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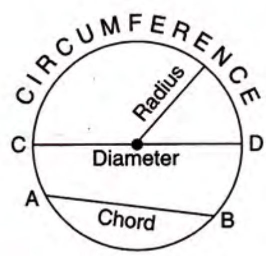
17 Circle

17.1 INTRODUCTION

Consider the shape of the wheel of a bicycle, car, etc. Each of it is said to be a **circle**.

Observe the path traced by the tip of minute-hand of a wall clock, the path traced is a **circle**.

17.2 CIRCLE



A **circle** is defined as the figure (closed curve) obtained by joining all those points in a plane which are at the same *fixed distance* from a fixed point in the same plane.

Infact, a circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point, in the same plane, always remains constant.

The *perimeter* of the circle is called its ***circumference.***

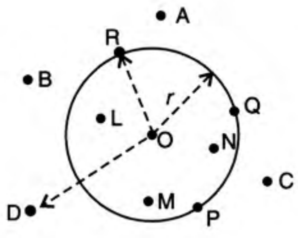
The *fixed point* is called the ***centre*** of the circle and the *fixed distance* is called the ***radius*** of the circle.

The line segment, joining any two points on the circumference of the circle, is called a ***chord***.

A chord which passes through the centre of the circle is called ***diameter***. It is the largest chord of a circle.

Also, diameter is twice the radius.

17.3 MORE ABOUT CIRCLE



1. Draw a circle with radius r and centre O as shown alongside :

   (i) ***Exterior point :*** A point lying in the same plane as that of the circle is called an exterior point, if its distance from the centre of the circle is *greater than* the radius of the circle.

In the given figure, points A, B, C and D are exterior points as each of OA, OB, OC and OD is greater than the radius of the circle.

   (ii) ***Interior point :*** A point lying in the same plane as that of the circle is called an interior point, if its distance from the centre of the circle is *less than* the radius of the circle.

In the figure, given above, points L, M and N are interior points as each of OL, OM and ON is less than the radius of the circle.

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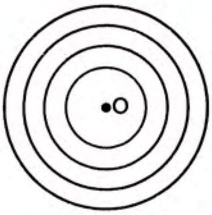
(iii) ***Point on the circumference of a circle :*** A point in the same plane as that of the circle will be on the circumference of the circle, if its distance from the centre of the circle is *equal to* the radius of the circle.

In the figure, given above, points P, Q and R are on the circumference of the circle as each of OP, OQ and OR is equal to the radius of the circle.

2. ***Concentric circles :***

Two or more circles are said to be concentric, if they have the same centre but different radii.

In the adjoining figure, O is the centre of each circle drawn; so the circles are called ***concentric circles***.

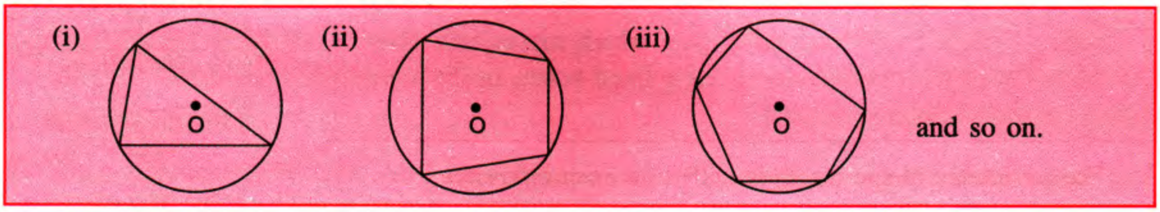


3. ***Equal circles :***

Circles are said to be equal if they have equal radii. Equal circles are also called ***congruent circles***.

4. ***Circumscribed circle :***

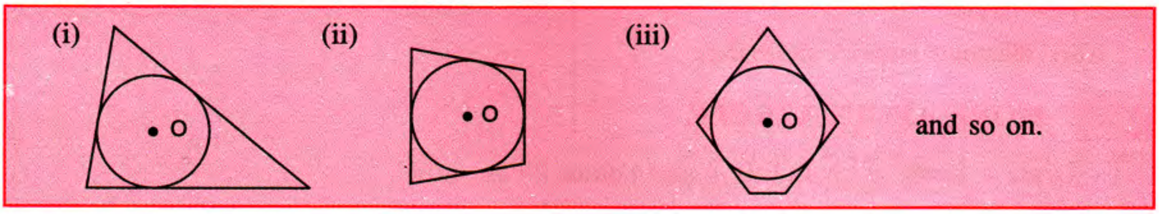
A circle that passes through all the vertices of a polygon is called a ***circumscribed circle.*** The centre of circumscribed circle is called ***circumcentre*** and the polygon is called ***incribed polygon***. Examples are shown below :



5. ***Inscribed circle :***

A circle that touches all the sides of a polygon is called an ***inscribed circle*** (or, in-circle) of the polygon.

The centre of inscribed circle is called ***incentre*** and the polygon is called ***circumscribed polygon***. Examples are shown below :

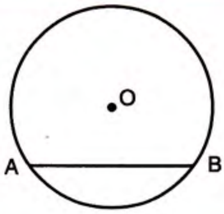


17.4 ARC, SEGMENT AND SECTOR

1. ***Arc :***

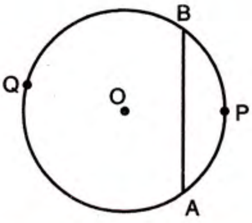
A part of the circumference of a circle is called its ***arc***.

The adjoining figure shows a chord AB of a circle with centre O.



The chord AB divides the circumference of the circle into two parts and each of these two parts is an arc.

If both the arcs are unequal in size, **smaller arc** (arc APB in the adjoining figure) is called **minor arc**, whereas the **bigger arc** (arc AQB in the adjoining figure) is called **major arc**.



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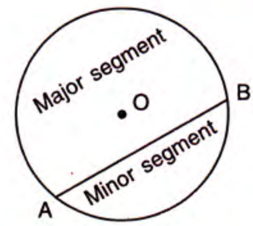
If both the arcs are equal, each is called a **semi-circle**.

1. A minor arc is always smaller than the semi-circle.
2. A major arc is always bigger than the semi-circle.
3. Unless otherwise stated, an arc stands for a minor arc.

2. ***Segment :***

*The part of the circle, bounded by an arc and a chord, is called a **segment**.*

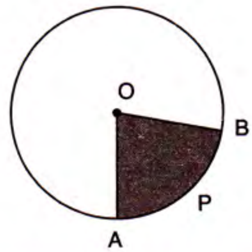


In the figure, shown alongside, chord AB divides the circle into two segments. The *smaller segment*, which is less than semi-circle, is called **minor segment** and the *bigger segment*, which is greater than semi-circle, is called **major segment**.

The centre of the circle lies in the major segment.

3. ***Sector :***

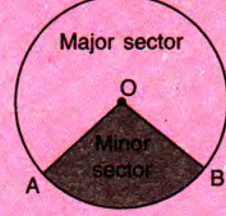
*The region bounded by an arc and two radii, joining the centre to the end points of the arc, is called a **sector**.*



The figure, given alongside, shows a circle with centre O. The arc APB, radii OA and OB form the sector AOBP.

Clearly, the region (shaded portion) bounded by arc APB and radii OA and OB is a sector.

1. The **minor arc** corresponds to the **minor sector** and **major arc** corresponds to the **major sector**.
2. When both the arcs are equal, the region enclosed by each arc alongwith its diameter is called a **semi-circle**.
3. In the case of a semi-circle, both the segments are equal and both the sectors are also equal.



17.5 CHORD PROPERTIES :

Theorem 22

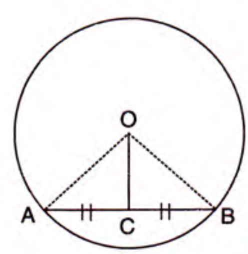
A straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is at right angles to the chord.

Given : A circle with centre O and OC bisects the chord AB.

To Prove : OC ⊥ AB.

Construction : Join OA and OB.

Proof :



Statement :	**Reason :**
1. In Δ OAC and Δ OBC;	
(i) OA = OB	Radii of the same circle
(ii) OC = OC	Common
(iii) AC = BC	OC bisects AB Given
∴ Δ OAC ≅ Δ OBC	S.S.S.

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2. And ∠OCA = ∠OCB	Corr. ∠s of congruent Δs are congruent.
3. But, ∠OCA + ∠OCB = 180°	ACB is a straight line.
∴ ∠OCA = ∠OCB = 90°	From (2) and (3).
⇒ **OC ⊥ AB.**	

Hence Proved

Theorem 23

(Converse of Theorem 22)

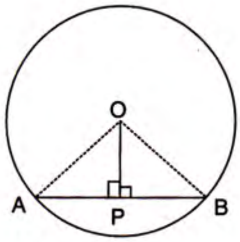
The perpendicular to a chord, from the centre of the circle, bisects the chord.

Given : A circle with centre O and OP is perpendicular to the chord AB.

To Prove : AP = BP.

Construction : Join OA and OB.

Proof :



Statement :	Reason :
In Δ OAP and Δ OBP;	
(i) OA = OB	Radii of the same circle.
(ii) OP = OP	Common
(iii) ∠OPA = ∠OPB = 90°	OP ⊥ AB Given
∴ Δ OAP ≅ Δ OBP	R.H.S.
⇒ **AP = BP**	Corr. parts of congruent Δs are congruent.

Hence Proved

Remember :

Greater is the size of a chord, smaller is its distance from the centre and vice-versa.

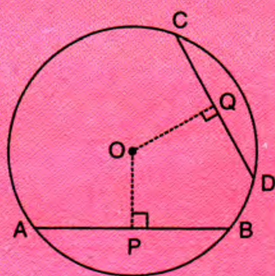
The adjoining figure shows a circle with centre O. OP is perpendicular to chord AB and OQ is perpendicular to chord CD.

Since, chord AB is greater than chord CD

⇒ AB is at smaller distance from the centre as compared to CD

i.e. OP < OQ.

Conversely, as OP < OQ ⇒ AB > CD.



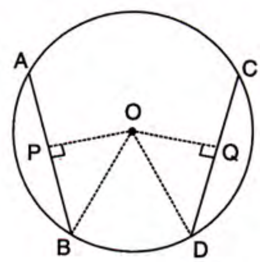
Theorem 24

Equal chords of a circle are equidistant from the centre.

Given : A circle with centre O in which chord AB = chord CD.

To Prove : Chords AB and CD are equidistant from the centre *i.e.* if OP ⊥ AB and OQ ⊥ CD, then to prove that OP = OQ.

Construction : Join OB and OD.



Proof :

Statement :	Reason :
1. BP = $\frac{1}{2}$AB	Perpendicular from the centre bisects the chord.

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2.		$DQ = \frac{1}{2}CD$	Perpendicular from the centre bisects the chord.
3.	But;	$AB = CD$	Given
4.	$\therefore$	$BP = DQ$	From (1), (2) and (3).
5.	In Δ OPB and Δ OQD,		
	(i)	$BP = DQ$	From (4)
	(ii)	$OB = OD$	Radii of the same circle
	(iii)	$\angle OPB = \angle OQD = 90°$	$OP \perp AB$ and $OQ \perp CD$
	$\therefore$	$\Delta OPB \cong \Delta OQD$	R.H.S.
	$\Rightarrow$	**$OP = OQ$**	Corresponding parts of congruent Δs

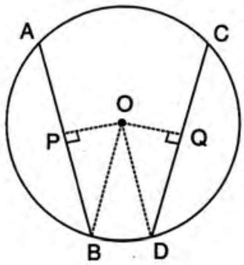
Hence Proved

Theorem 25

(Converse of Theorem 24)

Chords of a circle, equidistant from the centre of the circle, are equal.

Given : A circle with centre O in which chords AB and CD are equidistant from the centre.
i.e. if $OP \perp AB$ and $OQ \perp CD$, then $OP = OQ$.



To Prove : Chord AB = chord CD.

Construction : Join OB and OD.

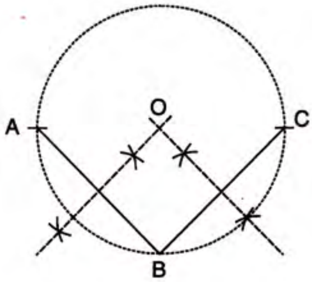
Proof :

Statement :			**Reason :**
1.	In Δ OPB and Δ OQD,		
	(i)	$OP = OQ$	Given
	(ii)	$OB = OD$	Radii of the same circle
	(iii)	$\angle OPB = \angle OQD = 90°$	$OP \perp AB$ and $OQ \perp CD$
	$\therefore$	$\Delta OPB \cong \Delta OQD$	R.H.S.
2.	$\therefore$	$PB = QD$	Corr. parts of congruent Δs
	$\Rightarrow$	$\frac{1}{2}AB = \frac{1}{2}CD$	$\perp$ from centre bisects the chord.
	$\Rightarrow$	**Chord AB = Chord CD**	

Hence Proved

Theorem 26

There is one and only one circle, which passes through three given points not in a straight line.



Given : Three points A, B and C, which are not in a straight line.

To Prove : One and only one circle can be drawn through A, B and C.

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Construction : Join AB and BC. Draw perpendicular bisectors of AB and BC. Let these perpendicular bisectors meet at point O.

Proof :

Statement :	**Reason :**
1. Since, O lies on perpendicular bisector of AB $\therefore$ OA = OB	Each point of the $\perp$ bisector is equidistant from the extremities of the line.
2. Since, O lies on $\perp$ bisector of BC $\therefore$ OB = OC	Each point of the $\perp$ bisector is equidistant from the extremities of the line.
3. OA = OB = OC	From (1) and (2)

$\Rightarrow$ O is equidistant from A, B and C.

$\Rightarrow$ If a circle is drawn with O as centre and OA as radius, the circle will pass through B and C also.

Since, perpendicular bisectors of AB and BC cut each other at point O only.

$\Rightarrow$ O is the only point equidistant from A, B and C.

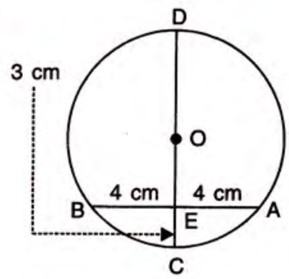
Therefore, **one and only one circle can be drawn through points A, B and C not in a straight line.**

Hence Proved

It must be noted here that :

1. Perpendicular bisector of every chord of a circle always passes through its centre.
2. Perpendicular bisectors of any two chords of a circle always intersect at the centre of the circle.

1 In the figure, given alongside, CD is a diameter which meets the chord AB at E, such that AE = BE = 4 cm. If CE is 3 cm, find the radius of the circle.



Solution :

Let radius of the circle be r cm,

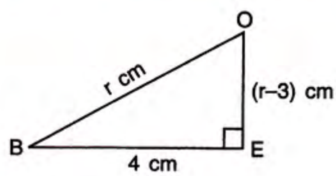
i.e. OB = OC = r cm.

$\therefore$ OE = $(r - 3)$ cm.

Also, as E is mid-point of chord AB,

$\angle OEB = 90°$ Line, joining the centre and mid-point of the chord, is $\perp$ to the chord.

Now in right triangle OEB,



$$OB^2 = OE^2 + BE^2$$

$$\Rightarrow \quad r^2 = (r-3)^2 + 4^2$$

On solving we get : $r = 4\frac{1}{6}$ cm **Ans.**

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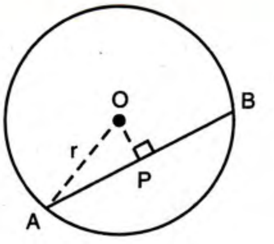
2 A chord of length 48 cm is at a distance of 10 cm from the centre of the circle. Another chord of length 20 cm is drawn in the same circle, find its distance from the centre of the circle.

Solution :



Let AB be the given chord of the circle with centre O and OP is perpendicular to AB.

Clearly, AB = 48 cm and OP = 10 cm

Since, perpendicular from the centre bisects the chord

$\therefore \quad AP = BP = \frac{1}{2} AB = \frac{1}{2} \times 48 \text{ cm} = 24 \text{ cm}$

Let radius of the circle be r cm $\Rightarrow$ OA = r cm

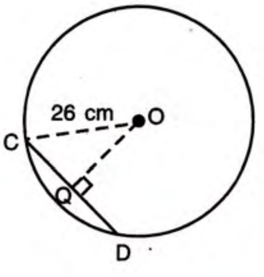
In right-angled triangle OAP,

$OA^2 = OP^2 + AP^2 \quad \Rightarrow \quad r^2 = 10^2 + 24^2 = 100 + 576 = 676$

$\Rightarrow \quad r = \sqrt{676} = 26$

$\therefore$ Radius of the circle = 26 cm

Let another chord of the circle be CD such that CD = 20 cm



If $OQ \perp CD \Rightarrow CQ = QD = \frac{20}{2}$ cm = 10 cm

Clearly, OC = radius of the circle = 26 cm

Required to find the distance of chord from the centre of the circle *i.e.* length of OQ.

In right-angled triangle OCQ,

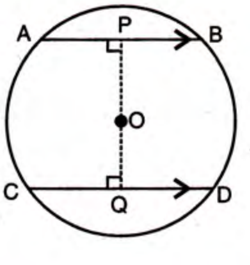
$OQ^2 + CQ^2 = OC^2 \quad \Rightarrow OQ^2 = 26^2 - 10^2 = 676 - 100 = 576$

$\Rightarrow \quad OQ = \sqrt{576} = 24$

$\therefore$ **Distance of the chord from the centre of the circle = 24 cm** **Ans.**

3 Chords AB and CD of a circle are parallel to each other and lie on opposite sides of the centre of the circle. If AB = 36 cm, CD = 48 cm and the distance between the chords is 42 cm; find the radius of the circle.

Solution :



According to the given statement, the figure will be as shown alongside :

Since; $OP \perp AB$, $OQ \perp CD$ and AB // CD ;

it can be shown that POQ is a straight line.

Clearly; AB = 36 cm, CD = 48 cm and PQ = 42 cm

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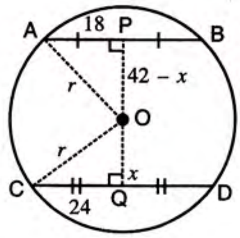
$\Rightarrow$ $AP = PB = \frac{1}{2}AB = 18$ cm, $CQ = DQ = \frac{1}{2}CD = 24$ cm



and if $OQ = x$ cm, $OP = (42 - x)$ cm

Join OA and OC.

$OA = OC = r$ (radius of the circle)

In right-angled Δ OAP,

$OA^2 = OP^2 + AP^2 \Rightarrow r^2 = (42 - x)^2 + 18^2$ I

In right-angled Δ OCQ,

$OC^2 = OQ^2 + CQ \Rightarrow r^2 = x^2 + 24^2$ II

From equations I and II, we get :

$$(42 - x)^2 + 18^2 = x^2 + 24^2$$

$$\Rightarrow \quad 1764 - 84x + x^2 + 324 = x^2 + 576$$

$$\Rightarrow \quad 84x = 1512 \text{ and } x = 18$$

$$\therefore \quad r^2 = x^2 + 24^2$$

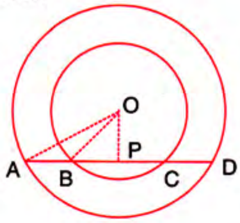
$$\Rightarrow \quad r^2 = 18^2 + 24^2 = 324 + 576 = 900$$

$$\Rightarrow \quad r = \sqrt{900} = 30$$

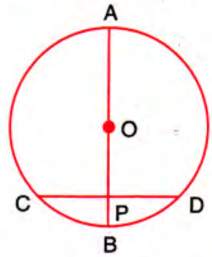
$\Rightarrow$ **Radius of the circle = 30 cm** **Ans.**

EXERCISE 17(A)

1. A chord of length 6 cm is drawn in a circle of radius 5 cm. Calculate its distance from the centre of the circle.
2. A chord of length 8 cm is drawn at a distance of 3 cm from the centre of a circle. Calculate the radius of the circle.
3. The radius of a circle is 17·0 cm and the length of perpendicular drawn from its centre to a chord is 8·0 cm. Calculate the length of the chord.
4. A chord of length 24 cm is at a distance of 5 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 12 cm from the centre.
5. In the following figure, AD is a straight line. $OP \perp AD$ and O is the centre of both the circles. If OA = 34 cm, OB = 20 cm and OP = 16 cm; find the length of AB.



6. In a circle of radius 17 cm, two parallel chords of lengths 30 cm and 16 cm are drawn. Find the distance between the chords, if both the chords are :
   (i) on the opposite sides of the centre,
   (ii) on the same side of the centre.
7. Two parallel chords are drawn in a circle of diameter 30·0 cm. The length of one chord is 24·0 cm and the distance between the two chords is 21·0 cm; find the length of the other chord.
8. A chord CD of a circle, whose centre is O, is bisected at P by a diameter AB.



Given OA = OB = 15 cm and OP = 9 cm. Calculate the lengths of :
(i) CD (ii) AD (iii) CB.

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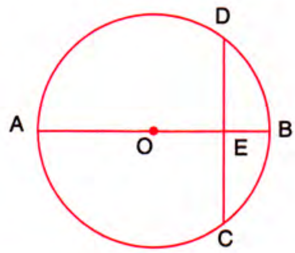
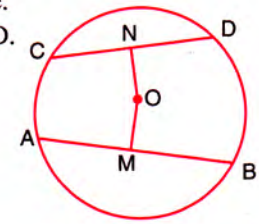
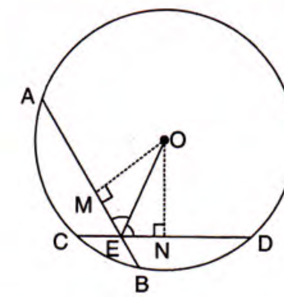
9. The figure, given below, shows a circle with centre O in which diameter AB bisects the chord CD at point E. If CE = ED = 8 cm and EB = 4 cm, find the radius of the circle.



10. In the given figure, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the :
(i) radius of the circle.
(ii) length of chord CD.



4 Chords AB and CD of a circle with centre O, intersect at a point E. If OE bisects angle AED, prove that chord AB = chord CD.



Solution :

Draw OM ⊥ AB and ON ⊥ CD.

In Δ OEM and Δ OEN,

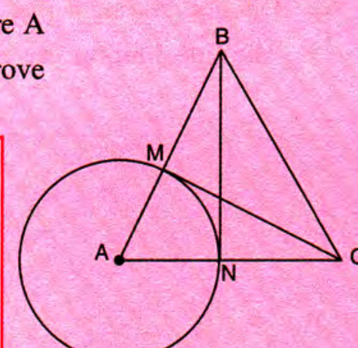
∠OME = ∠ONE = 90°	OM ⊥ AB and ON ⊥ CD
∠OEM = ∠OEN	Given, OE bisects ∠AED
∴ ∠MOE = ∠NOE	When two angles of two Δs are equal, each to each, their third angles are also equal
Also, OE = OE	Common
∴ Δ OEM ≅ Δ OEN	A.S.A.
⇒ OM = ON	Corresponding parts of congruent Δs

⇒ Chords AB and CD are equidistant from the centre.

∴ **Chord AB = Chord CD.** — Chords equidistant from the centre are equal

Hence Proved

5 ABC is an equilateral triangle. A circle is drawn with centre A so that it cuts AB and AC at points M and N respectively. Prove that BN = CM



Proof :

1. AB = AC [Sides of the equilateral Δ ABC]
2. AM = AN [Radii of the same circle]

∴ AB – AM = AC – AN

⇒ BM = CN

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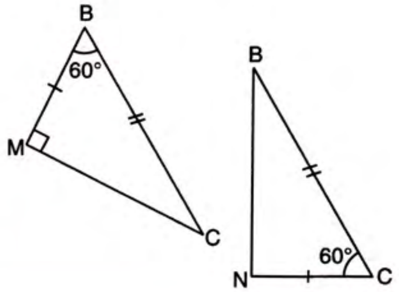
Now in Δ BMC and Δ CNB



BM = CN (Proved above)

BC = BC (Common)

and, ∠MBC = ∠BCN = 60° [Each angle of an equilateral Δ is 60°]

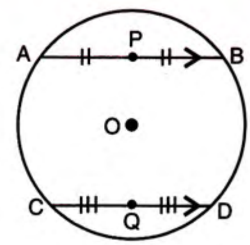
∴ Δ BMC ≅ Δ CNB [By SAS]

⇒ **BN = CM** [By C.P.C.T.C.]

Hence Proved.

6 The line segment joining the mid-points of two parallel chords of a circle passes through the centre. Prove it.

Solution :



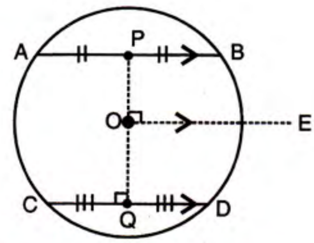
Let AB and CD be two parallel chords of a circle with centre O. P is mid-point of AB and Q is mid-point of CD.

We have to prove that the line joining the points P and Q passes through the centre O. *i.e.* ∠POQ = 180°.

Construction

Join OP and OQ. Also, draw OE parallel to AB and CD.

Proof :



Since, line segment joining the mid-point of the chord with the centre of the circle is perpendicular to the chord, therefore OP⊥AB and OQ⊥CD

⇒ ∠OPA = 90° and ∠OQC = 90°

Now, OE // AB and OP is transversal

∴ ∠POE = ∠OPA = 90° Alternate angles

Similarly, OE // CD and OQ is transversal

∴ ∠QOE = ∠OQC = 90° Alternate angles

∴ ∠POQ = ∠POE + ∠QOE

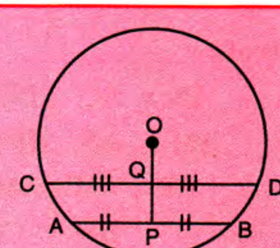
= 90° + 90° = 180°

⇒ POQ is a straight line

i.e. **the line joining the mid-points of two parallel chords passes through the centre.**

Hence Proved.

If the parallel chords are on the same side of the centre, then also the line through the mid-points of the chords passes through the centre.



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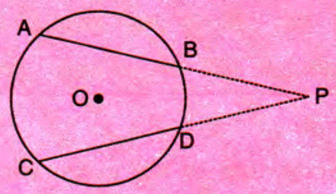
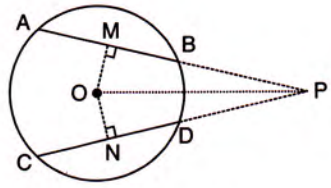
7 AB and CD are two equal chords of a circle with centre O. If AB and CD, on being produced, meet at a point P outside the circle, prove that :

(a) PA = PC (b) PB = PD



Solution :



Draw OM ⊥ AB, ON ⊥ CD and join OP.

Since, perpendicular from the centre bisects the chord.

$\therefore$ AM = BM = $\frac{1}{2}$ AB and CN = DN = $\frac{1}{2}$ CD

$\Rightarrow$ AM = BM = CN = DN ....I [$\because$ AB = CD]

In Δ OMP and Δ ONP,

(i) OM = ON [Equal chords are equidistant from the centre]

(ii) ∠OMP = ∠ONP [Each 90°]

(iii) OP = OP [Common]

$\therefore$ Δ OMP ≅ Δ ONP [By R.H.S.]

$\Rightarrow$ MP = NP ....II [By C.P.C.T.C.]

(a) **To prove :** PA = PB

$\because$ MP = NP [From equation II]

and, AM = CN [From equation I]

$\therefore$ AM + MP = CN + NP

$\Rightarrow$ **PA = PC**

Hence Proved.

(b) **To prove :** PB = PD

$\because$ MP = NP [From equation II]

and, BM = DN [From equation I]

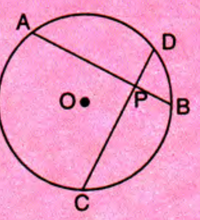
$\therefore$ MP – BM = NP – DN

$\Rightarrow$ **PB = PD**

Hence Proved.

If the two equal chords AB and CD meet at point P, inside the circle, then also we have :

(a) PA = PC and (b) PB = PD



In other words, if two equal chords of a circle intersect each other at a point, inside or outside the circle, the corresponding parts of the chords are equal.

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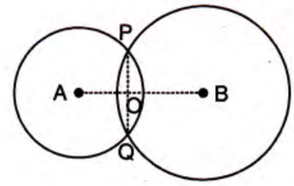
8 Two circles with centres A and B intersect each other at points P and Q. Prove that the centre-line AB bisects the common chord PQ perpendicularly.

Solution :

According to the given statement, the figure will be as shown alongside :

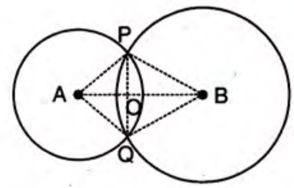


We have to prove that AB bisects common chord PQ perpendicularly *i.e.* OP = OQ and ∠AOP = 90°

Join PA, PB, QA and QB.

Now show that ΔPAB and ΔQAB are congruent by S.S.S.

⇒ ∠PAB = ∠QAB [By C.P.C.T.C.]



Now, in ΔPAO and ΔQAO,

(i) ∠PAO = ∠QAO [Proved above]

(ii) AP = AQ [Radii of the same circle]

(iii) OA = OA [Common]

∴ ΔPAO ≡ ΔQAO [By S.A.S.]

⇒ OP = OQ and ∠AOP = ∠AOQ

But, ∠AOP + ∠AOQ = 180°

⇒ ∠AOP = ∠AOQ = 90°

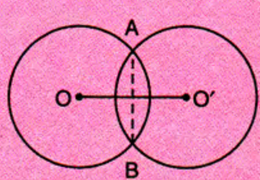
OP = OQ and ∠AOP = ∠AOQ = 90°

⇒ **AB bisects PQ perpendicularly**

Hence Proved.

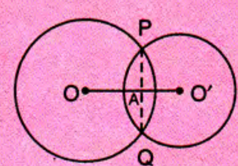
Whenever two circles, equal or unequal, intersect each other, the line joining their centres bisect the common chord.

(i)



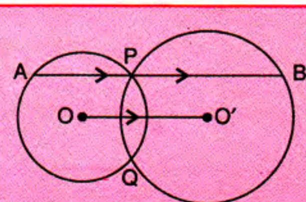
[OO′ is perpendicular bisector of AB]

(ii)



[AP = AQ and ∠OAP = 90°]

9 Two circles with centres O and O′ intersect each other at points P and Q. The straight line APB is parallel to centre-line OO′. Prove that : $OO' = \frac{1}{2}AB$.

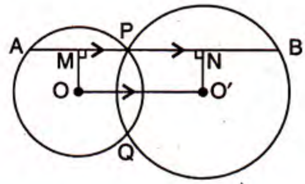


Solution :

Draw OM ⊥ AP and O′N ⊥ PB.

Show that quadrilateral OO′ NM is a rectangle as each angle of it is 90°.

Since, the opposite sides of a rectangle are equal, therefore OO′ = MN I



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We know, that the perpendicular from the centre bisects the chord, therefore :

$OM \perp AP \Rightarrow AM = PM = \frac{1}{2}AP$ and

$O'N \perp PB \Rightarrow BN = PN = \frac{1}{2}BP$

Now $\quad OO' = MN$

$= PM + PN$

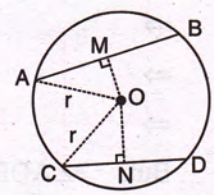
$= \frac{1}{2}AP + \frac{1}{2}BP$

$= \frac{1}{2}(AP + BP) = \frac{1}{2}AB$

Hence Proved.

10 Out of two unequal chords of a circle, the bigger chord is closer to the centre of the circle. Prove it.

Solution :



The given figure shows two chords AB and CD of a circle with centre O and radius r such that chord AB is bigger than chord CD. Then to prove that AB is closer to the centre *i.e.* if $OM \perp AB$ and $ON \perp CD$, we have to prove that OM is smaller than ON ($OM < ON$).

In right-angled ΔOAM, $AM^2 = OA^2 - OM^2 = r^2 - OM^2$ I

And, in right-angled ΔOCN, $CN^2 = OC^2 - ON^2 = r^2 - ON^2$ II

Since, perpendicular from the centre bisects the chord

$\therefore AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$

Given $AB > CD \Rightarrow AM > CN$

$\Rightarrow AM^2 > CN^2$

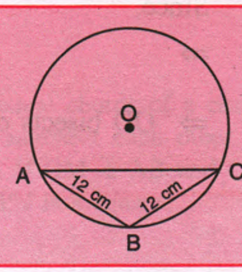
$\Rightarrow r^2 - OM^2 > r^2 - ON^2$ [From equations I and II]

$\Rightarrow ON^2 > OM^2$

$\Rightarrow ON > OM$ *i.e.* $OM < ON$

Hence Proved.

11 Three boys A, B and C are standing on the circumference of a circle with radius 10 cm and centre at point O. If the distance between A and B = distance between B and C = 12 cm; find the distance between A and C.



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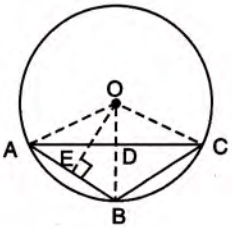
Solution :



Join OA, OB and OC. Let OB meet AC at point D. Also, draw OE ⊥ AB.

Since, perpendicular from the centre bisects the chord,

$$\therefore\ AE = EB = \frac{1}{2}AB = \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$$

In right angled triangle AOE,

$$OE^2 = OA^2 - AE^2$$

$$= 10^2 - 6^2 = 100 - 36 = 64 \ \Rightarrow\ OE = 8 \text{ cm}$$

$$\therefore\quad \text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OE$$

$$= \frac{1}{2} \times 12 \times 8 \text{ cm}^2 = 48 \text{ cm}^2$$

Since, $\Delta AOB \cong \Delta BOC$ [By SSS]

$\Rightarrow\ \angle OBA = \angle OBC$ [By CPCTC]

$\Rightarrow\ \Delta ADB \cong \Delta CDB$ [By SAS]

$\Rightarrow\ \angle ADB = \angle CDB$ [By CPCTC]

But, $\angle ADB + \angle CDB = 180°$

$$\Rightarrow\ \angle ADB = \angle CDB = \frac{180°}{2} = 90°$$

$\Rightarrow\ AD \perp OB$

$$\text{Now, area of } \Delta\, AOB = \frac{1}{2} \times OB \times AD$$

$$= \frac{1}{2} \times 10 \text{ cm} \times AD = 5 \text{ cm} \times AD$$

Since, area of Δ ADB = 48 cm² [Shown above]

$$\Rightarrow\ 5 \times AD = 48 \quad \textit{i.e.}\ AD = \frac{48}{5} \text{ cm} = 9{\cdot}6 \text{ cm}$$

and **AC** $= 2 \times AD$

$= 2 \times 9{\cdot}6$ cm = **19·2 cm** **Ans.**

Since, $\angle ADB = 90°$ [Proved above]

$\Rightarrow\ OD \perp AC$

$\Rightarrow$ OD bisects chord AC *i.e.* $AD = \frac{1}{2}AC$ and $AC = 2 \times AD$

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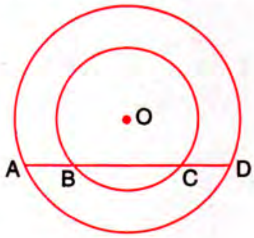
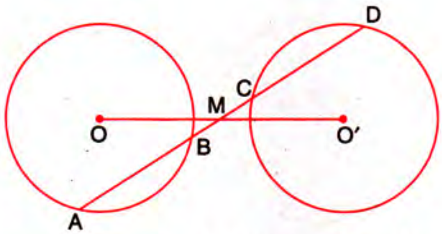
EXERCISE 17(B)

1. The figure shows two concentric circles and AD is a chord of larger circle.



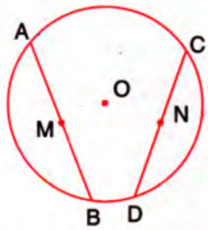
Prove that : AB = CD.

2. A straight line is drawn cutting two equal circles and passing through the mid-point M of the line joining their centres O and O′.



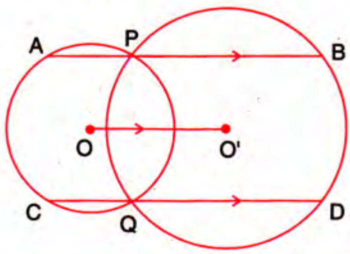
Prove that the chords AB and CD, which are intercepted by the two circles, are equal.

3. M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. Prove that :
   (i) ∠BMN = ∠DNM, (ii) ∠AMN = ∠CNM.

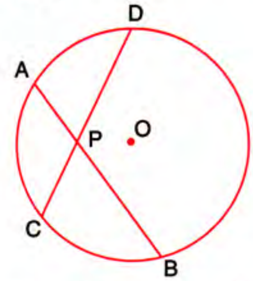


4. In the following figure; P and Q are the points of intersection of two circles with centres O and O′. If straight lines APB and CQD are parallel to OO′; prove that :

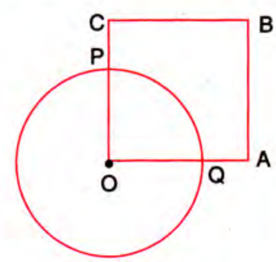
   (i) $OO' = \frac{1}{2} AB$, (ii) AB = CD



5. Two equal chords AB and CD of a circle with centre O, intersect each other at point P inside the circle. Prove that :
   (i) AP = CP,
   (ii) BP = DP



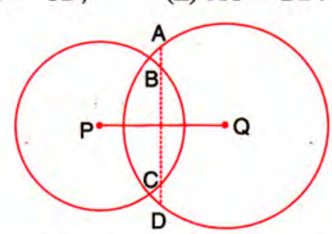
6. In the following figure, OABC is a square. A circle is drawn with O as centre which meets OC at P and OA at Q. Prove that :
   (i) Δ OPA ≅ Δ OQC, (ii) Δ BPC ≅ Δ BQA.



7. The length of common chord of two intersecting circles is 30 cm. If the diameters of these two circles be 50 cm and 34 cm, calculate the distance between their centres.

8. The line joining the mid-points of two chords of a circle passes through its centre. Prove that the chords are parallel.

9. In the following figure, the line ABCD is perpendicular to PQ; where P and Q are the centres of the circles. Show that :
   (i) AB = CD, (ii) AC = BD.



10. AB and CD are two equal chords of a circle with centre O which intersect each other at right angle at point P. If OM ⊥ AB and ON ⊥ CD; show that OMPN is a square.

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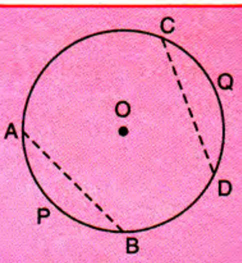
17.6 ARC AND CHORD PROPERTIES

Equal arcs cut equal chords and equal chords cut equal arcs :



1. In a circle, if two arcs are equal, they cut equal chords. Conversely, if two chords of a circle are equal, they cut equal arcs.

   In the given figure,

   arc APB = arc CQD ⇒ chord AB = chord CD

   and chord AB = chord CD ⇒ arc APB = arc CQD

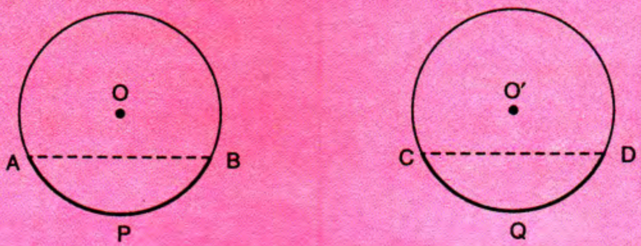
2. If arcs of two equal (congruent) circles are equal, they cut equal chords.

   Conversely, if chords of two equal (congruent) circles are equal, they cut equal arcs.

   Therefore, in the following figure,

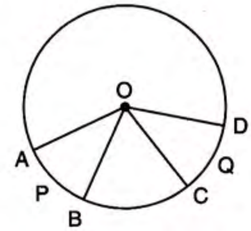
   if arc APB = arc CQD ⇒ chord AB = chord CD

   and, if chord AB = chord CD ⇒ arc APB = arc CQD



Theorem 27

If two arcs of the same circle subtend equal angles at the centre, they are equal.



Given : A circle with centre O. Arcs APB and CQD of this circle subtend equal angles at centre O.

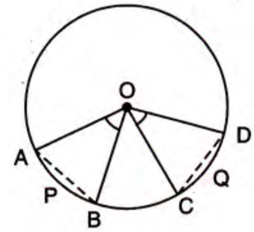
i.e. ∠AOB = ∠COD

To Prove : arc APB = arc CQD

Construction : Draw chords AB and CD

Proof :

In Δ AOB and Δ COD,



1.	OA = OC	[Radii of the same circle]
2.	OB = OD	[Radii of the same circle]
3.	∠AOB = ∠COD	[Given]
∴	Δ AOB = Δ COD	[By SAS]
⇒	Chord AB = Chord CD	[By CPCTC]
⇒	**arc APB = arc CQD**	[Equal chords of a circle cut equal arcs]

Hence Proved.

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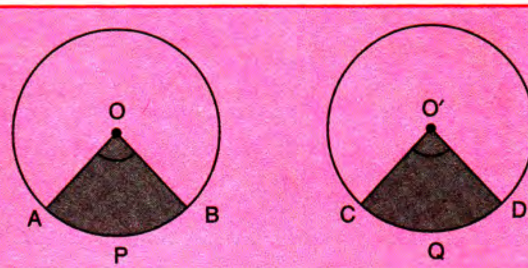
Similarly, in two equal (congruent) circles with centres O and O′, if arcs APB and CQD subtend equal angles at their centres, then these arcs are equal.

i.e. ∠AOB = ∠CO′D

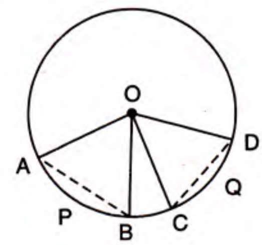
⇒ arc APB = arc CQD.



Theorem 28

[Converse of theorem 27]

If two arcs of a circle are equal, they subtend equal angles at the centre.



Given : A circle with centre O. Equal arcs APB and CQD subtend angles AOB and COD at the centre.

To Prove : ∠AOB = ∠COD

Construction : Draw chords AB and CD.

Proof :

Since, equal arcs of a circle cut equal chords

∴ arc APB = arc CQD ⇒ chord AB = chord CD

In Δ AOB and Δ COD,

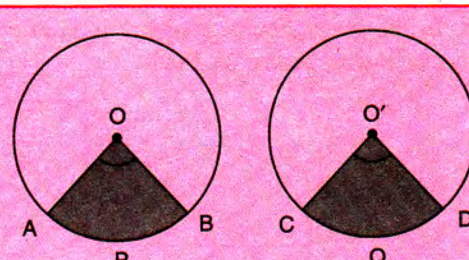
1.	OA = OC	[Radii of the same circle]
2.	OB = OD	[Radii of the same circle]
3.	AB = CD	[Proved above]
⇒	Δ AOB ≡ Δ COD	[By SSS]
⇒	**∠AOB = ∠COD**	[By CPCTC]

Hence Proved.

Similarly, in two equal (congruent) circles with centres O and O′, if arc APB and arc CQD are equal, they subtend equal angles at the centre.

In the given figure; if arc APB = arc CQD

⇒ ∠AOB = ∠CO′D.



Important :

1. In a circle with centre O, for arcs APB and CQD, if
   (i) arc APB = arc CQD ⇒ ∠AOB = ∠COD
   (ii) arc APB = 2 × arc CQD ⇒ ∠AOB = 2∠COD
   (iii) arc APB : arc CQD = 5 : 7 ⇒ ∠AOB : ∠COD = 5 : 7
2. If an n-sided regular polygon is inscribed in a circle, then the angle subtended by each side (arm) of this polygon at the centre of the circle = $\frac{360°}{n}$.

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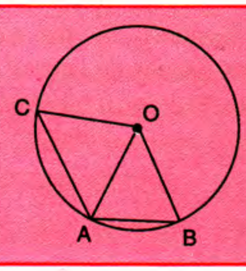
12 In the given figure, O is the centre of a circle, AB is a side of regular octagon and AC is a side of regular hexagon. Find :

(i) ∠AOB (ii) ∠AOC (iii) ∠BOC.



Solution :

(i) Since, AB is a side of regular octagon,

$$\angle AOB = \frac{360^\circ}{8} = 45^\circ \quad \textbf{Ans.}$$

(ii) Since, AC is a side of regular hexagon,

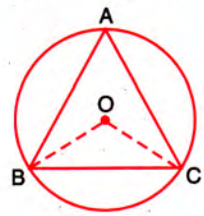
$$\angle AOC = \frac{360^\circ}{6} = 60^\circ \quad \textbf{Ans.}$$

(iii) $\angle BOC = \angle AOB + \angle AOC$

$= 45^\circ + 60^\circ = 105^\circ$ **Ans.**

EXERCISE 17(C)

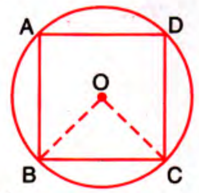
1. In the given figure, an equilateral triangle ABC is inscribed in a circle with centre O. Find : (i) ∠BOC (ii) ∠OBC



2. In the given figure, a square is inscribed in a circle with centre O. Find :

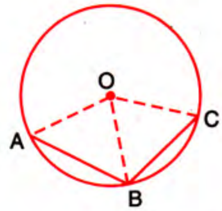
(i) ∠BOC
(ii) ∠OCB
(iii) ∠COD
(iv) ∠BOD

Is BD a diameter of the circle ?



3. In the given figure, AB is a side of regular pentagon and BC is a side of regular hexagon.

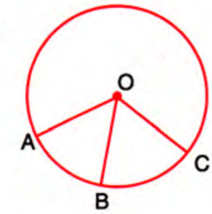
(i) ∠AOB
(ii) ∠BOC
(iii) ∠AOC
(iv) ∠OBA
(v) ∠OBC
(vi) ∠ABC



4. In the given figure, arc AB and arc BC are equal in length.

If ∠AOB = 48°, find:

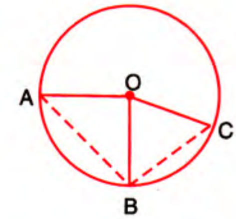
(i) ∠BOC
(ii) ∠OBC
(iii) ∠AOC
(iv) ∠OAC



5. In the given figure, the lengths of arcs AB and BC are in the ratio 3 : 2.

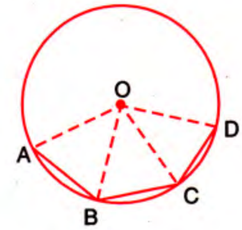
If ∠AOB = 96°, find:

(i) ∠BOC
(ii) ∠ABC



6. In the given figure, AB = BC = DC and ∠AOB = 50°.

(i) ∠AOC
(ii) ∠AOD
(iii) ∠BOD
(iv) ∠OAC
(v) ∠ODA



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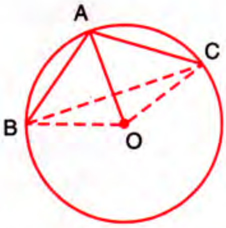
7. In the given figure, AB is a side of a regular hexagon and AC is a side of a regular eight sided polygon. Find :

(i) ∠AOB

(ii) ∠AOC

(iii) ∠BOC

(iv) ∠OBC

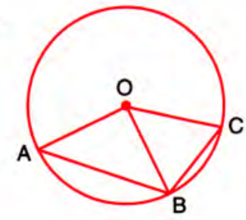


8. In the given figure, O is the centre of the circle and the length of arc AB is twice the length of arc BC.

If ∠AOB = 100°, find:

(i) ∠BOC

(ii) ∠OAC



EXERCISE 17(D)

1. The radius of a circle is 13 cm and the length of one of its chords is 24 cm. Find the distance of the chord from the centres.

2. Prove that equal chords of congruent circles subtend equal angles at their centre.

3. Draw two circles of different radii. How many points these circles can have in common? What is the maximum number of common points ?

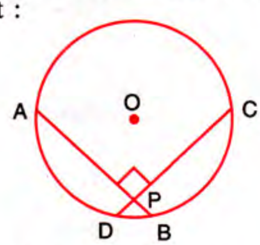
4. Suppose you are given a circle. Describe a method by which you can find the centre of this circle.

Draw any two chords of the given circle and then draw their perpendicular bisectors. The point of intersection of these perpendicular bisectors is the required centre of the given circle.

5. Given two equal chords AB and CD of a circle, with centre O, intersecting each other at point P. Prove that :

(i) AP = CP

(ii) BP = DP



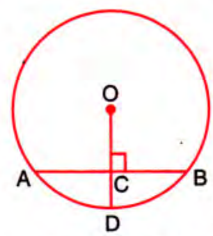
6. In a cricle of radius 10 cm, AB and CD are two parallel chords of lengths 16 cm and 12 cm respectively. Calculate the distance between the chords, if they are on :

(i) the same side of the centre.

(ii) the opposite sides of the centre.

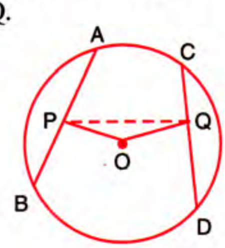
7. In the given figure, O is the centre of the circle with radius 20 cm and OD is perpendicular to AB.

If AB = 32 cm, find the length of CD.

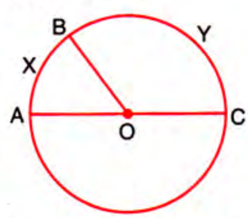


8. In the given figure, AB and CD are two equal chords of a circle, with centre O.

If P is the mid-point of chord AB, Q is the mid-point of chord CD and ∠POQ = 150°, find ∠APQ.



9. In the given figure, AOC is the diameter of the circle, with centre O.



If arc AXB is half of arc BYC, find ∠BOC.

10. The circumference of a circle, with centre O, is divided into three arcs APB, BQC and CRA such that :

$$\frac{\text{arc APB}}{2} = \frac{\text{arc BQC}}{3} = \frac{\text{arc CRA}}{4}$$

Find ∠BOC.

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