7

LINEAR EQUATIONS

7.1 LINEAR EQUATIONS

Equation. *A statement that two quantities are equal is called an* ***equation.***

Linear Expression. *An expression of the form* $ax + b$, $a \neq 0$, *is called a* ***linear*** *(or* ***first degree****)* ***expression*** *in the variable x.*

Linear Equation. *An equation of the type* $ax + b = 0$, $a \neq 0$, *is called a* ***linear equation*** *in the variable x.*

Solution. *Any value (or values) of the variable (or variables) which when substituted in an equation makes its both sides equal is called a* ***solution*** *(or* ***root****) of the equation.*

Thus, a number α is a root of the equation $ax + b = 0$ if and only if $a\alpha + b = 0$.

For example, when we substitute $x = 2$ in the equation $x + 3 = 5$, we get $2 + 3 = 5$ *i.e.* $5 = 5$, which is true, therefore, 2 is a solution (or root) of the equation $x + 3 = 5$.

To solve an equation is to find all its solutions, and the process of finding all the solutions is called **solving the equation.**

7.1.1 Two permissible rules

☆ **Addition – Subtraction Rule.**

If the same number or expression is added to or subtracted from both sides of an equation, the resulting equation has the same solution (or solutions) as the original.

☆ **Multiplication – Division Rule.**

If each side of an equation is multiplied or divided by the same non-zero number or expression, the resulting equation has the same solution (or solutions) as the original.

7.2 SOLVING LINEAR EQUATIONS IN ONE VARIABLE

For solving a linear equation in one variable, proceed as under :

(*i*) Simplify both sides by removing group symbols and collecting like terms.

(*ii*) Remove fractions (or decimals) by multiplying both sides by an appropriate factor (L.C.M. of fractions or a power of 10 in case of decimals).

(*iii*) Isolate all variable terms on one side and all constants on the other side. Collect like terms if possible.

(*iv*) Make the coefficient of the variable 1.

Remark

The solution may be checked by substituting in the original equation.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve for x :* $(x - 1) = \frac{3}{4}(x + 1) - \frac{1}{2}$.

Solution. Given $(x - 1) = \frac{3}{4}(x + 1) - \frac{1}{2}$.

Multiplying both sides by 4, L.C.M. of fractions, we obtain

$$4(x - 1) = 3(x + 1) - 2$$

$$\Rightarrow \quad 4x - 4 = 3x + 3 - 2$$

$$\Rightarrow \quad 4x - 3x = 3 - 2 + 4 \Rightarrow x = 5.$$

Example 2. *Solve for x :* $\sqrt{3}(x - 1) = x + 1$.

Solution. Given $\sqrt{3}(x - 1) = x + 1$

$$\Rightarrow \quad \sqrt{3}x - \sqrt{3} = x + 1$$

$$\Rightarrow \quad \sqrt{3}x - x = \sqrt{3} + 1$$

$$\Rightarrow \quad (\sqrt{3} - 1)x = \sqrt{3} + 1$$

$$\Rightarrow \quad x = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3+1+2\sqrt{3}}{3-1} = \frac{4+2\sqrt{3}}{2}$$

$$\Rightarrow \quad x = 2 + \sqrt{3}.$$

Example 3. *Solve for x :* $2 + \frac{2x-3}{2x+3} = \frac{3x+4}{x+2}$.

Solution. Given $2 + \frac{2x-3}{2x+3} = \frac{3x+4}{x+2}$.

Multiplying both sides by $(2x + 3)(x + 2)$, L.C.M. of fractions, we get

$$2(2x + 3)(x + 2) + (x + 2)(2x - 3) = (2x + 3)(3x + 4)$$

$$\Rightarrow \quad 2(2x^2 + 4x + 3x + 6) + (2x^2 - 3x + 4x - 6) = 6x^2 + 8x + 9x + 12$$

$$\Rightarrow \quad 4x^2 + 14x + 12 + 2x^2 + x - 6 = 6x^2 + 17x + 12$$

$$\Rightarrow \quad 6x^2 + 15x + 6 = 6x^2 + 17x + 12$$

$$\Rightarrow \quad 6x^2 - 6x^2 + 15x - 17x = 12 - 6$$

$$\Rightarrow \quad -2x = 6 \Rightarrow x = \frac{6}{-2} \Rightarrow x = -3.$$

Example 4. *Solve for x :* $\frac{4}{x+1} + \frac{5}{x+3} = \frac{9}{x+2}$.

Solution. Given $\frac{4}{x+1} + \frac{5}{x+3} = \frac{9}{x+2}$

Multiplying both sides by $(x + 1)(x + 3)(x + 2)$, L.C.M. of fractions, we get

$4(x + 3)(x + 2) + 5(x + 1)(x + 2) = 9(x + 1)(x + 3)$

$\Rightarrow$ $4(x^2 + 2x + 3x + 6) + 5(x^2 + 2x + x + 2) = 9(x^2 + 3x + x + 3)$

$\Rightarrow$ $4x^2 + 20x + 24 + 5x^2 + 15x + 10 = 9x^2 + 36x + 27$

$\Rightarrow$ $9x^2 + 35x + 34 = 9x^2 + 36x + 27$

$\Rightarrow$ $9x^2 - 9x^2 + 35x - 36x = 27 - 34$

$\Rightarrow$ $-x = -7 \Rightarrow x = 7.$

Example 5. *Solve :* $\frac{2y+1}{10} - \frac{y-2}{6} = \frac{3-2y}{15}$. *Hence find x if* $\frac{1}{x} + \frac{1}{y} = 2.$

Solution. Given $\frac{2y+1}{10} - \frac{y-2}{6} = \frac{3-2y}{15}$.

Multiplying both sides by 30, L.C.M. of fractions, we get

$3(2y + 1) - 5(y - 2) = 2(3 - 2y)$

$\Rightarrow$ $6y + 3 - 5y + 10 = 6 - 4y$

$\Rightarrow$ $y + 13 = 6 - 4y$

$\Rightarrow$ $y + 4y = 6 - 13$

$\Rightarrow$ $5y = -7 \Rightarrow y = -\frac{7}{5}$.

Now $\frac{1}{x} + \frac{1}{y} = 2 \Rightarrow \frac{1}{x} = 2 - \frac{1}{y}$

$\Rightarrow$ $\frac{1}{x} = 2 - \left(-\frac{5}{7}\right)$ $\left(\because y = -\frac{7}{5} \Rightarrow \frac{1}{y} = -\frac{5}{7}\right)$

$\Rightarrow$ $\frac{1}{x} = 2 + \frac{5}{7} = \frac{14+5}{7} = \frac{19}{7} \Rightarrow x = \frac{7}{19}$.

Example 6. *If* $p = x + 1$ *and* $\frac{4p-3}{2} - \frac{3x+2}{5} = \frac{3}{2}$, *find x.*

Solution. Given $p = x + 1$ ...(*i*)

and $\frac{4p-3}{2} - \frac{3x+2}{5} = \frac{3}{2}$ ...(*ii*)

Substituting the value of p from (*i*) in (*ii*), we get

$\frac{4(x+1)-3}{2} - \frac{3x+2}{5} = \frac{3}{2} \Rightarrow \frac{4x+1}{2} - \frac{3x+2}{5} = \frac{3}{2}$.

Multiplying both sides by 10, L.C.M. of fractions, we get

$5(4x + 1) - 2(3x + 2) = 15$

$\Rightarrow$ $20x + 5 - 6x - 4 = 15$

$\Rightarrow$ $14x = 15 - 5 + 4 \Rightarrow 14x = 14$

$\Rightarrow$ $x = 1.$

Example 7. *If* $m = \frac{3x-4}{5}$, $n = \frac{x-7}{3}$ *and* $3(m + n) = 13$, *find the value of x.*

Solution. Given $m = \frac{3x-4}{5}$ ...(*i*)

$n = \frac{x-7}{3}$ ...(*ii*)

and $3(m + n) = 13$ ...(*iii*)

Substituting the values of m and n from (i) and (ii) in (iii), we get

$$3\left(\frac{3x-4}{5}+\frac{x-7}{3}\right) = 13 \Rightarrow \frac{3x-4}{5}+\frac{x-7}{3} = \frac{13}{3}.$$

Multiplying both sides by 15, L.C.M. of fractions, we get

$$3(3x-4)+5(x-7) = 5 \times 13$$

$$\Rightarrow \quad 9x - 12 + 5x - 35 = 65$$

$$\Rightarrow \quad 14x = 65 + 12 + 35 \Rightarrow 14x = 112$$

$$\Rightarrow \quad x = 8.$$

Example 8. *If* $\frac{3x}{p}+\frac{y}{3q} = 7$ *and* $2x + 1 = y + 1 = p = 5$, *find the value of* q.

Solution. Given $2x + 1 = y + 1 = p = 5$

$$\Rightarrow \quad 2x + 1 = 5,\ y + 1 = 5 \text{ and } p = 5$$

$$\Rightarrow \quad 2x = 5 - 1,\ y = 5 - 1 \text{ and } p = 5$$

$$\Rightarrow \quad 2x = 4,\ y = 4 \text{ and } p = 5$$

$$\Rightarrow \quad x = 2,\ y = 4 \text{ and } p = 5.$$

$$\therefore \quad \frac{3x}{p}+\frac{y}{3q} = 7 \Rightarrow \frac{3\times 2}{5}+\frac{4}{3q} = 7$$

$$\Rightarrow \quad \frac{4}{3q} = 7 - \frac{6}{5} \Rightarrow \frac{4}{3q} = \frac{29}{5}$$

$$\Rightarrow \quad q = \frac{4}{3}\times\frac{5}{29} \Rightarrow q = \frac{20}{87}.$$

Exercise 7

Solve the following equations (1 to 10) :

1. (i) $\frac{x}{2} = 3 + \frac{x}{3}$ (ii) $x + 1 = \sqrt{2}\,(1 - x)$.
2. (i) $3(2x - 1) = 5 - (3x - 2)$ (ii) $-2[x - 3(x - 5)] = 6 - 5x$.
3. (i) $\frac{x+1}{3} - \frac{x+7}{2} = 1$ (ii) $\frac{x+7}{3} = 1 + \frac{3x-2}{5}$.
4. (i) $\frac{4x-5}{4} - \frac{3x-1}{3} = \frac{11x}{12}$ (ii) $\frac{1}{3}(x-1) - \frac{1}{4}(x-2) = 1$.
5. (i) $\frac{1-x}{6} + \frac{2x}{3} - \frac{1-7x}{4} = 2\frac{1}{6}$ (ii) $\frac{x-4}{4} + \frac{3x}{2} = 5 - \frac{x-2}{4}$.
6. (i) $\frac{x-1}{3} + \frac{2x+3}{6} = \frac{3x-5}{9} - \frac{2x-7}{2}$ (ii) $\frac{5x-11}{4} + \frac{1}{2}(3x-7) = \frac{4x-7}{3} + x - 1$.
7. (i) $\frac{4}{5}\left(x+\frac{5}{6}\right) - \frac{2}{3}\left(x-\frac{1}{4}\right) = 1\frac{1}{9}$ (ii) $\frac{7x-1}{4} - \frac{1}{3}\left(2x - \frac{1-x}{2}\right) = 6\frac{1}{3}$.
8. (i) $\frac{2x+5}{2} - \frac{5x}{x-1} = x$ (ii) $\frac{2x-3}{2x-1} = \frac{3x-1}{3x+1}$.

9. (*i*) $\frac{x+2}{x-2} - \frac{x-2}{x+2} = 1 - \frac{x^2}{x^2-4}$ (*ii*) $\frac{4}{x-3} + \frac{2}{x-2} = \frac{6}{x}$.

10. $0{\cdot}5\ (4x + 1) = 0{\cdot}3\ (2x + 1) + 1{\cdot}6$.

11. If $x = p + 1$, find the value of p from the equation

$\frac{1}{2}(5x - 30) - \frac{1}{3}(1 + 7p) = \frac{1}{4}$.

12. Solve $\frac{x+3}{3} - \frac{x-2}{2} = 1$. Hence find p if $\frac{1}{x} + p = 1$.

13. Solve $\frac{x-3}{3} - \frac{2-x}{2} = x - 1$. Hence find the value of a if $\frac{x}{a} + 3 = 0$.

14. Solve $\frac{3x+3}{4} - \frac{2x-1}{3} = 1$. Hence find y if $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$.

15. If $x = \frac{1}{2}$ is a solution of the equation $ax^2 + (a - 1)x + 3 = a$, find the value of a.

16. If $p = 3x + 1$, $q = \frac{1}{3}(9x + 13)$ and $p : q = 6 : 5$, find x.

17. If $\frac{C}{5} = \frac{1}{9}(F - 32)$, find C when F = 86.

18. If $p = \frac{x-4}{3}$, $q = \frac{x+2}{4}$ and $2(p - q) + 3 = 0$, find the value of x.

19. If $7 - 2(x - 3) = 9$, find the value of $x^2 - 5x + 6$.

20. If two sides of an equilateral triangle are $\frac{1}{4}(3x - 1)$ cm and $\frac{1}{6}(5x - 3)$ cm, find the perimeter of the triangle.

Hint

$\frac{1}{4}(3x - 1) = \frac{1}{6}(5x - 3)$, solving for x, we get $x = 3$.

$\therefore$ Each side $= \frac{1}{4}(3 \times 3 - 1)$ cm = 2 cm.

CHAPTER TEST

Solve the following equations (1 to 7) :

1. $2(x + 1) + 3(x + 2) = 33.$
2. $(3x - 2)(2x + 1) = (6x + 1)(x - 2) + 5.$
3. $\frac{4}{5x} + \frac{3}{2x} = \frac{1}{x} + \frac{13}{20}.$
4. $(3x + 1)^2 + (4x + 1)^2 - (5x + 1)^2 = 5.$
5. $(7x - 1) - \left(x - \frac{1 - x}{2}\right) = 5x + \frac{1}{2}.$
6. $5x + 4 = \sqrt{5}\,(2 - x) + 8.$
7. $\frac{4}{x - 2} + \frac{3}{x - 1} = \frac{7}{x - 3}.$
8. If $(2ax + 1)(5x - 1) = 6a(3x - 1)$ and $x = 1$, find a.
9. If $p = x + 2$ and $\frac{3p - 1}{2} - \frac{x + 3}{5} = \frac{p - x}{4}$, find x.
10. If $\frac{2x + 7}{x + 2} = \frac{4x + 3}{2x - 7}$, find the value of $x^3 + x^2 + x + 1$.
11. Equal sides of an isosceles triangle are $3(3x - 4)$ cm and $\left(\frac{2x - 1}{3} + 55\right)$ cm; find x. Hence show that the triangle whose sides are $(3x - 4)$ cm, $(2x + 4)$ cm and $\left(\frac{x - 2}{3} + \frac{9x}{4}\right)$ cm is equilateral.
12. If a side of a square is $\frac{1}{2}(x + 1)$ cm and its diagonal is $\frac{3 - x}{\sqrt{2}}$ cm, find the perimeter of the square.

Hint

Length of diagonal of a square = $\sqrt{2}$ × side of the square,

$\therefore \frac{3 - x}{\sqrt{2}} = \sqrt{2} \times \frac{1}{2}(x + 1)$, solving for x, we get $x = 1$.

$\therefore$ Each side $= \frac{1}{2}(1 + 1)$ cm = 1 cm.