20 Area and Perimeter of Plane Figures

UNIT 6 : Mensuration

20.1 INTRODUCTION

1. Perimeter

The *perimeter* of a plane figure is the *length of its boundary*.

The *unit of perimeter* is the same as the *unit of length, i.e.* cm, m, etc.

2. Area

The *area* of a plane figure is the *measure of the surface enclosed by its boundary*.

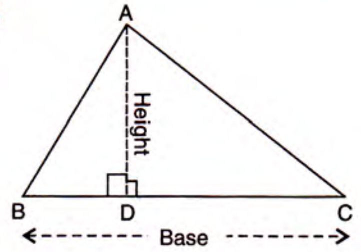
The *unit of area* is cm^2 (square centimetre); m^2 (square metre), etc.

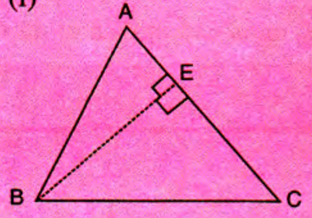
> Students should know the difference between "*square metre*" and "metre square". x *square metre* means an area and x *metre square* means a square each of whose sides is x metre long and so its area = $x \times x = x^2$ square metre.

20.2 AREA AND PERIMETER OF TRIANGLES

Area = $\frac{1}{2}$ base × corresponding height (altitude).

$= \frac{1}{2}$ BC × AD



> 1. Corresponding height (or altitude) of a triangle means *length of perpendicular* from the opposite vertex to the base.
> 2. In a triangle, any of its sides can be considered as base *e.g.*
>
> (i)
>
> 
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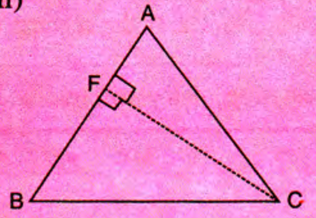
>
> If side AC is taken as the base, the length of perpendicular BE is the corresponding height (altitude).
>
> ∴ Area = $\frac{1}{2}$ base × height
>
> $= \frac{1}{2}$ AC × BE
>
> (ii)
>
> 
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>
> If side AB is taken as the base, the length of perpendicular CF is the corresponding height.
>
> Area = $\frac{1}{2}$ AB × CF

Heron's formula :

If a, b and c are three sides of a triangle, then its perimeter $(2s) = a + b + c$

and semi-perimeter $(s) = \dfrac{a+b+c}{2}$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Remember :

Unit of :	*In C.G.S.*	*In M.K.S. (S.I.)*	*Relation :*
1. *Length*	Centimetre (*cm*)	Metre (*m*)	$1\ cm = \frac{1}{100}\ m$ and $1\ m = 100\ cm$
2. *Perimeter*	*cm*	*m*	*same as above*
3. *Area*	Square *cm* (cm^2)	Square *m* (m^2)	$1\ cm^2 = \frac{1}{100\times100}\ m^2$ and $1\ m^2 = 100 \times 100\ cm^2$

1 Find the area of a triangle :

(i) whose height is 6 cm and base is 10 cm.

(ii) whose three sides are 17 cm, 8 cm and 15 cm long.
Also, in part (ii) of this question; calculate the length of the altitude corresponding to the largest side of the triangle.

Solution :

(i) **Area of triangle** $= \dfrac{1}{2}$ base × height

$= \dfrac{1}{2} \times 10 \times 6\ cm^2 = \mathbf{30\ cm^2}$ **Ans.**

(ii) Let $a = 17$ cm, $b = 8$ cm and $c = 15$ cm

$\therefore s = \dfrac{a+b+c}{2} = \dfrac{17+8+15}{2}$ cm = 20 cm

$\therefore$ **Area** $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(20-17)(20-8)(20-15)} = \mathbf{60\ cm^2}$ **Ans.**

Since, the largest side of the triangle is 17 cm

and $\dfrac{1}{2} \times$ base × altitude = area

$\Rightarrow \dfrac{1}{2} \times 17 \times$ alt. $= 60$ $\therefore$ **alt.** $= \dfrac{60\times2}{17}$ cm = **7.06 cm** **Ans.**

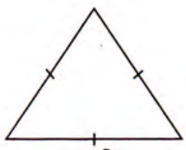
20.3 SOME SPECIAL TYPES OF TRIANGLES

1. Equilateral Triangle :

Let the length of each side of an equilateral triangle be a unit;

then, its perimeter = 3 × its side = $3a$

and its area $= \dfrac{\sqrt{3}}{4} \times (\text{side})^2 = \dfrac{\sqrt{3}}{4} \cdot a^2$



2 The area of an equilateral triangle is numerically equal to its perimeter. Find a side of the triangle [Take $\sqrt{3}$ = 1·73].

Solution :

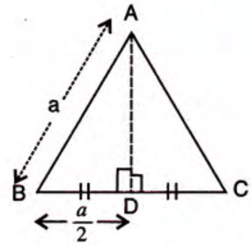
Given : Area = Perimeter [Numerically]

$\Rightarrow \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 3 \text{ side} \quad i.e. \quad \text{side} = \frac{3 \times 4}{\sqrt{3}}$

$\Rightarrow \quad$ **side** $= 4\sqrt{3} = 4 \times 1{\cdot}73$ unit = **6·92 unit** **Ans.**

3 Calculate the area of an equilateral triangle, whose height is 20 cm.

Solution :



Let ABC be the given equilateral triangle and AD is perpendicular to base BC; then clearly; AD = 20 cm

If each side of the given triangle be a cm; then AB = a cm

and, $BD = \frac{1}{2} BC$ [In equilateral Δ, perpendicular from vertex bisects the base]

$= \frac{1}{2} a$ cm

In right-angled triangle ABD :

$AD^2 + BD^2 = AB^2 \quad \Rightarrow (20)^2 + \left(\frac{a}{2}\right)^2 = a^2$ [Pythagoras Theorem]

On simplifying, we get : $a^2 = 400 \times \frac{4}{3} = \frac{1600}{3}$

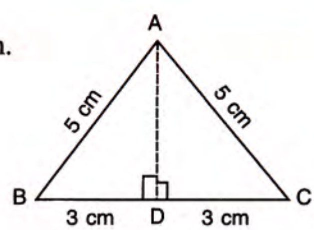
$\therefore$ **Area of the triangle** $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times \frac{1600}{3}$ cm² = **230·9 cm²** **Ans.**

2. Isosceles Triangle :

4 Find the area of an isosceles triangle whose equal sides are 5 cm each and base is 6 cm.

Solution :



In an isosceles triangle ABC, let AB = AC = 5 cm and BC = 6 cm.

Draw AD perpendicular to BC.

Since, the perpendicular from the vertex to the base of an isosceles triangle bisects the base, therefore

$BD = CD = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$

Applying Pythagoras Theorem in triangle ABD, we get :

$AD^2 = AB^2 - BD^2$

$= 5^2 - 3^2 = 25 - 9 = 16 \quad \Rightarrow AD = 4$ cm

$\therefore$ Area of $\Delta = \frac{1}{2}$ base × height

$= \frac{1}{2} BC \times AD = \frac{1}{2} \times 6 \times 4 \text{ cm}^2 =$ **12 cm²** **Ans.**

Alternative method :

Since, the sides of the given isosceles triangle are 5 cm, 5 cm and 6 cm

$\therefore s = \frac{5+5+6}{2}$ cm $= 8$ cm

and **area of Δ** $= \sqrt{8(8-5)(8-5)(8-6)}$ cm²

$= \sqrt{8 \times 3 \times 3 \times 2}$ cm² = **12 cm²** **Ans.**

Third method :

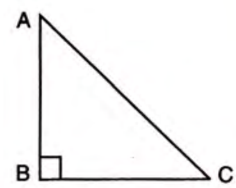
Area of an isosceles triangle

$= \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$; where, a = length of each equal side and, b = length of base.

$= \frac{1}{4} \times 6 \times \sqrt{4 \times 5^2 - 6^2}$ = **12 cm²** **Ans.**

3. Right-angled Triangle :

The area of a right-angled triangle is equal to half the product of the sides containing the right angle.



In the given figure; Area of Δ ABC $= \frac{1}{2}$ AB × BC

5 The sides of a triangle containing the right angle are $5x$ cm and $(3x - 1)$ cm. If the area of the triangle is 60 cm², calculate the lengths of the sides of the triangle.

Solution :

Since, area of a right-angled triangle $= \frac{1}{2}$ × product of its sides containing the right angle

$\therefore \quad 60 = \frac{1}{2} \times 5x \times (3x - 1)$

$\Rightarrow \quad 120 = 15x^2 - 5x$ *i.e.,* $3x^2 - x - 24 = 0$ [Dividing each term by 5]

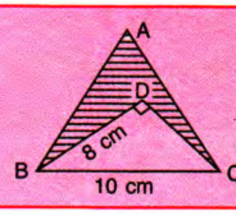
On solving the quadratic equation; we get : $x = 3$ and $x = \frac{-8}{3}$

Since, $x = \frac{-8}{3}$ will give negative values of the sides of the triangle, which is impossible; therefore, $x = 3$.

$\therefore$ **Lengths of the sides** $= 5x$ cm and $(3x - 1)$ cm

$= 5 \times 3$ cm and $(3 \times 3 - 1)$ cm = **15 cm and 8 cm** **Ans.**

6 The given figure shows an equilateral triangle ABC whose each side is 10 cm and a right-angled triangle BDC whose side BD = 8 cm and ∠D = 90°. Find the area of the shaded portion.



Solution :

In right-angled Δ BDC,

$$BD^2 + CD^2 = BC^2 \Rightarrow 8^2 + CD^2 = 10^2$$
$$\Rightarrow CD = 6 \text{ cm}$$

$\therefore$ Area of Δ BDC $= \frac{1}{2} \times BD \times CD = \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$

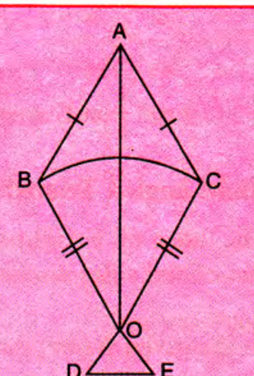
Also, area of equilateral Δ ABC $= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{1{\cdot}732}{4} \times 10^2 \text{ cm}^2 = 43{\cdot}3 \text{ cm}^2$

$\therefore$ **The area of the shaded portion**

= Area of Δ ABC – Area of Δ BDC

$= 43{\cdot}3 \text{ cm}^2 - 24 \text{ cm}^2 =$ **$19{\cdot}3 \text{ cm}^2$** **Ans.**

7 A kite is made as shown alongside in which ABC is an equilateral triangle with side 20 cm, BOC is an isosceles triangle with OB = OC = 26 cm and ODE is an isosceles triangle with base DE = 8 cm and height 6 cm. Find the whole area of the kite.



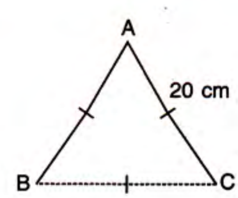
Solution :

Area of ΔABC $= \frac{\sqrt{3}}{4} \times (20)^2$ sq. cm

$= \frac{\sqrt{3}}{4} \times 20 \times 20$ sq. cm

$= 100 \times 1{\cdot}732$ sq cm

$= 173{\cdot}2$ sq. cm



Join BC and draw OP ⊥ BC.

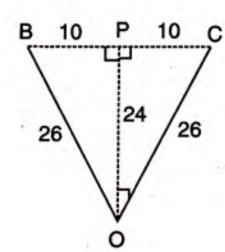
Since, BOC is an isosceles triangle, OP will bisect BC

$\Rightarrow$ BP = CP = 10 cm

In right angle Δ OBP,

BP = 10 cm and OB = 26 cm

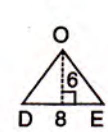
$\Rightarrow$ OP = 24 cm [Using Pythagoras Theorem]



$\therefore$ Area of Δ BOC $= \frac{1}{2} \times BC \times OP$

$= \frac{1}{2} \times 20 \times 24$ sq cm = 240 sq cm

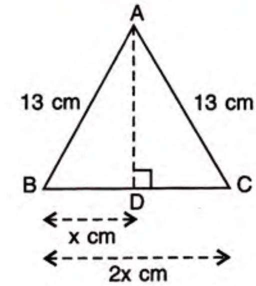
Area of Δ ODE $= \frac{1}{2} \times 8 \times 6$ sq cm = 24 sq cm



$\therefore$ **Required area** $= (173{\cdot}2 + 240 + 24)$ sq cm = **$437{\cdot}2$ sq cm.** **Ans.**

8 If the area of an isosceles triangle is 60 cm^2 and the length of each of its equal sides is 13 cm, find its base.

Solution :



Let base = $2x$ cm *i.e.* BC = $2x$ cm

$\because$ In an isosceles triangle, the perpendicular from the vertex bisects the base

$\Rightarrow$ BD = CD = x cm

In right-triangle ABD,

$$AD^2 + BD^2 = AB^2$$

$\Rightarrow$ $AD^2 + x^2 = 13^2$

i.e. $AD^2 = 169 - x^2$ and $AD = \sqrt{169 - x^2}$ cm

Given, area of the triangle = 60 cm^2

$\Rightarrow$ $\frac{1}{2}BC \times AD = 60$ *i.e.* $\frac{1}{2} \times 2x \times \sqrt{169 - x^2} = 60$

$\Rightarrow$ $x\sqrt{169 - x^2} = 60$ *i.e.* $x^2(169 - x^2) = 3600$

$\Rightarrow$ $x^4 - 169x^2 + 3600 = 0$ *i.e.* $x^4 - 144x^2 - 25x^2 + 3600 = 0$

$\Rightarrow$ $x^2(x^2 - 144) - 25(x^2 - 144) = 0$ *i.e.* $(x^2 - 144)(x^2 - 25) = 0$

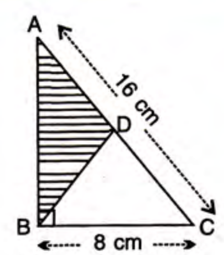
$\Rightarrow$ $x^2 - 144 = 0$ or $x^2 - 25 = 0$ *i.e.* $x = 12$ or $x = 5$

$x = 12$ $\Rightarrow$ **base** = $2x$ cm = 2×12 cm = **24 cm** **Ans.**

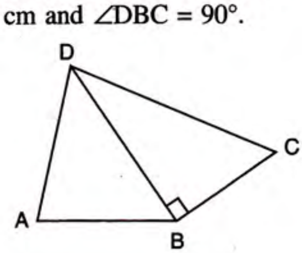
$x = 5$ $\Rightarrow$ **base** = $2x$ cm = 2×5 cm = **10 cm** **Ans.**

EXERCISE 20(A)

1. Find the area of a triangle whose sides are 18 cm, 24 cm and 30 cm.
   Also, find the length of altitude corresponding to the largest side of the triangle.
2. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5. Find the area of the triangle if its perimeter is 144 cm.
3. ABC is a triangle in which AB = AC = 4 cm and $\angle A = 90°$. Calculate :
   (i) the area of Δ ABC,
   (ii) the length of perpendicular from A to BC.
4. The area of an equilateral triangle is $36\sqrt{3}$ sq. cm. Find its perimeter.
5. Find the area of an isosceles triangle with perimeter 36 cm and base 16 cm.
6. The base of an isosceles triangle is 24 cm and its area is 192 sq. cm. Find its perimeter.
7. The given figure shows a right-angled triangle ABC and an equilateral triangle BCD.
   Find the area of the shaded portion.



8. Find the area and the perimeter of quadrilateral ABCD, given below; if, AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and $\angle DBC = 90°$.



9. The base of a triangular field is three times its height. If the cost of cultivating the field at ₹ 36·72 per 100 m^2 is ₹ 49,572; find its base and height.

10. The sides of a triangular field are in the ratio 5 : 3 : 4 and its perimeter is 180 m. Find :
   (i) its area.
   (ii) altitude of the triangle corresponding to its largest side.
   (iii) the cost of levelling the field at the rate of ₹ 10 per square metre.

11. Each of equal sides of an isosceles triangle is 4 cm greater than its height. If the base of the triangle is 24 cm; calculate the perimeter and the area of the triangle.

12. Calculate the area and the height of an equilateral triangle whose perimeter is 60 cm.

13. In triangle ABC; angle A = 90°, side AB = x cm, AC = $(x + 5)$ cm and area = 150 cm^2. Find the sides of the triangle.

14. If the difference between the sides of a right angled triangle is 3 cm and its area is 54 cm^2; find its perimeter.

15. AD is altitude of an isosceles triangle ABC in which AB = AC = 30 cm and BC = 36 cm. A point O is marked on AD in such a way that ∠BOC = 90°. Find the area of quadrilateral ABOC.

> Area of quadrilateral ABOC
> $= \text{Ar. }(\Delta\text{ ABC}) - \text{Ar. }(\Delta\text{ BOC})$
> $= \frac{1}{2} \times BC \times AD - \frac{1}{2} \times OB \times OC$
> It can easily be shown that Δ BOD ≡ Δ COD
> ⇒ OB = OC = x cm (let) and
> $OB^2 + OC^2 = BC^2$ as ∠BOC = 90°

20.4 AREA AND PERIMETER OF QUADRILATERALS

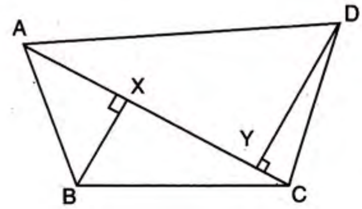
1. *When one diagonal and perpendiculars to this diagonal from the remaining vertices are given.*

In quadrilateral ABCD, the diagonal AC and perpendiculars BX and DY to AC from the remaining vertices B and D respectively are given; then the

Area of quad. ABCD = Δ ABC + Δ ADC

$$= \frac{1}{2} AC \times BX + \frac{1}{2} AC \times DY$$

$$= \frac{1}{2} AC \times (BX + DY)$$



∴ Area of quadrilateral = $\frac{1}{2}$ × one diagonal × sum of the lengths of the perpendiculars drawn on it from the remaining two vertices.

2. *When two diagonals of a quadrilateral cut each other at right angles;* then the

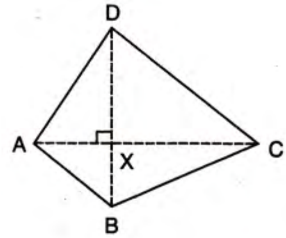
Area of quad. ABCD = Δ ABC + Δ ADC

$$= \frac{1}{2} AC \times BX + \frac{1}{2} AC \times DX$$

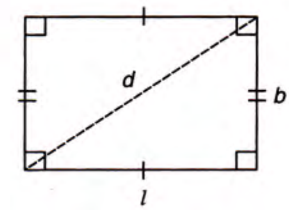
$$= \frac{1}{2} AC\ (BX + DX)$$

$$= \frac{1}{2} AC \times BD$$

$$= \frac{1}{2} \times \text{the product of the diagonals.}$$



20.5 SOME SPECIAL TYPES OF QUADRILATERALS



1. Rectangle :

Area = length × breadth = $l \times b$

Perimeter = 2 (length + breadth) = $2(l + b)$

Length of diagonal (d) = $\sqrt{l^2 + b^2}$

9 The perimeter of a rectangle is 25·5 m. Its length is 9·5 m. Calculate its area in sq. m (m^2).

Solution :

Given : $P = 25{\cdot}5$ m and $l = 9{\cdot}5$ m

Since, $P = 2(l + b)$

$\therefore \quad 25{\cdot}5 = 2(9{\cdot}5 + b)$ *i.e.,* $25{\cdot}5 = 19 + 2b$

$\Rightarrow \quad b = 3{\cdot}25$ m

$\therefore$ **Area** $= 9{\cdot}5 \times 3{\cdot}25 \text{ m}^2$ [$\because$ Area = $l \times b$]

$= \mathbf{30{\cdot}875 \text{ m}^2}$ **Ans.**

10 A room is 8 m long and 5 m broad. Find the cost of covering the floor of the room with 80 cm wide carpet at the rate of ₹ 225 per metre.

Solution :

Area of floor of the room = $8 \times 5 \text{ m}^2 = 40 \text{ m}^2$

Let the length of the carpet be l m.

$\therefore$ Area of carpet = length × breadth

$= l \times \frac{80}{100} \text{ m}^2 = 0{\cdot}80\, l \text{ m}^2$

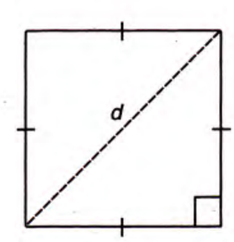
Since, Area of carpet = Area of floor of the room

$\therefore \quad 0{\cdot}80\, l = 40$

$\Rightarrow \quad l = \frac{40}{0{\cdot}80} \text{ m} = 50 \text{ m}$

$\therefore$ **Cost of carpet** = 50 × ₹ 225 = **₹ 11,250** **Ans.**

2. Square :



Area = $(\text{side})^2 = a^2$

Perimeter = $4 \times \text{side} = 4a$

Diagonal (d) = $\sqrt{a^2 + a^2} = a \,.\, \sqrt{2}$

Also, diagonal $d = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2 \times \text{Area}}$

11 The area of a square field is 484 m^2. Find :

(i) the length of its one side;

(ii) the length of its diagonal, correct to two places of decimal.

Solution :

(i) Since, area $= (\text{side})^2 = 484 \text{ m}^2$

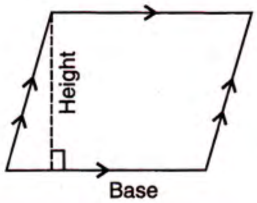
$\therefore$ **Side** $= \sqrt{484}$ m = **22 m** **Ans.**

(ii) **Diagonal** $= \text{side} \times \sqrt{2}$

$= 22 \times 1{\cdot}414$ m = **31·11 m** **Ans.**

3. Parallelogram :

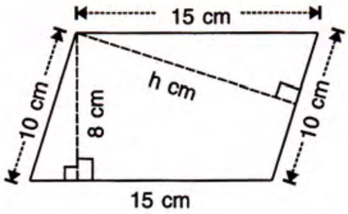
Area = base × height



The height of a parallelogram is the distance between its base and side opposite to the base.

12 Two adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between 15 cm sides is 8 cm; find the distance between 10 cm sides.

Solution :



Let the distance between 10 cm sides be h cm.

Since, area = base × height

$\therefore$ $15 \times 8 = 10 \times h$

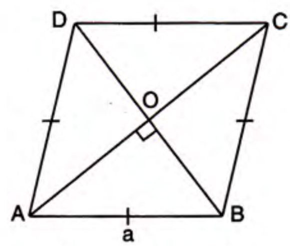
$\Rightarrow$ **h = 12 cm** **Ans.**

4. Rhombus :

Perimeter = 4 × side = 4 a

Area $= \frac{1}{2} \times$ (the product of diagonals)

$= \frac{1}{2} \times AC \times BD.$



Since, the diagonals of a rhombus bisect each other at right angle, therefore in Δ AOB

$AB^2 = OA^2 + OB^2$

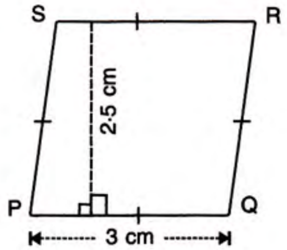
or $(\text{side})^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$; where $AC = d_1$ and $BD = d_2$.

13 PQRS is a rhombus.

(i) If it is given that PQ = 3 cm, calculate the perimeter of PQRS.

(ii) If the height of the rhombus is 2·5 cm, calculate the area.

Solution :



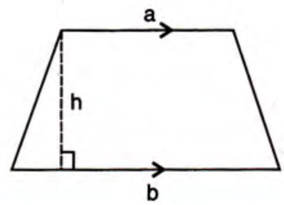
(i) **Perimeter** = 4 × 3 cm = **12 cm** **Ans.**

(ii) Since, rhombus is a parallelogram also, therefore its area can be obtained by using the formula

$$A = \text{base} \times \text{height}$$

$$= 3 \times 2{\cdot}5 \text{ sq. cm} = \mathbf{7{\cdot}5 \text{ sq. cm}}$$ **Ans.**

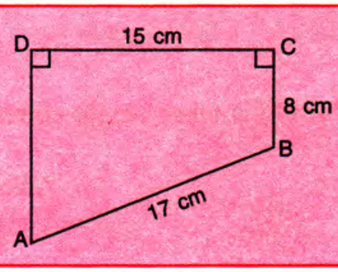
5. Trapezium :



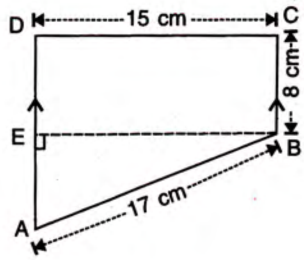
$$\text{Area} = \frac{1}{2} \times (\text{sum of parallel sides}) \times (\text{distance between parallel sides})$$

$$= \frac{1}{2}(a + b) \times h$$

14 The given figure shows a trapezium ABCD in which AB = 17 cm, BC = 8 cm and CD = 15 cm. Find area and perimeter of the trapezium.



Solution :



Draw BE perpendicular to AD.

In rectangle BCDE, BE = DC = 15 cm and DE = CB = 8 cm

In right-angled triangle ABE,

$$AE^2 = AB^2 - BE^2$$

$$= 17^2 - 15^2 = 64$$

$\Rightarrow$ $AE = 8$ cm

Also, $AD = AE + DE = 8 \text{ cm} + 8 \text{ cm} = 16 \text{ cm}$

$\therefore$ **Area of trapezium** $= \frac{1}{2}(AD + BC) \times BE$

$$= \frac{1}{2}(16 \text{ cm} + 8 \text{ cm}) \times 15 \text{ cm} = \mathbf{180 \text{ cm}^2}$$ **Ans.**

and, **perimeter** = AB + BC + CD + DA

= (17 + 8 + 15 + 16) cm = **56 cm** **Ans.**

15 Find the area of the trapezium whose parallel sides are 15 cm and 23 cm; whereas the non-parallel sides are 10 cm and 8 cm.

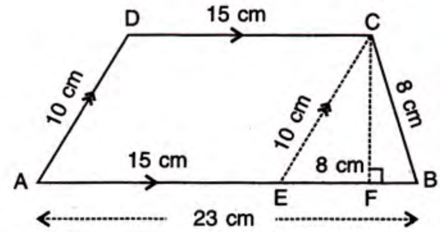
Solution :

Let the given trapezium is ABCD as shown ahead.

Clearly; DC // AB,

DC = 15 cm, AB = 23 cm;

AD = 10 cm and BC = 8 cm.



Draw CE parallel to DA which meets AB at point E.

Since, opposite sides of the quadrilateral AECD are parallel, it is parallelogram

Also, draw CF perpendicular to EB

In triangle EBC,

Let a = EB = AB – AE = AB – DC = (23 – 15) cm = 8 cm

b = CE = DA = 10 cm and c = BC = 8 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{8+10+8}{2} \text{ cm} = 13 \text{ cm}$$

$$\text{Area of } \Delta \text{ EBC} = \sqrt{13(13-8)(13-10)(13-8)} \text{ cm}^2$$

$$= \sqrt{13\times5\times3\times5} \text{ cm}^2 = 5\sqrt{39} \text{ cm}^2$$

If CF is taken height corresponding to the base BE,

$$\text{Area of } \Delta \text{ EBC} = \frac{1}{2} \times \text{EB} \times \text{CF cm}^2$$

$$\therefore \quad \frac{1}{2} \times 8 \times \text{CF} = 5\sqrt{39} \quad \Rightarrow \text{CF} = \frac{5\sqrt{39}}{4} \text{ cm}$$

Clearly, distance between the parallel sides AB and DC is the length of CF.

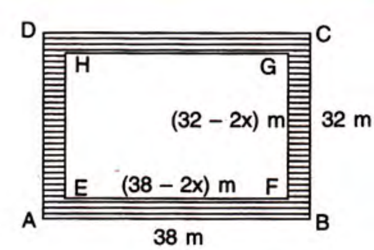
$\therefore$ **Area of given trapezium** $= \frac{1}{2}$(sum of parallel sides) × distance between them

$$= \frac{1}{2}(15 + 23) \times \text{CF}$$

$$= \frac{1}{2} \times 38 \times \frac{5\sqrt{39}}{4} \text{ cm}^2 = \mathbf{148{\cdot}32 \text{ cm}^2}$$ **Ans.**

16 A footpath of uniform width runs all around the inside of a rectangular field 38 m long and 32 m wide. If the path occupies 600 m^2, find its width.

Solution :



Let the given rectangular field be ABCD with length AB = 38 m and width BC = 32 m.

If the width of the uniform path = x m; the length of rectangle excluding path is EF = (38 – 2x) m and the width of rectangle excluding path is FG = (32 – 2x) m.

$$\text{Area of rect. ABCD} - \text{Area of rect. EFGH} = \text{Area of path}$$

$$\Rightarrow \quad 38 \times 32 - (38 - 2x)(32 - 2x) = 600$$

$$\Rightarrow \quad 38 \times 32 - 38 \times 32 + 76x + 64x - 4x^2 = 600$$

$$\Rightarrow \quad 4x^2 - 140x + 600 = 0 \quad i.e. \quad x^2 - 35x + 150 = 0$$

$\Rightarrow \quad x^2 - 30x - 5x + 150 = 0 \quad i.e. \quad (x - 30)(x - 5) = 0$

$\Rightarrow \quad x = 30$ or $x = 5$

Rejecting $x = 30$ (because it does not satisfy the calculation of the area of the path *i.e.* 600 m^2); we get : $x = 5$

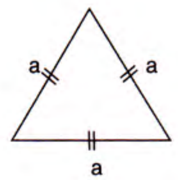
$\Rightarrow$ **The width of the path = 5 m** **Ans.**

17 A wire is bent in the form of an equilateral triangle of largest area. If it encloses an area of $49\sqrt{3}$ cm^2, find the largest area enclosed by the same wire when bent to form :

(i) a square (ii) a rectangle of length 12 cm.

Solution :

Let the side of the given equilateral $\Delta = a$ cm



Given : Its area $= 49\sqrt{3}$

$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 = 49\sqrt{3} \quad \Rightarrow a^2 = 49\sqrt{3} \times \frac{4}{\sqrt{3}} = 49 \times 4$

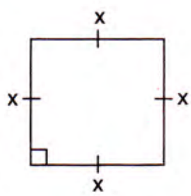
$\Rightarrow \quad a = 7 \times 2 = 14$ cm

Clearly, the length of the wire

= The perimeter of the triangle

$= 3a = 3 \times 14$ cm $= 42$ cm

(i) Let the side of the square $= x$ cm



$\therefore$ The perimeter of the square formed

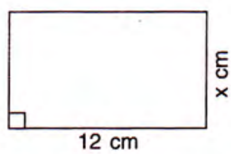
= The length of the wire

$\Rightarrow \quad 4x = 42 \Rightarrow x = 10{\cdot}5$ cm

And, **the area enclosed** $= (\text{side})^2$

$= (10{\cdot}5)^2$ cm^2 = **110·25 cm^2** **Ans.**

(ii) Let the width of the rectangle formed $= x$ cm



$\therefore$ The perimeter of the rectangle formed = The length of the wire

$\Rightarrow \quad 2(12 + x) = 42 \Rightarrow x = 9$ cm

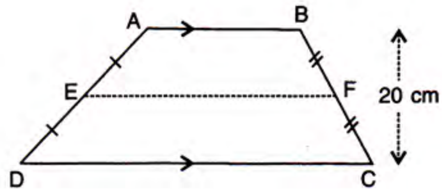
$\therefore$ **The area of the rectangle** = Its length × its width

= 12 cm × 9 cm = **108 cm^2** **Ans.**

18 The distance between parallel sides of a trapezium is 20 cm and the length of the line segment joining the mid-points of its non-parallel sides is 53 cm. Find the area of the trapezium.

Solution :

Let the given trapezium be as shown alongside in which distance between parallel sides AB and DC is 20 cm, *i.e.* height (h) of the trapezium = 20 cm. E and F are the mid-points of non-parallel sides AD and BC respectively.



Given : EF = 53 cm

$\Rightarrow \quad \frac{AB + DC}{2} = 53$ cm *i.e.* AB + DC = 106 cm

$\therefore$ **Area of the given trapezium** $= \frac{1}{2} \times (AB + DC) \times h$

$= \frac{1}{2} \times 106 \times 20 \text{ cm}^2$

$=$ **1060 cm²** **Ans.**

19 Area of a square is same as that of a rectangle. The length and the breadth of the rectangle are respectively 5 cm more and 4 cm less than the side of the square. Find the side of the square.

Solution :

Let the side of the square = x cm.

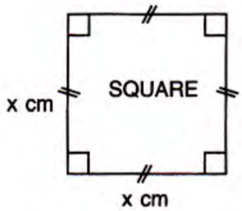
$\therefore$ Sides of the rectangle are $(x + 5)$ cm and $(x - 4)$ cm.

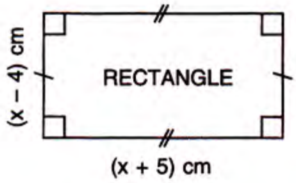
Given : Area of the square = Area of the rectangle

$\Rightarrow \quad x^2 = (x + 5)(x - 4)$

i.e. $\quad x^2 = x^2 - 4x + 5x - 20$ *i.e.* $x = 20$

$\therefore$ **The side of the square = 20 cm** **Ans.**

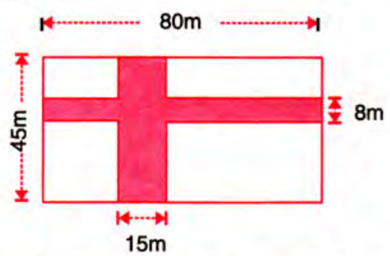




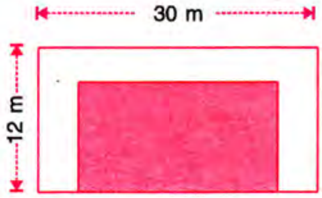
EXERCISE 20(B)

1. Find the area of a quadrilateral one of whose diagonals is 30 cm long and the perpendiculars from the other two vertices are 19 cm and 11 cm respectively.
2. The diagonals of a quadrilateral are 16 cm and 13 cm. If they intersect each other at right angles; find the area of the quadrilateral.
3. Calculate the area of quadrilateral ABCD, in which $\angle ABD = 90°$, triangle BCD is an equilateral triangle of side 24 cm and AD = 26 cm.
4. Calculate the area of quadrilateral ABCD in which AB = 32 cm, AD = 24 cm, $\angle A = 90°$ and BC = CD = 52 cm.
5. The perimeter of a rectangular field is $\frac{3}{5}$ km. If the length of the field is twice its width; find the area of the rectangle in sq. metres.

6. A rectangular plot 85 m long and 60 m broad is to be covered with grass leaving 5 m all around. Find the area to be laid with grass.
7. The length and the breadth of a rectangle are 6 cm and 4 cm respectively. Find the height of a triangle whose base is 6 cm and area is 3 times that of the rectangle.
8. How many tiles, each of area 400 cm^2, will be needed to pave a footpath which is 2 m wide and surrounds a grass plot 25 m long and 13 m wide ?
9. The cost of enclosing a rectangular garden with a fence all round, at the rate of 75 paise per metre, is ₹ 300. If the length of the garden is 120 metres, find the area of the field in square metres.
10. The width of a rectangular room is $\frac{4}{7}$ of its length, x, and its perimeter is y. Write an equation connecting x and y. Find the length of the room when the perimeter is 4400 cm.
11. The length of a rectangular verandah is 3 m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.
    (i) Taking x as the breadth of the verandah, write an equation in x that represents the above statement.
    (ii) Solve the equation obtained in (i) above and hence find the dimensions of the verandah.
12. The diagram, given below, shows two paths drawn inside a rectangular field 80 m long and 45 m wide. The widths of the two paths are 8 m and 15 m as shown. Find the area of the shaded portion.

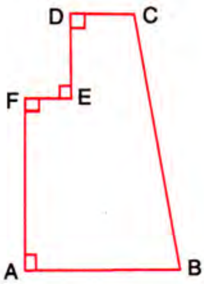


13. The rate for a 1·20 m wide carpet is ₹ 40 per metre; find the cost of covering a hall 45 m long and 32 m wide with this carpet. Also, find the cost of carpeting the same hall if the carpet, 80 cm wide, is at ₹ 25 per metre.
14. Find the area and perimeter of a square plot of land, the length of whose diagonal is 15 metres. Give your answer correct to 2 places of decimals.
15. The shaded region of the given diagram represents the lawn in the form of a house. On the three sides of the lawn there are flower-beds having a uniform width of 2 m.

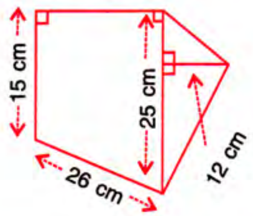


    (i) Find the length and the breadth of the lawn.
    (ii) Hence, or otherwise, find the area of the flower-beds.
16. A floor which measures 15m × 8m is to be laid with tiles measuring 50 cm × 25cm. Find the number of tiles required.

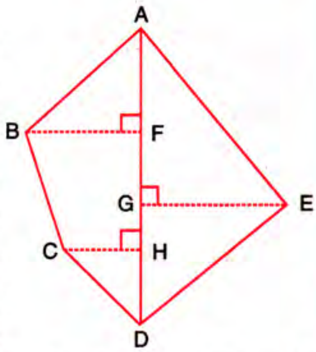
    Further, if a carpet is laid on the floor so that a space of 1 m exists between its edges and the edges of the floor, what fraction of the floor is left uncovered.
17. Two adjacent sides of parallelogram are 24 cm and 18 cm. If the distance between the longer sides is 12 cm; find the distance between the shorter sides.
18. Two adjacent sides of a parallelogram are 10 cm and 12 cm. If one diagonal of it is 16 cm long; find area of the parallelogram. Also, find the distance between its shorter sides.
19. The area of a rhombus is 216 sq. cm. If its one diagonal is 24 cm; find :
    (i) length of its other diagonal,
    (ii) length of its side,
    (iii) perimeter of the rhombus.
20. The perimeter of a rhombus is 52 cm. If one diagonal is 24 cm; find :
    (i) the length of its other diagonal,
    (ii) its area.
21. The perimeter of a rhombus is 46 cm. If the height of the rhombus is 8 cm; find its area.
22. The figure given below shows the cross-section of a concrete structure. Calculate the area of cross-section if AB = 1·8 m, CD = 0·6 m, DE = 0·8 m, EF = 0·3 m and AF = 1·2 m.



23. Calculate the area of the figure given below : which is not drawn to scale.



24. The following diagram shows a pentagonal field ABCDE in which the lengths of AF, FG, GH and HD are 50 m, 40 m, 15 m and 25 m respectively; and the lengths of perpendiculars BF, CH and EG are 50 m, 25 m and 60 m respectively. Determine the area of the field.

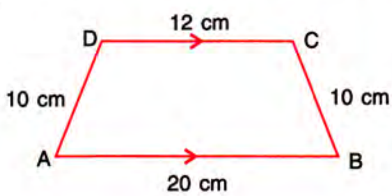


25. A footpath of uniform width runs all around the outside of a rectangular field 30 m long and 24 m wide. If the path occupies an area of 360 m^2, find its width.

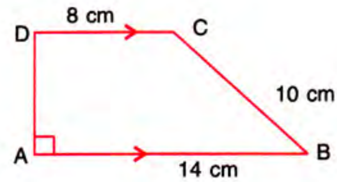
26. A wire when bent in the form of a square encloses an area of 484 m^2. Find the largest area enclosed by the same wire when bent to form :
    (i) an equilateral triangle.
    (ii) a rectangle of breadth 16 m.

27. For each trapezium given below; find its area.

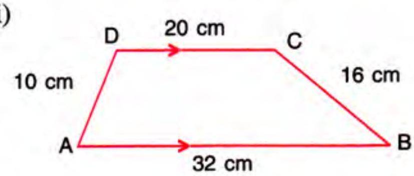
(i)



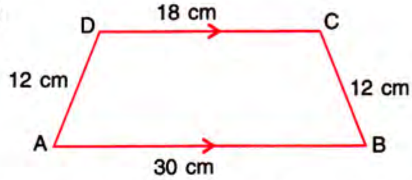
(ii)



(iii)



(iv)



28. The perimeter of a rectangular board is 70 cm. Taking its length as x cm, find its width in terms of x.
If the area of the rectangular board is 300 cm^2; find its dimensions.

29. The area of a rectangle is 640 m^2. Taking its length as x m; find, in terms of x, the width of the rectangle. If the perimeter of the rectangle is 104 m; find its dimensions.

30. The length of a rectangle is twice the side of a square and its width is 6 cm greater than the side of the square. If area of the rectangle is three times the area of the square; find the dimensions of each.

31. ABCD is a square with each side 12 cm. P is a point on BC such that area of Δ ABP : area of trapezium APCD = 1 : 5. Find the length of CP.

32. A rectangular plot of land measures 45m × 30m. A boundary wall of height 2·4 m is built all around the plot at a distance of 1m from the plot. Find the area of the inner surface of the boundary wall.

33. A wire when bent in the form of a square encloses an area = 576 cm^2. Find the largest area enclosed by the same wire when bent to form:
    (i) an equilateral triangle.
    (ii) a rectangle whose adjacent sides differ by 4 cm.

34. The area of a parallelogram is y cm^2 and its height is h cm. The base of another parallelogram is x cm more than the base of the first parallelogram and its area is twice the area of the first. Find, in terms of y, h and x,

the expression for the height of the second parallelogram.

Base of 1st parallelogram = $\frac{y}{h}$ cm

$\Rightarrow$ Base of 2nd parallelogram = $\left(\frac{y}{h}+x\right)$ cm

and $\left(\frac{y}{h}+x\right)$ × height = $2y$

$\Rightarrow$ height = $\frac{2hy}{y+xh}$

35. The distance between parallel sides of a trapezium is 15 cm and the length of the line segment joining the mid-points of its non-parallel sides is 26 cm. Find the area of the trapezium.

36. The diagonal of a rectangular plot is 34 m and its perimeter is 92 m. Find its area.

20.6 CIRCUMFERENCE OF A CIRCLE :

It is the length of the boundary of a circle.

For every circle, the ratio between its circumference and its diameter is always constant (same). This constant ratio is represented by Greek letter π (pie).

$\therefore$ For any circle, $\frac{\text{its circumference}}{\text{its diameter}} = \pi$

$\Rightarrow$ Its circumference = π × its diameter

$\Rightarrow$ $C = \pi \times d$ [d = diameter = 2 × radius]

$= \pi \times 2r = 2\pi r$

The value of $\pi = \frac{22}{7} = 3{\cdot}14$ (approximately) and is same for all the circles.

20 The circumference of a circle exceeds its diameter by 270 cm. Find its diameter.

Solution :

Let radius of the circle be r cm

$\therefore$ Its circumference = $2\pi r$ and diameter = $2r$

Given : $2\pi r - 2r = 270$ $\Rightarrow$ $2r(\pi - 1) = 270$

$\Rightarrow$ $2r\left[\frac{22}{7}-1\right] = 270$

$\Rightarrow$ $r = \frac{270\times7}{2\times15}$ cm = 63 cm

$\therefore$ **Diameter** = 2r

= 2 × 63 cm = **126 cm** **Ans.**

21 Find the diameter of the circle whose circumference is equal to the sum of the circumference of circles with radii 5 cm, 8 cm and 10 cm.

Solution :

For the circle with radius 5 cm, $C = 2\pi r = 2 \times \frac{22}{7} \times 5$ cm $= \frac{220}{7}$ cm

For the circle with radius 8 cm, $C = 2 \times \frac{22}{7} \times 8 \text{ cm} = \frac{352}{7} \text{ cm}$

For the circle with radius 10 cm, $C = 2 \times \frac{22}{7} \times 10 \text{ cm} = \frac{440}{7} \text{ cm}$

Sum of the circumference of these circles $= \frac{220}{7} \text{ cm} + \frac{352}{7} \text{ cm} + \frac{440}{7} \text{ cm}$

$= \frac{1012}{7} \text{ cm}$

Let the radius of the new circle = R cm

$\therefore$ Its circumference = $2\pi R$ cm

Given : $2\pi R = \frac{1012}{7} \Rightarrow 2 \times \frac{22}{7} \times R = \frac{1012}{7}$

$\Rightarrow$ **R** $= \frac{1012}{44}$ cm = **23 cm** **Ans.**

Direct method :

Let the radius of the resulting circle = R cm

$\therefore$ Circumference of the circle = Sum of the circumferences of the given circles

$\Rightarrow 2\pi R = 2\pi \times 5 \text{ cm} + 2\pi \times 8 \text{ cm} + 2\pi \times 10 \text{ cm}$

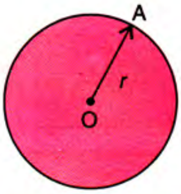
$\Rightarrow R = 5 + 8 + 10$ [Dividing each term by 2π]

$= 23$

$\therefore$ **Required diameter** = 2R

= 2 × 23 cm = **46 cm** **Ans.**

20.7 AREA OF A CIRCLE



The measurement of a surface, enclosed by the perimeter of the circle, is called its *area*.

If radius of a circle = r unit

Its *area* = πr^2 sq. unit

22 The area of a circle is numerically equal to its circumference. Find its area (Take $\pi = 3{\cdot}14$).

Solution :

Let radius of the given circle be r unit.

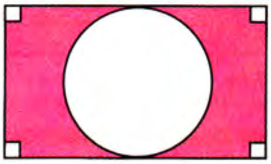
Given : $\pi r^2 = 2\pi r$ [Numerically]

$\Rightarrow r = 2$ unit

$\therefore$ **Its area** $= \pi r^2 = 3{\cdot}14 \times 2^2$ sq. unit = **12·56 sq. unit** **Ans.**

23 A rectangular sheet of paper is 35 cm long and 28 cm wide. Find the area of the largest circle that can be cut from this sheet.

Solution :



It is clear from the adjoining figure, that the diameter of the circle cut = 28 cm and so its radius = $\frac{28}{2}$ cm = 14 cm.

$\therefore$ **Area of the circle cut** $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14$ cm^2 = **616 cm^2** **Ans.**

24 Find the perimeter of a circle whose area is equal to sum of areas of the circles with diameters 10 cm and 24 cm. *Give your answer correct to two decimal places.*

Solution :

Since, the diameters of the given cirlces are 10 cm and 24 cm

$\Rightarrow$ Their radii are $\frac{10}{2}$ cm = 5 cm and $\frac{24}{2}$ cm = 12 cm

Sum of the area of these two circles

$= \pi \times 5^2$ sq. cm + $\pi \times 12^2$ sq. cm

$= \pi(25 + 144)$ sq. cm = 169π sq. cm

If the radius of the required circle = R cm, its area = πR^2 sq. cm

Given : $\pi R^2 = 169\pi \Rightarrow R^2 = 169 \Rightarrow R = 13$ cm

$\therefore$ **Required perimeter** $= 2\pi R$

$= 2 \times \frac{22}{7} \times 13$ cm = **81·71 cm** **Ans.**

25 The radii of two circles are in the ratio 5 : 8. If the difference between their areas is 351π sq. cm; find the area of the bigger circle. Take $\pi = 3{\cdot}14$.

Solution :

Let the radii of the two circles be $5x$ cm and $8x$ cm respectively.

Given : $\pi \times (8x)^2 - \pi \times (5x)^2 = 351\,\pi$

$\Rightarrow$ $64x^2 - 25x^2 = 351$

$\Rightarrow$ $39x^2 = 351$ and $x = 3$

$\because$ Radius of the bigger circle $= 8x = 8 \times 3$ cm = 24 cm

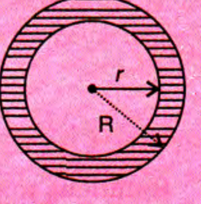
$\therefore$ **Required area** $= \pi \times (24)^2$ sq. cm

$= 3{\cdot}14 \times 24 \times 24$ sq. cm = **1808·64 sq. cm** **Ans.**

26 A uniform circular track is the area bounded by two concentric circles. If the area of the track is 1144 m^2 and its width is 14 m; find the diameters of the two circles.

Solution :

A circular running track, a circular ring, etc. have the shape as given alongside. *It is the area bounded by two concentric circles.*



Clearly, R = radius of the outer (external) circle,

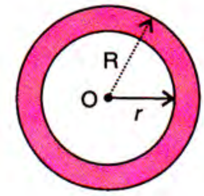
r = radius of the inner circle

and, **area bounded by two circles**

= Ext. area – Int. area

$= \pi R^2 - \pi r^2 = \boldsymbol{\pi(R^2 - r^2)}$

According to the given statement, if the radius of the outer circle = R m



then $\qquad R - r = 14$

and, $\qquad \pi(R^2 - r^2) = 1144$

$\Rightarrow \qquad \frac{22}{7}(R + r)(R - r) = 1144$

$\Rightarrow \qquad \frac{22}{7}(R + r) \times 14 = 1144$ and $R + r = 26$,

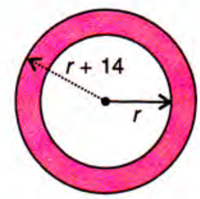
On solving $R - r = 14$ and $R + r = 26$, we get, $R = 20$ m and $r = 6$ cm

$\therefore$ Diameters of the two circles = 2R m and 2r m

= 2 × 20 m and 2 × 6 cm = **40 m** and **12 m** **Ans.**

Alternative method :

Let the radii of the two circles be r m $(r + 14)$ m



$\therefore \qquad \pi(r + 14)^2 - \pi r^2 = 1144$

$\Rightarrow \qquad \frac{22}{7}[r^2 + 28r + 196 - r^2] = 1144$

$\Rightarrow \qquad 28r + 196 = 1144 \times \frac{7}{22} = 364$

$\Rightarrow \qquad 28r = 364 - 196$ and $r = 6$

$\therefore$ Radii of the two circles = r m and $r + 14$ m

= 6 m and 20 m

$\Rightarrow$ **Diameters of the two circles** = 2 × 6 m and 2 × 20 m

= **12 m and 40 m** **Ans.**

27 The radius of the wheel of a car is 28 cm. Find the number of rotations made by the wheel in order to cover a distance of 4·4 km ?

Solution :

1. The distance covered by a wheel in 1 rotation
   = Circumference by the wheel = **C** [where $C = 2\pi r$]
2. Total distance covered by the wheel in n rotations = $2\pi r \times n$
   i.e. Distance covered = $2\pi r \times n = \mathbf{C \times n}$.

Since, radius of the wheel = 28 cm

its circumference (C) = $2\pi r$ $\qquad$ *i.e.* $C = 2\pi r$

$= 2 \times \frac{22}{7} \times 28$ cm = 176 cm

Given, distance covered = 4·4 km

= 4·4 × 1000 × 100 cm = 440000 cm.

Distance = $c \times n \Rightarrow 440000 = 176 \times n \Rightarrow n = 2500$

$\therefore$ **Rotations made by the wheel = 2500** **Ans.**

28 A circular wheel of radius 28 cm makes 300 revolutions per minute. Find the speed of the wheel in kilometre per hour.

Solution :

Since, distance covered in 1 round = circumference of the wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 28$ cm = 176 cm = 1·76 m

and, no. of revolutions made in one minute = 300

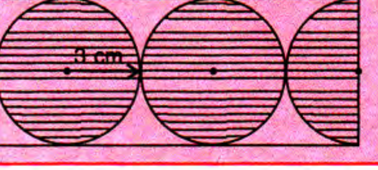
∴ Total distance covered by the wheel in one minute = 1·76 × 300 m = 528 m

Since, distance covered = 528 m and time = 1 minute *i.e.* 60 seconds

∴ **Speed** $= \frac{528}{60}$ m/s $= \frac{528}{60} \times \frac{18}{5}$ km/h

= **31·68 km/h** **Ans.**

29 In the given figure, find the area of the unshaded portion within the rectangle. (Take π = 3·14).



Solution :

The length of the given rectangle = (3 + 3 + 3 + 3 + 3) cm = 15 cm

and, its width = $2r$ = 6 cm

∴ Area of the rectangle = 15 × 6 cm² = 90 cm²

Area of the shaded portion = area of $2\frac{1}{2}$ circles each with radius 3 cm

$= \frac{5}{2} \times \pi r^2 = \frac{5}{2} \times 3{\cdot}14 \times 3^2$ cm² = 70·65 cm²

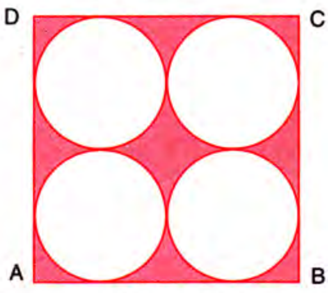
∴ **Required area of unshaded portion** = 90 cm² – 70·65 cm² = **19·35 cm²** **Ans.**

EXERCISE 20(C)

1. The diameter of a circle is 28 cm. Find its :
   (i) circumference (ii) area.
2. The circumference of a circular field is 308 m. Find its :
   (i) radius (ii) area.
3. The sum of the circumference and diameter of a circle is 116 cm. Find its radius.
4. The radii of two circles are 25 cm and 18 cm. Find the radius of the circle which has circumference equal to the sum of circumferences of these two circles.
5. The radii of two circles are 48 cm and 13 cm. Find the area of the circle which has its circumference equal to the difference of the circumferences of the given two circles.
6. The diameters of two circles are 32 cm and 24 cm. Find the radius of the circle having its area equal to sum of the areas of the two given circle.
7. The radius of a circle is 5 m. Find the circumference of the circle whose area is 49 times the area of the given circle.

8. A circle of largest area is cut from a rectangular piece of card-board with dimensions 55 cm and 42 cm. Find the ratio between the area of the circle cut and the area of the remaining card-board.

9. The following figure shows a square card-board ABCD of side 28 cm. Four identical circles of largest possible size are cut from this card as shown below.



Find the area of the remaining card-board.

10. The radii of two circles are in the ratio 3 : 8. If the difference between their areas is 2695π cm^2, find the area of the smaller circle.

11. The diameters of three circles are in the ratio 3 : 5 : 6. If the sum of the circumferences of these circles be 308 cm; find the difference between the areas of the largest and the smallest of these circles.

12. Find the area of a ring shaped region enclosed between two concentric circles of radii 20 cm and 15 cm.

13. The circumference of a given circular park is 55 m. It is surrounded by a path of uniform width 3·5 m. Find the area of the path.

14. There are two circular gardens A and B. The circumference of garden A is 1·760 km and the area of garden B is 25 times the area of garden A. Find the circumference of garden B.

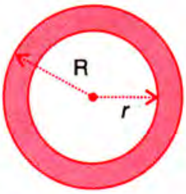
15. A wheel has diameter 84 cm. Find how many complete revolutions must it make to cover 3·168 km.

16. Each wheel of a car is of diameter 80 cm. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour ?

17. An express train is running between two stations with a uniform speed. If the diameter of each wheel of the train is 42 cm and each wheel makes 1200 revolutions per minute, find the speed of the train.

18. The minute hand of a clock is 8 cm long. Find the area swept by the minute hand between 8.30 a.m. and 9.05 a.m.

19. The shaded portion of the figure, given alongside, shows two concentric circles.



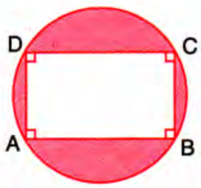
If the circumference of the two circles be 396 cm and 374 cm, find the area of the shaded portion.

20. In the figure, given above for question no. 19, the area of the shaded portion is 770 cm^2. If the circumference of the outer circle is 132 cm, find the width of the shaded portion.

21. The cost of fencing a circular field at the rate of ₹ 240 per metre is ₹ 52,800. The field is to be ploughed at the rate of ₹ 12.50 per m^2. Find the cost of ploughing the field.

22. Two circles touch each other externally. The sum of their areas is 58π cm^2 and the distance between their centres is 10 cm. Find the radii of the two circles.

23. The given figure shows a rectangle ABCD inscribed in a circle as shown alongside.



If AB = 28 cm and BC = 21 cm, find the area of the shaded portion of the given figure.

24. A square is inscribed in a circle of radius 7 cm. Find the area of the square.

25. A metal wire, when bent in the form of an equilateral triangle of largest area, encloses an area of $484\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle of largest area, find the area of this circle.

26. The diameters of the front and the rear wheels of a tractor are 63 cm and 1·54 m respectively. The rear wheel is rotating at $24\frac{6}{11}$ revolutions per minute. Find :
    (i) the revolutions per minute made by the front wheel.
    (ii) the distance travelled by the tractor in 40 minutes.

27. Two circles touch each other externally. The sum of their areas is 74π cm^2 and the distance between their centres is 12 cm. Find the diameters of the circle.

Let the radii of two circles be R cm and r cm

$\therefore \quad \pi R^2 + \pi r^2 = 74\pi$ and $R + r = 12$ cm

$\Rightarrow \quad R^2 + r^2 = 74$ and $R + r = 12$ cm

First method :

$R + r = 12 \quad \Rightarrow \quad r = 12 - R$

$\therefore \quad R^2 + r^2 = 74$ *i.e.* $(12 - r)^2 + r^2 = 74$

Solve this quadratic equation to get the value of r and then value of R.

Second method :

$(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$\Rightarrow \quad 12^2 + (R - r)^2 = 2 \times 74$

$\Rightarrow \quad (R - r)^2 = 4$

$\Rightarrow \quad R - r = 2$

Now solve $R - r = 2$ and $R + r = 12$.

28. If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.

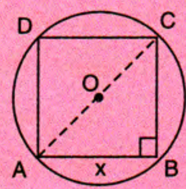
If AB = x; AC = $x\sqrt{2}$

Diameter of the circle = diagonal of the square

$\Rightarrow \quad 2r = x\sqrt{2}$

i.e. $\quad r = \dfrac{x\sqrt{2}}{2}$

Required $= \dfrac{\pi r^2}{x^2}$.



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