3

COMPOUND INTEREST

INTEREST

It is the additional money besides the original money paid by the borrower to the moneylender (bank, financial agency or individual) in lieu of the money used by him.

Principal. *The money borrowed (or the money lent) is called* ***principal.***

Amount. *The sum of the principal and the interest is called* ***amount.***

Thus, **amount = principal + interest.**

Rate. *It is the interest paid on* ₹ *100 for a specified period.*

Time. *It is the time for which the money is borrowed.*

Simple Interest. *It is the interest calculated on the original money (principal) for any given time and rate.*

Formula : Simple Interest $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$.

3.1 COMPOUND INTEREST

At the end of the first year (or any other fixed period), if the interest accrued is not paid to the moneylender but is added to the principal, then this amount becomes the principal for the next year (or any other fixed period) and so on. This process is repeated until the amount for the whole time is found.

The difference between the final amount and the (original) principal is called ***compound interest.***

Remark

In the case of simple interest, the principal remains constant for the whole time but in the case of compound interest, the principal keeps on changing every year (or any other fixed period).

If the interest is *compounded annually,* the principal changes after every year and if the interest is *compounded half-yearly* (or *any other fixed period*), the principal changes after every six months (or any other fixed period).

ILLUSTRATIVE EXAMPLES

Example 1. *Find the amount and the compound interest on ₹ 15000 for 2 years at 8% per annum.*

Solution. Principal for the first year = ₹ 15000.

$$\text{Interest for the first year} = ₹\ \frac{15000 \times 8 \times 1}{100} = ₹\ 1200.$$

Amount after one year = ₹ 15000 + ₹ 1200 = ₹ 16200.

Principal for the second year = ₹ 16200.

$$\text{Interest for the second year} = ₹\ \frac{16200 \times 8 \times 1}{100} = ₹\ 1296.$$

Amount after 2 years = ₹ 16200 + ₹ 1296 = ₹ 17496.

Compound interest for 2 years = final amount – (original) principal

= ₹ 17496 – ₹ 15000 = ₹ 2496.

Note

The compound interest may also be obtained by adding together the interest of consecutive years.

Thus, in the above example,

compound interest = interest of first year + interest of second year

= ₹ 1200 + ₹ 1296 = ₹ 2496.

Example 2. *Find the amount and the compound interest on ₹ 25000 for 3 years at 12% per annum, compounded annually.*

Solution. Principal for the first year = ₹ 25000.

$$\text{Interest for the first year} = ₹\ \frac{25000 \times 12 \times 1}{100} = ₹\ 3000.$$

Amount after one year = ₹ 25000 + ₹ 3000 = ₹ 28000.

Principal for the second year = ₹ 28000.

$$\text{Interest for the second year} = ₹\ \frac{28000 \times 12 \times 1}{100} = ₹\ 3360.$$

Amount after 2 years = ₹ 28000 + ₹ 3360

= ₹ 31360.

Principal for the third year = ₹ 31360.

$$\text{Interest for the third year} = ₹\ \frac{31360 \times 12 \times 1}{100} = ₹\ 3763{\cdot}20$$

Amount after 3 years = ₹ 31360 + ₹ 3763·20

= ₹ 35123·20

Compound interest for 3 years = final amount – (original) principal

= ₹ 35123·20 – ₹ 25000

= ₹ 10123·20.

Example 3. *Find the compound interest to the nearest rupee on ₹ 7500 for 2 years 4 months at 12% per annum reckoned annually.*

Solution. Principal for the first year = ₹ 7500.

$$\text{Interest for the first year} = ₹ \frac{7500 \times 12 \times 1}{100} = ₹ 900.$$

Amount after one year = ₹ 7500 + ₹ 900 = ₹ 8400.

Principal for the second year = ₹ 8400.

$$\text{Interest for the second year} = ₹ \frac{8400 \times 12 \times 1}{100} = ₹ 1008.$$

Amount after 2 years = ₹ 8400 + ₹ 1008 = ₹ 9408.

Remaining time = 4 months = $\frac{4}{12}$ year = $\frac{1}{3}$ year.

Principal for the next $\frac{1}{3}$ year = ₹ 9408.

$$\text{Interest for the next } \frac{1}{3} \text{ year} = ₹ \frac{9408 \times 12 \times \frac{1}{3}}{100} = ₹ 376{\cdot}32$$

Amount after 2 years 4 months = ₹ 9408 + ₹ 376·32

= ₹ 9784·32

∴ Compound interest for 2 years 4 months

= final amount – original principal

= ₹ 9784·32 – ₹ 7500 = ₹ 2284·32

= ₹ 2284 (to the nearest rupee).

Example 4. *Find the amount and the compound interest on ₹ 16000 for $1\frac{1}{2}$ years at 10% per annum, the interest being compounded half-yearly.*

Solution. Since the rate of interest is 10% per annum, therefore, the rate of interest half-yearly = $\frac{1}{2}$ of 10% = 5%.

Principal for the first half-year = ₹ 16000.

$$\text{Interest for the first half-year} = ₹ \frac{16000 \times 5 \times 1}{100} = ₹ 800.$$

Amount after the first half-year = ₹ 16000 + ₹ 800 = ₹ 16800.

Principal for the second half-year = ₹ 16800.

$$\text{Interest for the second half-year} = ₹ \frac{16800 \times 5 \times 1}{100} = ₹ 840.$$

Amount after one year = ₹ 16800 + ₹ 840 = ₹ 17640.

Principal for the third half-year = ₹ 17640.

$$\text{Interest for the third half-year} = ₹ \frac{17640 \times 5 \times 1}{100} = ₹ 882.$$

Amount after $1\frac{1}{2}$ years = ₹ 17640 + ₹ 882 = ₹ 18522.

∴ Compound interest for $1\frac{1}{2}$ years = final amount – original principal

= ₹ 18522 – ₹ 16000 = ₹ 2522.

Example 5. *Calculate the amount due and the compound interest on ₹ 37500 for 2 years when the rate of interest on successive years is 8% and 9% respectively.*

Solution. Principal for the first year = ₹ 37500, rate = 8% p.a.

Interest for the first year = ₹ $\frac{37500 \times 8 \times 1}{100}$ = ₹ 3000.

Amount after one year = ₹ 37500 + ₹ 3000 = ₹ 40500.

Principal for the second year = ₹ 40500, rate = 9% p.a.

Interest for the second year = ₹ $\frac{40500 \times 9 \times 1}{100}$ = ₹ 3645.

Amount due after 2 years = ₹ 40500 + ₹ 3645 = ₹ 44145.

Compound interest for 2 years = final amount – original principal

= ₹ 44145 – ₹ 37500 = ₹ 6645.

Example 6. *Kiran invests ₹ 8000 for 2 years at a certain rate of interest, compounded annually. At the end of one year, this sum amounts to ₹ 8960. Calculate :*

(i) the rate of interest per annum.

(ii) the amount due at the end of second year.

(iii) the compound interest earned in 2 years.

Solution. (*i*) Given P(original principal) = ₹ 8000,

amount after one year = ₹ 8960.

∴ Interest for the first year = ₹ 8960 – ₹ 8000 = ₹ 960.

Let the rate of interest be R% per annum.

Using Simple Interest = $\frac{P \times R \times T}{100}$, we get

$960 = \frac{8000 \times R \times 1}{100} \Rightarrow R = \frac{960 \times 100}{8000} = 12.$

Hence, the rate of interest = 12% per annum.

(*ii*) Principal for the second year = ₹ 8960.

Interest for the second year = ₹ $\frac{8960 \times 12 \times 1}{100}$ = ₹ 1075·20

∴ The amount due at the end of second year = ₹ 8960 + ₹ 1075·20

= ₹ 10035·20.

(*iii*) Compund interest earned in 2 years = final amount – (original) principal

= ₹ 10035·20 – ₹ 8000

= ₹ 2035·20

Example 7. *The simple interest on a certain sum of money for 3 years at 10% is ₹ 3600. Find the amount due and the compound interest on this sum of money at the same rate after 3 years, interest is compounded annually.*

Solution. Given simple interest for 3 year = ₹ 3600, rate = 10% p.a.

Let the original sum (principal) be ₹ P.

Using Simple Interest $= \frac{P \times R \times T}{100}$, we get

$$3600 = \frac{P \times 10 \times 3}{100} \Rightarrow P = \frac{3600 \times 100}{10 \times 3} = 12000.$$

$\therefore$ Principal for the first year = ₹ 12000.

Interest for the first year = ₹ $\frac{12000 \times 10 \times 1}{100}$ = ₹ 1200.

Amount after one year = ₹ 12000 + ₹ 1200 = ₹ 13200.

Principal for the second year = ₹ 13200.

Interest for the second year = ₹ $\frac{13200 \times 10 \times 1}{100}$ = ₹ 1320.

Amount after 2 years = ₹ 13200 + ₹ 1320

= ₹ 14520.

Principal for the third year = ₹ 14520.

Interest for the third year = ₹ $\frac{14520 \times 10 \times 1}{100}$ = ₹ 1452.

Amount due after 3 years = ₹ 14520 + ₹ 1452

= ₹ 15972.

Compound interest for 3 years = final amount – original principal

= ₹ 15972 – ₹ 12000

= ₹ 3972.

Exercise 3.1

1. Find the amount and the compound interest on ₹ 10000 for 2 years at 6% per annum.
2. A person invests ₹ 12500 for 2 years at 8% per annum compound interest. Calculate :
   (*i*) the interest for the first year.
   (*ii*) the amount at the end of the first year.
   (*iii*) the interest for the second year.
   (*iv*) the amount due at the end of second year.
   (*v*) the compound interest earned in 2 years.
3. Julie invests ₹ 15000 for three years at the rate of 10% per annum compound interest. Find :
   (*i*) the sum due to Julie at the end of first year.
   (*ii*) the interest she earns for the second year.
   (*iii*) the total amount due to her at the end of three years.
4. Calculate the compound interest for the second year on ₹ 14000 invested for 3 years at 10% per annum. Also find the sum due at the end of third year.
5. Suraj invests ₹ 46875 in State Bank of India at 8% per annum compound interest for 3 years. Calculate :
   (*i*) the interest for the first year.
   (*ii*) the amount standing to his credit at the end of second year.

(*iii*) the interest for the third year.

(*iv*) the amount due to him at the end of third year.

6. To renovate his shop, Mohanti borrowed ₹ 50000 from Bank of Baroda for 3 years at the rate of 12% per annum, compounded annually. What amount will he pay to the bank to clear his debt after 3 years?

7. A sum of ₹ 16000 is invested for three years at 10% per annum interest compounded annually.

(*i*) What is the sum due at the end of first year?

(*ii*) What is the sum due at the end of second year?

(*iii*) What is the compound interest earned in 2 years?

(*iv*) What is the interest earned in the second year?

(*v*) What is the sum due at the end of three years?

8. Find the compound interest to the nearest rupee on ₹ 5000 for 2 years 6 months at 6% per annum reckoned annually.

9. Find the amount and the compound interest on ₹ 40000 for $1\frac{1}{2}$ years at 10% per annum, the interest being compounded semi-annually.

10. Calculate the amount and the compound interest on ₹ 15000 in 2 years when the rate of interest for the successive years is 8% and 9% per annum respectively.

11. Calculate the amount and the compound interest on ₹ 20000 in 3 years when the rate of interest for the successive years is 10%, 10% and 12% per annum respectively.

12. A man invests ₹ 12000 for two years at a certain rate of interest, compounded annually. At the end of one year, it amounts to ₹ 13200. Calculate :

(*i*) the rate of interest per annum.

(*ii*) the amount at the end of second year.

(*iii*) the compound interest earned in 2 years.

13. Rohit invests ₹ 15625 for three years at a certain rate of interest, compounded annually. At the end of one year, it amounts to ₹ 16875. Calculate :

(*i*) the rate of interest per annum.

(*ii*) the interest accrued in the second year.

(*iii*) the amount due to him at the end of third year.

(*iv*) the interest earned in the third year.

(*v*) the interest earned in 3 years.

14. The simple interest on a certain sum of money for 2 years at 10% per annum is ₹ 4000. Find the amount due and the compound interest on this sum of money at the same rate after 3 years, the interest being compounded annually.

3.2 FORMULA FOR COMPOUND INTEREST

Compound interest *(abbreviated* ***C.I.****) can be easily calculated by the following formula :*

Formula : $\mathbf{A = P\left(1+\frac{r}{100}\right)^{n}}$

where A is the final amount, P is the principal, r is the rate of interest compounded yearly and n is the number of years.

$$\text{C.I.} = \text{A} - \text{P} = \text{P}\left(1+\frac{r}{100}\right)^n - \text{P} = \text{P}\left[\left(1+\frac{r}{100}\right)^n - 1\right].$$

Remark

If the interest is calculated for any other fixed period (like 6 months), then the principal keeps on changing every term of the fixed period (like 6 months).

The time from one specified interest period to the next period is called a ***conversion period.*** If this specified period is one year (*i.e.* the interest is compounded annually), then there is *one conversion period* in a year; if this period is six months (*i.e.* the interest is compounded semi-annually), then there are *two conversion periods* in a year; if this period is three months (*i.e.* the interest is compounded quarterly), then there are *four conversion periods* in a year. In view of this discussion, we can restate the formula as :

Formula : $$\text{A} = \text{P}\left(1+\frac{r}{100}\right)^n$$

where A is the final amount, P is the principal, r is the rate of interest per conversion period and n is the number of conversion periods.

Note

Obviously, if the rate of interest per annum is 10% and if the interest is compounded semi-annually, then the rate of interest per conversion period is $\frac{1}{2}$ of 10% *i.e.* 5%. If the interest is compounded quarterly, then the rate of interest per conversion period is $\frac{1}{4}$ of 10% *i.e.* 2·5%.

3.2.1 In solving problems on compound interest, remember the following :

1. $\text{A} = \text{P}\left(1+\frac{r}{100}\right)^n$ and $\text{C.I.} = \text{P}\left[\left(1+\frac{r}{100}\right)^n - 1\right]$
   where A is the amount, P is the principal, r is the rate of interest per conversion period and n is the number of conversion periods.
2. S.I. (simple interest) and C.I. are equal for the first conversion period on the same sum and at the same rate.
3. C.I. of 2nd conversion period is more than the C.I. of 1st conversion period and C.I. of 2nd conversion period – C.I. of 1st conversion period = S.I. on the interest of the 1st conversion period.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the amount and compound interest on ₹ 40000 for 2 years at 9% per annum, interest being payable annually.*

Solution. Given P = ₹ 40000, $r = 9$ and $n = 2$.

Using the formula, $\text{A} = \text{P}\left(1+\frac{r}{100}\right)^n$, we get

$$A = ₹\,40000\left(1+\frac{9}{100}\right)^2 = ₹\,40000 \times \left(\frac{109}{100}\right)^2$$

$$= ₹\left(40000 \times \frac{109}{100} \times \frac{109}{100}\right) = ₹\,47524.$$

$$\text{C.I.} = A - P = ₹\,47524 - ₹\,40000 = ₹\,7524.$$

Example 2. *Find the amount and compound interest on ₹ 16000 for 3 years at 15% per annum, interest compounded annually.*

Solution. Here $P = ₹\,16000$, $r = 15$ and $n = 3$.

Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we get

$$A = ₹\,16000\left(1+\frac{15}{100}\right)^3 = ₹\,16000 \times \left(\frac{115}{100}\right)^3$$

$$= ₹\,16000 \times \left(\frac{23}{20}\right)^3 = ₹\left(16000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{23}{20}\right)$$

$$= ₹\,24334.$$

$$\text{C.I.} = A - P = ₹\,24334 - ₹\,16000 = ₹\,8334.$$

Example 3. *Find the amount and compound interest on ₹ 30000 for 4 years at 10% per annum, interest compounded yearly.*

Solution. Here $P = ₹\,30000$, $r = 10$ and $n = 4$.

Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we get

$$A = ₹\,30000\left(1+\frac{10}{100}\right)^4 = ₹\,30000 \times \left(\frac{11}{10}\right)^4$$

$$= ₹\left(30000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}\right) = ₹\,43923.$$

$$\text{C.I.} = A - P = ₹\,43923 - ₹\,30000 = ₹\,13923.$$

Example 4. *Calculate the interest earned and the amount due if a sum of ₹ 12500 is invested for 1 year at 12% per annum, interest being compounded semi-annually.*

Solution. Since the rate of interest is 12% per annum, therefore, the rate of interest per conversion period (half-yearly) $= \frac{1}{2}$ of $12\% = 6\%$.

As the money is invested for 1 year, therefore,

n(the number of conversion periods) $= 2$.

Here, $P = ₹\,12500$, $r = 6$ and $n = 2$.

Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we get

$$A = ₹\,12500\left(1+\frac{6}{100}\right)^2 = ₹\,12500 \times \left(\frac{106}{100}\right)^2$$

$$= ₹\left(12500 \times \frac{53}{50} \times \frac{53}{50}\right) = ₹\,14045.$$

$$\therefore \quad \text{C.I.} = A - P = ₹\,14045 - ₹\,12500 = ₹\,1545.$$

Example 5. *Find the amount and the compound interest on ₹ 24000 at 10% per annum for $1\frac{1}{2}$ years, compound interest reckoned half-yearly.*

Solution. Since the rate of interest is 10% per annum, therefore, the rate of interest per conversion period (half-yearly) = $\frac{1}{2}$ of 10% = 5%.

As the money is invested for $1\frac{1}{2}$ year, therefore,

n(the number of conversion periods) = 3.

Here, P = ₹ 24000, r = 5 and n = 3.

Using the formula, $A = P\left(1+\frac{r}{100}\right)^n$, we get

$$A = ₹\,24000\left(1+\frac{5}{100}\right)^3 = ₹\,24000 \times \left(\frac{21}{20}\right)^3$$

$$= ₹\left(24000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right) = ₹\,27783.$$

$\therefore$ C.I. = A − P = ₹ 27783 − ₹ 24000 = ₹ 3783.

Exercise 3.2

1. Find the amount and the compound interest on ₹ 8000 for 2 years at 8% per annum, interest payable yearly.
2. Find the amount and the compound interest on ₹ 12000 for 2 years at 7·5% per annum, interest reckoned yearly.
3. Find the amount and the compound interest on ₹ 50000 for 3 years at 12% per annum, interest compounded annually.
4. Swati took a loan of ₹ 16000 from State Bank of India for 3 years at the rate of 12·5% per annum, compounded annually. Find the amount and the compound interest she has to pay at the end of 3 years to clear her debt, to the nearest rupee.
5. Find the amount and the compound interest on ₹ 25000 for 4 years at 10% per annum, interest compounded annually.
6. If the interest is compounded half-yearly, calculate the amount and the compound interest when the principal is ₹ 3750, the rate of interest is 8% and the duration is one year.
7. Find the amount and the compound interest on ₹ 8000 at 10% per annum for $1\frac{1}{2}$ years, compound interest reckoned semi-annually.
8. Find the difference between compound interest and simple interest on ₹ 5000 for 2 years at 8% per annum payable yearly.

CHAPTER TEST

1. A man invests ₹ 15000 at 11% per annum for 2 years, interest compounded annually. Calculate :
   (*i*) the interest for the first year.
   (*ii*) the amount at the end of the first year.
   (*iii*) the interest for the second year.
   (*iv*) the amount at the end of second year.
   (*v*) the interest earned in 2 years.
2. Vikram invests ₹ 14000 for 3 years at the rate of 10% per annum, interest reckoned yearly. Calculate :
   (*i*) the sum due to Vikarm at the end of first year.
   (*ii*) the interest he earns for the second year.
   (*iii*) the amount due to him at the end of three years.
   (*iv*) the interest he earns in 3 years.
   (*v*) the interest he earns for the third year.
3. A man invests ₹ 10000 for 3 years at a certain rate of interest, compounded annually. At the end of one year it amounts to ₹ 10800. Calculate :
   (*i*) the rate of interest per annum.
   (*ii*) the interest accrued in the second year.
   (*iii*) the amount at the end of the third year (to the nearest rupee).
4. The simple interest on a certain sum of money for 2 years at 5% is ₹ 1600. Find the amount due and the compound interest at the same rate after 3 years, interest compounded annually.
5. Jacob invests ₹ 25000 for 3 years at 8% per annum, interest compounded annually. Find the amount and the compound interest due to him after 3 years.
6. ₹ 20000 was lent for one year at 12% per annum. By how much more will the interest be if the sum was lent at 12% per annum, interest being compounded half-yearly.
7. Find the amount and the compound interest on ₹ 37500 at 8% per annum for $1\frac{1}{2}$ years, interest compounded half-yearly.