8

SIMULTANEOUS LINEAR EQUATIONS

8.1 SIMULTANEOUS LINEAR EQUATIONS

Linear equation in two variables. *An equation of the form* $ax + by + c = 0$*, where* a*,* b *and* c *are arbitrary constants and* a *and* b *are non-zero is called a* ***general linear equation*** *in the two variables* x *and* y*.*

For example, $x + y - 3 = 0$ is a linear equation in the two variables (unknowns) x and y.

Every linear equation in two variables has an unlimited number of solutions.

For example, $x = 0$, $y = 3$; $x = 1$, $y = 2$; $x = 2$, $y = 1$; $x = 3$, $y = 0$ and $x = 7$, $y = -4$ etc. are *all solutions* of the equation $x + y - 3 = 0$.

System of simultaneous linear equations. Let us consider two linear equations in two variables,

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

These two equations are said to form a **system of simultaneous linear equations.**

For example,

$$x + y - 3 = 0$$
$$2x - 5y + 1 = 0$$

is a system of two simultaneous linear equations in the two variables x and y.

A ***solution*** *to a system of two simultaneous linear equations in two variables is an ordered pair of numbers which satisfy both the equations.*

For the above example, $x = 2$, $y = 1$ is a solution to the system of simultaneous linear equations. We can check this by substituting $x = 2$, $y = 1$ into each of these two equations.

If there is *only one* such solution, then the system of linear equations is said to be **consistent** and **independent.** In this book, we shall be dealing only with such a system of simultaneous linear equations.

There are two methods to solve a system of simultaneous linear equations.

8.2 SUBSTITUTION METHOD

Procedure :

(*i*) Solve one of the given equations for one of the variables.

(*ii*) Substitute that value of the variable in the *other* equation.

(*iii*) Solve the resulting single variable equation. Substitute this value into either of the two *original* equations, and solve it to find the value of the second variable.

Remark

The solution may be checked by substituting in both the original equations.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the following system of linear equations :*

$$4x - 3y = 8$$
$$x - 2y = -3.$$

Solution. The given equations are

$$4x - 3y = 8 \quad \ldots(i)$$
$$x - 2y = -3 \quad \ldots(ii)$$

We can solve either equation for either variable. But to avoid fractions, we solve the second equation for x,

$$x = 2y - 3 \quad \ldots(iii)$$

Substituting this value of x in equation (*i*), we get

$$4(2y - 3) - 3y = 8$$
$$\Rightarrow 8y - 12 - 3y = 8$$
$$\Rightarrow 5y = 20 \Rightarrow y = 4.$$

Substituting this value of y in (*ii*), we get

$$x - 2 \times 4 = -3 \Rightarrow x - 8 = -3 \Rightarrow x = 5.$$

Hence the solution is $x = 5$, $y = 4$.

Example 2. *Solve the following system of linear equations :*

$$8x + 5y = 9$$
$$3x + 2y = 4.$$

Solution. The given system of simultaneous linear equations is

$$8x + 5y = 9 \quad \ldots(i)$$
$$3x + 2y = 4 \quad \ldots(ii)$$

From equation (*ii*), we get

$$2y = 4 - 3x \Rightarrow y = \frac{4 - 3x}{2}.$$

Substituting this value of y in equation (*i*), we get

$$8x + 5 \cdot \frac{4 - 3x}{2} = 9$$
$$\Rightarrow 16x + 20 - 15x = 18 \Rightarrow x + 20 = 18$$
$$\Rightarrow x = -2.$$

Substituting this value of x in equation (*ii*), we get

$$3 \times (-2) + 2y = 4$$
$$\Rightarrow -6 + 2y = 4 \Rightarrow 2y = 10 \Rightarrow y = 5.$$

Hence the solution is $x = -2$, $y = 5$.

Remark

An alternative method of solving system of simultaneous linear equations that is perhaps superior to substitution for system such as

$4x - 3y - 11 = 0$

$6x + 7y - 5 = 0$

is known as **elimination** (or **addition-subtraction**) **method.** Solving the above system of equations by using the substitution method usually involves fractions regardless of our choice of variable and choice of equation. The elimination method enables us to avoid fractions.

8.3 ELIMINATION METHOD

Procedure :

(*i*) Multiply one or both equations (if necessary) by suitable numbers to transform them so that addition or subtraction will eliminate one variable.

(*ii*) Solve the resulting single variable equation and substitute this value into either of the two original equations, and solve it to find the value of the second variable.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the system of simultaneous linear equations :*

$4x - 3y - 11 = 0$

$6x + 7y - 5 = 0.$

Solution. The given system of simultaneous equations is

$4x - 3y - 11 = 0$ ...(*i*)

$6x + 7y - 5 = 0$ ...(*ii*)

Multiplying the first equation by 3 and the second equation by 2, making both coefficients of the x terms 12 (L.C.M. of 4 and 6), we get

$12x - 9y - 33 = 0$ ...(*iii*)

$12x + 14y - 10 = 0$ ...(*iv*)

Subtracting (*iv*) from (*iii*), we get

$-23y - 23 = 0 \Rightarrow -23y = 23 \Rightarrow y = -1.$

Substituting $y = -1$ in equation (*i*), we get

$4x - 3(-1) - 11 = 0 \Rightarrow 4x + 3 - 11 = 0$

$\Rightarrow 4x - 8 = 0 \Rightarrow 4x = 8 \Rightarrow x = 2.$

Hence the solution is $x = 2$, $y = -1$.

Note

In particular, if the coefficient of x in the first equation is numerically equal to the coefficient of y in the second equation and the coefficient of y in the first equation is numerically equal to the coefficient of x in the second equation, it is better to proceed as in example 2 below.

Example 2. *Solve* $83x - 67y = 383$

$67x - 83y = 367.$

Solution. Given $83x - 67y = 383$ ...(*i*)

$67x - 83y = 367$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$150x - 150y = 750$$

$$\Rightarrow \quad x - y = 5 \quad \ldots(iii)$$

Subtracting (*ii*) from (*i*), we get

$$16x + 16y = 16$$

$$\Rightarrow \quad x + y = 1 \quad \ldots(iv)$$

On adding (*iii*) and (*iv*), we get

$$2x = 6 \quad \Rightarrow \quad x = 3.$$

Substituting $x = 3$ in equation (*iv*), we get

$$3 + y = 1 \quad \Rightarrow \quad y = -2.$$

Hence the solution is $x = 3$, $y = -2$.

Example 3. *Solve :* $\frac{3x-7}{2} - \frac{2y-8}{3} = -1$, $\frac{5-x}{3} - \frac{3-2y}{7} = 1$.

Solution. Given $\frac{3x-7}{2} - \frac{2y-8}{3} = -1 \quad \ldots(i)$

$$\frac{5-x}{3} - \frac{3-2y}{7} = 1 \quad \ldots(ii)$$

To clear fractions, multiplying equation (*i*) by 6 and equation (*ii*) by 21, we get

$$3(3x - 7) - 2(2y - 8) = -6 \Rightarrow 9x - 4y + 1 = 0 \quad \ldots(iii)$$

$$7(5 - x) - 3(3 - 2y) = 21 \Rightarrow -7x + 6y + 5 = 0 \quad \ldots(iv)$$

On multiplying equation (*iii*) by 3 and equation (*iv*) by 2, we get

$$27x - 12y + 3 = 0 \quad \ldots(v)$$

$$-14x + 12y + 10 = 0 \quad \ldots(vi)$$

On adding (*v*) and (*vi*), we get

$$13x + 13 = 0 \Rightarrow 13x = -13 \Rightarrow x = -1.$$

Substituting $x = -1$ in (*iv*), we get

$$-7(-1) + 6y + 5 = 0 \Rightarrow 6y = -12 \Rightarrow y = -2.$$

Hence the solution is $x = -1$, $y = -2$.

Example 4. *Solve :* $4x + \frac{6}{y} = 15$, $3x - \frac{4}{y} = 7$.

Solution. Given $4x + \frac{6}{y} = 15 \quad \ldots(i)$

$$3x - \frac{4}{y} = 7 \quad \ldots(ii)$$

Multiplying (*i*) by 2 and (*ii*) by 3, we get

$$8x + \frac{12}{y} = 30 \quad \ldots(iii)$$

$$9x - \frac{12}{y} = 21 \quad \ldots(iv)$$

Adding (*iii*) and (*iv*), we get

$$17x = 51 \Rightarrow x = 3.$$

Substituting $x = 3$ in (*i*), we get

$$4 \times 3 + \frac{6}{y} = 15 \Rightarrow \frac{6}{y} = 3 \Rightarrow y = \frac{6}{3} = 2.$$

Hence the solution is $x = 3$, $y = 2$.

Example 5. *Solve :* $\frac{8}{x}-\frac{5}{y}=34,\ \frac{3}{x}-\frac{2}{y}=13.$

Solution. Given $\frac{8}{x}-\frac{5}{y}=34$ ...(*i*)

$\frac{3}{x}-\frac{2}{y}=13$ ...(*ii*)

Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$, then the given equations become

$8a-5b=34$ ...(*iii*)

$3a-2b=13$ ...(*iv*)

On multiplying (*iii*) by 2 and (*iv*) by 5, we get

$16a-10b=68$ ...(*v*)

$15a-10b=65$ ...(*vi*)

Subtracting (*vi*) from (*v*), we get $a=3$.

Substituting $a=3$ in (*iv*), we get

$$3\times 3-2b=13 \Rightarrow -2b=4 \Rightarrow b=-2.$$

$$\therefore \quad \frac{1}{x}=3 \text{ and } \frac{1}{y}=-2 \Rightarrow x=\frac{1}{3} \text{ and } y=-\frac{1}{2}.$$

Hence the solution is $x=\frac{1}{3},\ y=-\frac{1}{2}$.

Example 6. *Solve :* $\frac{2}{x}+\frac{5}{y}=1,\ \frac{60}{x}-\frac{20}{y}=13.$ *Hence find the value of k if* $y=kx-2$.

Solution. Given $\frac{2}{x}+\frac{5}{y}=1$ ...(*i*)

$\frac{60}{x}-\frac{20}{y}=13$ ...(*ii*)

Let $\frac{1}{x}=a$ and $\frac{1}{y}=b$, then the given equations become

$2a+5b=1$ ...(*iii*)

$60a-20b=13$ ...(*iv*)

Multiplying (*iii*) by 4, we get

$8a+20b=4$ ...(*v*)

Adding (*iv*) and (*v*), we get

$$68a=17 \Rightarrow a=\frac{1}{4}.$$

Substituting this value of a in (*iii*), we get

$$2\times\frac{1}{4}+5b=1 \Rightarrow \frac{1}{2}+5b=1 \Rightarrow 5b=1-\frac{1}{2}$$

$$\Rightarrow 5b=\frac{1}{2} \Rightarrow b=\frac{1}{10}.$$

$$\therefore \quad \frac{1}{x}=\frac{1}{4} \text{ and } \frac{1}{y}=\frac{1}{10} \Rightarrow x=4 \text{ and } y=10.$$

Hence the solution is $x=4,\ y=10$.

To find k

Putting $x=4$ and $y=10$ in $y=kx-2$, we get

$$10=4k-2 \Rightarrow 4k=12 \Rightarrow k=3.$$

Example 7. *Solve* : $4x + 9y = 30xy$, $5y - 3x = xy$.

Solution. The given system of simultaneous equations is

$$4x + 9y = 30xy \quad \ldots(i)$$

$$5x - 3y = xy \quad \ldots(ii)$$

First, we note that $x = 0$, $y = 0$ is a solution of the given system of equations. Now, when $x \neq 0$, $y \neq 0$, then dividing both sides of each equation by xy, we get

$$\frac{4}{y} + \frac{9}{x} = 30 \quad \ldots(iii)$$

$$\frac{5}{x} - \frac{3}{y} = 1 \quad \ldots(iv)$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$, then equations (*iii*) and (*iv*) become

$$9a + 4b = 30 \quad \ldots(v)$$

$$5a - 3b = 1 \quad \ldots(vi)$$

On multiplying (*v*) by 3 and (*vi*) by 4, we get

$$27a + 12b = 90 \quad \ldots(vii)$$

$$20a - 12b = 4 \quad \ldots(viii)$$

Adding (*vii*) and (*viii*), we get

$$47a = 94 \Rightarrow a = 2.$$

Substituting $a = 2$ in (*v*), we get

$$9 \times 2 + 4b = 30 \Rightarrow 4b = 12 \Rightarrow b = 3.$$

$\therefore \quad \frac{1}{x} = 2$ and $\frac{1}{y} = 3 \Rightarrow x = \frac{1}{2}$ and $y = \frac{1}{3}$.

Hence the solutions of the given system of equations are $x = 0$, $y = 0$; $x = \frac{1}{2}$, $y = \frac{1}{3}$.

Example 8. *Solve* : $\frac{20}{x+y} + \frac{3}{x-y} = 7$, $\frac{8}{x-y} - \frac{15}{x+y} = 5$.

Solution. Let $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$, then the given equations become

$$20a + 3b = 7 \quad \ldots(i)$$

$$8b - 15a = 5 \quad \ldots(ii)$$

Multiplying (*i*) by 3 and (*ii*) by 4, we get

$$60a + 9b = 21 \quad \ldots(iii)$$

$$-60a + 32b = 20 \quad \ldots(iv)$$

Adding (*iii*) and (*iv*), we get

$$41b = 41 \Rightarrow b = 1.$$

Substituting $b = 1$ in (*i*), we get

$$20a + 3 \times 1 = 7 \Rightarrow 20a = 4 \Rightarrow a = \frac{1}{5}.$$

$\therefore \quad \frac{1}{x+y} = \frac{1}{5}$ and $\frac{1}{x-y} = 1$

$\Rightarrow \quad x + y = 5,\ x - y = 1.$

Adding these equations, we get

$$2x = 6 \Rightarrow x = 3.$$

$$\therefore \quad 3 + y = 5 \Rightarrow y = 2.$$

Hence the solution of the given linear equations is $x = 3$, $y = 2$.

Example 9. *Can the following equations hold simultaneously?*

$$\frac{x}{2} + \frac{5y}{3} = 12, \frac{5x}{4} - \frac{y}{6} = 4 \text{ and } 7x - 3y = 10.$$

If so, find x and y.

Solution. The given equations are

$$\frac{x}{2} + \frac{5y}{3} = 12 \quad \ldots(i)$$

$$\frac{5x}{4} - \frac{y}{6} = 4 \quad \ldots(ii)$$

$$7x - 3y = 10 \quad \ldots(iii)$$

Let us solve the first two equations simultaneously. To clear fractions, multiplying equation (*i*) by 6 and equation (*ii*) by 12, we get

$$3x + 10y = 72 \quad \ldots(iv)$$

$$15x - 2y = 48 \quad \ldots(v)$$

On multiplying equation (*v*) by 5, we get

$$75x - 10y = 240 \quad \ldots(vi)$$

On adding (*iv*) and (*vi*), we get

$$78x = 312 \Rightarrow x = 4.$$

Substituting $x = 4$ in (*iv*), we get

$$3 \times 4 + 10y = 72 \Rightarrow 10y = 60 \Rightarrow y = 6.$$

Thus $x = 4$ and $y = 6$ is the solution of (*i*) and (*ii*).

Putting $x = 4$ and $y = 6$ in equation (*iii*), we get

$$7 \times 4 - 3 \times 6 = 10 \Rightarrow 28 - 18 = 10 \Rightarrow 10 = 10,$$

which is true.

Therefore, the three given equations can hold simultaneously *i.e.* they are **consistent** and the solution is $x = 4$ and $y = 6$.

Note

If the values of x and y obtained from two equations do not satisfy the third, then the three equations cannot hold simultaneously and we conclude that the three equations are **inconsistent**.

Exercise 8.1

Solve the following systems of simultaneous linear equations (1 to 14) :

1. (*i*) $x + y - 5 = 0$
$y - 2 = 2x$

(*ii*) $2x = 5y + 4$
$3x - 2y = -16.$

2. (*i*) $a + 3b = 5$
$7a - 8b = 6$
(*ii*) $5x + 4y - 4 = 0$
$x - 20 = 12y.$

3. (*i*) $2x - \frac{3y}{4} = 3$
$5x - 2y - 7 = 0$
(*ii*) $2x + 3y = 23$
$5x - 20 = 8y.$

4. (*i*) $\frac{3}{4}x - \frac{2}{3}y = 1$
$\frac{3}{8}x - \frac{1}{6}y = 1$
(*ii*) $2x - 3y - 3 = 0$
$\frac{2x}{3} + 4y + \frac{1}{2} = 0.$

5. (*i*) $15x - 14y = 117$
$14x - 15y = 115$
(*ii*) $41x + 53y = 135$
$53x + 41y = 147.$

6. (*i*) $\frac{x}{6} = y - 6$
$\frac{3x}{4} = 1 + y$
(*ii*) $x - \frac{2}{3}y = \frac{8}{3}$
$\frac{2x}{5} - y = \frac{7}{5}.$

7. (*i*) $9 - (x - 4) = y + 7$
$2(x + y) = 4 - 3y$
(*ii*) $2x + \frac{x - y}{6} = 2$
$x - \frac{2x + y}{3} = 1.$

8. (*i*) $103x + 51y = 617$
$97x + 49y = 583$
(*ii*) $\frac{7 + x}{5} - \frac{2x - y}{4} = 3y - 5$
$\frac{5y - 7}{2} + \frac{4x - 3}{6} = 18 - 5x.$

9. (*i*) $4x + \frac{x - y}{8} = 17$
$2y + x - \frac{5y + 2}{3} = 2$
(*ii*) $x - 3y = 3x - 1 = 2x - y.$

Hint

(*ii*) $x - 3y = 3x - 1,\ 3x - 1 = 2x - y.$

10. (*i*) $\frac{3}{x} + 4y = 7$
$\frac{5}{x} + 6y = 13$
(*ii*) $5x - 9 = \frac{1}{y}$
$x + \frac{1}{y} = 3.$

11. (*i*) $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$
$\frac{2}{x} - \frac{1}{y} = 1$
(*ii*) $\frac{3}{2x} + \frac{2}{3y} = 5$
$\frac{5}{x} - \frac{3}{y} = 1.$

12. (*i*) $3x + 14y = 5xy$
$21y - x = 2xy$
(*ii*) $3x + 5y = 4xy$
$2y - x = xy.$

13. (*i*) $\frac{20}{x + 1} + \frac{4}{y - 1} = 5$
$\frac{10}{x + 1} - \frac{4}{y - 1} = 1$
(*ii*) $\frac{3}{x + y} + \frac{2}{x - y} = 3$
$\frac{2}{x + y} + \frac{3}{x - y} = \frac{11}{3}.$

14. (*i*) $\dfrac{1}{2(2x+3y)} + \dfrac{12}{7(3x-2y)} = \dfrac{1}{2}$

$\dfrac{7}{2x+3y} + \dfrac{4}{3x-2y} = 2$

(*ii*) $\dfrac{1}{2(x+2y)} + \dfrac{5}{3(3x-2y)} = -\dfrac{3}{2}$

$\dfrac{5}{4(x+2y)} - \dfrac{3}{5(3x-2y)} = \dfrac{61}{60}$.

15. Solve $2x + y = 35$, $3x + 4y = 65$. Hence find the value of $\dfrac{x}{y}$.

16. Solve the simultaneous equations $3x - y = 5$, $4x - 3y = -1$. Hence find p, if $y = px - 3$.

17. Solve $2x + y = 23$, $4x - y = 19$. Hence find the values of $x - 3y$ and $5y - 2x$.

18. The expression $ax + by$ has value 7 when $x = 2$, $y = 1$. When $x = -1$, $y = 1$, it has value 1, find a and b.

19. The sides of an equilateral triangle are $(6x + 5y)$ cm, $(7x + 3y + 1)$ cm and $2(x + 6y - 1)$ cm respectively. Find the length of each side.

Hint

$6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$. Solve for x and y.

20. Can the following equations hold simultaneously?

$3x - 7y = 7$

$11x + 5y = 87$

$5x + 4y = 43$.

If so, find x and y.

8.4 GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

The graph of a linear equation in two variables is always a straight line.

To draw graphs of linear equations in two variables x and y, proceed as under :

(*i*) Rewrite the given equation with y as the subject.

(*ii*) Select any three convenient values of x and find the corresponding values of y for each of the selected value of x.

(*iii*) Make table of values.

(*iv*) Draw the axes on the graph paper and choose suitable scale.

(*v*) Select same scale on both axes.

(*vi*) Plot the points on the graph paper (coordinate plane).

(*vii*) Connect any two points by a straight line and check that the third point lies on it.

ILLUSTRATIVE EXAMPLES

Example 1. *Draw the graph of* $3x - 2y - 2 = 0$.

Solution. The given equation is $3x - 2y - 2 = 0$, it can be written as

$2y = 3x - 2$ or $y = \dfrac{3}{2}x - 1$ ...(*i*)

Select any three values of x, say 0, 2, 4, and find the corresponding values of y by using equation (i).

When $x = 0$, $\quad y = \frac{3}{2} . 0 - 1 = -1,$

$x = 2$, $\quad y = \frac{3}{2} . 2 - 1 = 2,$

$x = 4$, $\quad y = \frac{3}{2} . 4 - 1 = 5.$

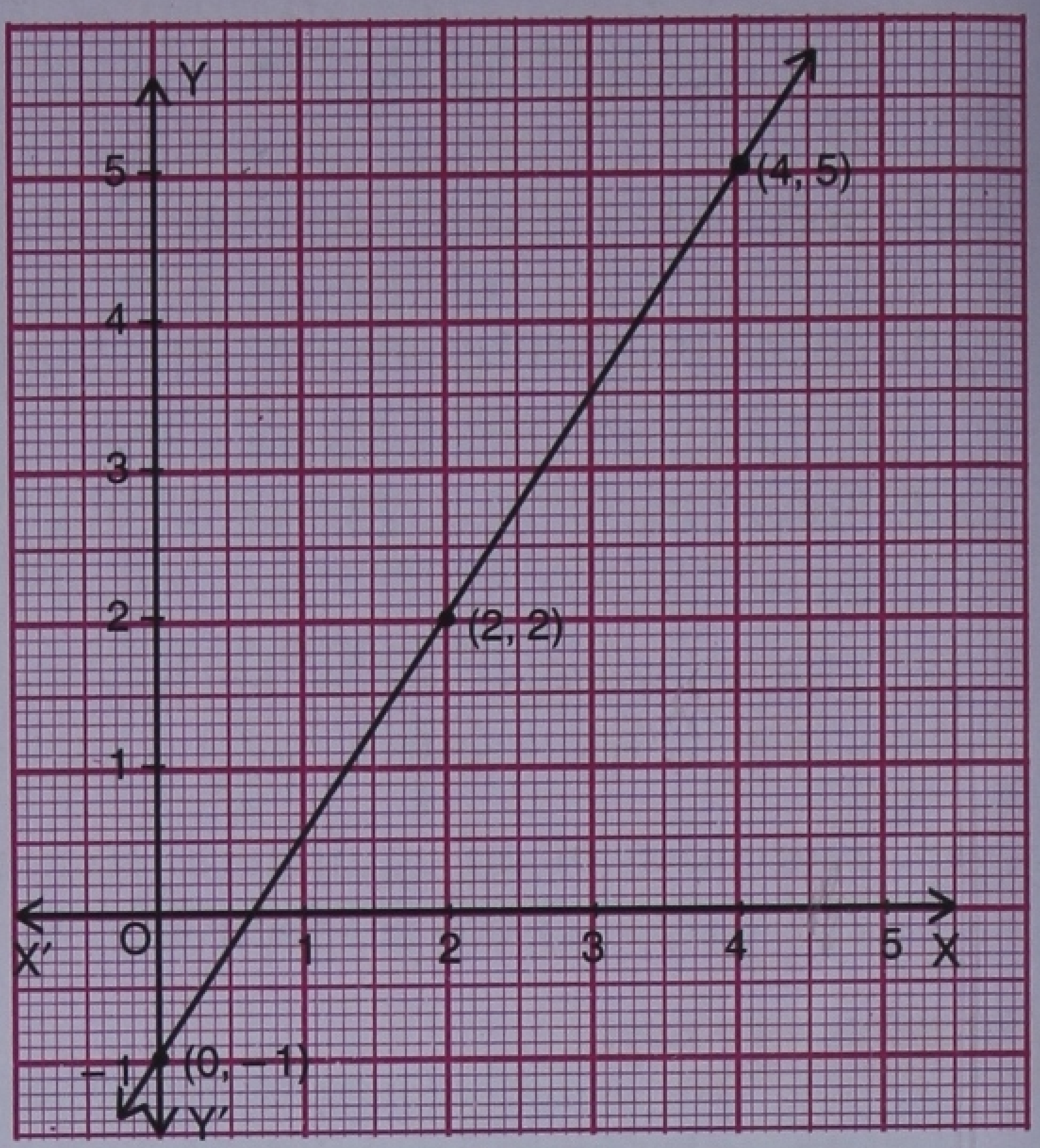


Table of values

x	0	2	4
y	-1	2	5

Select coordinate axes and take 1 cm = 1 unit on both the axes.

Plot the points (0, – 1), (2, 2) and (4, 5) on the graph paper (coordinate plane). Connect any two points by a straight line. The graph of the given linear equation is shown in the adjoining figure.

Observe that the third point lies on the straight line.

Remark

Select values of x in such a manner that the points to be plotted are not too close.

Example 2. *Draw the graph of $2x + 3y = 6$ and use it to find the area of the triangle formed by the line and the coordinate axes. Take 1 cm = 1 unit on both the axes.*

Solution. The given equation is $2x + 3y = 6$, it can be written as

$$3y = -2x + 6 \quad \text{or} \quad y = -\frac{2}{3}x + 2 \qquad \ldots(i)$$

Select any three values of x, say 0, 3, – 3 and find the corresponding values of y by using equation (i).

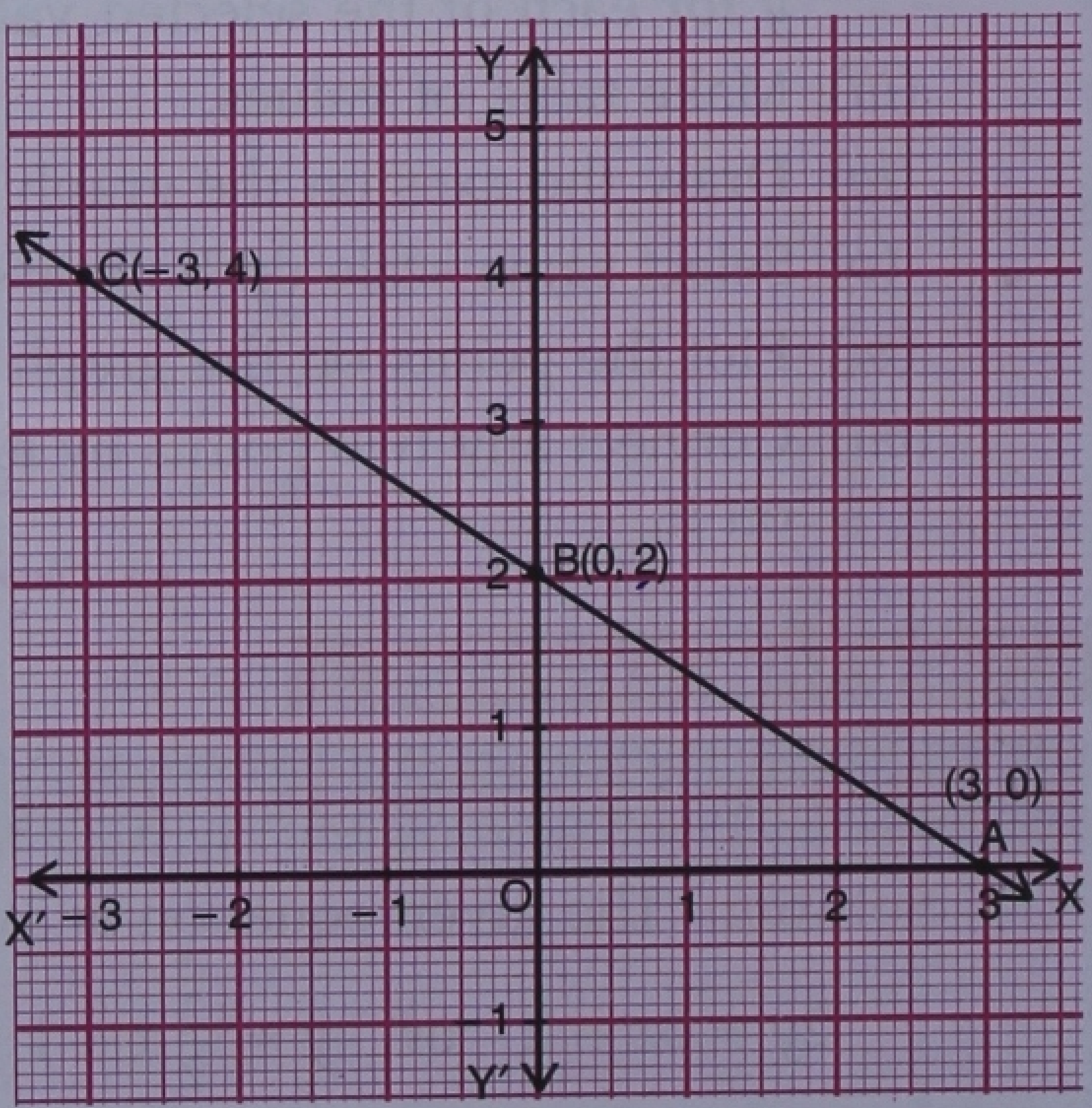
When $x = 0$, $\quad y = -\frac{2}{3} . 0 + 2 = 2,$

$x = 3$, $\quad y = -\frac{2}{3} . 3 + 2 = 0,$

$x = -3$, $\quad y = -\frac{2}{3}(-3) + 2 = 4.$

Table of values

x	0	3	-3
y	2	0	4



Select coordinate axes and take 1 cm = 1 unit on both the axes. Plot the points A(3, 0), B(0, 2) and C(– 3, 4) on the graph paper. Connect any points by a straight line. The graph of the given equation is shown in the adjoining figure. Observe that the third point lies on the straight line.

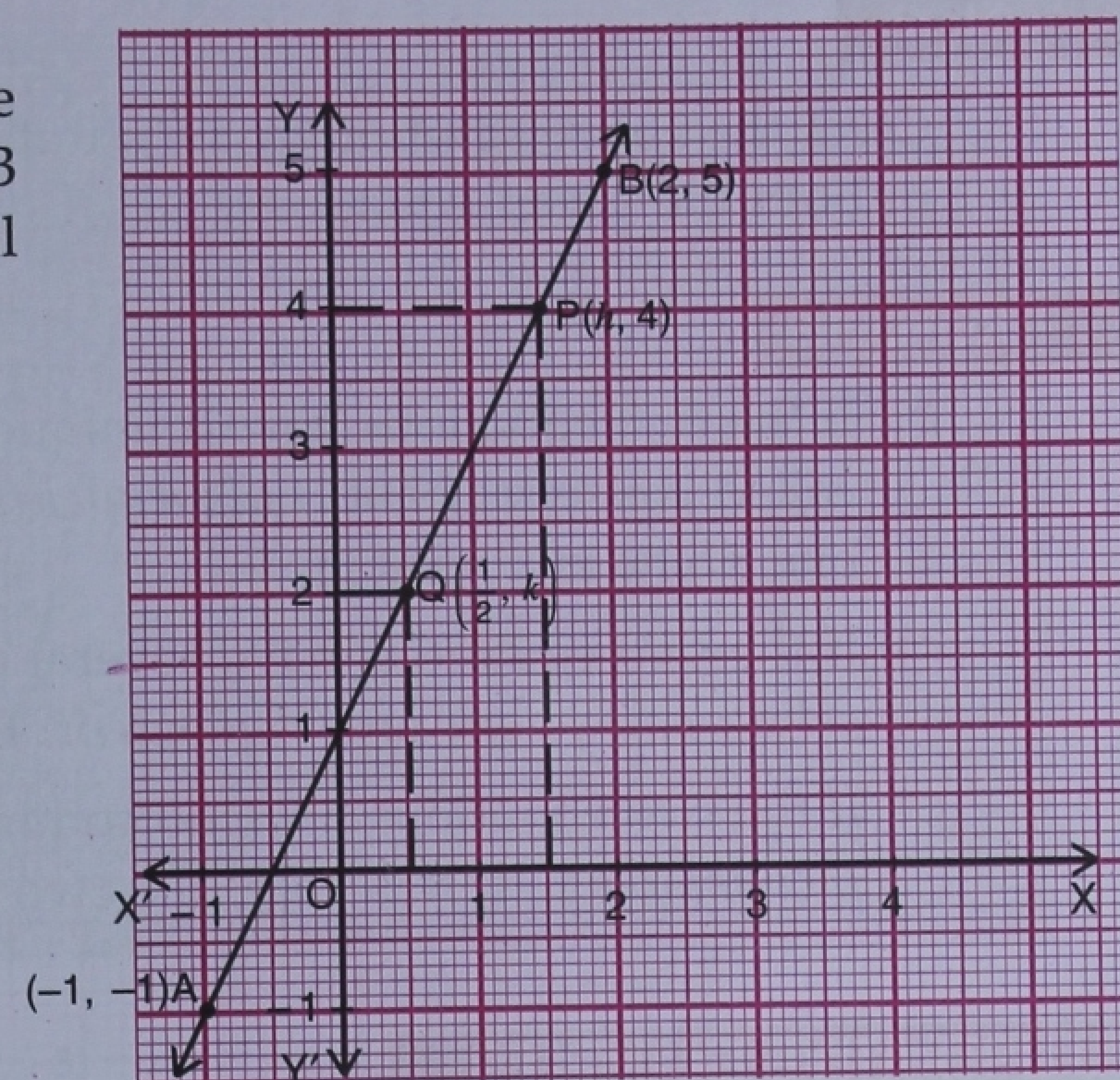
Area of the triangle formed by the line and the coordinate axes

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 3 \times 2 = 3$ sq. units.

Example 3. *The graph of a linear equation in x and y passes through A(–1, –1) and B(2, 5). Find the values of h and k if the graph passes through (h, 4) and* $\left(\frac{1}{2}, k\right)$.

Solution. Select coordinate axes and take 1 cm = 1 unit on both axes. Plot the points A(– 1, – 1) and B(2, 5) on the graph paper and draw a straight line passing through these points.



Through $y = 4$, draw a horizontal line to meet the graph of the straight line AB at the point P. Through P, draw a vertical line which meets x-axis at $x = \frac{3}{2}$

$\Rightarrow \quad h = \frac{3}{2}$.

Similarly, through $x = \frac{1}{2}$, draw a vertical line to meet the graph of the straight AB at Q. Through Q, draw a horizontal line which meets y-axis at $y = 2 \Rightarrow k = 2$.

Hence $h = \frac{3}{2}$ and $k = 2$.

Exercise 8.2

1. Draw the graphs of the following linear equations :
   (*i*) $2x + y + 3 = 0$ (*ii*) $x - 5y - 4 = 0$.
2. Draw the graph of $3y = 12 - 2x$. Take 2 cm = 1 unit on both axes.
3. Draw the graph of $5x + 6y - 30 = 0$ and use it to find the area of the triangle formed by the line and the coordinate axes.
4. Draw the graph of $4x - 3y + 12 = 0$ and use it to find the area of the triangle formed by the line and the coordinate axes. Take 2 cm = 1 unit on both axes.
5. Draw the graph of the equation $y = 3x - 4$. Find graphically
   (*i*) the value of y when $x = -1$ (*ii*) the value of x when $y = 5$.
6. The graph of a linear equation in x and y passes through (4, 0) and (0, 3). Find the value of k if the graph passes through (k, 1·5).
7. Use the table given alongside to draw the graph of a straight line. Find, graphically, the values of a and b.

x	1	2	3	a
y	– 2	b	4	– 5

8.5 GRAPHICAL SOLUTION OF A PAIR OF LINEAR EQUATIONS

To solve graphically a system of two simultaneous linear equations in two variables x and y, proceed as under :

(*i*) Draw graph (straight line) for each of the given linear equation.

(*ii*) Find the coordinates of the point of intersection of the two lines drawn.

(*iii*) The coordinates of the point of intersection of the two lines will be the *common solution* of the given equations.

(*iv*) *Write the values of x and y.*

Note

Check the above solution by substituting the values of x and y (obtained above) in both the given equations.

Remarks

- If the two equations have a unique common solution, then the equations are called ***consistent*** and ***independent***. In this case, the lines have one and only one point in common.
- If the two equations have several common solutions, then the equations are called ***consistent*** and ***dependent***. In this case, the two lines will coincide.
- If the two equations have no common solution, then the equations are called ***inconsistent***. In this case, the two lines will be parallel.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the following system of equations graphically :*

$$4x - y = 5, \quad 5y - 4x = 7.$$

Solution. The given equations can be written as

$y = 4x - 5$ ... (*i*) and $y = \frac{1}{5}(4x + 7)$ ...(*ii*)

Table of values for equation (*i*)

x	1	0	3
y	– 1	– 5	7

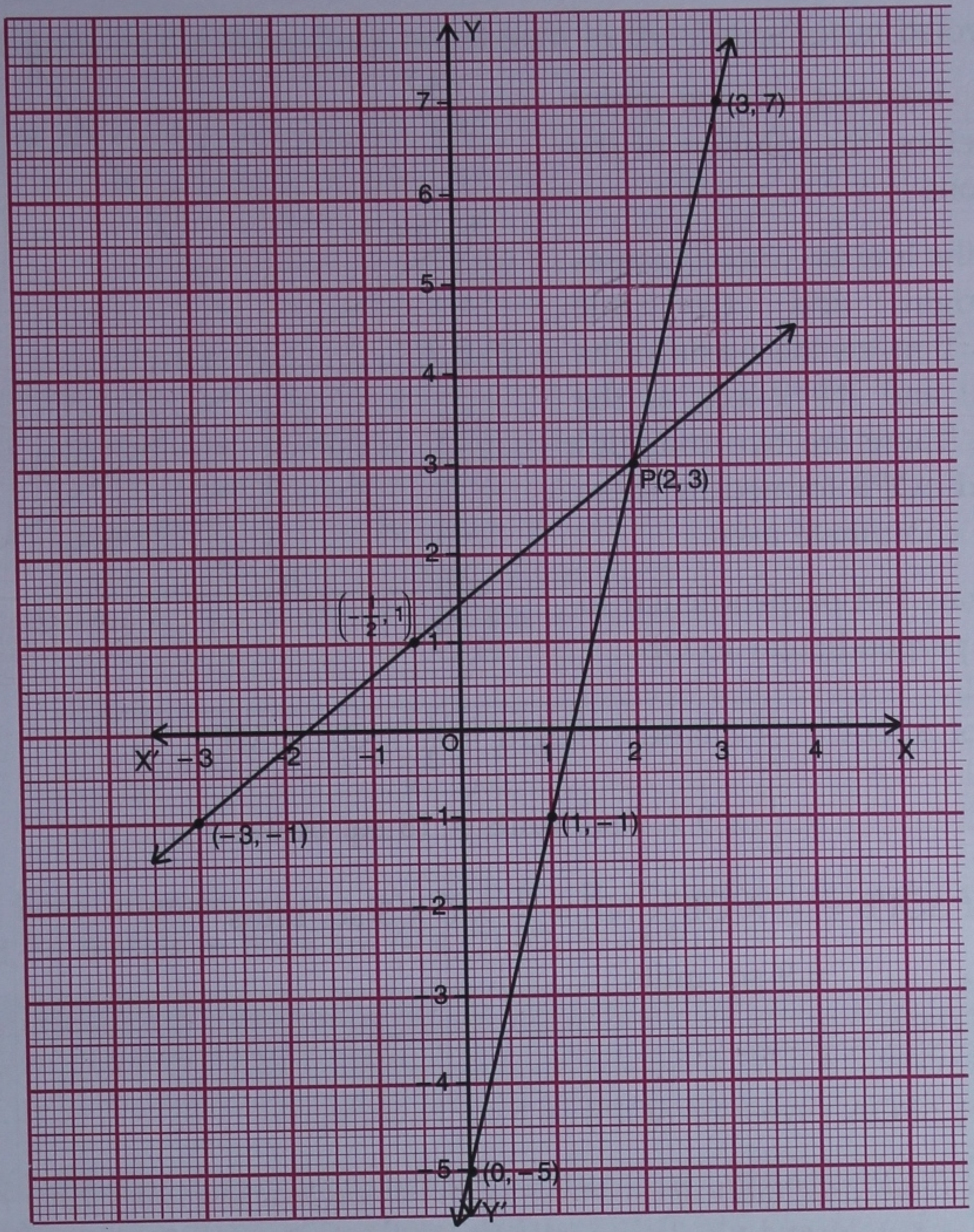
Table of values for equation (*ii*)

x	– 3	$-\frac{1}{2}$	2
y	– 1	1	3

Select coordinate axes and take 1 cm = 1 unit on both the axes. Plot the points (1, – 1), (0, – 5) and (3, 7) on a graph paper. Connect any two points by a straight line.

Plot the points (– 3, – 1), $\left(-\frac{1}{2}, 1\right)$ and (2, 3) on the same graph paper. Connect any two points by a straight line. The graphs of both the straight lines are shown in the figure given below.

The lines intersect at the point P(2, 3). Therefore, the solution of the given equations is $x = 2$, $y = 3$.



Check. On substituting $x = 2$, $y = 3$ in the given equations, we find that it satisfies both the given equations.

Example 2. *Solve graphically the equations $4x - 3y = 0$ and $2x + 3y - 18 = 0$. Also find the ratio of the areas of the triangles formed by these lines and the coordinate axes.*

Solution. The given equations can be written as

$$y = \frac{4}{3}x \qquad \ldots(i)$$

and $$y = \frac{18 - 2x}{3} \qquad \ldots(ii)$$

Table of values for equation (*i*)

x	0	3	6
y	0	4	8

Table of values for equation (*ii*)

x	0	3	6
y	6	4	2

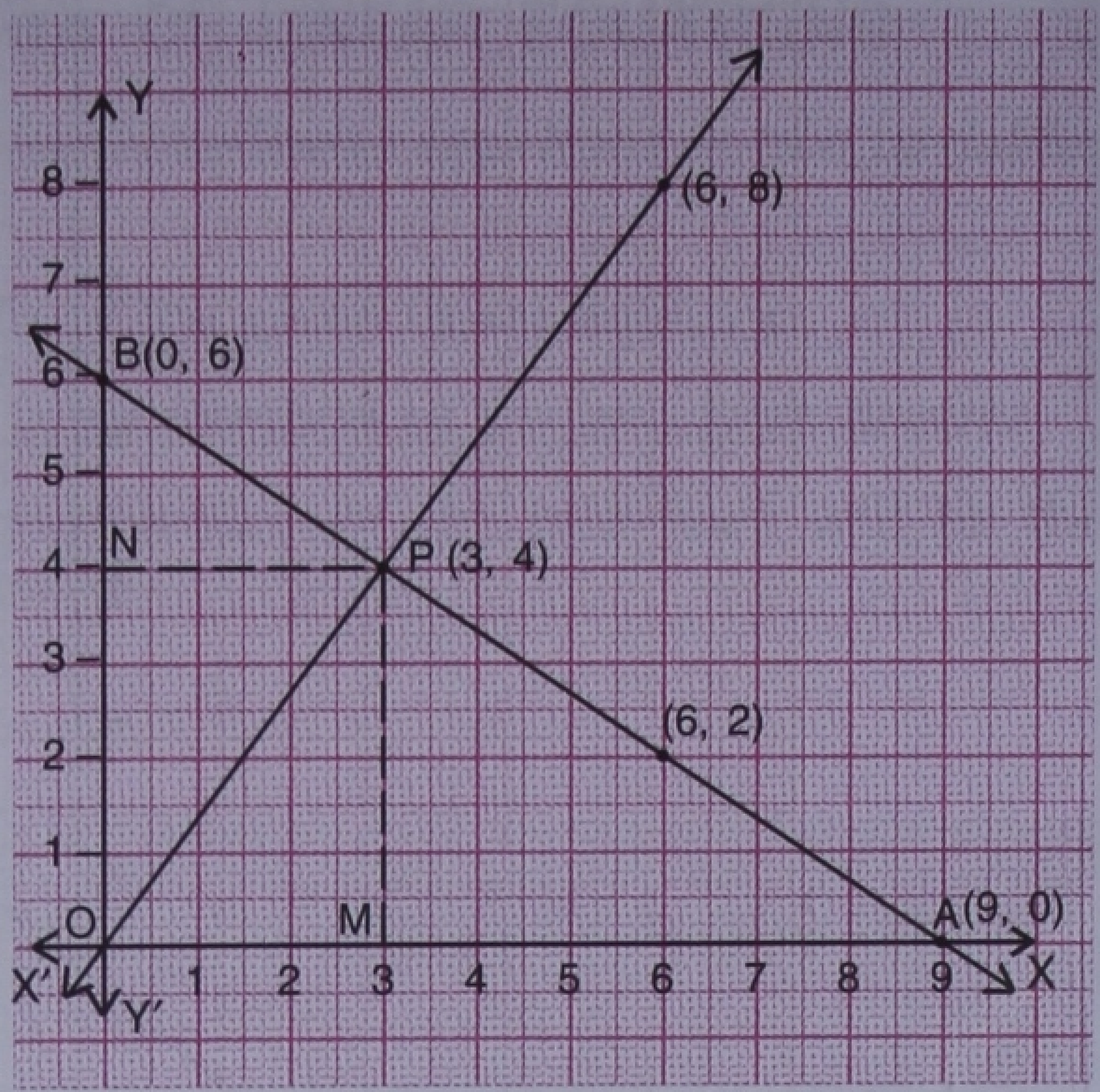
Select coordinate axes on the graph paper (as shown) and take 1 cm = 1 unit on both the axes.

Plot the points (0, 0), (3, 4) and (6, 8) on the graph paper. Connect any two points by a straight line.

Plot the points (0, 6), (3, 4) and (6, 2) on the graph paper. Connect any two points by a straight line. The graphs of both the straight lines is shown in the graph given below.

The lines intersect at the point P(3, 4). Therefore, the solution of the given equations is $x = 3$, $y = 4$.

Note that the line $4x - 3y = 0$ passes through origin and the line $2x + 3y - 18 = 0$ meets the x-axis at the point A(9, 0) and the y–axis at the point B(0, 6). There are two triangles *i.e.* ΔPOA and ΔOPB formed by these lines and the coordinate axes.



From P, draw PM perpendicular to OA, and PN perpendicular to OB.

$$\text{Area of } \Delta POA = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times OA \times MP$$

$$= \frac{1}{2} \times 9 \times 4 \text{ sq. units} = 18 \text{ sq. units}$$

and area of $\Delta OPB = \frac{1}{2} \times OB \times NP = \frac{1}{2} \times 6 \times 3$ sq. units

$$= 9 \text{ sq. units}$$

$$\therefore \quad \frac{\text{Area of } \Delta POA}{\text{Area of } \Delta OPB} = \frac{18}{9} = \frac{2}{1}$$

$\Rightarrow$ Area of ΔPOA : Area of ΔOPB = 2 : 1.

Example 3. *Find graphically the vertices of the triangle whose sides have equations $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$ respectively. Take 1 cm = 1 unit on both axes.*

Solution. The given equations can be written as

$$y = \frac{1}{2}(x + 8) \qquad \ldots(i)$$

$$y = \frac{1}{5}(x + 14) \qquad \ldots(ii)$$

$$y = 2x + 1 \qquad \ldots(iii)$$

Select coordinate axes and take 1 cm = 1 unit on both the axes. Plot the points (0, 4), (2, 5) and (– 2, 3) on the graph paper (coordinate plane). Connect any two points by a straight line.

Table of values for equation (*i*)

x	0	2	– 2
y	4	5	3

Plot the points (– 4, 2), (1, 3) and (6, 4) on the same graph paper and connect any two points by a straight line.

Table of values for equation (*ii*)

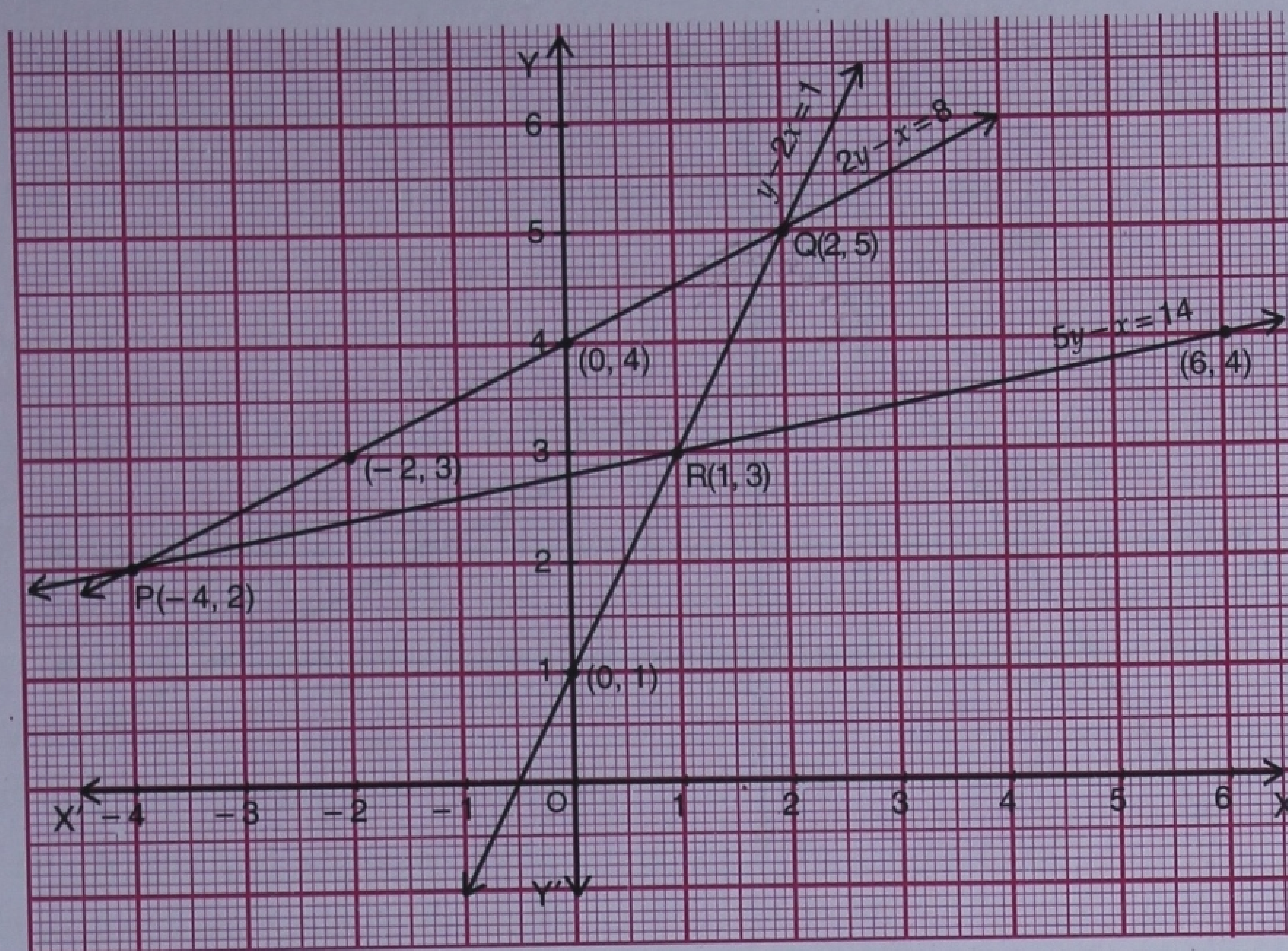
x	– 4	1	6
y	2	3	4

Plot the points (0, 1), (1, 3) and (2, 5) on the same graph paper and connect any two points by a straight line.

Table of values for equation (*iii*)

x	0	1	2
y	1	3	5

The graphs of the three lines are shown in the figure given below.



The lines intersect at the points P, Q, R. Therefore, the vertices of the triangle formed by the given lines are (– 4, 2), (2, 5) and (1, 3).

Example 4. *A triangle is formed by the lines $x + 2y - 3 = 0$, $3x - 2y + 7 = 0$ and $y + 1 = 0$. Find graphically*

(i) the coordinates of the vertices of the triangle.

(ii) the area of the triangle.

Solution. The given equations can be written as

$$y = -\frac{1}{2}(x - 3) \quad \ldots(i)$$

$$y = \frac{1}{2}(3x + 7) \quad \ldots(ii)$$

$$y = -1 \quad \ldots(iii)$$

Select coordinate axes and take 1 cm = 1 unit on both the axes. Plot the points (3, 0), (1, 1) and (– 1, 2) on the graph paper (coordinate plane). Connect any two points by a straight line.

Table of values for equation (*i*)

x	3	1	– 1
y	0	1	2

Plot the points (– 1, 2), $\left(-2, \frac{1}{2}\right)$ and (– 3, – 1) on the same graph paper and connect any two points by a straight line.

Table of values for equation (*ii*)

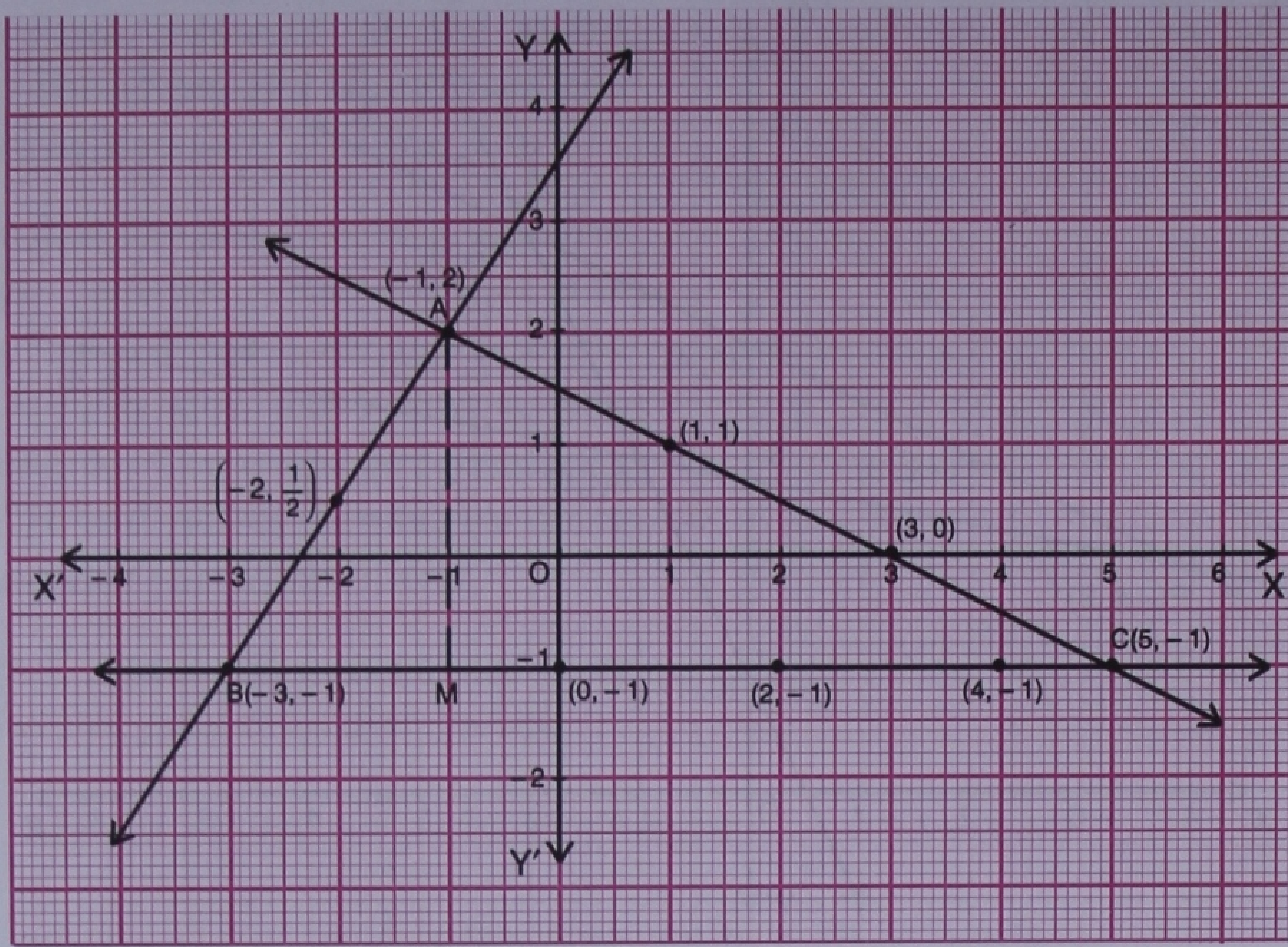
x	– 1	– 2	– 3
y	2	$\frac{1}{2}$	– 1

Plot the points (0, – 1), (2, – 1) and (4, – 1) on the same graph paper and connect any two points by a straight line.

Table of values for equation (*iii*)

x	0	2	4
y	– 1	– 1	– 1

The graphs of the three lines are shown in the figure below.



Let the lines intersect at the points A, B and C.

(*i*) The coordinates of the vertices of the triangle formed by the given lines are A(− 1, 2), B(− 3, − 1) and C(5, − 1).

(*ii*) From A, draw AM perpendicular to BC.

Area of ABC $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AM$

$= \frac{1}{2} \times 8 \times 3$ sq. units = 12 sq. units.

Exercise 8.3

1. Solve the following equations graphically : $3x - 2y = 4$, $5x - 2y = 0$.
2. Solve the following pair of equations graphically. Plot at least 3 points for each straight line.
   $2x - 7y = 6$, $5x - 8y = -4$.
3. Using the same axes of coordinates and the same unit, solve graphically.
   $x + y = 0$, $3x - 2y = 10$.
4. Take 1 cm to represent 1 unit on each axis to draw the graphs of the equations $4x - 5y = -4$ and $3x = 2y - 3$ on the same graph sheet (same axes). Use your graph to find the solution of the above simultaneous equations.
5. Solve the following simultaneous equations graphically.
   $x + 3y = 8$, $3x = 2 + 2y$.
6. Solve graphically the simultaneous equations $3y = 5 - x$, $2x = y + 3$.
   (Take 2 cm = 1 unit on both axes).
7. Use graph paper for this question. Take 2 cm = 1 unit on both axes.
   (*i*) Draw the graphs of $x + y + 3 = 0$ and $3x - 2y + 4 = 0$. Plot only three points per line.

(*ii*) Write down the coordinates of the point of intersection of the lines.

(*iii*) Measure and record the distance of the point of intersection of the lines from the origin in cm.

8. Solve the following simultaneous equations, graphically :

$2x - 3y + 2 = 4x + 1 = 3x - y + 2.$

Hint

$2x - 3y + 2 = 4x + 1, \ 4x + 1 = 3x - y + 2.$

9. Use graph paper for this question.

(*i*) Draw the graphs of $3x - y - 2 = 0$ and $2x + y - 8 = 0$. Take 1 cm = 1 unit on both axes and plot three points per line.

(*ii*) Write down the coordinates of the point of intersection and the area of the triangle formed by the lines and the x-axis.

10. Solve the following system of linear equations graphically :

$2x - y - 4 = 0, x + y + 1 = 0.$

Hence, find the area of the triangle formed by these lines and the y-axis.

11. Solve graphically the following equations : $x + 2y = 4, \ 3x - 2y = 4.$

Take 2 cm = 1 unit on each axis. Write down the area of the triangle formed by the lines and the x-axis.

12. On graph paper, take 2 cm to represent one unit on both the axes, draw the lines : $x + 3 = 0, \ y - 2 = 0, \ 2x + 3y = 12.$

Write down the coordinates of the vertices of the triangle formed by these lines.

13. Find graphically the coordinates of the vertices of the triangle formed by the lines $y = 0$, $y = x$ and $2x + 3y = 10$. Hence find the area of the triangle formed by these lines.

CHAPTER TEST

Solve the following simultaneous linear equations (1 to 4) :

1. (*i*) $2x - \frac{3}{4}y = 3$
$5x - 2y = 7$
(*ii*) $2(x - 4) = 9y + 2$
$x - 6y = 2.$

2. (*i*) $97x + 53y = 177$
$53x + 97y = 573$
(*ii*) $x + y = 5{\cdot}5$
$x - y = 0{\cdot}9.$

3. (*i*) $x + y = 7xy$
$2x - 3y + xy = 0$
(*ii*) $\frac{30}{x-y} + \frac{44}{x+y} = 10$
$\frac{40}{x-y} + \frac{55}{x+y} = 13.$

4. (*i*) $ax + by = a - b$
$bx - ay = a + b$
(*ii*) $3x + 2y = 2xy$
$\frac{1}{x} + \frac{2}{y} = 1\frac{1}{6}.$

5. Solve $2x - \frac{3}{y} = 9$, $3x + \frac{7}{y} = 2$. Hence find the value of k if $x = ky + 5$.

6. Solve $\frac{1}{x+y} - \frac{1}{2x} = \frac{1}{30}$, $\frac{5}{x+y} + \frac{1}{x} = \frac{4}{3}$. Hence find the value of $2x^2 - y^2$.

7. Can x, y be found to satisfy the following equations simultaneously?
$\frac{2}{y} + \frac{5}{x} = 19$, $\frac{5}{y} - \frac{3}{x} = 1$, $3x + 8y = 5.$
If so, find them.

8. The lengths of the sides of a triangle (in centimetres) are $4\left(x + \frac{y}{3}\right)$, $3x + 2y$ and $2(x + 3) + \frac{1}{4}(3y - 1)$. If the triangle is equilateral, find the lengths of its sides.

9. Solve graphically, the simultaneous equations : $2x - 3y = 7$; $x + 6y = 11$.

10. Solve the following system of equations graphically, taking 2 cm to represent 1 unit on both the axes : $x - 2y - 4 = 0$, $2x + y - 3 = 0$.

11. Using a scale of 1 cm to 1 unit for both the axes, draw the graphs of the following equations : $6y = 5x + 10$, $y = 5x - 15$. From the graph, find
(*i*) the coordinates of the point where the two lines intersect.
(*ii*) the area of the triangle between the lines and the x-axis.

12. Find, graphically, the coordinates of the vertices of the triangle formed by the lines :
$8y - 3x + 7 = 0$, $2x - y + 4 = 0$ and $5x + 4y = 29$.

13. Find graphically the coordinates of the vertices of the triangle formed by the lines $y - 2 = 0$, $2y + x = 0$ and $y + 1 = 3(x - 2)$. Hence find the area of the triangle formed by these lines.