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CHANGING THE SUBJECT OF A FORMULA

6.1 FORMULA

Formula. *A **formula** is an algebraic rule. It is written in the form of an equation (called a **literal equation**) which contains two or more variables.*

For example ,

(*i*) A = *lb*, where A is the area of a rectangle, *l* its length and *b* its breadth.

(*ii*) *d* = *ut*, where *d* is the distance covered by a body moving in a straight line with uniform speed *u* in time *t*.

(*iii*) $I = \frac{P \times R \times T}{100}$, where I is the simple interest, P the principal, R the rate percent per annum and T is the time.

Each one of these is a formula.

6.2 CHANGE OF SUBJECT OF A FORMULA

Subject of a formula. ***Subject*** *of a formula is the variable which is expressed in terms of the other variables involved in the formula.*

For example, in the formula $I = \frac{P \times R \times T}{100}$, I is the subject of the formula.

Change of subject of a formula

The formula $I = \frac{P \times R \times T}{100}$ can also be written as

(*i*) $P = \frac{I \times 100}{R \times T}$, here P is the subject of formula.

(*ii*) $R = \frac{I \times 100}{P \times T}$, here R is the subject of formula.

(*iii*) $T = \frac{I \times 100}{P \times R}$, here T is the subject of formula.

ILLUSTRATIVE EXAMPLES

Example 1. *Make d as the subject of the formula* $S = \frac{n}{2}[2a + (n-1)d]$.

Solution. Given $S = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 2S = 2an + n(n-1)d$

$\Rightarrow \quad 2S - 2an = n(n-1)d$

$\Rightarrow \quad n(n-1)d = 2(S - an)$

$\Rightarrow \quad d = \frac{2(S - an)}{n(n-1)}$.

Example 2. *Make g as the subject of the formula* $T = 2\pi\sqrt{\frac{l}{g+k}}$.

Solution. Given $T = 2\pi\sqrt{\frac{l}{g+k}}$

$\Rightarrow \quad \frac{T}{2\pi} = \sqrt{\frac{l}{g+k}}$

$\Rightarrow \quad \frac{T^2}{4\pi^2} = \frac{l}{g+k}$ [Squaring both sides]

$\Rightarrow \quad gT^2 + kT^2 = 4\pi^2 l$

$\Rightarrow \quad gT^2 = 4\pi^2 l - kT^2$

$\Rightarrow \quad g = \frac{4\pi^2 l - kT^2}{T^2}$.

Example 3. *Given* $a + b = 2$, *express* $a^2 - 2b - 4$ *in terms of b.*

Solution. Given $a + b = 2 \Rightarrow a = 2 - b$.

$\therefore \quad a^2 - 2b - 4 = (2 - b)^2 - 2b - 4$

$= (4 - 4b + b^2) - 2b - 4$

$= b^2 - 6b = b(b - 6)$.

Example 4. *If* $x + y = m$ *and* $\frac{1}{x} + \frac{1}{y} = \frac{1}{r}$, *find a formula for m in terms of x and r.*

Solution. Given $\frac{1}{x} + \frac{1}{y} = \frac{1}{r} \Rightarrow \frac{1}{y} = \frac{1}{r} - \frac{1}{x}$

$\Rightarrow \quad \frac{1}{y} = \frac{x - r}{rx} \Rightarrow y = \frac{rx}{x - r}$ ...(*i*)

But $x + y = m \Rightarrow y = m - x$.

Substituting this value of y in (*i*), we get

$m - x = \frac{rx}{x - r}$

$\Rightarrow \quad m = x + \frac{rx}{x - r} \Rightarrow m = \frac{x^2 - rx + rx}{x - r}$

$\Rightarrow \quad m = \frac{x^2}{x - r}$.

Example 5. *If* $x = 2y + \sqrt{y^2 + t}$, *express t in terms of x and y.*

Solution. Given $x = 2y + \sqrt{y^2 + t}$

$\Rightarrow \quad x - 2y = \sqrt{y^2 + t}$.

On squaring both sides, we get

$$y^2 + t = (x - 2y)^2$$

$$\Rightarrow \quad t = (x - 2y)^2 - y^2$$

$$\Rightarrow \quad t = (\overline{x - 2y} + y)(\overline{x - 2y} - y)$$

$$\Rightarrow \quad t = (x - y)(x - 3y).$$

Exercise 6.1

Change the subject of each of the following (1 to 6) formulae to the letter given against them :

1. (*i*) $v^2 = u^2 + 2as$; s (*ii*) $s = ut + \frac{1}{2} at^2$; a.

2. (*i*) $S = 2\pi rh + 2\pi r^2$; h (*ii*) $P = \frac{m}{m+n}$; m.

3. (*i*) $A = \pi (R^2 - r^2)$; R (*ii*) $W = pq + \frac{1}{2} N x^2$; x.

4. (*i*) $T = 2\pi \sqrt{\frac{I}{MH}}$; I (*ii*) $c = \frac{1 - t^2}{1 + t^2}$; t.

5. (*i*) $t = \frac{ax - b}{x + b}$; x (*ii*) $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$; r_1.

6. (*i*) $\frac{1}{l} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$; n (*ii*) $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$; c.

7. Given $\frac{p}{2} - r = \frac{q - r}{5}$, express p in terms of q and r.

8. Given $l = a + (n - 1) d$, express n in terms of l, a, d.

9. If $p = \frac{am + b}{cm + d}$, express m in terms of p, a, b, c and d.

10. Given $x = \frac{ny}{3 - nt}$, express n in terms of x, y and t.

11. Given $2ax + 3b^2 = 2bx + 3a^2$, express x in terms of a and b. Give the result in the simplest form.

12. Given that $y = \frac{2x}{x - 2}$, and $z = \frac{3y - 4}{4y + 3}$, find an expression for z in terms of x, in its simplest form.

6.3 EVALUATION OF THE SUBJECT OF A FORMULA

Procedure. Substitute the values of all the variables (except the subject) in the formula and solve it to find the value of the subject.

ILLUSTRATIVE EXAMPLES

Example 1. *If $s = \frac{n}{2}(a + l)$, express a in terms of s, n and l. Find the value of a when $s = 40$, $n = 4$ and $l = 20$.*

Solution. Given $s = \frac{n}{2}(a + l) \Rightarrow \frac{2s}{n} = a + l$

$\Rightarrow \quad a = \frac{2s}{n} - l.$

When $s = 40, n = 4$ and $l = 20,$

$a = \frac{2 \times 40}{4} - 20 = 20 - 20 = 0.$

Example 2. *Given $\frac{x}{1-x} = \frac{P}{Q}$, express x in terms of P and Q. Evaluate x as a decimal when P = 0·7 and Q = 0·3.*

Solution. Given $\frac{x}{1-x} = \frac{P}{Q}$

$\Rightarrow \quad Qx = P(1 - x) \Rightarrow Qx = P - Px$

$\Rightarrow \quad Qx + Px = P \Rightarrow (P + Q)x = P$

$\Rightarrow \quad x = \frac{P}{P+Q}.$

When $P = 0{\cdot}7$ and $Q = 0{\cdot}3,$

$x = \frac{0{\cdot}7}{0{\cdot}7 + 0{\cdot}3} = \frac{0{\cdot}7}{1} = 0{\cdot}7.$

Example 3. *Given $x - 2y = 5$, find the value of $x^2 - 2xy$ when $x = -3$.*

Solution. Given $x - 2y = 5$ and $x = -3$

$\therefore \quad x^2 - 2xy = x(x - 2y)$

$= (-3)(5) = -15.$

Example 4. *A formula for changing temperature in degrees Fahrenheit (F) to degrees in Celsius (C) is given by $F = \frac{9}{5}C + 32$.*

(i) Express C in terms of F.

(ii) Find C if F = 50.

(iii) Is it possible for C and F to have the same value ?

Solution. (*i*) Given $F = \frac{9}{5}C + 32$

$\Rightarrow \quad F - 32 = \frac{9}{5}C \Rightarrow C = \frac{5}{9}(F - 32)$ ...(1)

(*ii*) When $F = 50$, from (1), we get

$C = \frac{5}{9}(50 - 32) = \frac{5}{9} \times 18 = 10.$

(*iii*) When $F = C$, from (1), we get

$C = \frac{5}{9}(C - 32) \Rightarrow 9C = 5C - 160$

$\Rightarrow \quad 4C = -160 \Rightarrow C = -40.$

Hence, C and F both have equal value at – 40.

Example 5. *A swimming pool is 50 m long. Its depth d metres at any point x metres from the edge at the shallow end is given by the formula* $d = 1 + \frac{x}{10}$.

(i) Find its depth at the middle and at the deepest end.

(ii) A man whose nose height is 180 cm walks in the pool. Find the farthest distance, from the edge at the shallow end, he can go without getting his nose wet. Assume that he keeps his head vertical all the time.

Solution. (*i*) At the middle, $x = \frac{50}{2}$ m [$\because$ total length of pool = 50 m]

$\Rightarrow x = 25$ m

$$\therefore \quad d = \left(1 + \frac{25}{10}\right) \text{m} = \left(1 + \frac{5}{2}\right) \text{m} = \frac{7}{2} \text{ m} = 3{\cdot}5 \text{ m}.$$

At the deepest end, $x = 50$ m,

$$\therefore \quad d = \left(1 + \frac{50}{10}\right) \text{ m} = (1 + 5) \text{ m} = 6 \text{ m}.$$

(*ii*) Since $d = 1 + \frac{x}{10} \Rightarrow d - 1 = \frac{x}{10}$

$\Rightarrow \quad x = 10(d - 1).$

As the nose height of the man is 180 cm *i.e.* 1·8 m, he can go up to the distance where depth is 1·8 m.

$\therefore \quad x = 10(1{\cdot}8 - 1) \text{ m} = 10 \times {\cdot}8 \text{ m} = 8 \text{ m}.$

$\therefore$ The farthest distance which the man can go from the shallow end is 8 m.

Exercise 6.2

1. If $y = \frac{2x - 1}{x + 3}$, find x in terms of y. Also find x when $y = 1$.
2. Given $W = \frac{H}{2} - 20$, find H in terms of W. Also find H when W = 70.
3. Given $A = 2\pi r(r + h)$, make h as the subject. Find h when A = 704, $\pi = \frac{22}{7}$ and $r = 7$.
4. If the total cost (T) of a number of items (n) is given, find a formula for the cost per item (c). What is the value of c when T is ₹ 52·50 and n is 35 ?
5. Given $a = b(1 + c\,t)$, express c in terms of a, b and t, and evaluate it as a decimal if $a = 1100$, $b = 1000$ and $t = 100$.
6. If $\frac{1}{p} = \frac{qr}{q + r}$, make r as the subject of the formula. Also calculate r if $p = 1\frac{1}{7}$ and $q = 5\frac{1}{4}$.
7. If $a = \frac{v - u}{t_2 - t_1}$, make u as the subject of the formula. Also find u when $a = 5$, $v = 27$, $t_2 = 6$ and $t_1 = 2$.

8. If $I = \frac{nE}{R + nr}$, find n if $I = 4$, $r = 4$, $E = 24$ and $R = 10$.

9. Given $\frac{p}{q} = \frac{a^2 + 2b}{a^2 - 2b}$, make a as the subject of the formula. Hence find a if $b = 2$, $p = 5$ and $q = 4$.

10. Make v the subject of the formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. Find the value of v when $f = 6$ and $u = 14$.

11. Given $s = \frac{n}{2}[2a + (n - 1)d]$, express d in terms of s, a and n. Hence find d if $n = 5$, $a = n + 1$ and $s = 20$.

12. The number x is increased by $c\%$ of itself and the number y is decreased by $c\%$ of itself. If the results are equal, express c in terms of x and y. Hence find c when $y = 3x$.

13. Frame a formula for the following statement :

'The number of diagonals, d, that can be drawn from one vertex of an n sided polygon to all the other vertices, is equal to the number of sides of the polygon less 3'. Hence find

(*i*) the number of diagonals from a vertex of a polygon of 10 sides.

(*ii*) the number of sides of the polygon if the number of diagonals that can be drawn from a vertex is 5.

CHAPTER TEST

1. (*i*) Write down a formula for the statement "The time period T of a pendulum is equal to 2π times the square root of quotient of length l and the gravitational constant g."

   (*ii*) Change the subject of the above formula to l.

   (*iii*) What is the time period of a pendulum having a length 39·2 cm ? Given that $\pi = 3\frac{1}{7}$ and $g = 9{\cdot}8$ m/sec^2.

2. Rearrange the formula $a = cx^2 - b$ so that x become the new subject. Hence find the value of x when $a = 2$, $b = -3$ and $c = -\frac{1}{2}$.

3. Given that $y = \frac{x+3}{x-1}$, express x in terms of y. Hence find x for which $y = x$.

4. Make x the subject of the formula $y = \frac{kx - a}{x - 1}$. Hence find x when $y = 93$, $a = 3$ and $k = 48$.

5. Given $x = \frac{y+1}{y-1}$ and $y = \frac{2z+1}{2z-1}$, express z in terms of x. Hence find the value of z when $x = 5$.