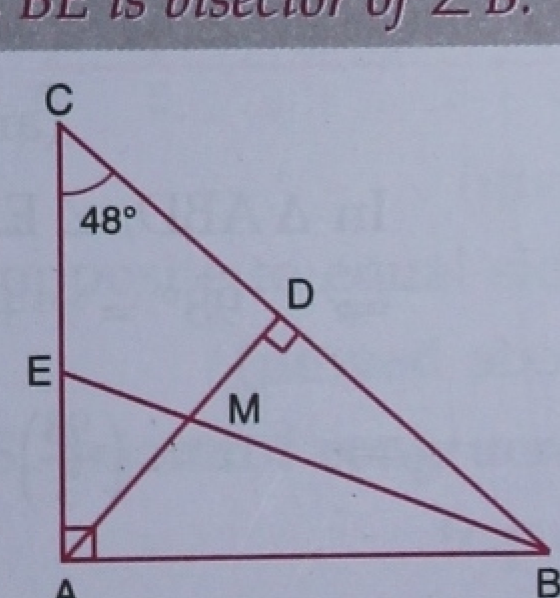
$\Rightarrow \angle ABC + 70° + 70° = 180°$ [Using (*i*) and (*ii*)]

$\Rightarrow \angle ABC = 180° - 140° = 40°$.

Example 2. *In the adjoining figure, $\angle A = \angle D = 90°$, $\angle C = 48°$, BE is bisector of $\angle B$. AD and BE intersect at M.*

(a) Calculate (i) $\angle BMD$ (ii) $\angle AEM$.

(b) Prove that AE = AM.



Solution. (*a*) (*i*) In ΔABC,

$\angle B + 90° + 48° = 180°$

$\Rightarrow \quad \angle B = 180° - 138° = 42°$.

As BE bisects $\angle B$, $\angle MBD = \frac{1}{2} \angle B = \frac{1}{2} . 42° = 21°$.

In Δ BMD, $\angle BMD + 90° + 21° = 180°$

$\Rightarrow \quad \angle BMD = 180° - 111° = 69°$.

(*ii*) As BE bisects $\angle B$, $\angle ABE = \angle MBD = 21°$.

In Δ ABE, $\angle AEB + 90° + 21° = 180°$

$\Rightarrow \quad \angle AEM = 180° - 111° = 69°$.

(*b*) $\quad \angle AME = \angle BMD$ (vert. opp. $\angle$s)

But $\quad \angle BMD = 69°$ (obtained above)

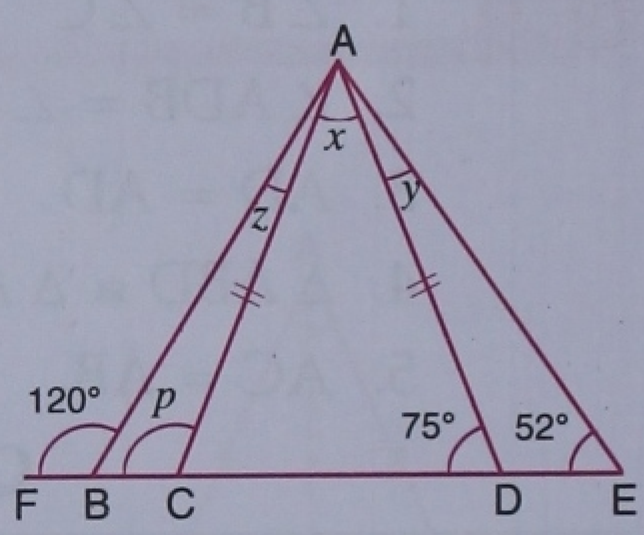
$\therefore \quad \angle AME = \angle AEM$ (each = 69°)

$\therefore \quad$ AE = AM (sides opp. to equal $\angle$s are equal)

Example 3. *Find the measure of each lettered angle in the adjoining figure.*

Solution. In Δ ACD, $\angle ACD = \angle ADC$

(angles opp. equal sides are equal)



$\Rightarrow \quad \angle ACD = 75°$ ($\because \angle ADC = 75°$ given)

$\therefore \quad x + 75° + 75° = 180°$ (angles of Δ ACD)

$\Rightarrow \quad x = 180° - 75° - 75° = 30°$.

In Δ ADE, $75° = y + 52°$

(ext. $\angle$ = sum of two opp. int. $\angle$s)

$\Rightarrow \quad y = 75° - 52° = 23°$.

As BCD is a st. line, $p + \angle ACD = 180°$

$\Rightarrow \quad p + 75° = 180° \Rightarrow p = 105°$.

In Δ ABC, $120° = z + p$ (ext. $\angle$ = sum of two opp. int. $\angle$s)

$\Rightarrow \quad z = 120° - p = 120° - 105° = 15°$.

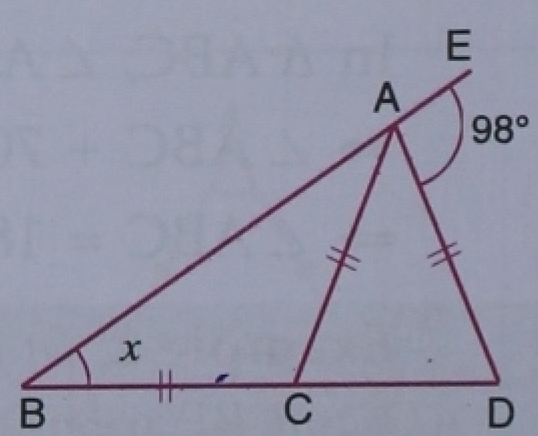
Example 4. *From the adjoining diagram, find the value of x.*

Solution. In Δ ABC, BC = AC (given)

$\therefore \quad \angle BAC = x$ (angles opp. equal sides are equal)

In Δ ACD, $\angle ACD = x + x$

(ext. $\angle$ = sum of two opp. int. $\angle$s)



$\Rightarrow \quad \angle ACD = 2x$.

In Δ ACD, AC = AD (given)

$\therefore \quad \angle ADC = 2x$

(angles opp. equal sides are equal)

In Δ ABD, $\angle EAD = \angle ABD + \angle ADB$ (ext. $\angle$ = sum of two opp. int. $\angle$s)

$\Rightarrow \quad 98° = x + 2x \Rightarrow 3x = 98°$

$\Rightarrow \quad x = \left(\frac{98}{3}\right)^{\circ} = 32°40'$.

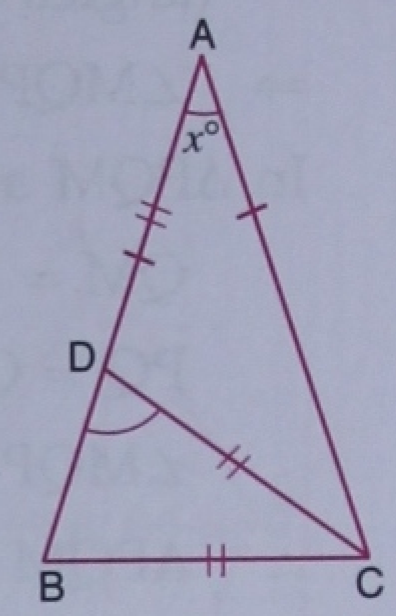
Example 5. *In ΔABC, $AB = AC$ and D is a point on AB such that $AD = DC = BC$. Show that $\angle BAC = 36°$.*

Solution. Let $\angle BAC = x°$.

In ΔADC, $AD = DC$, $\therefore \angle ACD = x°$.

$\therefore \quad \angle BDC = x° + x°$ (sum of two opp. int. $\angle$s)

$= 2x°$.



In ΔBDC, $DC = BC$, $\therefore \angle DBC = 2x°$.

In ΔABC, $AB = AC$, $\therefore \angle ACB = 2x°$.

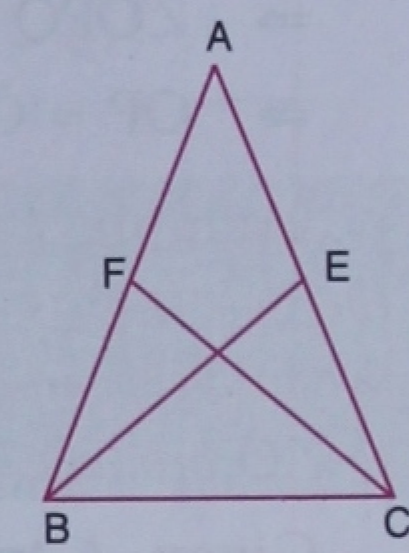
Also $\angle BAC + \angle ABC + \angle ACB = 180°$

(angles of a triangle)

$\Rightarrow \quad x° + 2x° + 2x° = 180° \Rightarrow 5x = 180 \Rightarrow x = 36$.

$\therefore \quad \angle BAC = 36°$, as required.

Example 6. *Prove that the medians on the equal sides of an isosceles triangle are equal.*

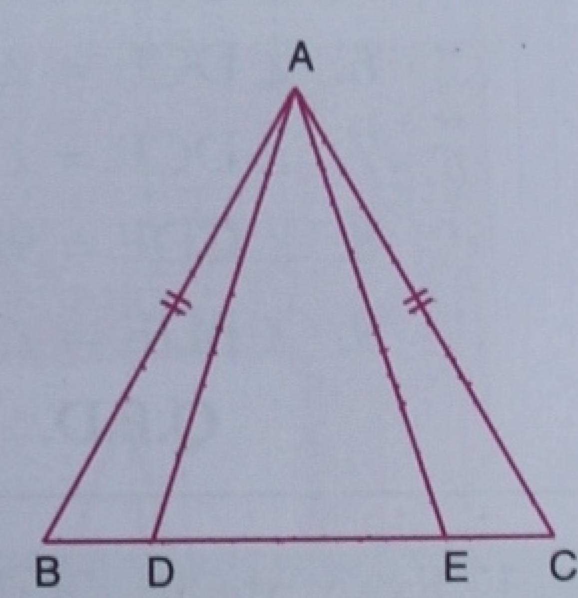


Given. A triangle ABC, $AB = AC$. BE is median to AC and CF is median to AB.

To prove. $BE = CF$.

Proof. *Statements*	*Reasons*
In Δs BCE and CBF	
1. $CE = BF$	1. Halves of equal sides AB, AC.
2. $\angle BCE = \angle CBF$	2. Angles opp. to equal sides.
3. $BC = BC$	3. Common.
4. $\Delta BCE \cong \Delta CBF$	4. S.A.S. (Axiom of congruency).
5. $BE = CF$	5. 'c.p.c.t.'
Q.E.D.	

Example 7. *In the figure given alongside, $AB = AC$, D and E are point on BC such that $BE = DC$. Prove that $AD = AE$.*



Solution. Given $BE = DC$

$\Rightarrow \quad BE - DE = DC - DE$

$\Rightarrow \quad BD = EC$.

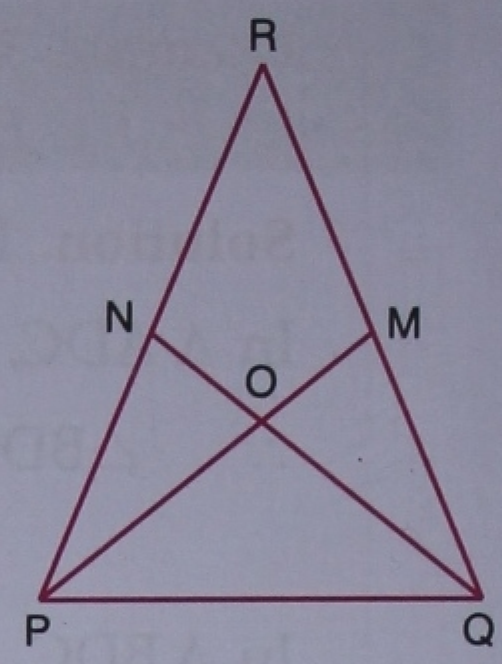
In Δs ABD and ACE, we have

1. $AB = AC$ (given)
2. $\angle B = \angle C$ (angles opposite to equal sides)
3. $BD = EC$ (proved above)

$\therefore \quad \Delta ABD \cong \Delta ACE$ (S.A.S. axiom of congruency)

$\Rightarrow \quad AD = AE$ (c.p.c.t.)

Example 8. *In the adjoining figure, RP = RQ and M, N are points on the sides RQ, RP respectively of ΔPQR such that QM = PN. Prove that OP = OQ, where O is the point of intersection of PM and QN.*



Solution. Given RP = RQ

⇒ ∠RQP = ∠RPQ

(angles opposite to equal sides are equal)

⇒ ∠MQP = ∠NPQ.

In ΔPQM and ΔQPN,

QM = PN (given)

PQ = QP (common)

∠MQP = ∠NPQ (proved above)

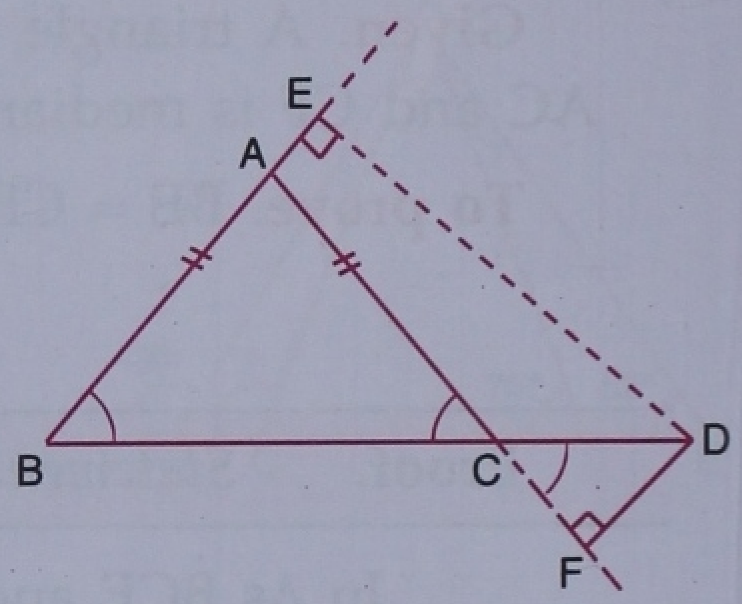
∴ ΔPQM ≅ ΔQPN (SAS axion of congruency)

∴ ∠MPQ = ∠NQP (c.p.c.t.)

⇒ ∠OPQ = ∠OQP

⇒ OP = OQ.

Example 9. *In a Δ ABC, the sides AB, AC are equal and the base BC is produced to any point D. From D, DE is drawn perpendicular to BA produced and DF perpendicular to AC produced. Prove that BD bisects ∠ EDF.*

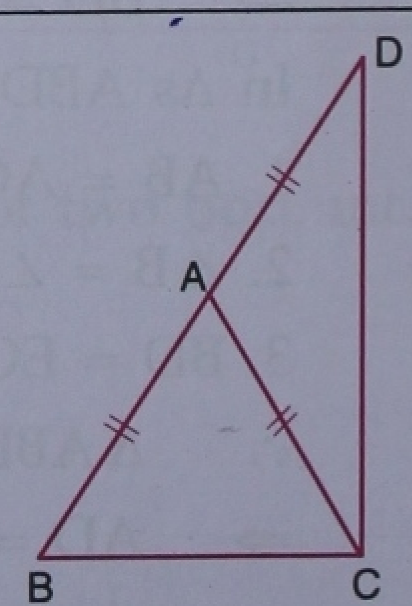


Given. A triangle ABC, AB = AC, D is a point on BC produced. DE ⊥ BA (produced) and DF ⊥ AC (produced).

To prove. ∠ BDE = ∠ CDF.

Proof. *Statements*	*Reasons*
1. ∠ B = ∠ C	1. AB = AC.
2. ∠ BED = 90°	2. DE ⊥ BA.
3. ∠ BDE = 90° − ∠ B	3. In Δ BED, ∠ BDE + ∠ B + 90° = 180°.
4. ∠ CFD = 90°	4. DF ⊥ AC.
5. ∠ CDF = 90° − ∠ DCF	5. In Δ CFD, ∠ CDF + ∠ DCF + 90° = 180°.
6. ∠ DCF = ∠ C	6. Vert. opp. ∠ s.
7. ∠ DCF = ∠ B	7. From 1, ∠ C = ∠ B.
8. ∠ CDF = 90° − ∠ B	8. From 5 and 7.
9. ∠ BDE = ∠ CDF	9. From 3 and 8.
Q.E.D.	

Example 10. *In the adjoining figure, AB = AC = AD. Prove that ∠ BCD is a right angle.*



Solution. ∠ ACB = ∠ ABC ...(*i*)

(AB = AC, angles opposite equal sides are equal)

∠ ACD = ∠ ADC ...(*ii*)

(AC = AD, angles opp. equal sides are equal)

Adding (*i*) and (*ii*)

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \quad \angle BCD = \angle ABC + \angle ADC$ ...(*iii*)

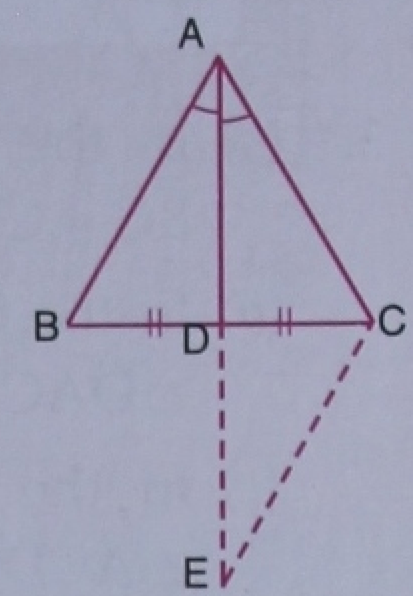
But $\angle BCD + \angle ABC + \angle ADC = 180°$ (sum of angles of Δ BCD)

$\Rightarrow \quad \angle BCD + \angle BCD = 180°$ (using (*iii*))

$\Rightarrow \quad 2\angle BCD = 180° \Rightarrow \angle BCD = 90°$.

Hence $\angle BCD$ is a right angle.

Example 11. *If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.*



Given. A traingle ABC, AD is bisector of $\angle A$ and D is mid-point of BC.

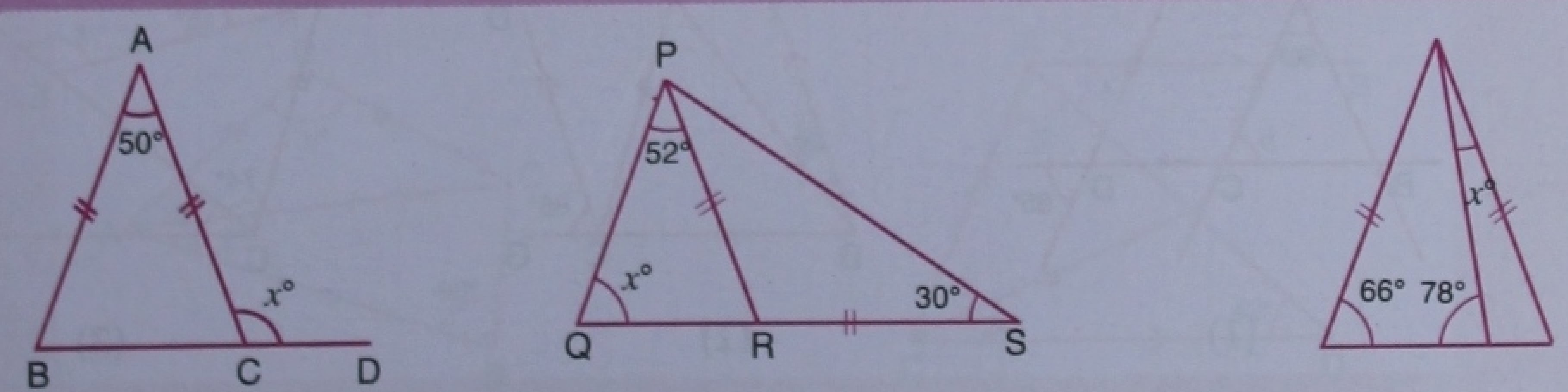
To prove. AB = AC

Construction. Produce AD to a point E such that DE = AD. Join C and E.

Proof. *Statements*	*Reasons*
In Δs ABD and EDC	
1. BD = DC	1. Given.
2. AD = DE	2. By construction.
3. $\angle ADB = \angle EDC$	3. Vert. opp. $\angle$s
4. $\Delta ABD \cong \Delta EDC$	4. S.A.S. (axiom of congruency)
5. AB = EC	5. c.p.c.t.
6. $\angle BAD = \angle DEC$	6. c.p.c.t.
7. $\angle BAD = \angle DAC$	7. AD is bisector of $\angle A$ (given)
8. $\angle DEC = \angle DAC$	8. From 6 and 7.
9. AC = EC	9. Side opp. equal angles are equal.
10. AB = AC	10. From 5 and 9.
Q.E.D.	

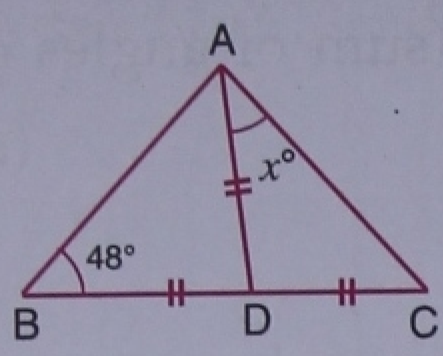
Exercise 12.2

1. In the following diagrams, find the value of x :

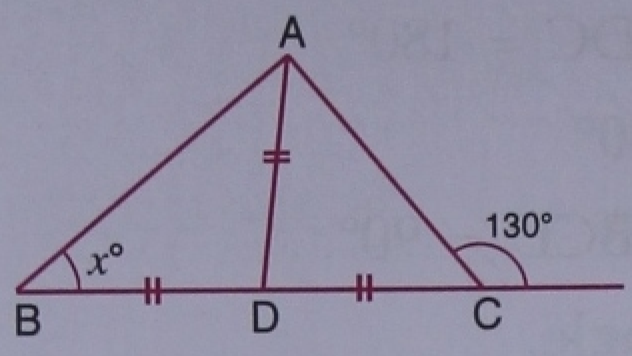


(*i*) (*ii*) (*iii*)

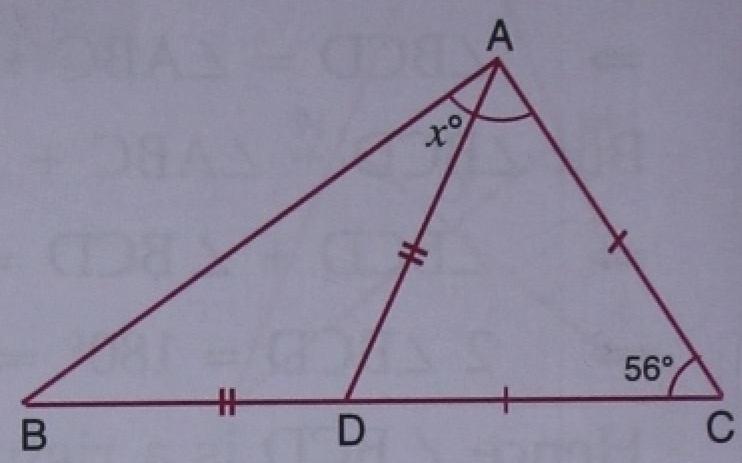
2. In the following diagrams, find the value of x :



(*i*)



(*ii*)

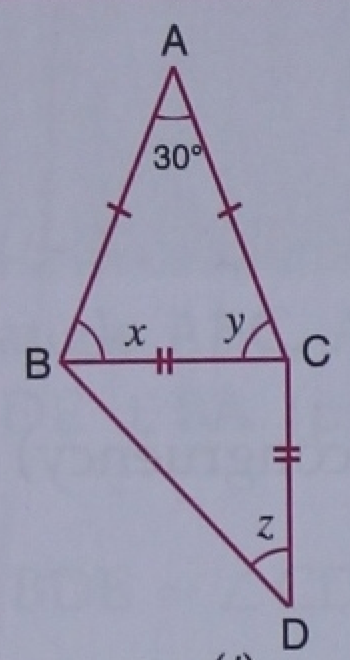


(*iii*)

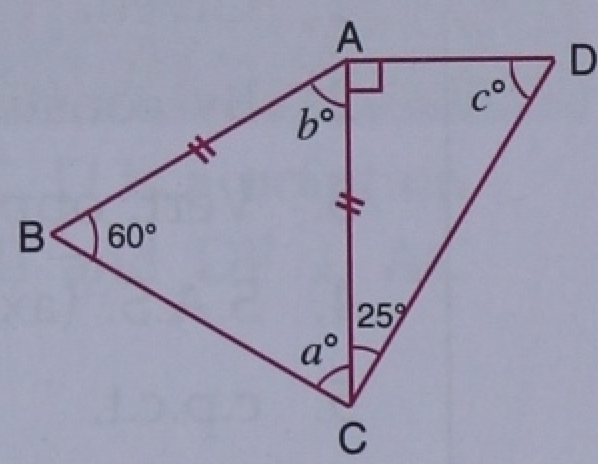
3. (*a*) In the figure (*i*) given below, ∠BAC = 30°, ∠BCD = 90°, AB = AC, BC = CD. Find the values of x, y and z.

(*b*) In the figure (*ii*) given below, AB = AC, ∠ABC = 60°, ∠ACD = 25° and ∠DAC is a right angle. Calculate the values of a, b and c.

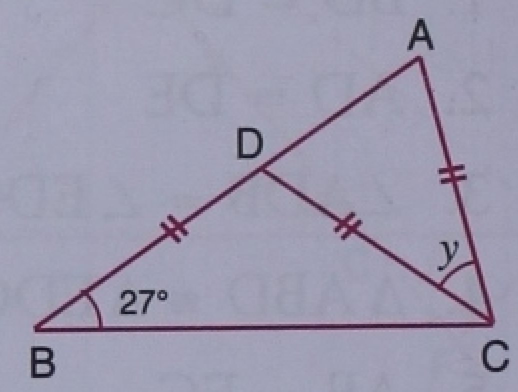
(*c*) In the figure (*iii*) given below, BD = CD = CA, ∠ABC = 27° and ∠ACD = y. Find the value of y. No proof is required but the essential steps of working must be shown.



(*i*)



(*ii*)

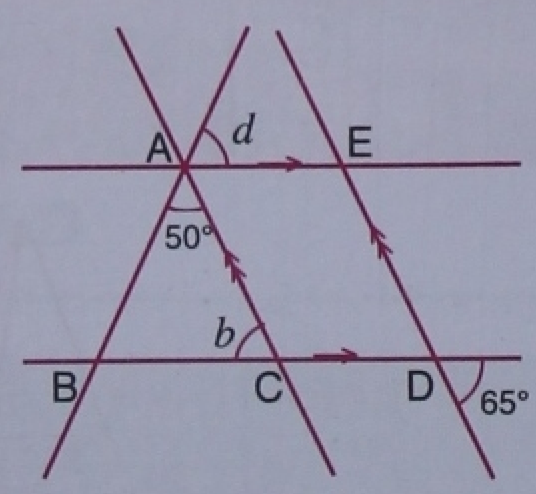


(*iii*)

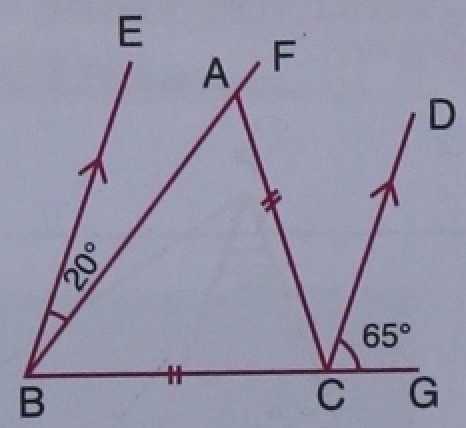
4. (*a*) In the figure (1) given below, AE ∥ BD, CA ∥ DE. Find ∠b and ∠d. Is AB = AC? No proof is required but the essential steps of working must be shown.

(*b*) In the figure (2) given below, BE is parallel to CD, CA = BC, ∠ABE = 20° and ∠DCG = 65°. Calculate ∠FAC.

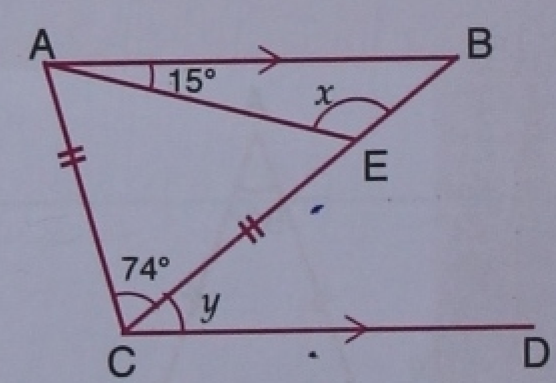
(*c*) In the figure (3) given below, AB ∥ CD and CA = CE. If ∠ACE = 74° and ∠BAE = 15°, find the values of x and y.



(1)



(2)

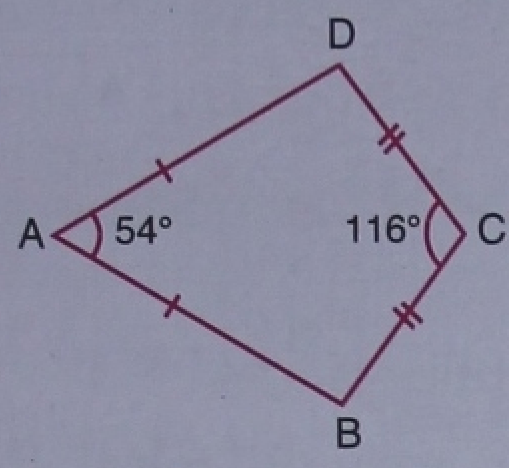


(3)

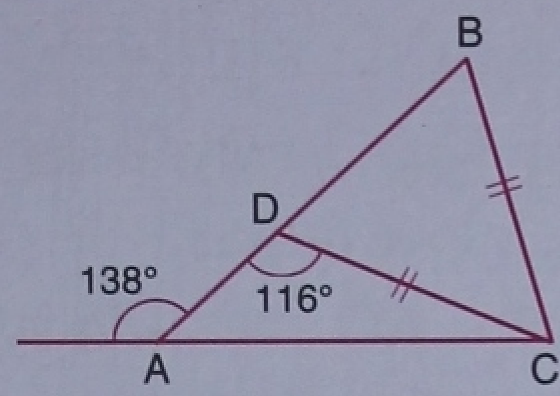
5. (*a*) In the figure (1) given below, AB = AD, BC = DC. Find ∠ABC.

(*b*) In the figure (2) given below, BC = CD. Find ∠ACB.

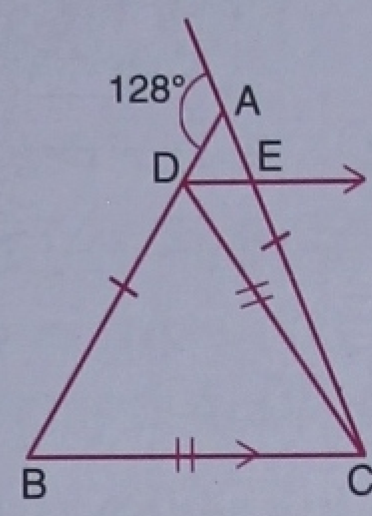
(*c*) In the figure (3) given below, AB = AC, BC = CD and DE ∥ BC. Find ∠CDE and ∠DCE.



(1)



(2)



(3)

Hint

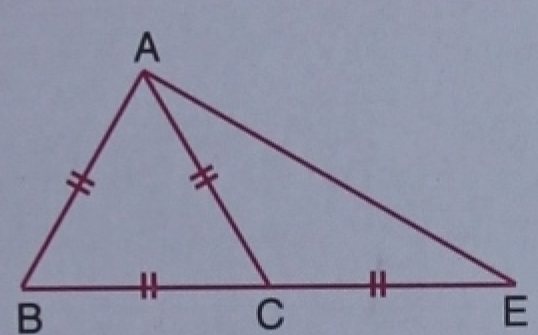
(*a*) Join BD.

(*b*) ∠ACD = 22° (why?), ∠BCD + $(90° - \frac{1}{2}$ ∠BCD$)$ = 116° (why ?)

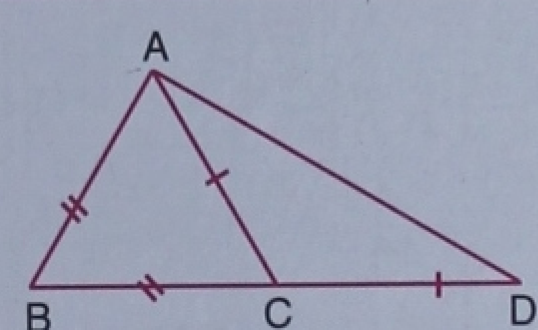
6. In Δ ABC, AB = AC, ∠A = $(5x + 20)°$ and each of the base angle is $\frac{2}{5}$ th of ∠A. Find the measure of ∠A.

7. (*a*) In the figure (1) given below, ABC is an equilateral triangle. Base BC is produced to E, such that BC = CE. Calculate ∠ACE and ∠AEC.

(*b*) In the figure (2) given below, prove that ∠BAD : ∠ADB = 3 : 1.



(1)

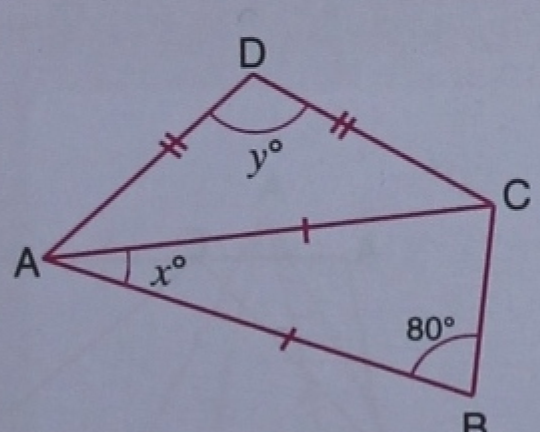


(2)

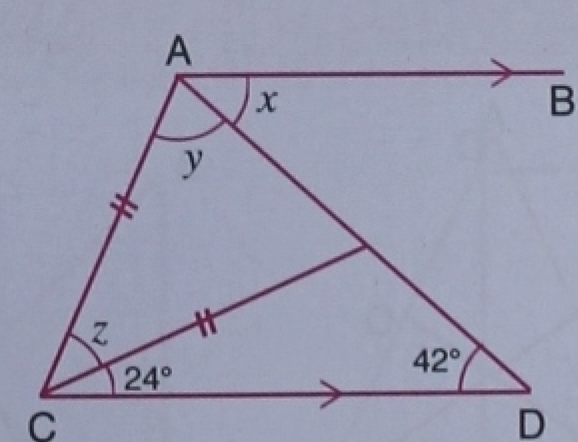
8. ABC is an equilateral triangle, BC is produced to D such that BC = CD. Prove that AD ⊥ AB.

9. (*a*) In the figure (1) given below, AC is bisector of ∠A. Find the values of x and y.

(*b*) In the figure (2) given below, AB ∥ CD. Find the values of x, y and z.



(1)



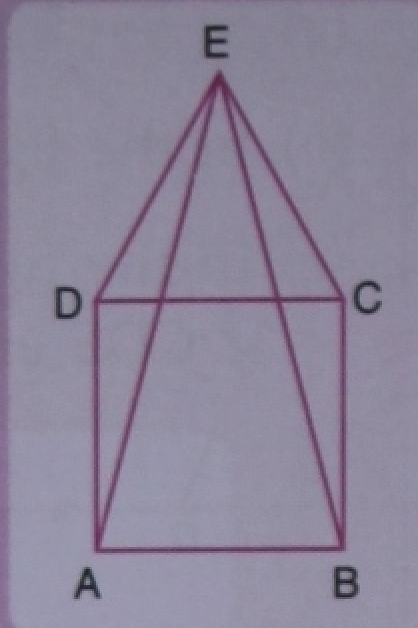
(2)

10. In the figure given alongside, ABCD is a square and EDC is an equilateral triangle. Prove that :

(i) $\Delta ADE \cong \Delta BCE$

(ii) AE = BE

(iii) $\angle DEA = 15°$.



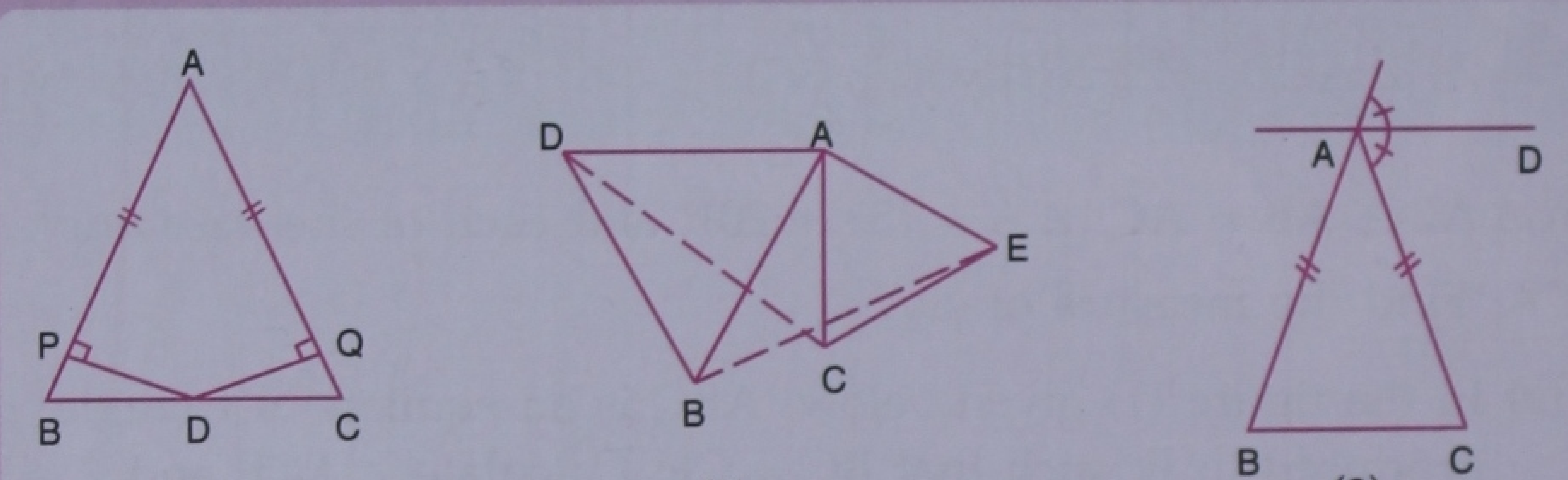
11. (a) In the figure (1) given below, AB = AC and D is mid-point of BC. $DP \perp AB$ and $DQ \perp AC$. Prove that

(i) DP = DQ (ii) AP = AQ (iii) AD bisects $\angle A$.

(b) In the figure (2) given below, Δ s ABD and ACE are equilateral. Prove that

(i) $\angle CAD = \angle BAE$ (ii) CD = BE.

(c) In the figure (3) given below, AB = AC and AD bisects exterior angle at A. Prove that AD is parallel to BC.



(1) (2) (3)

Hint

(b) (i) $\angle CAD = \angle CAB + 60° = \angle BAE$.

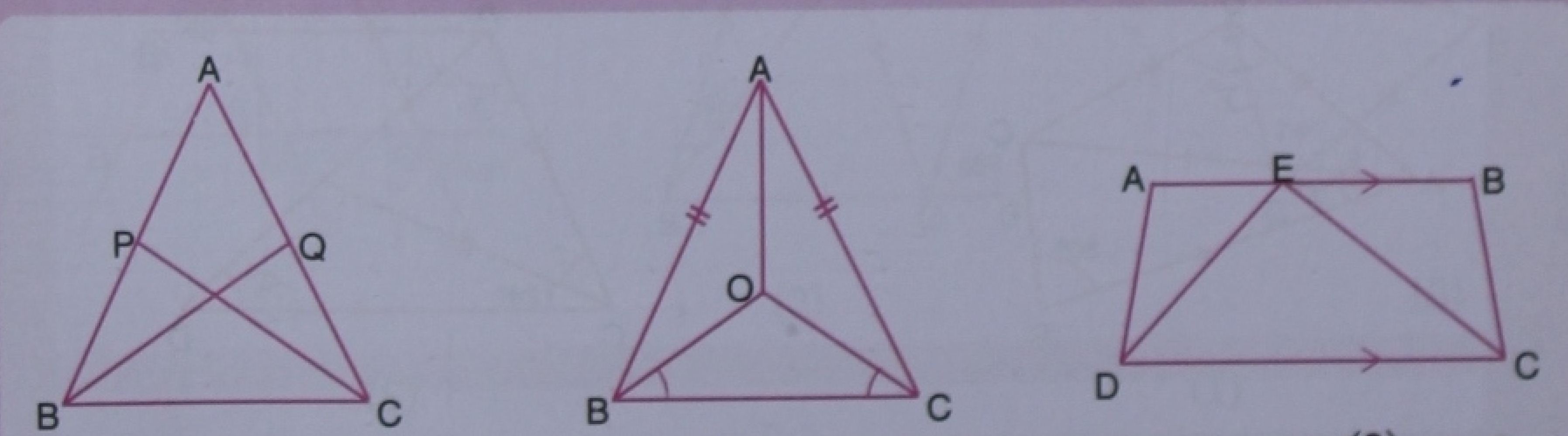
(ii) In Δs ACD and ABE, $\angle CAD = \angle BAE$ (from above) AD = AB and AC = AE

$\Rightarrow \Delta ACD \cong \Delta ABE$.

12. (a) In the figure (1) given below, AB = AC and AP = AQ. Prove that BQ = CP.

(b) In the figure (2) given below, AB = AC. O is a point in the interior of ΔABC such that $\angle OBC = \angle OCB$. Prove that AO bisects $\angle A$.

(c) In the figure (3) given below, AB || CD. CE and DE bisects $\angle BCD$ and $\angle ADC$ respectively. Prove that AB = AD + BC.



(1) (2) (3)

13. If the bisector of the exterior vertical angle of a triangle is parallel to the base, prove that the triangle is isosceles.

14. ABC is a triangle and D is mid-point of BC. If the perpendiculars from D to the sides AB and AC are equal, prove that the triangle ABC is issoceles.

Hint

Show that $\Delta BDE \cong \Delta CDF$.

15. Prove that bisectors of the base angles of an isosceles triangle are equal.

16. (*a*) In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet at O. If OB = OC, prove that Δ ABC is an isosceles triangle.

(*b*) In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Show that

(*i*) OB = OC (*ii*) OA bisects $\angle A$.

17. ABC is an equilateral triangle. If AD is a median, prove that AD bisects $\angle A$ and also AD is perpendicular to BC.

18. In a triangle ABC, AB = AC. BA is produced to D and AE is drawn to bisect $\angle DAC$. Prove that AE is parallel to BC.

19. In a ΔABC, $\angle A = 75°$, $\angle C = 35°$ and internal bisector of $\angle B$ meets AC at D. Prove that BD = CD.

Hint

$\angle B = 180° - 75° - 35° = 70°$, $\therefore \angle CBD = 35° = \angle C$.

20. In a Δ ABC, AB = AC and $\angle A = 36°$. If the internal bisector of $\angle C$ meets AB at D, prove that AD = BC.

Hint

$\angle B + \angle C = 180° - 36° = 144°$ but $\angle B = \angle C$, $\therefore \angle B = \angle C = 72°$,

$\therefore \angle ACD = 36° = \angle A \Rightarrow AD = CD$. Also $\angle CDB = 72° = \angle B \Rightarrow CD = BC$.

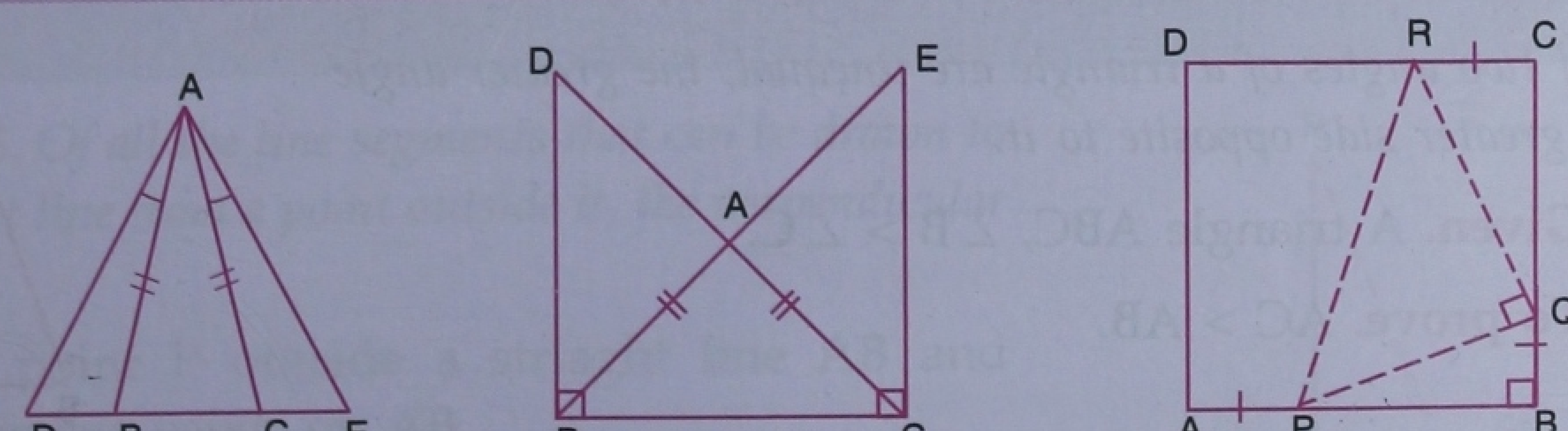
21. (*a*) In the figure (1) given below, AB = AC, $\angle BAD = \angle CAE$. Prove that AD = AE.

(*b*) In the figure (2) given below, AB = AC and BD $\perp$ BC, CE $\perp$ BC. Prove that

(*i*) BD = CE (*ii*) AD = AE.

(*c*) In the figure (3) given below, ABCD is a square. P, Q, R are points on the sides AB, BC and CD such that AP = BQ = CR and $\angle PQR = 90°$. Prove that :

(*i*) $\Delta PBQ \cong \Delta QCR$ (*ii*) PQ = QR (*iii*) $\angle RPQ = 45°$.



(1) (2) (3)

Hint

(*a*) $\angle ABD = \angle ACE$, $\Delta ABD \cong \Delta ACE$.

(*b*) $\angle B = \angle C \Rightarrow \angle ABD = 90° - B = 90° - C = \angle ACE$, $\Delta ABD \cong \Delta ACE$.

(*c*) $AB = BC$, $AP = BQ \Rightarrow AB - AP = BC - BQ \Rightarrow PB = QC$.

22. In a ΔABC, $\angle B = 90°$ and D is a point on AC such that $\angle DBC = \angle DCB$. Prove that $BD = AD$.

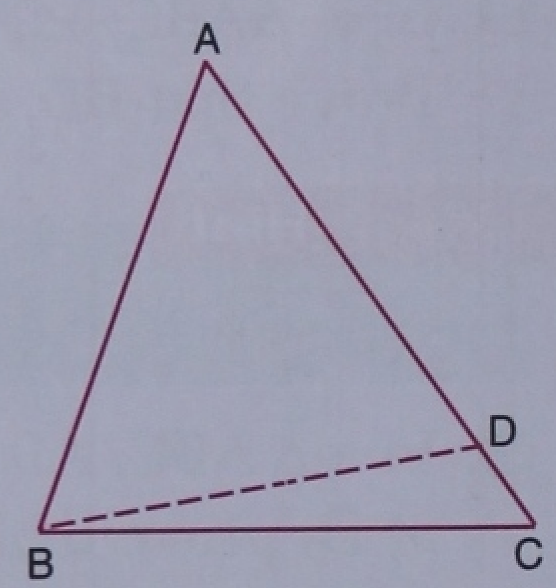
Hint

Let $\angle DBC = \angle DCB = x$ (say), then $\angle ABD = 90° - x$. Also $\angle BAD = 90° - \angle C = 90° - x \Rightarrow \angle ABD = \angle BAD \Rightarrow BD = AD$.

23. In a ΔABC, AD is the internal bisector of $\angle A$. CE is drawn parallel to DA to meet BA produced at E. Prove that ΔCAE is isosceles.

12.4 INEQUALITIES

Theorem 3. *If two sides of a triangle are unequal, the greater side has greater angle opposite to it.*



Given. A triangle ABC, $AC > AB$.

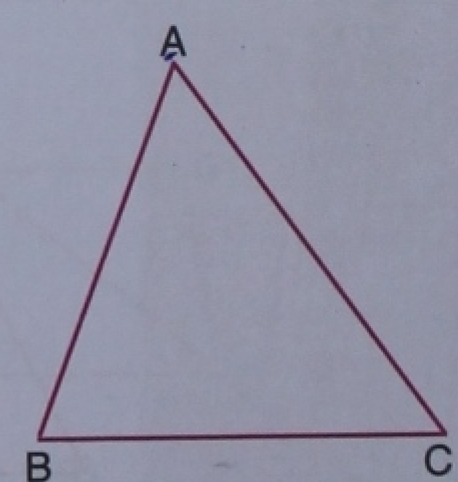
To prove. $\angle ABC > \angle BCA$.

Construction. Take a point D on AC such that $AD = AB$. Join B and D.

Proof. *Statements*	*Reasons*
1. $AD = AB$	1. By construction.
2. $\angle ABD = \angle BDA$	2. Angles opp. to equal sides.
3. $\angle BDA > \angle BCD$	3. $\angle BDA$ is an ext. $\angle$ of ΔBCD, and ext. $\angle$ is greater than each of opp. int. $\angle$.
4. $\angle ABD > \angle BCD$	4. From 2 and 3.
5. $\angle ABC > \angle ABD$	5. Since $\angle ABD$ is a part of $\angle ABC$.
6. $\angle ABC > \angle BCD$	6. From 4 and 5.
Hence $\angle ABC > \angle BCA$. **Q.E.D.**	

Theorem 4. (*Converse of theorem 3*)

If two angles of a triangle are unequal, the greater angle has greater side opposite to it.

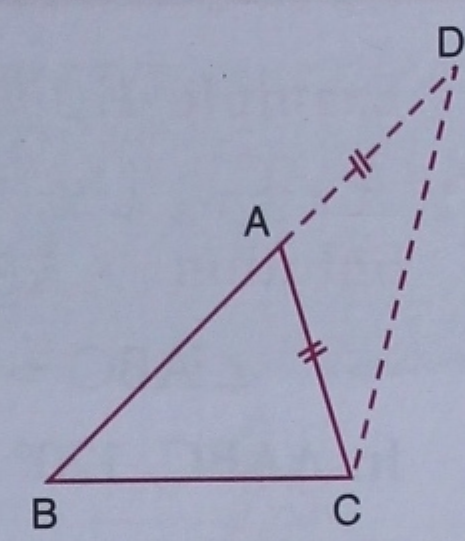


Given. A triangle ABC, $\angle B > \angle C$.

To prove. $AC > AB$.

Proof. *Statements*	*Reasons*
If AC is not greater than AB, then either (*i*) AC = AB or (*ii*) AC < AB.	Order property of real numbers.
Case I. If AC = AB, then ∠B = ∠C which is contrary to what is given.	Angles opp. to equal sides.
Case II. If AC < AB, then ∠B < ∠C which is contrary to what is given.	Greater side has greater angle opposite to it.
Hence AC > AB. **Q.E.D.**	

Corollary 1. *The sum of any two sides of a triangle is greater than the third side.*



Given. A triangle ABC.

To prove. BA + AC > BC.

Construction. Produce BA to D such that AD = AC. Join C and D.

Proof. *Statements*	*Reasons*
1. ∠ACD = ∠ADC	1. AD = AC by construction, and angles opp. to equal sides are equal.
2. ∠BCD > ∠ACD	2. Whole is greater than a part.
3. ∠BCD > ∠ADC	3. From 1 and 2.
4. BD > BC	4. Greater angle has greater side opposite to it.
5. BA + AD > BC	5. BAD is a st. line, BD = BA + AD
6. BA + AC > BC	6. AD = AC, by construction.
Q.E.D.	

Corollary 2. *The difference between any two sides of a triangle is less than the third side.*

Hint

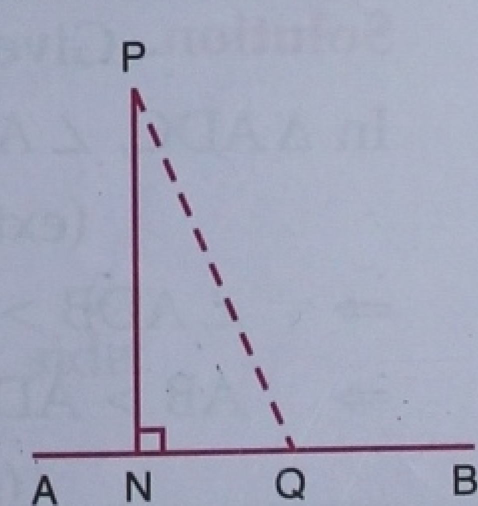
Let AC > AB. Since AB + BC > AC ⇒ BC > AC – AB
⇒ AC – AB < BC.

Remarks

From theorems 3 and 4, it follows that in a triangle

- *the greatest side has the greatest angle opposite to it and conversely.*
- *the smallest side has the smallest angle opposite to it and conversely.*

Theorem 5. *Of all the line segments that can be drawn to a given straight line from a point outside it, the perpendicular is the shortest.*



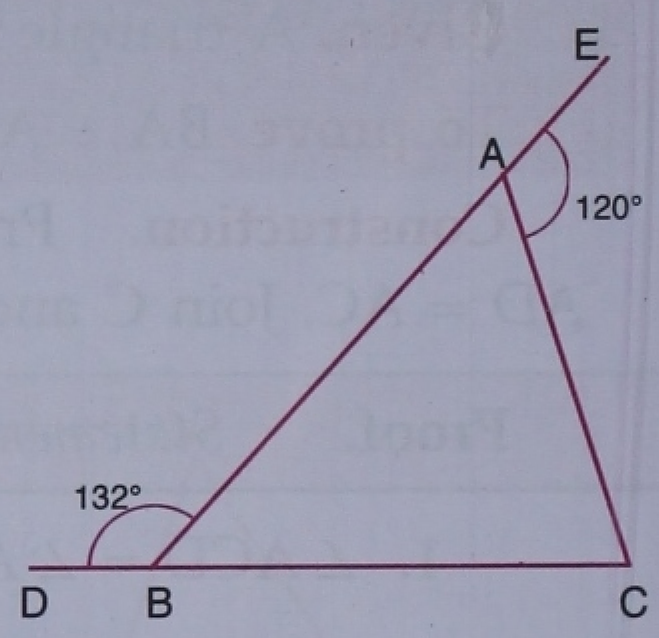
Given. A point P outside a striaght line AB and PN ⊥ AB. Q is *any point* on AB.

To prove. PN < PQ.

Proof. *Statements*	*Reasons*
1. $\angle PNQ = 90°$	1. $PN \perp AB$.
2. $\angle PQN < 90°$	2. $\angle PQN$ is acute angle, since sum of three $\angle$s in $\Delta PNQ = 180°$.
3. $\angle PQN < \angle PNQ$	3. From 1 and 2.
4. $PN < PQ$ **Q.E.D.**	4. Side opp. the smaller angle is smaller.

ILLUSTRATIVE EXAMPLES

Example 1. *In the adjoining figure, $\angle ABD = 132°$ and $\angle EAC = 120°$. Prove that $AB > AC$.*



Solution. $\angle ABC + 132° = 180°$ (linear pair)

$\Rightarrow$ $\angle ABC = 180° - 132° = 48°$.

In ΔABC, $120° = \angle ABC + \angle ACB$

(ext. $\angle$ = sum of two opp. int. $\angle$s)

$\Rightarrow$ $120° = 48° + \angle ACB$

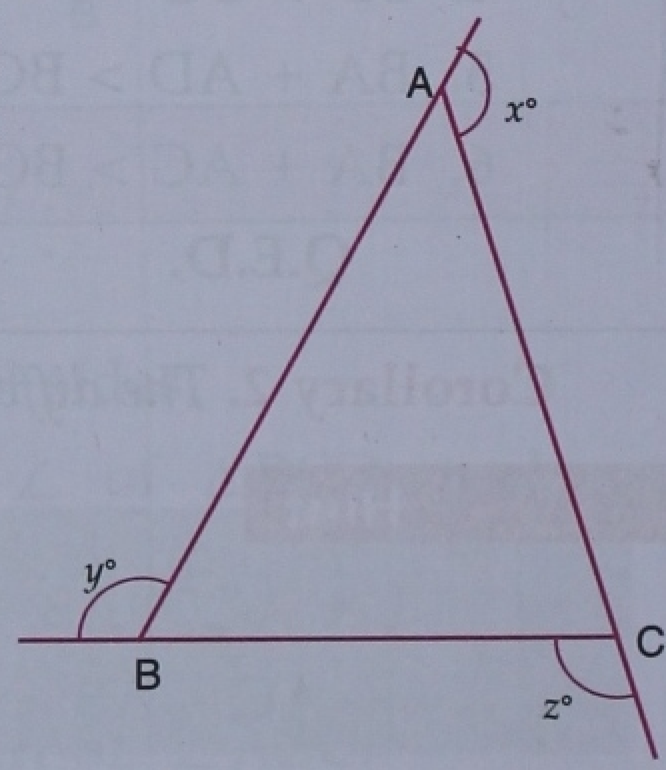
$\Rightarrow$ $\angle ACB = 120° - 48° = 72°$.

In ΔABC, we find that $\angle ACB > \angle ABC$

$\Rightarrow$ $AB > AC$

(side opposite to greater angle is greater)

Example 2. *In the adjoining figure, $AB = 7{\cdot}5$ cm, $BC = 5$ cm and $CA = 6{\cdot}2$ cm. Arrange x, y and z in ascending order.*



Solution. Given $AB = 7{\cdot}5$ cm, $BC = 5$ cm and $CA = 6{\cdot}2$ cm.

$\Rightarrow$ $AB > CA > BC$

$\Rightarrow$ $\angle C > \angle B > \angle A$

($\because$ angle opposite to greater side is greater)

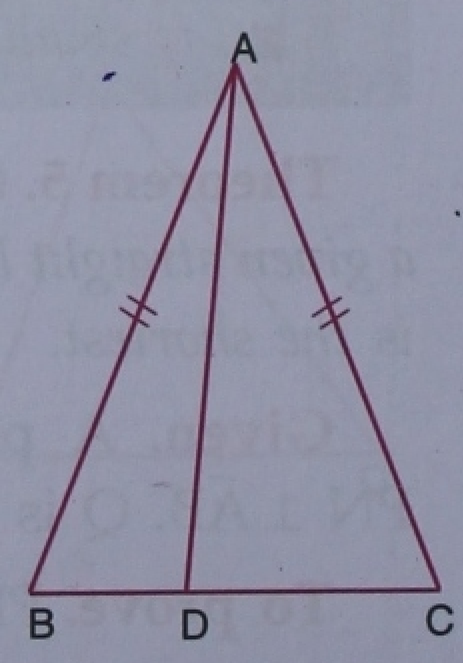
$\Rightarrow$ $-\angle C < -\angle B < -\angle A$

$\Rightarrow$ $(180° - \angle C) < (180° - \angle B) < (180° - \angle A)$

$\Rightarrow$ $z° < y° < x° \Rightarrow z < y < x$

$\Rightarrow$ $x > y > z$.

Example 3. *In the adjoining figure, $AB = AC$ and D is any point on BC. Prove that $AB > AD$.*



Solution. Given $AB = AC \Rightarrow \angle B = \angle C$.

In ΔADC, $\angle ADB > \angle C$

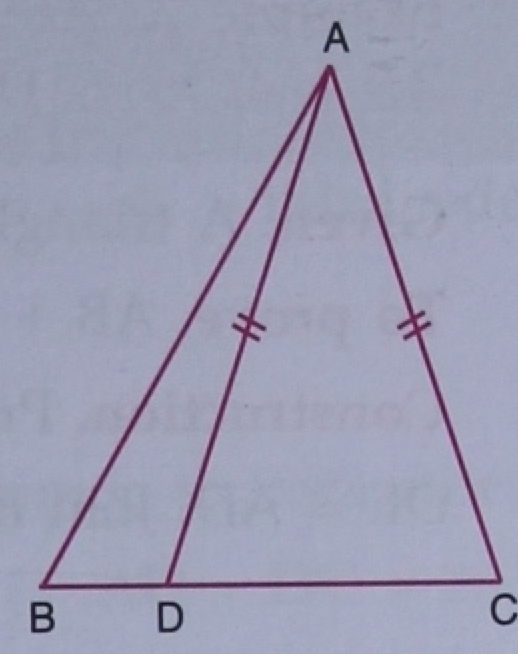
(ext. $\angle$ is greater than each opp. int. $\angle$s)

$\Rightarrow$ $\angle ADB > \angle B$ ($\because \angle C = \angle B$)

$\Rightarrow$ $AB > AD$

(side opposite greater angle is greater)

Example 4. *In the adjoining figure, D is a point on the side BC of ΔABC such that AD = AC. Show that AB > AD.*



Solution. Given AD = AC

⇒ ∠ACD = ∠ADC

(angles opp. equal sides are equal)

For ΔABD, ∠ADC is an exterior angle,

∴ ∠ADC > ∠ABD

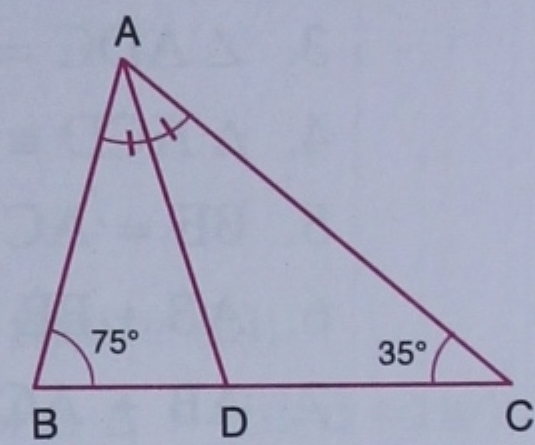
(ext. ∠ is greater than each opp. int ∠s)

⇒ ∠ACD > ∠ABD (∵ ∠ACD = ∠ADC)

⇒ AB > AC but AC = AD

∴ AB > AD.

Example 5. *In the adjoining figure, AD bisects ∠A. Arrange AB, BD and DC in ascending order.*



Solution. ∠A = 180° – 75° – 35° = 70°.

Since AD bisects ∠A,

∠BAD = ∠DAC = $\frac{1}{2}$. 70° = 35°.

∴ ∠ADB = ∠DAC + ∠C

[ext. ∠ = sum of opp. int. ∠s]

= 35° + 35° = 70°.

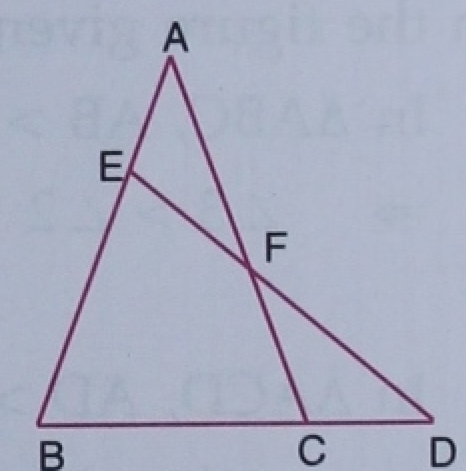
∴ In Δ ABD, ∠BAD < ∠ADB < ∠ABD

⇒ BD < AB < AD [side opp. to smaller angle is smaller] ...(*i*)

Also in Δ ADC, ∠DAC = 35° = ∠C ⇒ AD = DC ...(*ii*)

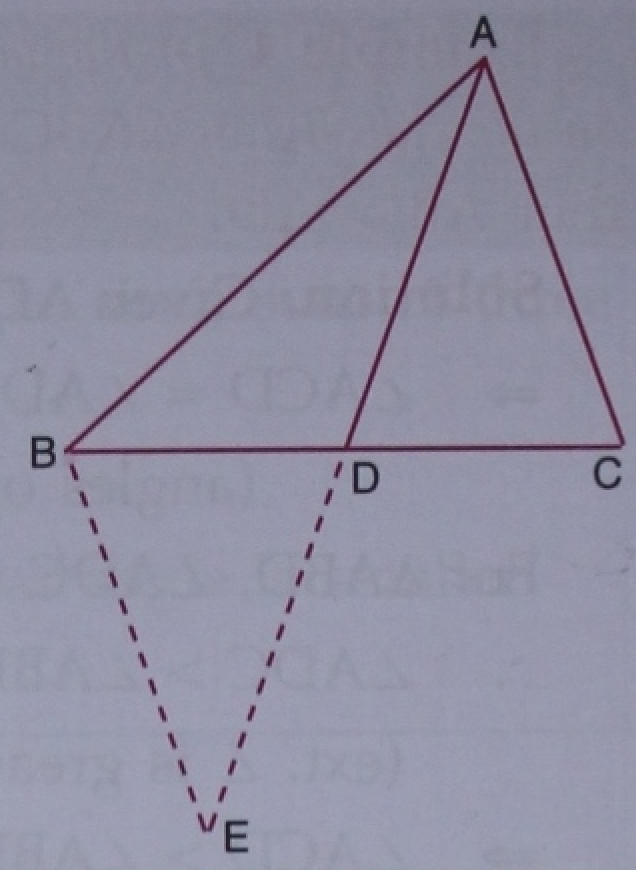
∴ BD < AB < DC [from (*i*) and (*ii*)]

Example 6. *In the adjoining figure, AB = AC. Prove that AF > AE.*



Proof. *Statements*	*Reasons*
1. ∠B = ∠C	1. AB = AC (given)
2. ∠AEF > ∠B	2. ∠AEF is ext. ∠ of Δ EBD, and ext. ∠ is > each opp. int. ∠.
3. ∠C > ∠CFD	3. ∠C is ext. ∠ of Δ CFD, and ext. ∠ is > each opp. int. ∠.
4. ∠B > ∠CFD	4. ∠B = ∠C.
5. ∠AEF > ∠CFD	5. From 2 and 4.
6. ∠EFA = ∠CFD	6. Vert. opp. ∠s.
7. ∠AEF > ∠EFA	7. From 5 and 6.
8. AF > AE **Q.E.D.**	8. Greater angle has greater opp. side.

Example 7. *AD is median of Δ ABC. Prove that AB + AC > 2AD.*



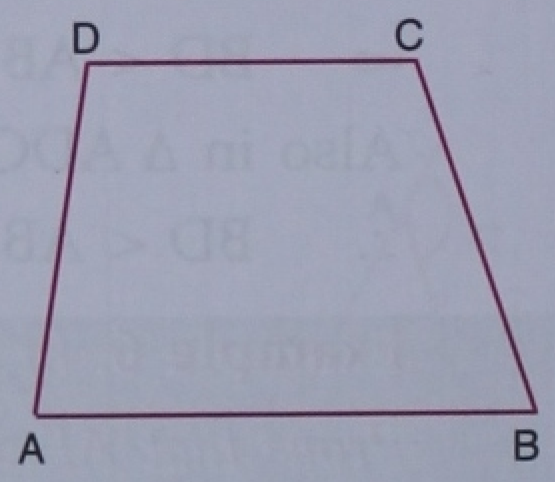
Given. A triangle ABC and AD is a median.

To prove. AB + AC > 2AD.

Construction. Produce AD to E such that DE = AD. Join B and E.

Proof. *Statements*	*Reasons*
In Δs ACD, EBD	
1. BD = DC	1. AD is median, so D is mid-point of BC.
2. AD = DE	2. By construction.
3. ∠ ADC = ∠ BDE	3. Vert. opp. ∠s.
4. Δ ACD ≅ Δ EBD	4. S.A.S. (Axiom of congruency).
5. BE = AC	5. 'c.p.c.t.'
6. AB + BE > AE	6. In Δ ABE, sum of two sides > third side.
7. AB + AC > 2AD	7. Using 5 and 2.
Q.E.D.	

Example 8. *In the adjoining quadrilateral ABCD, AB is the longest side and DC is the shortest side. Prove that (i) ∠C > ∠A (ii) ∠D > ∠B.*

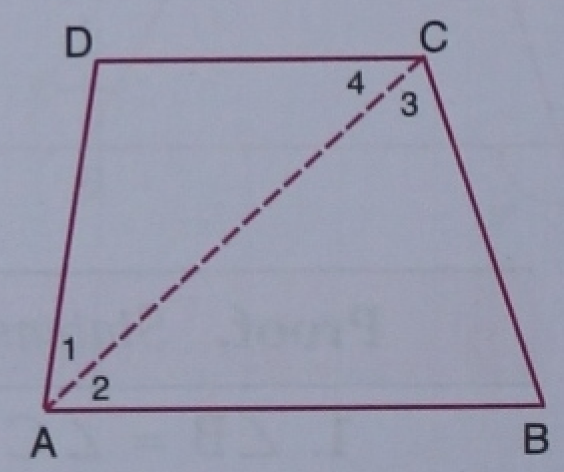


Solution. (*i*) Join A and C, and label the angles as shown in the figure given below.

In ΔABC, AB > BC (given AB is longest side)

⇒ ∠3 > ∠2 ...(1)

(angle opposite greater side is greater)



In ΔACD, AD > DC (given DC is shortest side)

⇒ ∠4 > ∠1 ...(2)

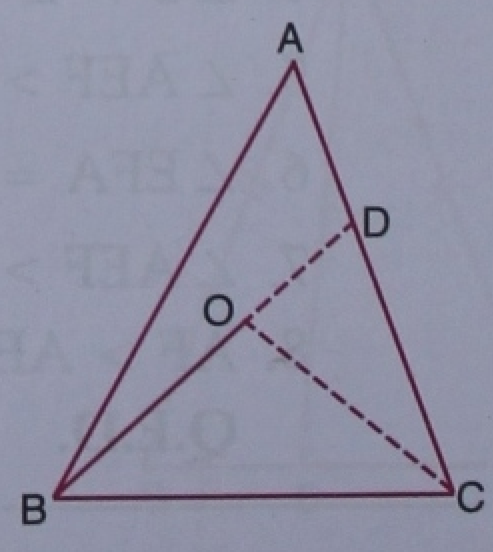
(angle opposite greater side is greater)

Adding (1) and (2), we get

∠3 + ∠4 > ∠2 + ∠1 ⇒ ∠C > ∠A.

(*ii*) Similarly, on joining B and D, we shall obtain ∠D > ∠B.

Example 9. *O is any point in the interior of a triangle ABC. Prove that OB + OC < AB + AC.*



Solution. Given O is any point in the interior of a triangle ABC.

Produce BO to intersect AC at D.

From Δ ABD, AB + AD > BD.

(∵ sum of two sides of a triangle > third side)

$\Rightarrow \quad AB + AD > OB + OD$ ...(*i*)

From Δ OCD, $OD + DC > OC$ ...(*ii*)

($\because$ sum of two sides of a triangle > third side)

On adding (*i*) and (*ii*), we get

$AB + AD + OD + DC > OB + OD + OC$

$\Rightarrow \quad AB + AD + DC > OB + OC$ ($\because$ OD is common to both sides)

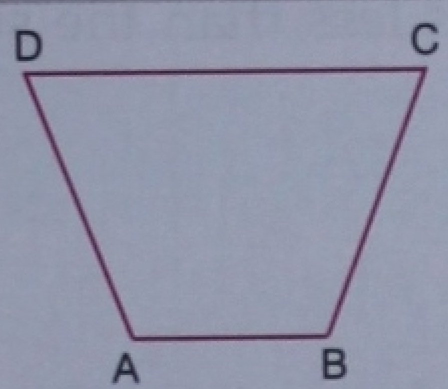
$\Rightarrow \quad AB + AC > OB + OC$ ($\because$ AD + DC = AC)

$\Rightarrow \quad OB + OC < AB + AC.$

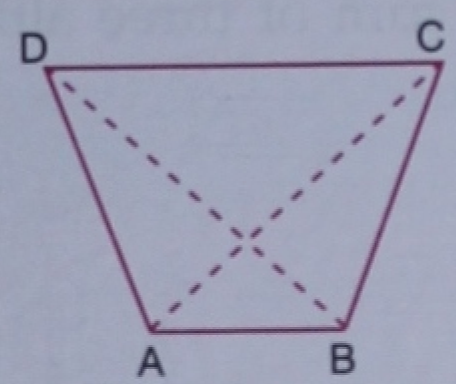
Example 10. *In the figure (1) given below, prove that*

(*i*) $AB + BC + CD > DA$ (*ii*) $AB + BC + CD + DA > 2AC$

(*iii*) $AB + BC + CD + DA > 2BD$ (*iv*) $AB + BC + CD + DA > AC + BD.$



(1)



(2)

Solution. Join A and C, also B and D (as shown in the above figure (2)).

(*i*) In ΔABC, $AB + BC > AC$ ...(1) (sum of two sides of a Δ > third side)

In ΔACD, $AC + CD > DA$ ...(2) (same reason)

Adding (1) and (2), we get

$AB + BC + AC + CD > AC + DA$

$\Rightarrow \quad AB + BC + CD > DA$ ($\because$ AC is common to both sides)

(*ii*) ΔACD, $CD + DA > AC$ ...(3)

On adding (1) and (3), we get

$AB + BC + CD + DA > 2AC$ ...(4)

(*iii*) In ΔABD, $DA + AB > BD$ ...(5)

In ΔBCD, $BC + CD > BD$ ...(6)

On adding (5) and (6), we get

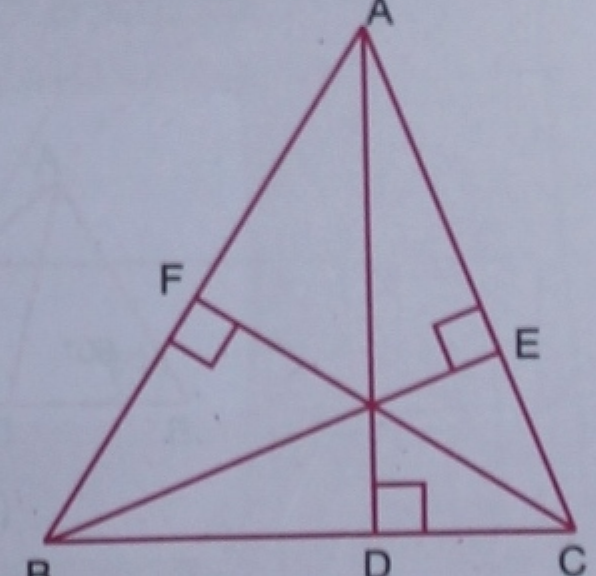
$AB + BC + CD + DA > 2BD$ ...(7)

(*iv*) On adding (4) and (7), we get

$2(AB + BC + CD + DA) > 2(AC + BD)$

$\Rightarrow \quad AB + BC + CD + DA > AC + BD.$

Example 11. *Prove that the sum of the three altitude of a triangle is less than the sum of three sides of the triangle.*



Given. A triangle ABC and $AD \perp BC$, $BE \perp AC$, $CF \perp AB$.

To prove. $AD + BE + CF < AB + BC + AC.$

Proof. We know that of all the line segments that can be drawn to a given straight line from a point outside it, the perpendicular is the shortest.

As AD ⊥ BC, therefore,

$AD < AB$ and $AD < AC$

$\Rightarrow \quad 2AD < AB + AC$ ...(*i*)

As BE ⊥ AC, therefore,

$BE < BC$ and $BE < AB$

$\Rightarrow \quad 2BE < BC + AB$ ...(*ii*)

As CF ⊥ AB, therefore,

$CF < BC$ and $CF < AC$

$\Rightarrow \quad 2CF < BC + AC$ ...(*iii*)

On adding (*i*), (*ii*) and (*iii*), we get

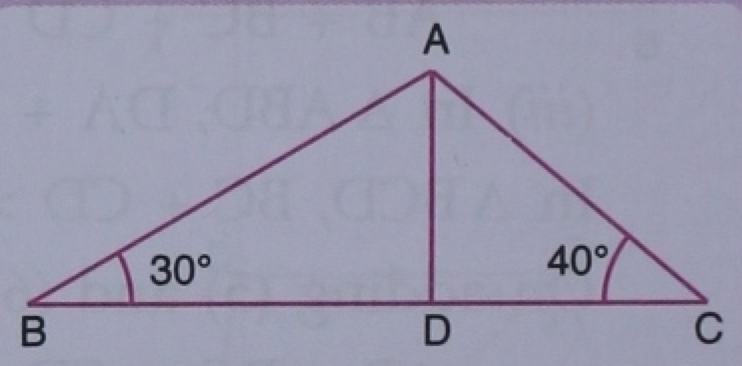
$2(AD + BE + CF) < 2(AB + BC + AC)$

$\Rightarrow \quad AD + BE + CF < AB + BC + AC.$

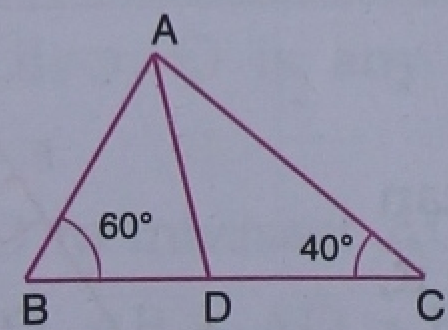
Hence the sum of three altitudes of a triangle is less than the sum of three sides of the triangle.

Exercise 12.3

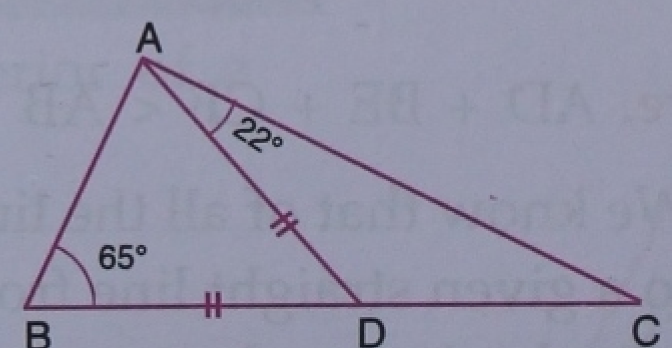
1. In Δ ABC, ∠A = 52°, ∠B = 63°. Name the greatest side of the Δ ABC.
2. PQR is a right angle triangle at Q and PQ : QR = 3 : 2. Which is the least angle ?
3. In Δ ABC, AB = 8 cm, BC = 5·6 cm and CA = 6·5 cm. Which is
   (*i*) the greatest angle? (*ii*) the smallest angle?
4. In a Δ PQR, PQ = 10 cm, QR = 7·5 cm. Find the greatest and the least length that RP can have.
5. Can you construct a triangle with sides 12 cm, 8·5 cm, 2·7 cm? Give reason.
6. In a Δ ABC, ∠A = 50°, ∠B = 60°. Arrange the sides of the triangle in ascending order.
7. In figure given alongside, ∠B = 30°, ∠C = 40° and the bisector of ∠A meets BC at D. Show that
   (*i*) BD > AD (*ii*) DC > AD
   (*iii*) AC > DC (*iv*) AB > BD.



8. In Δ ABC, ∠A = 53° and ∠B = 49°. The bisector of ∠C meets AB at D. Arrange the sides of Δ ADC in descending order.
9. (*a*) In the figure (1) given below, AD bisects ∠A. Arrange AB, BD and DC in the descending order of their lengths.
   (*b*) In the figure (2) given below, ∠ABD = 65°, ∠DAC = 22° and AD = BD. Calculate ∠ACD and state (giving reasons) which is greater : BD or DC ?



(1)

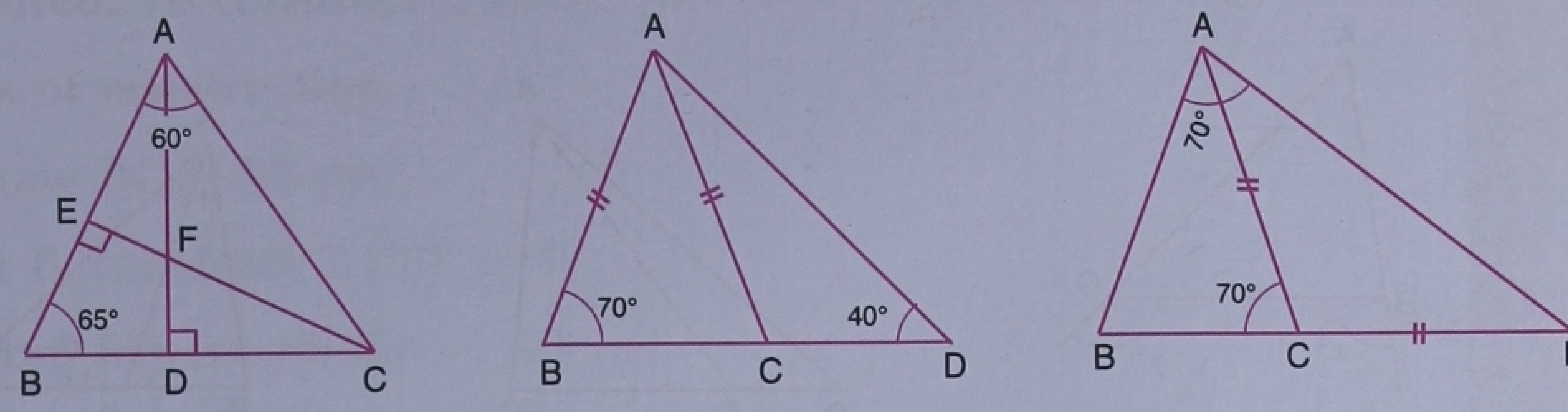


(2)

10. (*a*) In the figure (1) given below, prove that (*i*) CF > AF (*ii*) DC > DF.

(*b*) In the figure (2) given below, AB = AC. Prove that AB > CD.

(*c*) In the figure (3) given below, AC = CD. Prove that BC < CD.



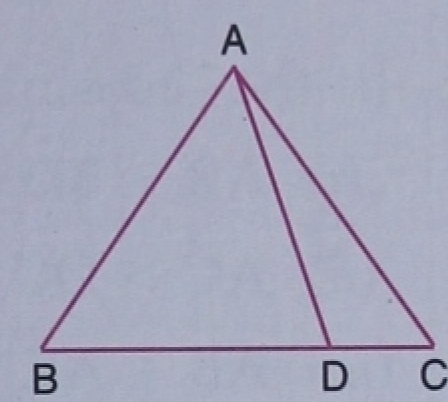
(1) (2) (3)

Hint

(*b*) In ΔACD, $\angle$CAD = 30°, AC > CD but AB = AC.

11. In a right angled triangle, prove that hypotenuse is the greatest.

12. In the adjoining figure, D is any point on the side BC of ΔABC. Prove that AB + BC + CA > 2 AD.



Hint

AB + BD > AD, DC + AC > AD

$\Rightarrow$ AB + BD + DC + AC > 2AD.

13. (*a*) In the figure (1) given below, AD ⊥ BC, prove that

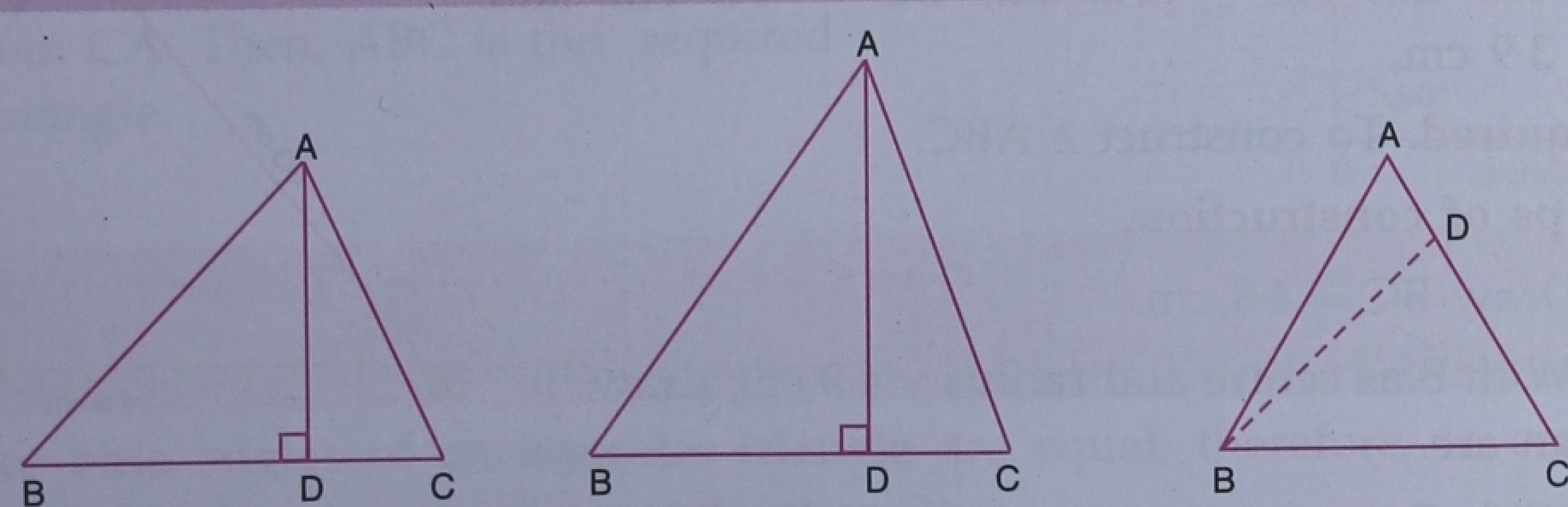
(*i*) AB > BD (*ii*) AC > CD (*iii*) AB + AC > BC.

(*b*) In the figure (2) given below, AD ⊥ BC. Prove that

(*i*) AB > AD (*ii*) AB + AC > 2AD.

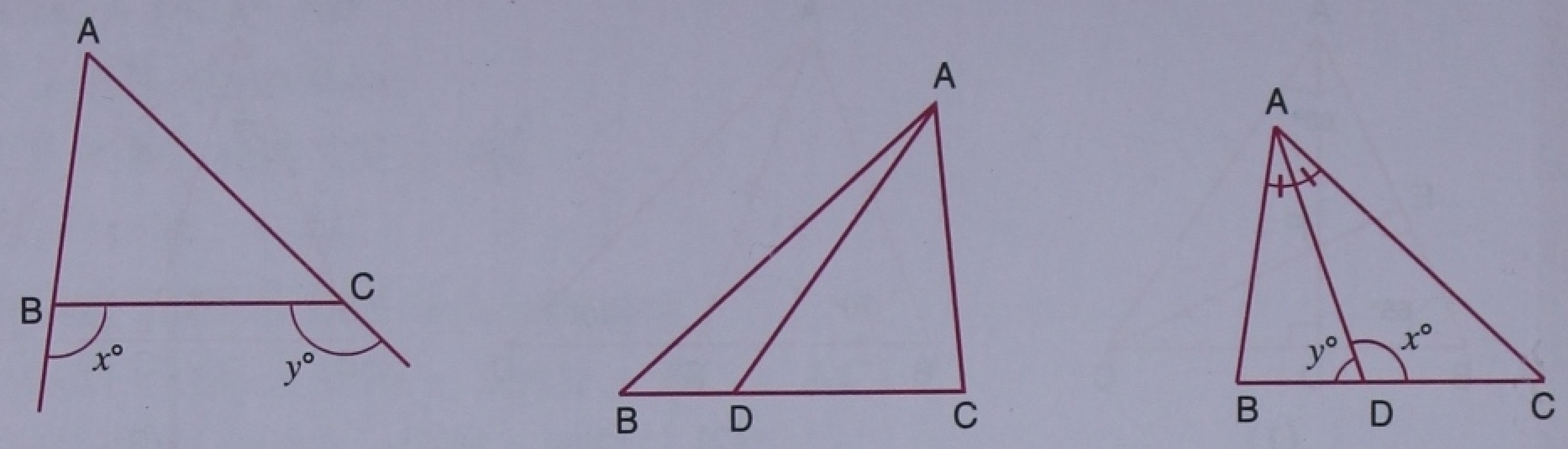
Hence prove that the perimeter of a triangle is greater than the sum of its altitudes.

(*c*) In the figure (3) given below, ABC is an equilateral triangle and D is any point in AC. Prove that (*i*) BD > AD (*ii*) BD > DC.



(1) (2) (3)

14. (*a*) In the figure (1) given below, $x < y$. Prove that AB < AC.
(*b*) In the figure (2) given below, AB > AC. Prove that AB > AD.
(*c*) In the figure (3) given below, AC > AB and AD bisects ∠A. Prove that $x > y$.



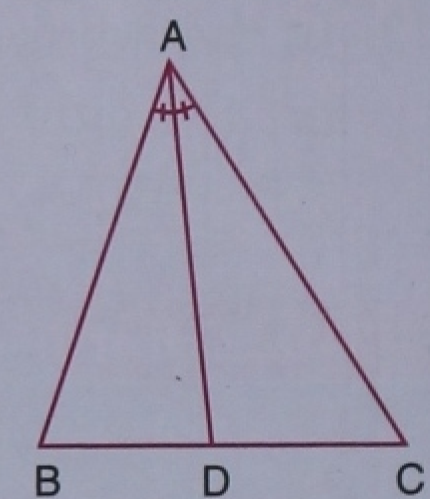
(1) (2) (3)

Hint

(*c*) $AC > AB \Rightarrow \angle B > \angle C \Rightarrow 180° - (\frac{1}{2}\angle A + y°) > 180° - (\frac{1}{2}\angle A + x°)$

$\Rightarrow -y° > -x° \Rightarrow y < x \Rightarrow x > y.$

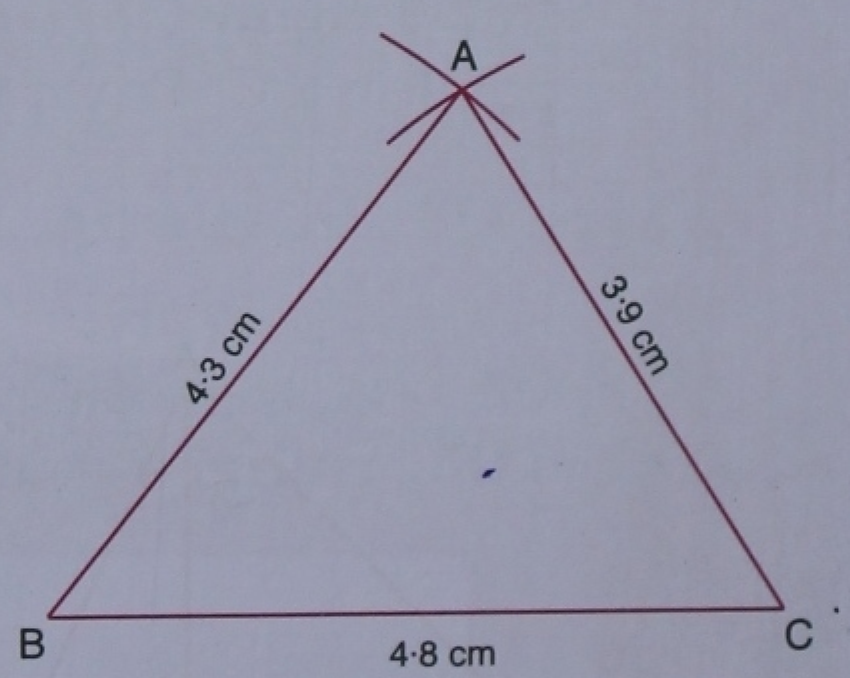
15. In the adjoining figure, AD bisects ∠ A. Prove that
(*i*) AB > BD
(*ii*) AC > DC
(*iii*) AB + AC > BC.



16. The sides AB and AC of a Δ ABC are produced, and the bisectors of the external angles B and C meet at O. If AB > AC, prove that OC > OB.
17. If AD is the median of Δ ABC, prove that AB + AC > 2AD. Hence prove that the perimeter of a triangle is greater than the sum of its medians.
18. If P is a point in the interior of a triangle ABC, prove that
AB + BC + CA < 2(PA + PB + PC).

12.5 CONSTRUCTION OF TRIANGLES

Construction 1. To construct a triangle when its three sides are given.



Given. Let AB = 4·3 cm, BC = 4·8 cm and CA = 3·9 cm.

Required. To construct Δ ABC.

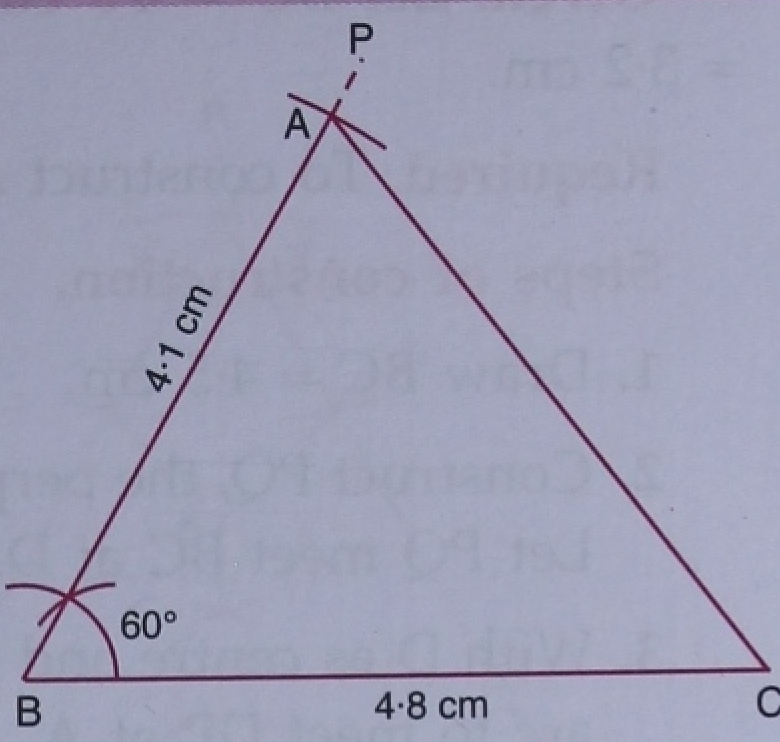
Steps of construction.

1. Draw BC = 4·8 cm.
2. With B as centre and radius = 4·3 cm, draw an arc.
3. With C as centre and radius = 3·9 cm, draw another arc to cut the previous arc at A.
4. Join AB and AC. Then, ABC is the required triangle.

Construction 2. To construct a triangle when two of its sides and the included angle are given.

Given. Let AB = 4·1 cm,

BC = 4·8 cm and ∠B = 60°.



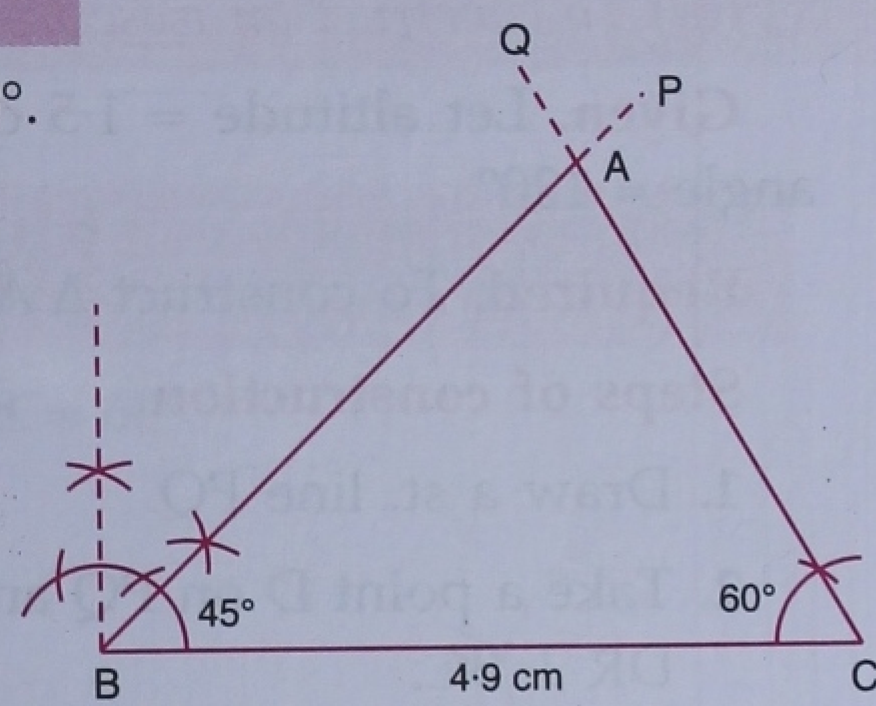
Required. To construct Δ ABC.

Steps of construction.

1. Draw BC = 4·8 cm.
2. At B, construct ∠CBP = 60°.
3. From BP, cut off BA = 4·1 cm.
4. Join AC. Then, ABC is the required triangle.

Construction 3. To construct a triangle when its two angles and the included side are given.

Given. Let BC = 4·9 cm, ∠B = 45° and ∠C = 60°.



Required. To construct Δ ABC.

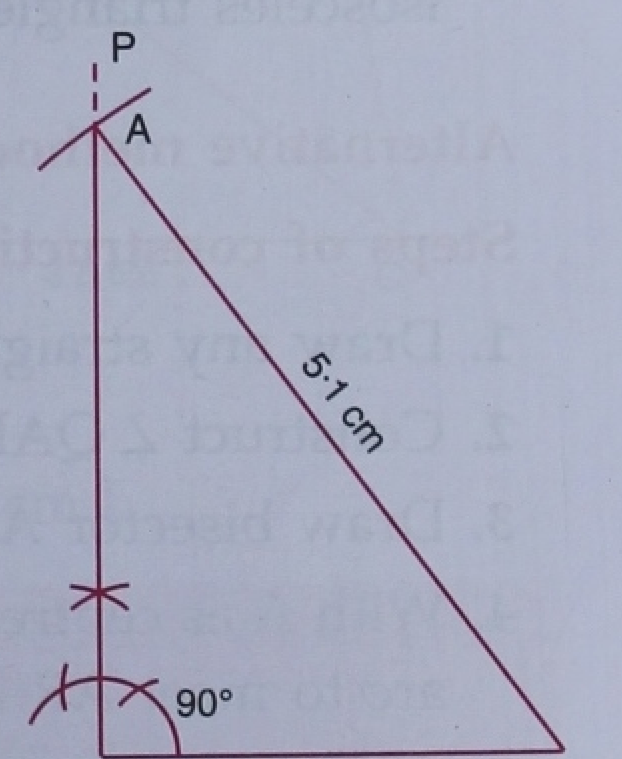
Steps of construction.

1. Draw BC = 4·9 cm.
2. At B, construct ∠CBP = 45°.
3. At C, construct ∠BCQ = 60°.
4. If BP and CQ intersect at A, then ABC is the required triangle.

Construction 4. To construct a right angled triangle when its hypotenuse and one side are given.

Given. Let BC = 3 cm, CA = 5·1 cm and ∠B = a right angle.

Required. To construct Δ ABC.



Steps of construction.

1. Draw BC = 3 cm.
2. At B, construct ∠CBP = 90°.
3. With C as centre and radius = 5·1 cm, draw an arc to meet BP at A.
4. Join CA. Then, ABC is the required triangle.

Construction 5. To construct isosceles triangles

(i) To construct an isosceles triangle when its base and one base angle are given.

Since base angles of an isosceles triangle are equal, therefore, to construct the required triangle proceed as in construction 3.

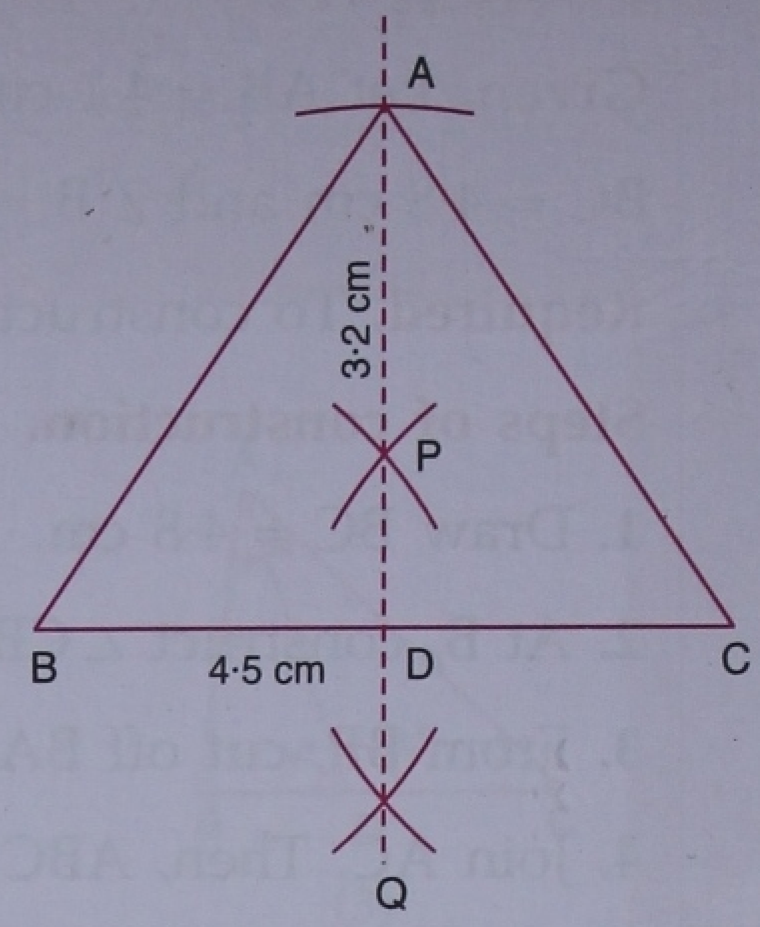
(ii) To construct an isosceles triangle when its base and altitude (from base) are given.

Given. Let BC = 4·5 cm and altitude (from BC) = 3·2 cm.

Required. To construct Δ ABC.

Steps of construction.

1. Draw BC = 4·5 cm.
2. Construct PQ, the perpendicular bisector of BC. Let PQ meet BC at D.
3. With D as centre and radius = 3·2 cm, draw an arc to meet DP at A.
4. Join AB and AC. Then, ABC is the required isosceles triangle.



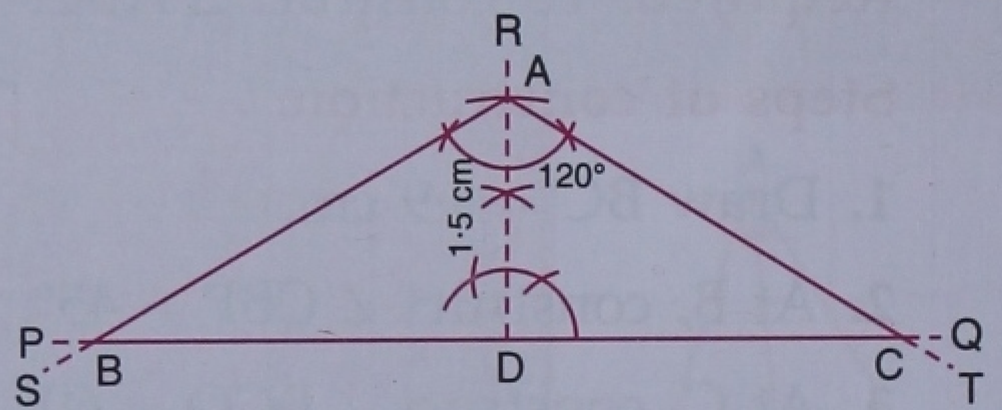
(iii) To construct an isosceles triangle when its altitude and the vertex angle are given.

Given. Let altitude = 1·5 cm and the vertex angle = 120°.

Required. To construct Δ ABC.

Steps of construction.

1. Draw a st. line PQ.
2. Take a point D on PQ and at D, draw DR ⊥ BC.
3. With D as centre and radius = 1·5 cm, draw an arc to meet DR at A.

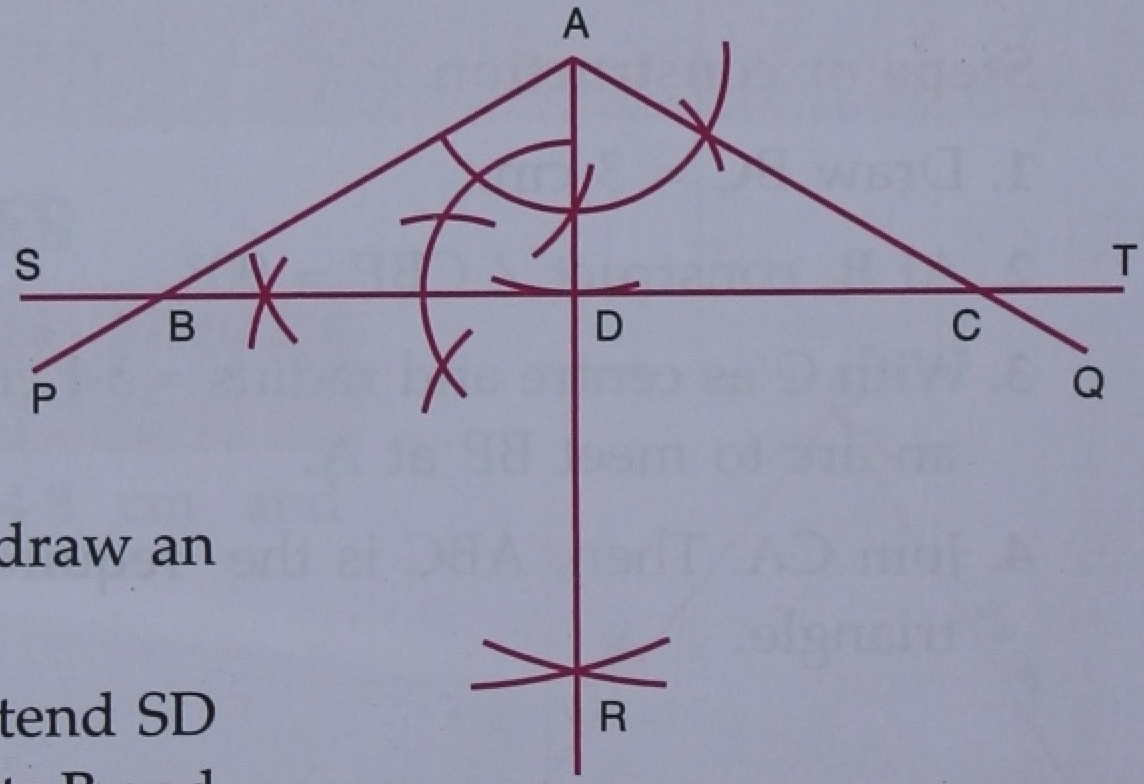


4. At A, construct ∠DAS = ∠DAT = $\frac{1}{2}$ × 120° = 60° on either side of AD. Let AS and AT meet PQ at points B and C respectively. Then, ABC is the required isosceles triangle.

Alternative method

Steps of construction.

1. Draw any straight line AP.
2. Construct ∠QAP = 120°.
3. Draw bisector AR of ∠QAP.
4. With A as centre and radius 1·5 cm, draw an arc to meet AR at D.
5. At D, construct ∠ADS = 90° and extend SD to T. Let ST meet AP and AQ at points B and C respectively. Then, ABC is the required isosceles triangle.



Construction 6. To construct equilateral triangles.

(i) To construct an equilateral triangle when its one side is given.

Since all the three sides of an equilateral triangle are equal, therefore, to construct the required triangle proceed as in construction 1.

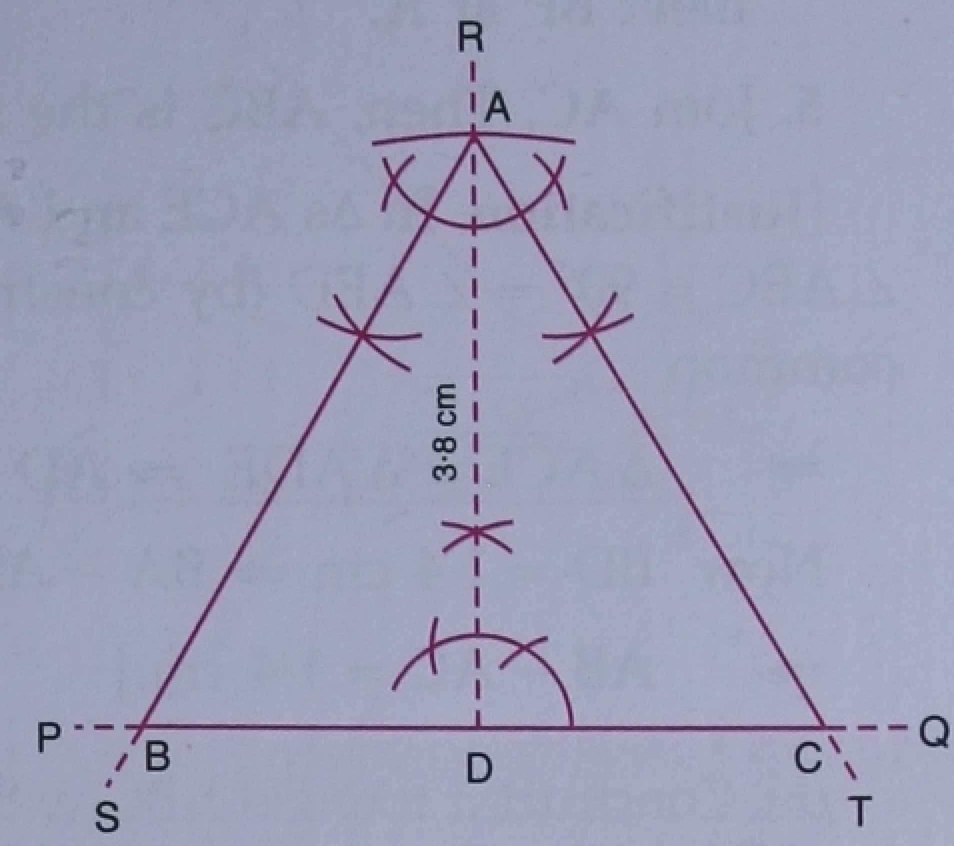
(ii) To construct an equilateral triangle when its altitude is given.

Given. Let an altitude = 3·8 cm.

Required. To construct Δ ABC.

Steps of construction.

1. Draw a st. line PQ.
2. Take a point D on PQ and at D, draw DR ⊥ PQ.
3. With D as centre and radius = 3·8 cm, draw an arc to meet DR at A.
4. At A, construct ∠DAS = ∠DAT = $\frac{1}{2}$ × 60° = 30° on either side of AD. Let AS and AT meet PQ at points B and C respectively. Then, ABC is the required equilateral triangle.



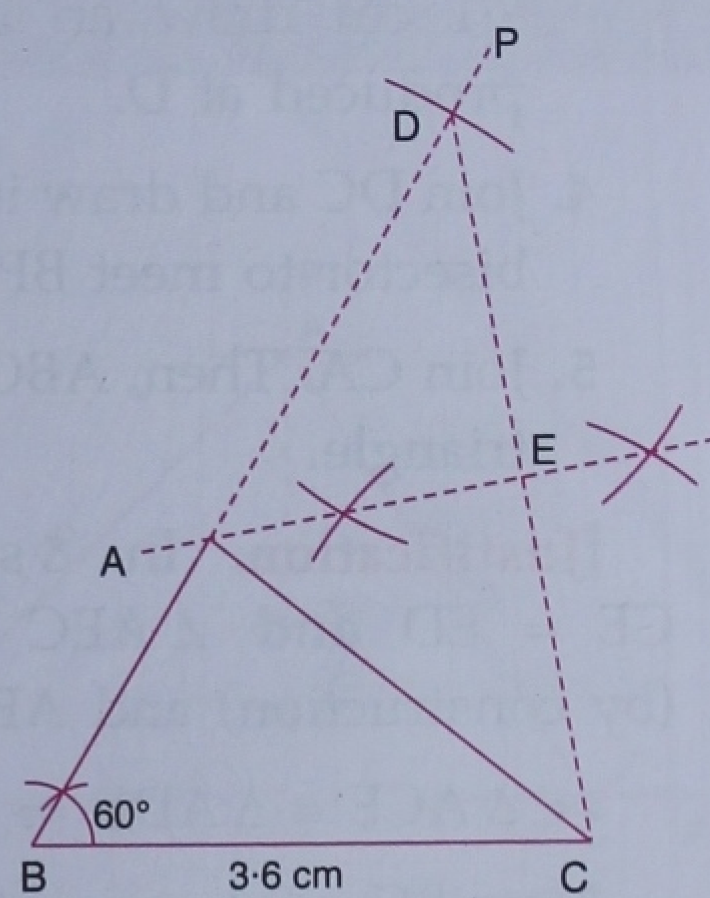
Construction 7. To construct a triangle when its base, the sum of lenghts of other two sides and one of the base angles are given.

Given. Let BC = 3·6 cm, AB + AC = 5·4 cm and ∠B = 60°.

Required. To construct Δ ABC.

Steps of construction.

1. Draw BC = 3·6 cm.
2. At B, construct ∠CBP = 60°.
3. From BP, cut off BD = 5·4 cm.
4. Join CD and draw its perpendicular bisector to meet BD at A.
5. Join AC. Then, ABC is the required triangle.



[**Justification.** In Δs ACE and ADE, CE = ED and ∠CEA = 90° = ∠AED (by construction) and AE is common

⇒ Δ ACE ≅ Δ ADE ⇒ AC = AD.

Now BD = 5·4 cm ⇒ BA + AD = 5·4 cm ⇒ AB + AC = 5·4 cm.]

Construction 8. To construct a triangle when its base, the difference of lengths of other two sides and one of the base angles are given.

(a) Construct a triangle ABC with BC = 3·2 cm, AB – AC = 1·4 cm and ∠B = 45°.

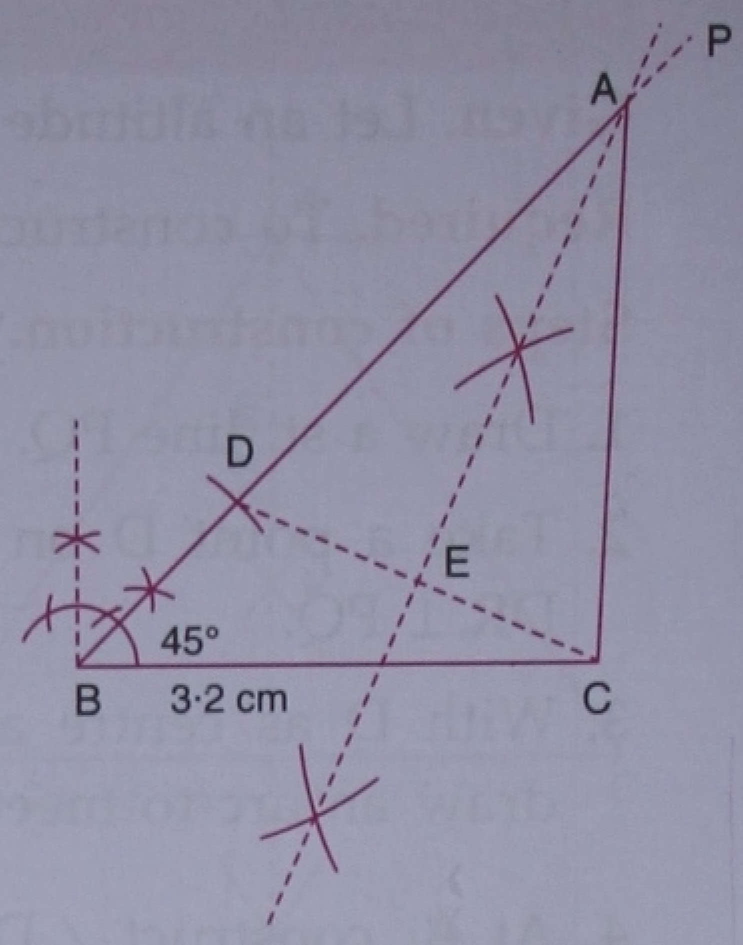
Given. BC = 3·2 cm, AB – AC = 1·4 cm and ∠B = 45°.

Required. To construct Δ ABC.

Steps of construction.

1. Draw BC = 3·2 cm.
2. At B, construct ∠CBP = 45°.

3. From BP, cut off BD = 1·4 cm.
4. Join CD and draw its perpendicular bisector to meet BP at A.
5. Join AC. Then, ABC is the required triangle.



[**Justification.** In Δs ACE and ADE, CE = ED and ∠AEC = 90° = ∠AED (by construction) and AE is common

⇒ Δ ACE ≅ Δ ADE ⇒ AD = AC.

Now BD = 1·4 cm ⇒ BA – AD = 1·4 cm

⇒ AB – AC = 1·4 cm.]

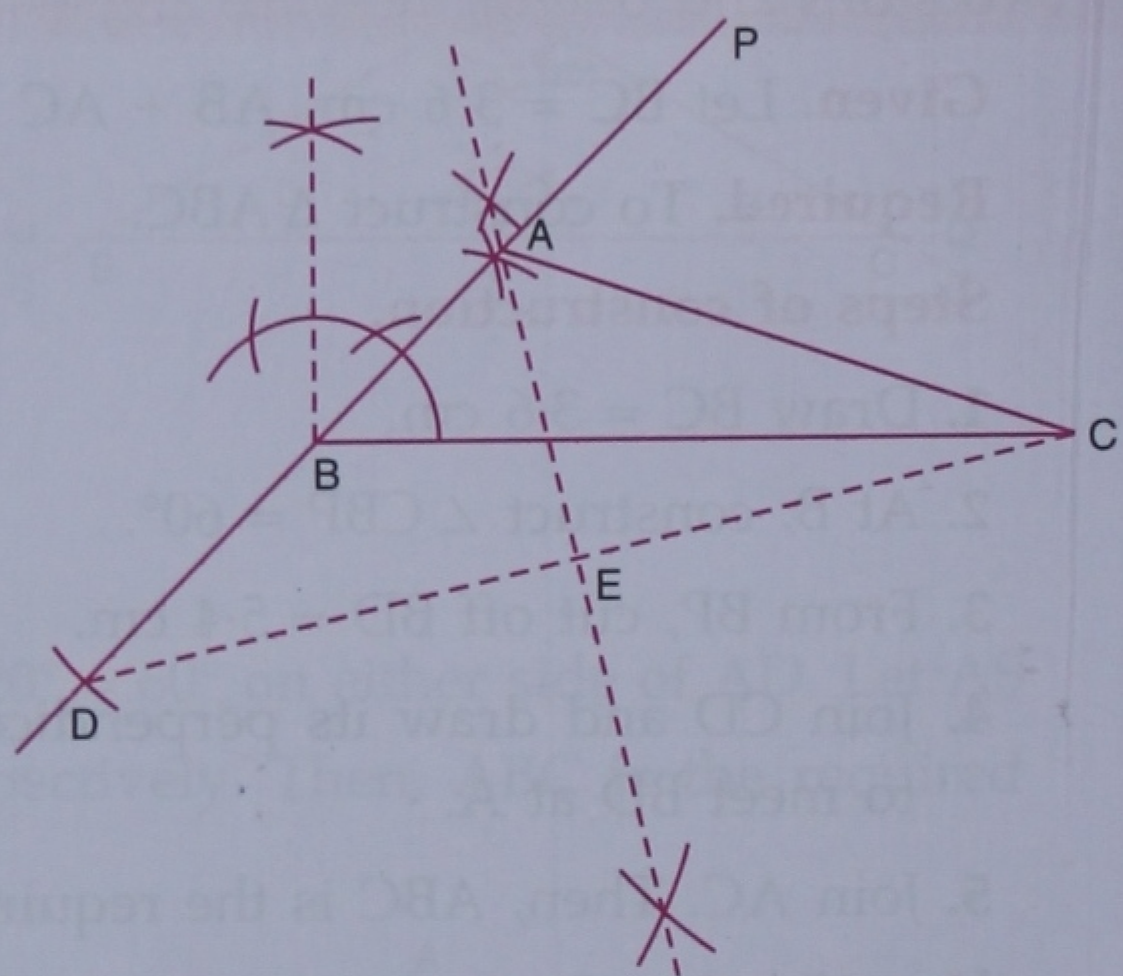
(*b*) Construct a triangle ABC with BC = 4·8 cm, AC – AB = 2·1 cm and ∠B = 45°.

Given. BC = 4·8 cm, AC – AB = 2·1 cm and ∠B = 45°.

Required. To construct Δ ABC.

Steps of construction.

1. Draw BC = 4·8 cm.
2. At B, construct ∠PBC = 45°.
3. With B as centre and radius 2·1 cm draw an arc to meet PB produced at D.
4. Join DC and draw its perpendicular bisector to meet BP at A.
5. Join CA. Then, ABC is the required triangle.



[**Justification.** In Δ s ACE and ADE, CE = ED and ∠AEC = 90° = ∠AED (by construction) and AE is common

⇒ Δ ACE ≅ Δ ADE ⇒ AC = AD.

Now BD = 2·1 cm ⇒ AD – AB = 2·1 cm

⇒ AC – AB = 2·1 cm.]

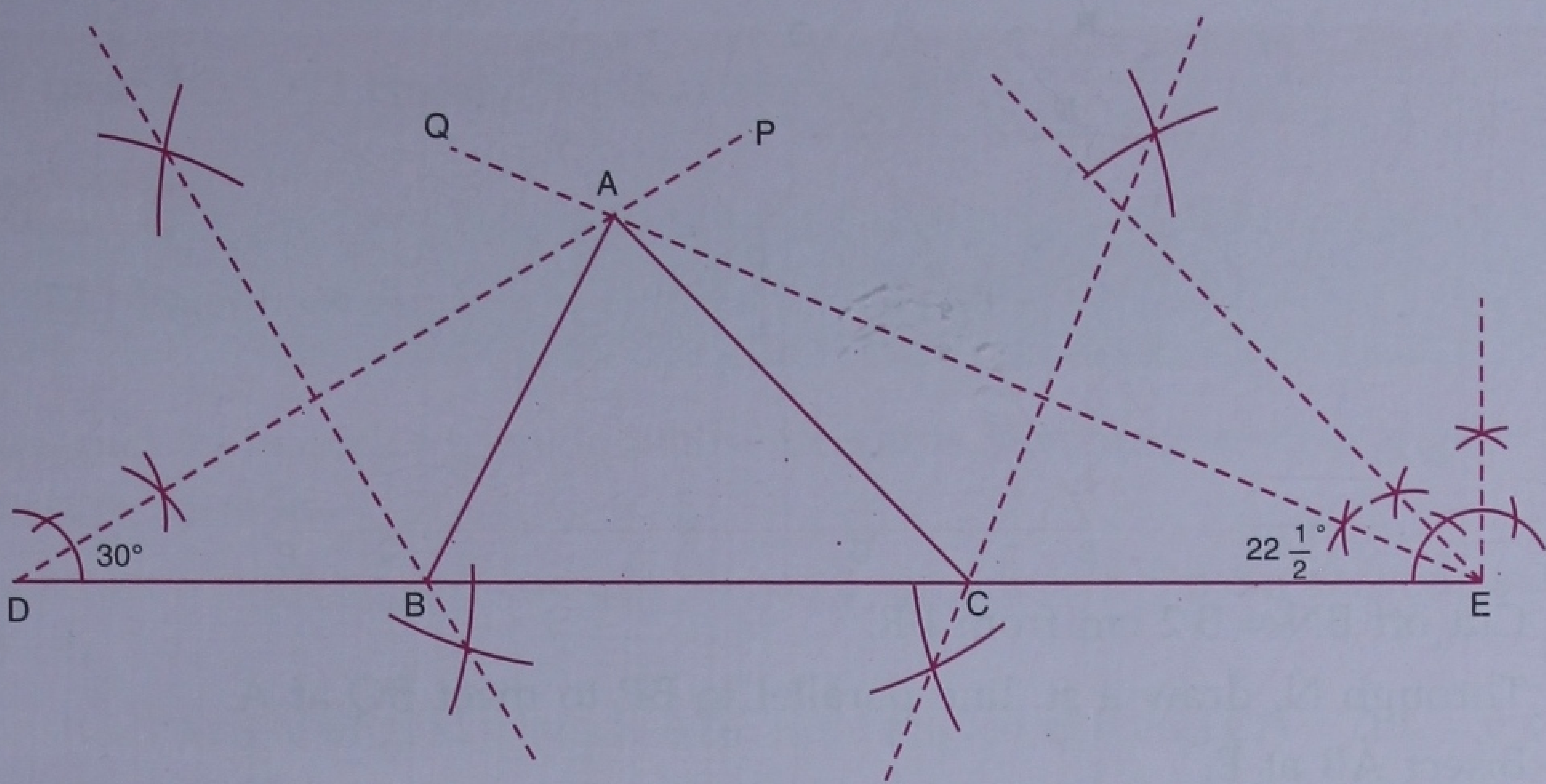
Construction 9. To construct a triangle when its perimeter and both the base angles are given.

Given. Let ∠B = 60°, ∠C = 45° and perimeter = 11·8 cm.

Required. To construct Δ ABC.

Steps of construction.

1. Draw DE = 11·8 cm.
2. At D, construct ∠EDP = $\frac{1}{2}$ of 60° = 30° and at E, construct ∠DEQ = $\frac{1}{2}$ of 45° = $22\frac{1}{2}^{\circ}$.



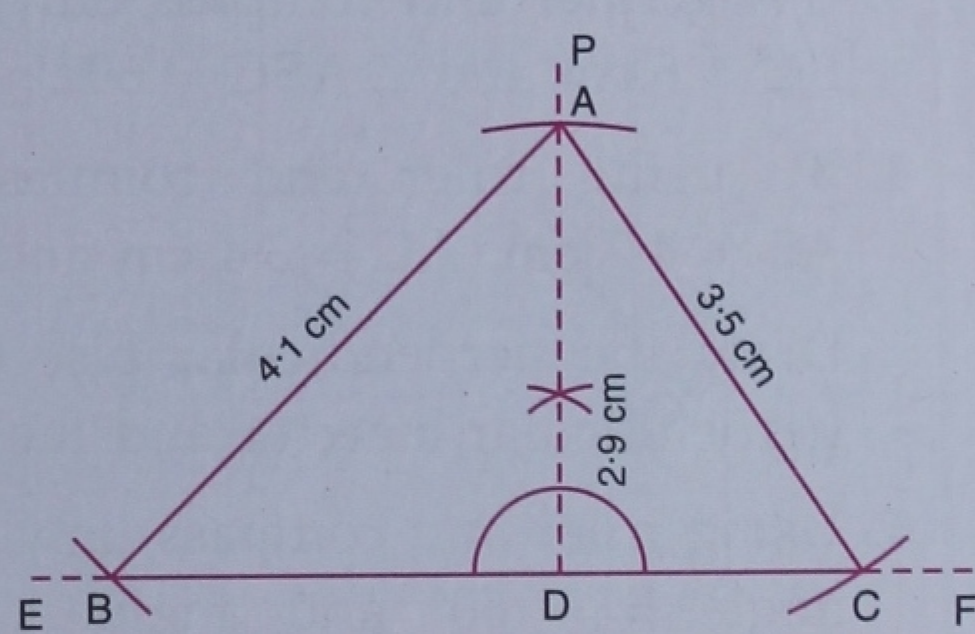
3. Let DP and EQ meet at A.
4. Draw perpendicular bisector of AD to meet DE at B.
5. Draw perpendicular bisector of AE to meet DE at C.
6. Join AB and AC. Then, ABC is the required triangle.

ILLUSTRATIVE EXAMPLES

Example 1. *Construct a Δ ABC in which AB = 4·1 cm, AC = 3·5 cm and the altitude AD = 2·9 cm.*

Solution. Steps of construction.

1. Draw a straight line EF.
2. Take a point D on EF and at D, draw PD ⊥ EF.
3. Cut off DA = 2·9 cm.
4. With A as centre and radius = 4·1 cm, draw an arc to meet EF at B.
5. With A as centre and radius = 3·5 cm, draw another arc to meet EF (on the other side of D) at C.



6. Join AB and AC. Then, ABC is the required triangle.

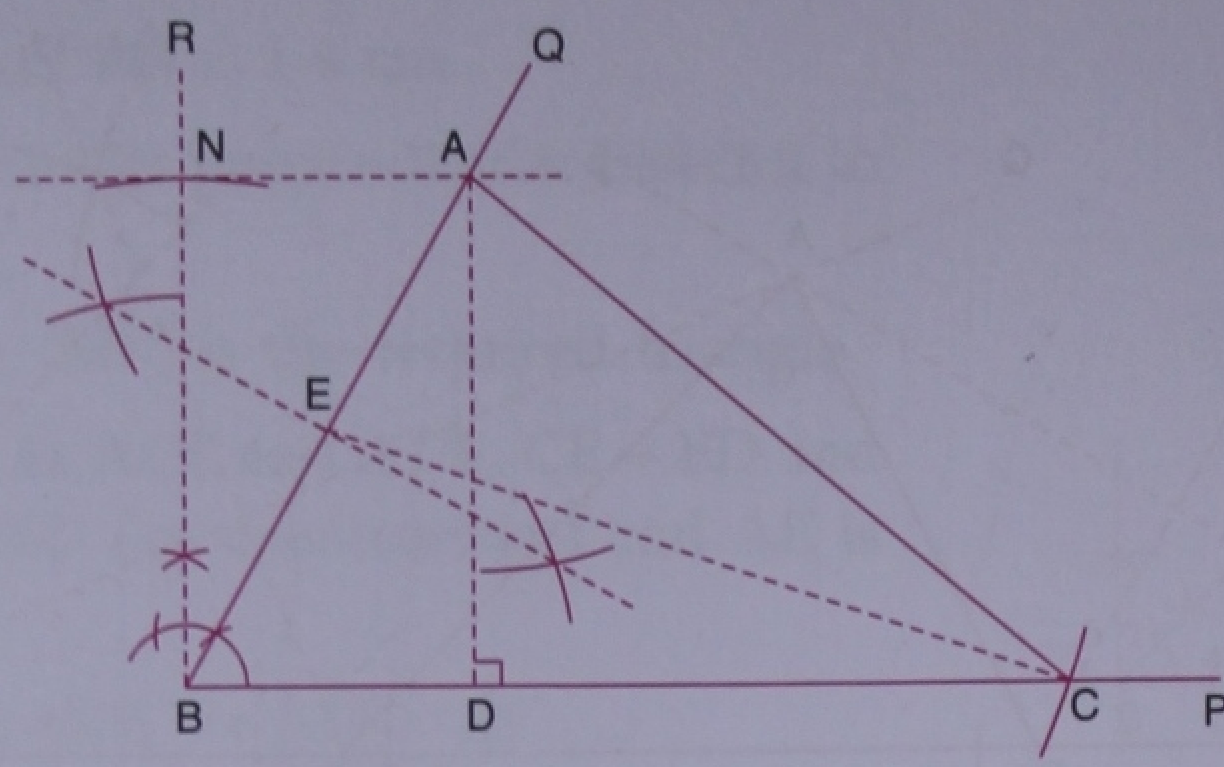
Remark

In step 5 (above), with A as centre and radius = 3·5 cm, we can draw arc to meet EF (on the same side of D) at C′. Then join AB and AC′. So we can have another triangle ABC′ with given data. Thus, we can construct two triangles.

Example 2. *Using ruler and compass only, construct a triangle ABC having given ∠ ABC = 60°, the altitude AD = 3·2 cm and the median CE = 4·9 cm. Measure the side BC.*

Solution. Steps of construction.

1. Draw a st. line BP.
2. At B, construct ∠ PBQ = 60°.
3. At B, draw BR ⊥ BP.



4. Cut off BN = 3·2 cm from BR.
5. Through N, draw a st. line parallel to BP to meet BQ at A.
6. Bisect AB at E.
7. With E as centre and radius 4·9 cm, draw an arc to meet BP at C.
8. Join AC, then ABC is the required triangle with given data. Measure BC. BC = 5·5 cm approximately.

Exercise 12.4

1. Construct a triangle ABC in which
   (*i*) AB = 8 cm, BC = 6 cm and CA = 5 cm
   (*ii*) BC = 5·4 cm, AB = 4·3 cm and CA = 8 cm.
2. Using ruler and compass, construct an equilateral triangle of side 4 cm.
3. Using ruler and compass only, construct a triangle ABC where AB = 5 cm, BC = 6 cm and $\angle$ ABC = 60°.
4. By using ruler and compass only, construct a triangle ABC having AB = 4·7 cm, AC = 3·4 cm and $\angle$BAC = 75°.
   Draw the perpendicular bisector of BC and the bisector of $\angle$BAC. If the perpendicular bisector and the bisector of $\angle$A meet at M, measure $\angle$BMC.
5. Using ruler and compass only, construct triangle ABC in which AB = 5 cm, angle B = 60° and angle A = 45°. Measure the length BC. Draw the perpendicular from C to AB and measure its length.
6. Using ruler and compass only, construct Δ ABC having $\angle$C = 135°, $\angle$B = 30° and BC = 5 cm. Bisect angles B and C and measure the distance of A from the point where the bisectors meet.
7. Construct an isosceles triangle ABC with base BC = 4·6 cm and $\angle$B = 75°.
8. Construct a Δ ABC having base AB = 7 cm, BC = 6·5 cm and $\angle$CAB = 60°.
9. Construct a triangle whose perimeter is 18 cm and the lengths of its sides are in the ratio 2 : 3 : 4.
10. Construct an equilateral triangle whose height is 4·3 cm.
11. Construct an isosceles triangle ABC in which
   (*i*) base BC = 4·6 cm and side AB = 5·3 cm.
   (*ii*) base BC = 5·6 cm and altitude = 4·7 cm.

(*iii*) base BC = 6·2 cm and vertical angle = 90°.

Hint

(*iii*) Each base angle = $\frac{1}{2}(180° - 90°) = 45°$

12. Construct an isosceles triangle whose height is 5 cm and vertical angle = 45°. Measure its sides.

13. Construct a right angled triangle whose hypotenuse is 5·8 cm and one side is 4·3 cm.

14. Construct a right angled triangle ABC right angled at B and CA = 2BC = 6·4 cm.

15. Using ruler and compass only, construct an isosceles right angled triangle whose hypotenuse is 5 cm. Measure and record the length of equal sides of the triangle.

16. Using ruler and compass only, construct a triangle ABC, given that base BC = 7 cm, ∠ ABC = 60° and AB + AC = 12 cm.

17. Construct a triangle ABC, given BC = 7 cm, AB – AC = 1 cm and ∠ ABC = 45°. Measure the lengths of AB and AC.

18. Construct a triangle ABC, given BC = 6·5 cm, AC – AB = 2·8 cm and ∠ B = 60°.

19. Using ruler and compass only, construct a Δ ABC from the following data : AB + BC + CA = 12 cm, ∠ B = 45° and ∠ C = 60°. Measure BC.

20. Using ruler and compass only, construct an isosceles triangle of height 2 cm and perimeter 8 cm. Measure the base of the triangle. State your steps of construction briefly.

Hint

Draw PQ = 8 cm. Bisect it at D, draw DR ⊥ PQ and cut off DA = 2 cm. Join AP, AQ. Let perpendicular bisectors of AP, AQ meet PQ at B, C respectively. Δ ABC is the required triangle.

21. Construct a triangle ABC in which BC = 4·6 cm, AB = AC and AB + AD = 7 cm where AD is the altitude. Measure AC.

Hint

Draw BC = 4·6 cm. Bisect it at D. Draw DP ⊥ BC. Cut off DQ = 7 cm. Join BQ, let perpendicular bisector of BQ meet DQ at A.

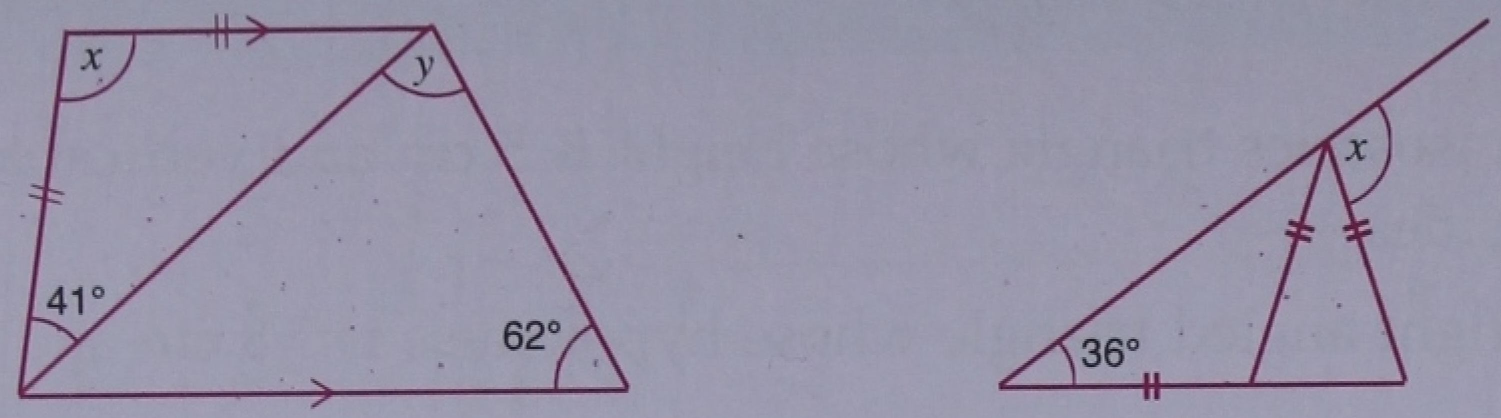
22. Construct a Δ ABC in which BC = 6 cm, ∠B = 60° and the length of perpendicular from vertex A to base = 3·5 cm.

Hint

Draw BC = 6 cm, ∠CBP = 60°. Draw a st. line parallel to BC at a distance 3·5 cm, let it meet BP at A. Join AC.

CHAPTER TEST

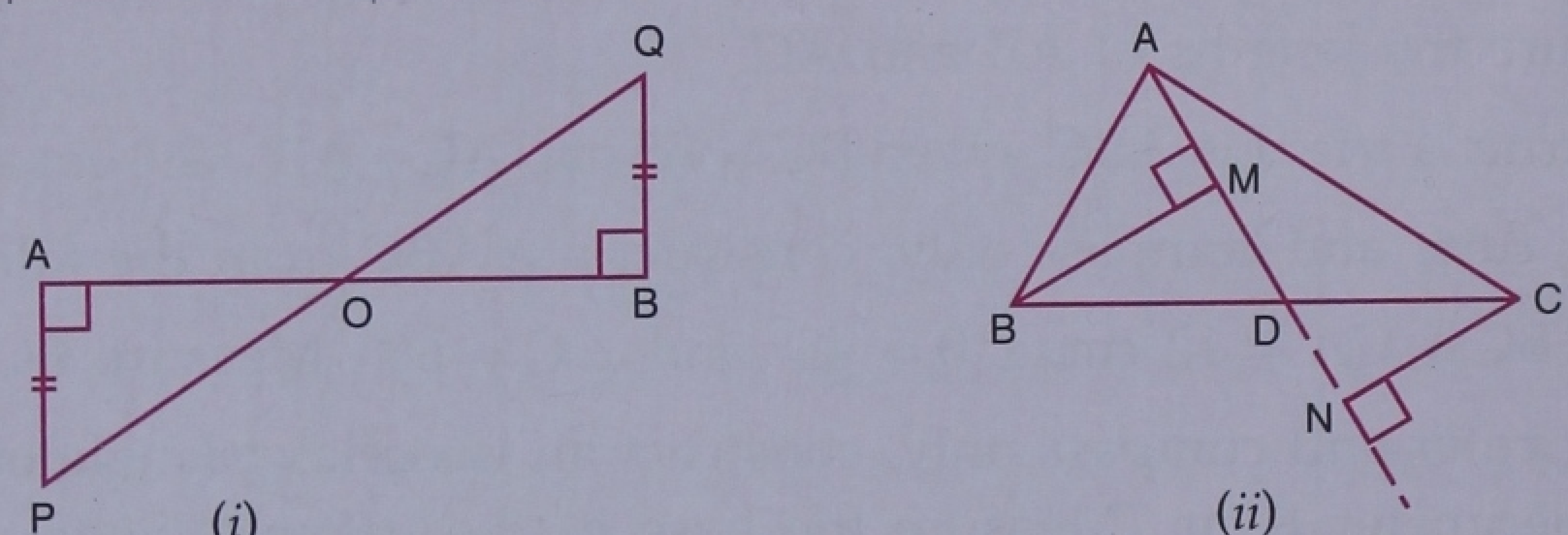
1. Calculate the size of each lettered angle in the following figures :



(*i*) (*ii*)

2. (*a*) In the figure (*i*) given below, AP and BQ are perpendiculars to the line segment AB and AP = BQ. Prove that O is the mid-point of the line segments AB and PQ.

(*b*) In the figure (*ii*) given below, AD is a median of Δ ABC and BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that BM = CN.



(*i*) (*ii*)

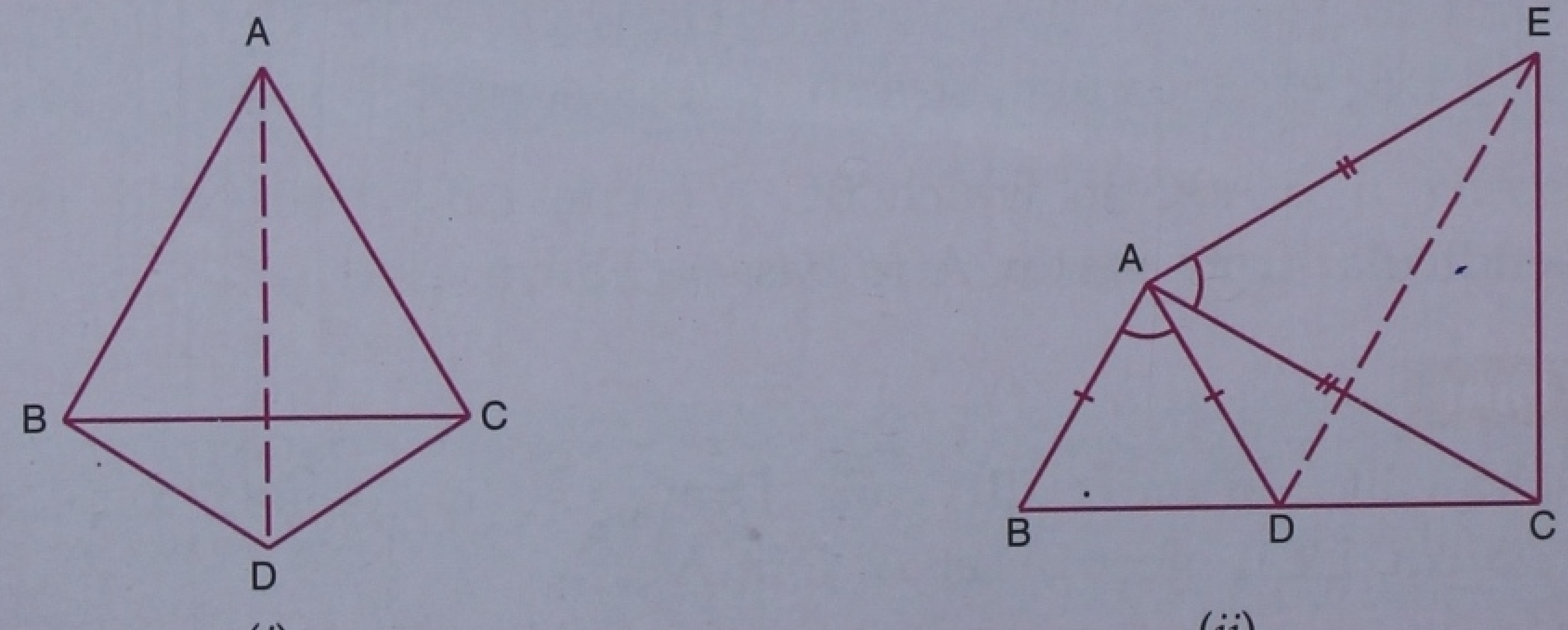
Hint

(*a*) Δ OAP ≅ Δ OBQ

(*b*) Δ BMD ≅ Δ CND.

3. (*a*) In the figure (*i*) given below, ABC and DBC are two isosceles triangles on the same base BC such that AB = AC and DB = DC. Prove that ∠ ABD = ∠ ACD and AD is bisector of ∠ A.

(*b*) In the figure (*ii*) given below, AB = AD, AC = AE and ∠ BAD = ∠ CAE. Prove that BC = ED.



(*i*) (*ii*)

Hint

(*a*) $\Delta ABD \cong \Delta ACD$

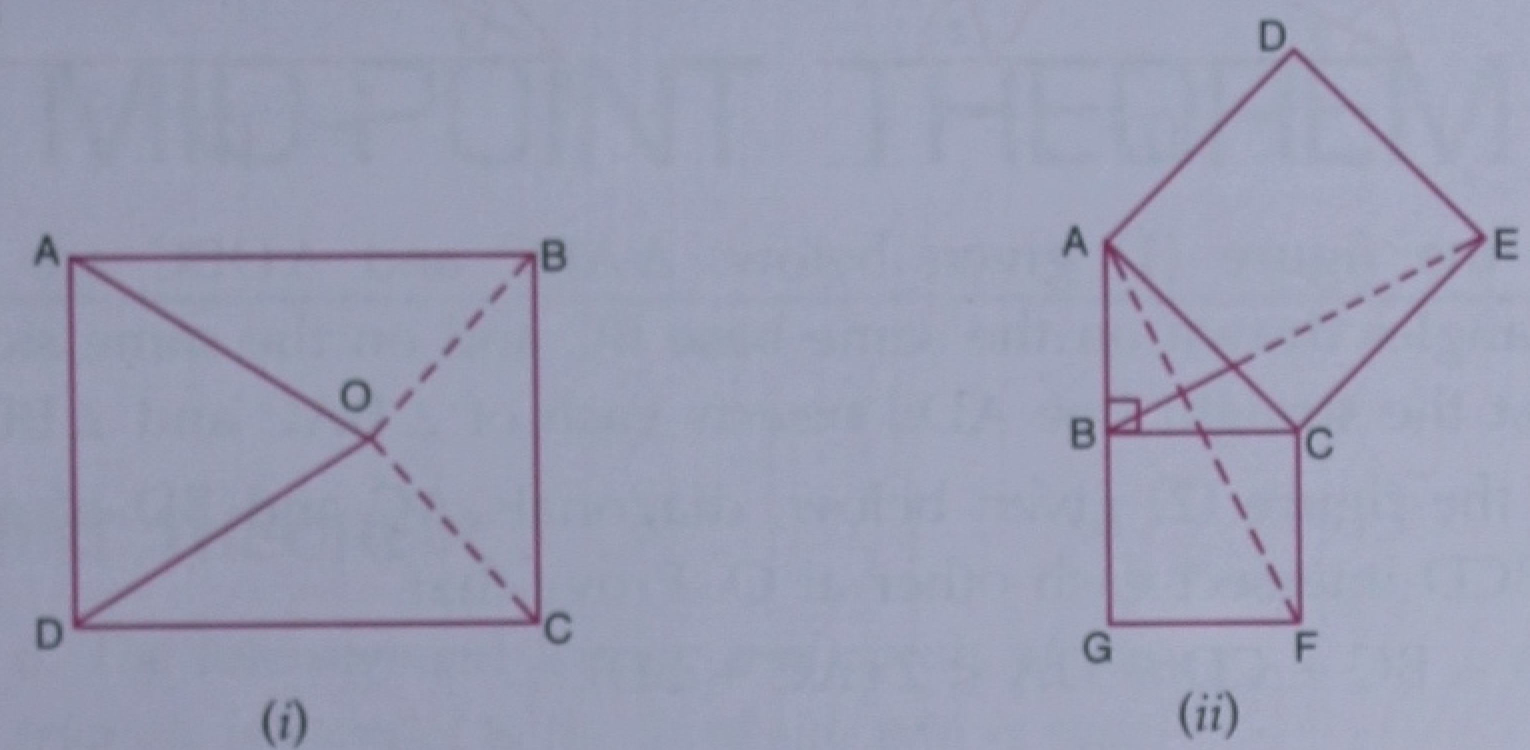
(*b*) $\angle BAD = \angle CAE$ (given)

$\Rightarrow \angle BAD + \angle DAC = \angle DAC + \angle CAE \Rightarrow \angle BAC = \angle DAE.$

$\Delta ABC \cong \Delta ADE.$

4. (*a*) In the figure (*i*) given below, ABCD is a rectangle and ΔAOD is an equilateral triangle. Prove that ΔCOB is an isosceles triangle.

(*b*) In the figure (*ii*) given below, ABC is a right angled triangle at B. ADEC and BCFG are squares. Prove that AF = BE.



Hint

(*a*) $\angle OAB = \angle DAB - \angle DAO = 90° - 60° = 30°$. Also $\angle ODC = 30°$. $\Delta OAB \cong \Delta ODC.$

(*b*) $\angle BCE = \angle BCA + 90°$, $\angle ACF = \angle BCA + 90° \Rightarrow \angle BCE = \angle ACF.$ $\Delta BCE \cong \Delta FCA.$

5. (*a*) In the figure (1) given below, AB = AC and AD is median of ΔABC. Prove that

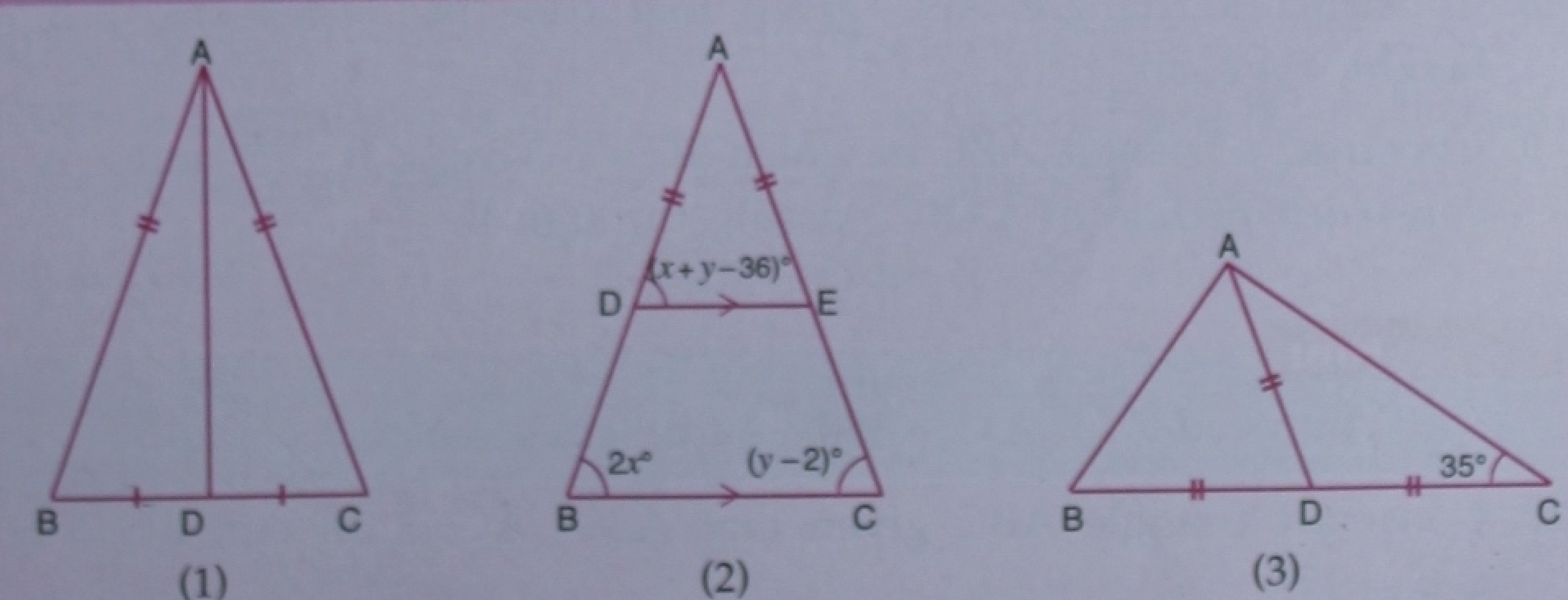
(*i*) AD is bisector of $\angle A$ (*ii*) AD is perpendicular to BC.

(*b*) In the figure (2) given below, AB = AC and DE ∥ BC. Calculate

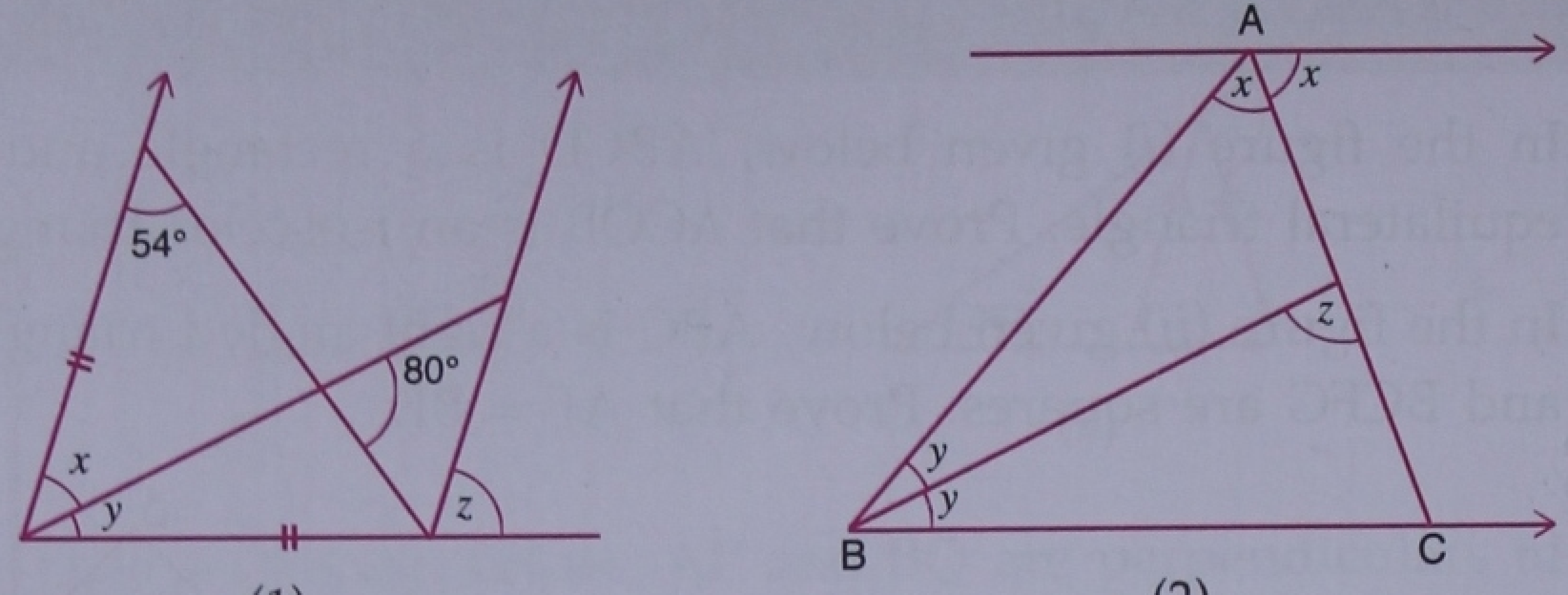
(*i*) x (*ii*) y (*iii*) $\angle BAC$.

(*c*) In the figure (3) given below, AD = BD = DC and $\angle ACD = 35°$. Show that

(*i*) AC > DC (*ii*) AB > AD.

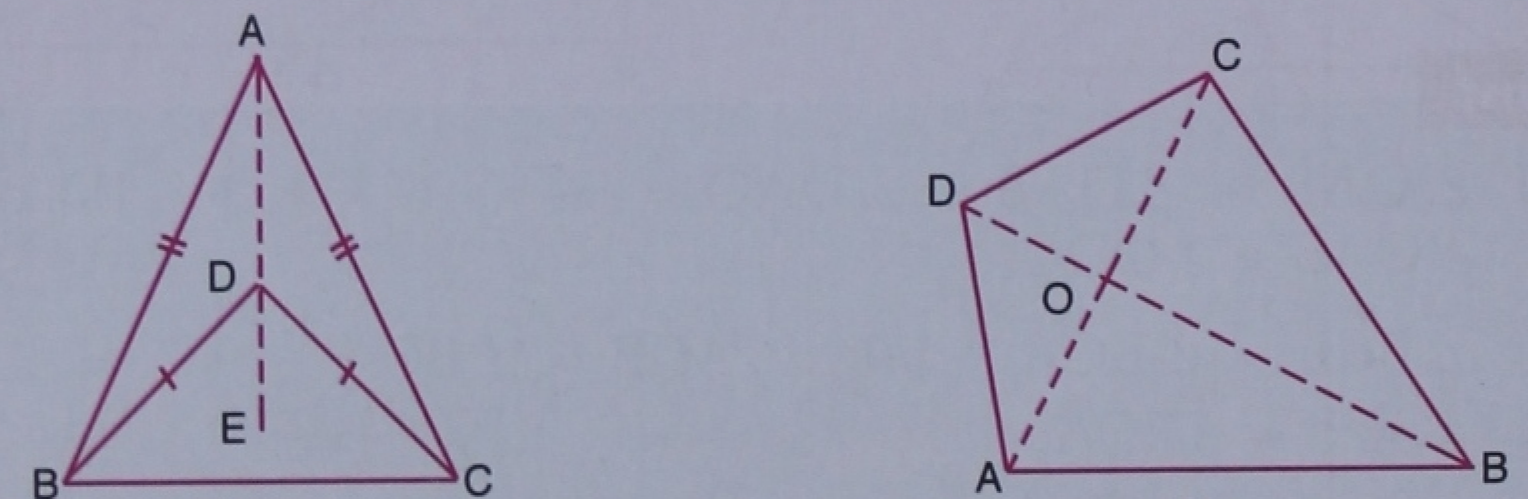


6. (*a*) In the figure (1) given below, calculate the size of each lettered angle.
   (*b*) In the figure (2) given below, prove that
   (*i*) $x + y = 90°$ (*ii*) $z = 90°$ (*iii*) AB = BC.



(1) (2)

7. (*a*) In the figure (1) given below, ΔABC and ΔDBC are two isosceles triangles drawn on the same base BC and on the same side of it. Prove that the straight line ADE bisects each of ∠BAC and ∠BDC.
   (*b*) In the figure (2) given below, diagonals AC and BD of a quadrilateral ABCD intersect each other at O. Prove that
   AB + BC + CD + DA < 2 (AC + BD).



(1) (2)

Hint

(*a*) AB = AC ⇒ ∠ABC = ∠ACB, BD = DC ⇒ ∠DBC = ∠DCB.
∴ ∠ABC – ∠DBC = ∠ACB – ∠DCB ⇒ ∠ABD = ∠ACD.
Prove that ΔABD ≅ ΔADC, so ∠BAD = ∠CAD.
Also ∠BDE = ∠ABD + ∠BAD = ∠DCA + ∠CAD = ∠CDE.
(*b*) AB < OA + OB etc.

8. Construct a triangle ABC given that base BC = 5·5 cm, ∠B = 75° and height = 4·2 cm.
9. Construct a triangle ABC in which BC = 6·5 cm, ∠B = 75° and ∠A = 45°. Also construct median of ΔABC passing through B.

Hint

You need ∠C; ∠C = 180° – ∠B – ∠A = 60°.

10. Construct triangle ABC given that AB – AC = 2·4 cm, BC = 6·5 cm and ∠B = 45°.