UNIT 5 : TRIGONOMETRY

22

Trigonometrical Ratios

POINTS TO REMEMBER

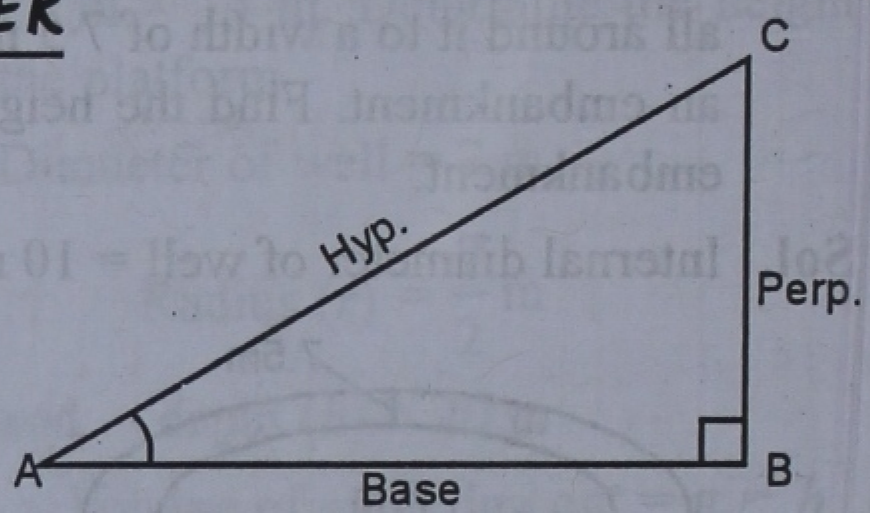
1. Trigonometrical Ratios (T-Ratios) of An Angle

In ΔABC, let $\angle B = 90°$ and let $\angle A$ be acute.

For $\angle A$, we have

Base = AB, **Perp.** = BC and **Hyp.** = AC.

*The **T-ratios** for $\angle A$ are defined as :*



(*i*) **Sine A** = $\frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC}$, written as **sin A.**

(*ii*) **Cosine A** = $\frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC}$, written as **cos A.**

(*iii*) **Tangent A** = $\frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB}$, written as **tan A.**

(*iv*) **Cosecant A** = $\frac{\text{Hyp.}}{\text{Perp.}} = \frac{AC}{BC}$, written as **cosec A.**

(*v*) **Secant A** = $\frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{AB}$, written as **sec A.**

(*vi*) **Cotangent A** = $\frac{\text{Base}}{\text{Perp.}} = \frac{AB}{BC}$, written as **cot A.**

2. Reciprocal Relations :

(*i*) $\text{cosec } A = \frac{1}{\sin A}$ (*ii*) $\sec A = \frac{1}{\cos A}$ (*iii*) $\cot A = \frac{1}{\tan A}$

Thus, we have :

(*i*) **sin A cosec A = 1** (*ii*) **cos A sec A = 1** (*iii*) **tan A cot A = 1.**

3. Quotient Relations in T-Ratios

(*i*) $\frac{\sin\theta}{\cos\theta} = \tan\theta$ (*ii*) $\frac{\cos\theta}{\sin\theta} = \cot\theta$

4. **Table for T-Ratios of some standard angles :**

θ	sin θ	cos θ	tan θ	cosec θ	sec θ	cot θ
0°	0	1	0	not defined	1	not defined
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	not defined	1	not defined	0

5. **T-Ratios of complementary angles :**

(*i*) $\sin(90^\circ - \theta) = \cos\theta$

(*ii*) $\cos(90^\circ - \theta) = \sin\theta$

(*iii*) $\tan(90^\circ - \theta) = \cot\theta$

(*iv*) $\cot(90^\circ - \theta) = \tan\theta$

(*v*) $\text{cosec}(90^\circ - \theta) = \sec\theta$

(*vi*) $\sec(90^\circ - \theta) = \text{cosec}\,\theta$

6. $\sqrt{2} = 1{\cdot}414$ or $1{\cdot}41$

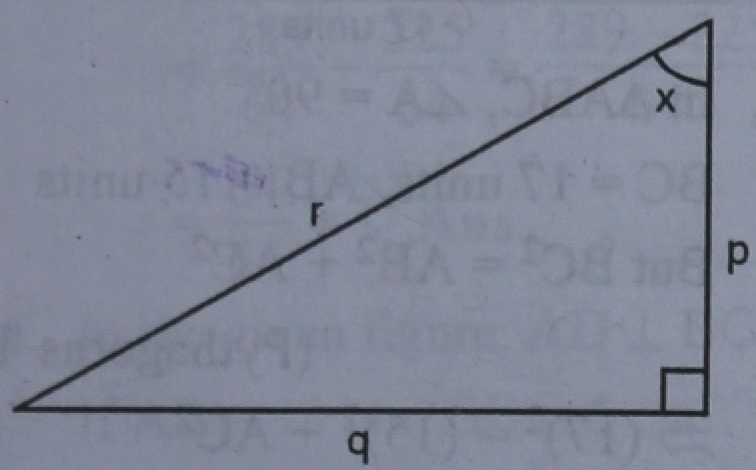
$\sqrt{3} = 1{\cdot}732$ or $1{\cdot}73$

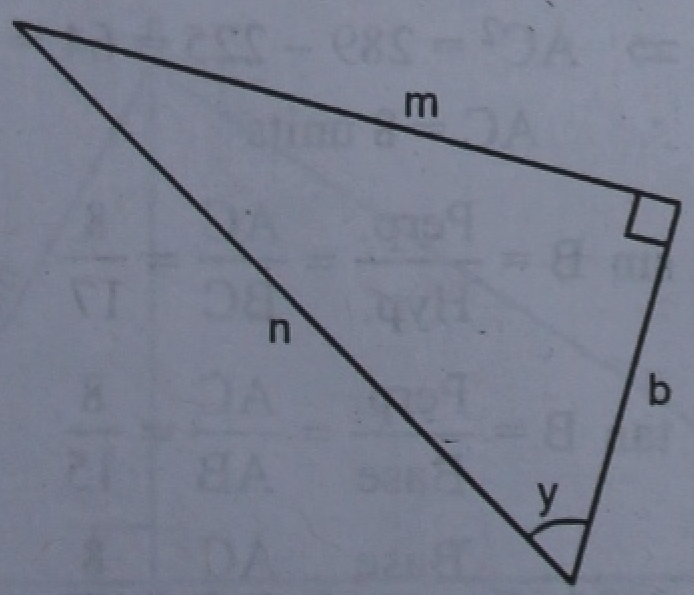
7. $(\sin\theta)^2$ is written as $\sin^2\theta$

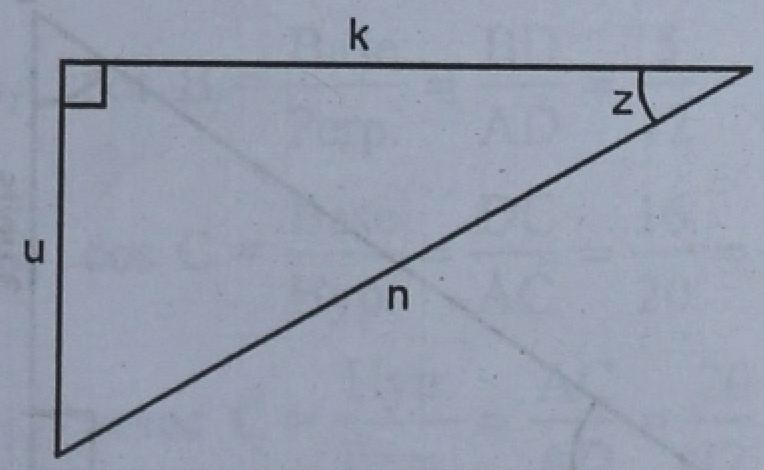
Similarly $(\cos\theta)^2$ is written as $\cos^2\theta$ and $(\tan\theta)^2$ is written as $\tan^2\theta$ and so on.

EXERCISE 22 (A)

Q. 1. Look at the figures given below :







From these figures, write down the values of :

(*i*) $\sin x$ (*ii*) $\tan x$
(*iii*) $\sec x$ (*iv*) $\cos y$
(*v*) $\cot y$ (*vi*) $\text{cosec}\, y$
(*vii*) $\sin z$ (*viii*) $\cos z$
(*ix*) $\tan z$

Sol. (*i*) $\sin x = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{q}{r}$

(*ii*) $\tan x = \frac{\text{Perp.}}{\text{Base}} = \frac{q}{p}$

(*iii*) $\sec x = \frac{\text{Hyp.}}{\text{Base}} = \frac{r}{p}$

(*iv*) $\cos y = \frac{\text{Base}}{\text{Hyp.}} = \frac{b}{n}$

(*v*) $\cot y = \frac{\text{Base}}{\text{Perp.}} = \frac{b}{m}$

(*vi*) $\text{cosec } y = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{n}{m}$

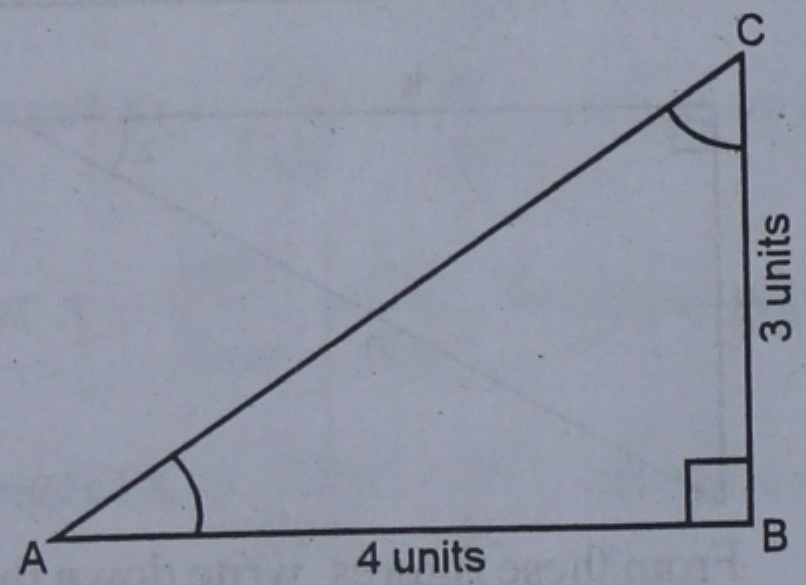
(*vii*) $\sin z = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{u}{n}$

(*viii*) $\cos z = \frac{\text{Base}}{\text{Hyp.}} = \frac{k}{n}$

(*xi*) $\tan z = \frac{\text{Perp.}}{\text{Base}} = \frac{u}{k}$ **Ans.**

Q. 2. In the given figure, $\angle B = 90^\circ$, AB = 4 units and BC = 3 units. Find :

(*i*) sin A (*ii*) cos A
(*iii*) cot A (*iv*) sin C
(*v*) sec C (*vi*) tan C



Sol. In ΔABC, $\angle B = 90^\circ$

AB = 4 units and BC = 3 units

But $AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$= (4)^2 + (3)^2$

$= 16 + 9$

$= 25 = (5)^2$

$\therefore$ AC = 5 units

Now

(*i*) $\sin A = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{3}{5}$

(*ii*) $\cos A = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{4}{5}$

(*iii*) $\cot A = \frac{\text{Base}}{\text{Perp.}} = \frac{AB}{BC} = \frac{4}{3}$

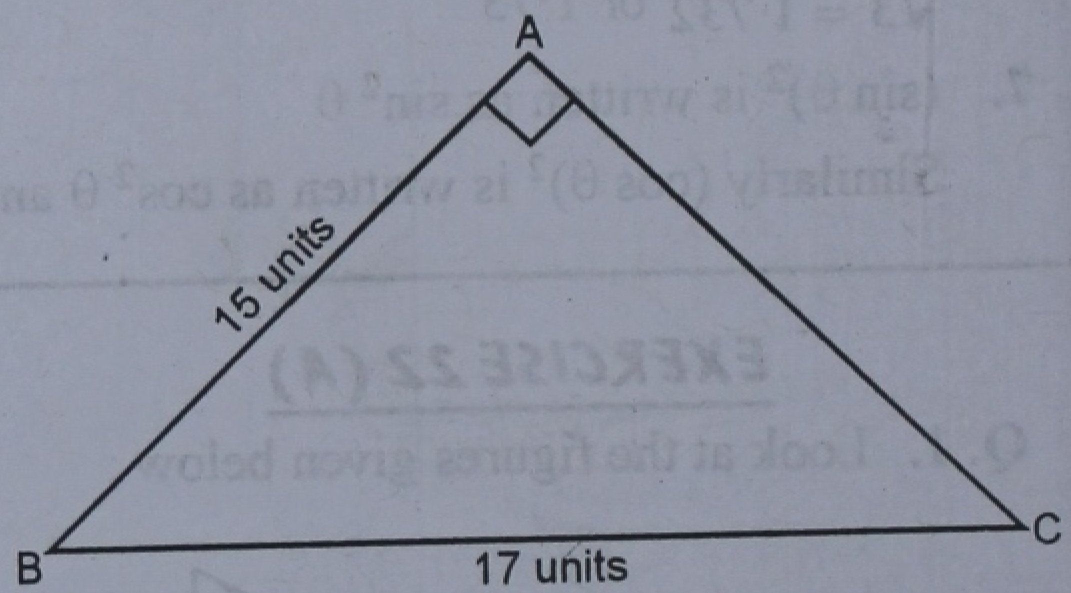
(*iv*) $\sin C = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{4}{5}$

(*v*) $\sec C = \frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{BC} = \frac{5}{3}$

(*vi*) $\tan C = \frac{\text{Perp.}}{\text{Base}} = \frac{AB}{BC} = \frac{4}{3}$ **Ans.**

Q. 3. From the given figure, write down the values of :

(*i*) sin B (*ii*) tan B
(*iii*) cos C (*iv*) cot C
(*v*) (sin B cos C + cos B sin C)
(*vi*) ($\sec^2 C - \tan^2 C$)



Sol. In ΔABC, $\angle A = 90^\circ$

BC = 17 units, AB = 15 units

But $BC^2 = AB^2 + AC^2$ (Pythagoras Theorem)

$\Rightarrow (17)^2 = (15)^2 + AC^2$

$\Rightarrow 289 = 225 + AC^2$

$\Rightarrow AC^2 = 289 - 225 = 64 = (8)^2$

$\therefore$ AC = 8 units

(*i*) $\sin B = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{AC}{BC} = \frac{8}{17}$

(*ii*) $\tan B = \frac{\text{Perp.}}{\text{Base}} = \frac{AC}{AB} = \frac{8}{15}$

(*iii*) $\cos C = \frac{\text{Base}}{\text{Hyp.}} = \frac{AC}{BC} = \frac{8}{17}$

(*iv*) $\cot C = \frac{\text{Base}}{\text{Perp.}} = \frac{AC}{AB} = \frac{8}{15}$

(*v*) $\sin B \cos C + \cos B \sin C$

$\sin B = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{AC}{BC} = \frac{8}{17}$

$\cos C = \frac{\text{Base}}{\text{Hyp.}} = \frac{AC}{BC} = \frac{8}{17}$

$\cos B = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{BC} = \frac{15}{17}$

$\sin C = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{AB}{BC} = \frac{15}{17}$

Now $\sin B \cos C + \cos B \sin C$

$= \frac{8}{17} \times \frac{8}{17} + \frac{15}{17} \times \frac{15}{17} = \frac{64}{289} + \frac{225}{289}$

$= \frac{64+225}{289} = \frac{289}{289} = 1$ **Ans.**

(*vi*) $\sec^2 C - \tan^2 C$

Now, $\sec C = \frac{BC}{AC} = \frac{17}{8}$

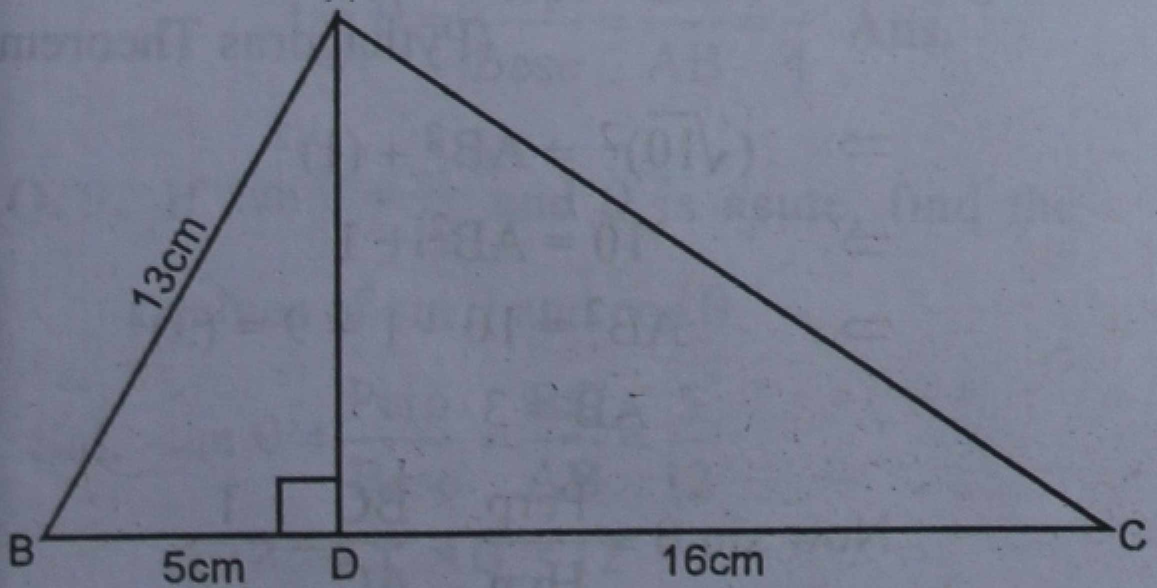
and $\tan C = \frac{AB}{AC} = \frac{15}{8}$

$\therefore \sec^2 C - \tan^2 C = \left(\frac{17}{8}\right)^2 - \left(\frac{15}{8}\right)^2$

$= \frac{289}{64} - \frac{225}{64} = \frac{289-225}{64}$

$= \frac{64}{64} = 1$ **Ans.**

Q. 4. In the given figure, AD ⊥ BC. If AB = 13 cm, BD = 5 cm and DC = 16 cm, find the values of



(*i*) sin B (*ii*) sec B
(*iii*) cot B (*iv*) cos C
(*v*) cosec C (*vi*) tan C

Sol. In ΔABC, AD ⊥ BC :

AB = 13 cm, BD = 5 cm, DC = 16 cm

In ΔABD, ∠D = 90°

$\therefore AB^2 = BD^2 + AD^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = (5)^2 + AD^2$

$\Rightarrow 169 = 25 + AD^2$

$\Rightarrow AD^2 = 169 - 25 = 144 + (12)^2$

$\therefore AD = 12$ cm

Similarly in ΔADC, ∠D = 90°

$AC^2 = AD^2 + DC^2$

$= (12)^2 + (16)^2 = 144 + 256$

$= 400 = (20)^2$

$\therefore AC = 20$ cm

(*i*) $\sin B = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{AD}{AB} = \frac{12}{13}$

(*ii*) $\sec B = \frac{\text{Hyp.}}{\text{Base}} = \frac{AB}{BD} = \frac{13}{5}$

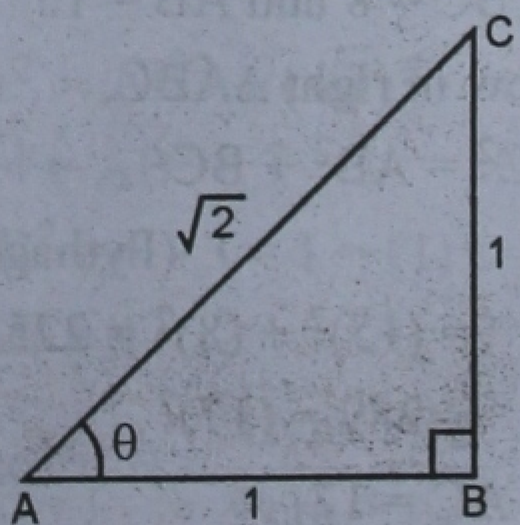
(*iii*) $\cot B = \frac{\text{Base}}{\text{Perp.}} = \frac{BD}{AD} = \frac{5}{12}$

(*iv*) $\cos C = \frac{\text{Base}}{\text{Hyp.}} = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$

(*v*) $\text{cosec } C = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{AC}{AD} = \frac{20}{12} = \frac{5}{3}$

(*vi*) $\tan C = \frac{\text{Perp.}}{\text{Base}} = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$ **Ans.**

Q. 5. If $\sin \theta = \frac{1}{\sqrt{2}}$, find the values of other trigonometrical ratios for θ.



Sol. $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}}$

$= \frac{BC}{AC}$ (from the fig.)

$= \frac{1}{\sqrt{2}}$

$\therefore$ BC = 1 and AC = $\sqrt{2}$

Now in right ΔABC, $\angle B = 90°$

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (\sqrt{2})^2 = (AB)^2 + (1)^2$

$\Rightarrow 2 = AB^2 + 1$

$\Rightarrow AB^2 = 2 - 1 = 1 = (1)^2$

$\therefore AB = 1$

Now $\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{1}{1} = 1$

$\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{1}{1} = 1$

$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{\sqrt{2}}{1} = \sqrt{2}$ **Ans.**

Q. 6. If $\tan\theta = \frac{8}{15}$, find the values of other trigonometrical ratios for θ.

Sol. $\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB} = \frac{8}{15}$

$\therefore$ BC = 8 and AB = 15

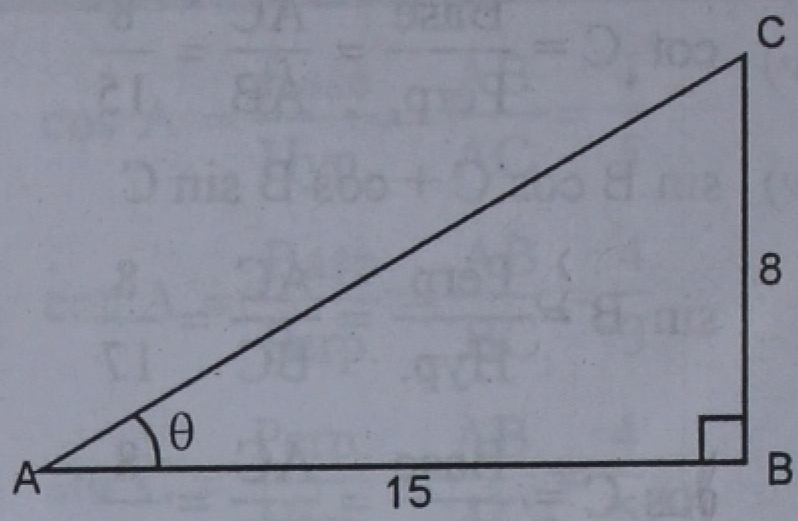
Now in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$= (15)^2 + (8)^2 = 225 + 64$

$= 289 = (17)^2$

$\therefore$ AC = 17



Now $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{8}{17}$

$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{15}{17}$

$\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{AB}{BC} = \frac{15}{8}$

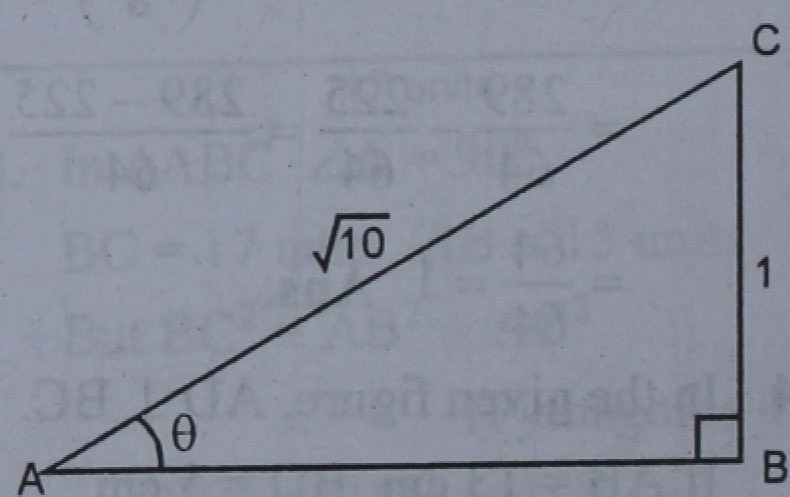
$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{AB} = \frac{17}{15}$

$\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{AC}{BC} = \frac{17}{8}$ **Ans.**

Q. 7. If $\text{cosec}\,\theta = \sqrt{10}$, find the values of other trigonometrical ratios for θ.

Sol. $\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{AC}{BC} = \frac{\sqrt{10}}{1}$

$\therefore$ AC = $\sqrt{10}$ and BC = 1



Now in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (\sqrt{10})^2 = AB^2 + (1)^2$

$\Rightarrow 10 = AB^2 + 1$

$\Rightarrow AB^2 = 10 - 1 = 9 = (3)^2$

$\therefore AB = 3$

Now $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{1}{\sqrt{10}}$

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{3}{\sqrt{10}}$$

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB} = \frac{1}{3}$$

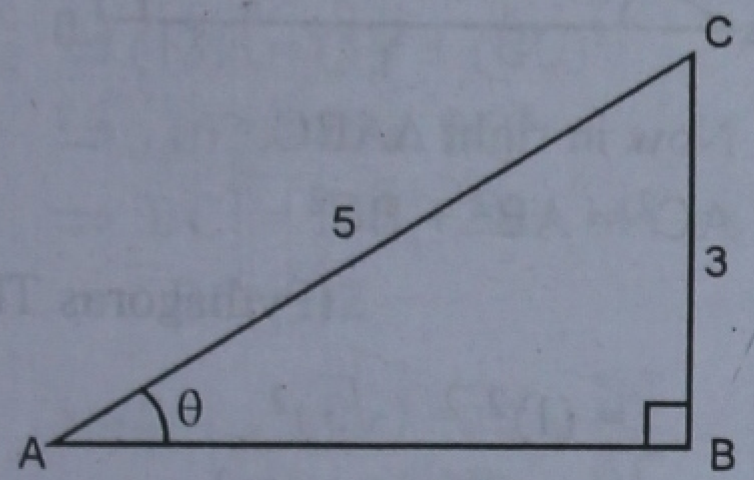
$$\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{AB}{BC} = \frac{3}{1}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{AB} = \frac{\sqrt{10}}{3} \quad \textbf{Ans.}$$

Q. 8. If $\sin\theta = \frac{3}{5}$ and θ is an acute angle, find the values of cos θ and tan θ.

Sol. $\sin\theta = \frac{3}{5} = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC}$

$\therefore BC = 3$ and $AC = 5$



Now in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (5)^2 = AB^2 + (3)^2$

$\Rightarrow 25 = AB^2 + 9$

$\Rightarrow AB^2 = 25 - 9 = 16 = (4)^2$

$\therefore AB = 4$

Now $\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{4}{5}$

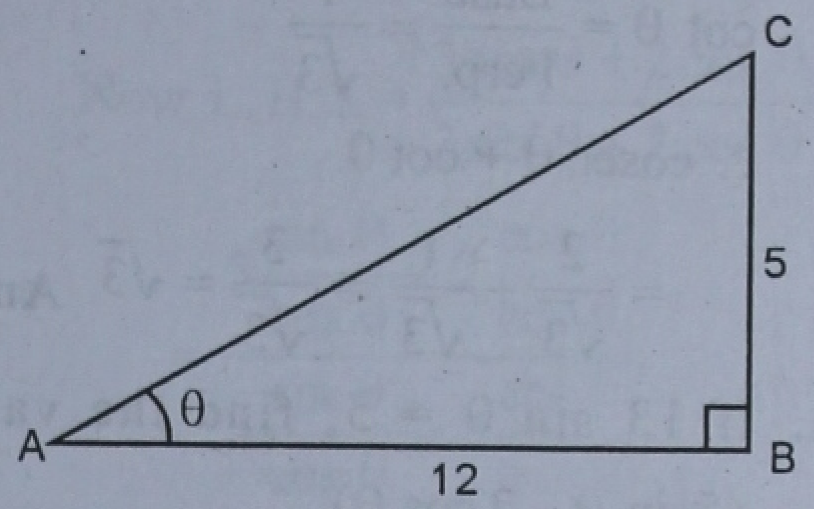
$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB} = \frac{3}{4}$ **Ans.**

Q. 9. If $\tan\theta = \frac{5}{12}$ and θ is acute, find the values of sin θ and cos θ.

Sol. $\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB} = \frac{5}{12}$

$\therefore BC = 5, AB = 12$

Now in right ΔABC,



$AC^2 = AB^2 + BC^2$

(Pythagoras Theorem)

$= (12)^2 + (5)^2 = 144 + 25 = 169$

$= (13)^2$

$\therefore AC = 13$

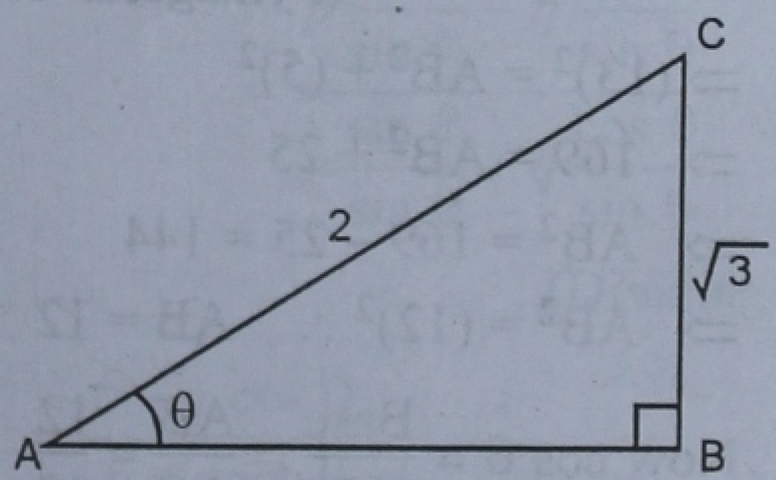
Now $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{5}{13}$

$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{12}{13}$ **Ans.**

Q. 10. If $\sin\theta = \frac{\sqrt{3}}{2}$, find the value of (cosec θ + cot θ).

Sol. $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

$\therefore BC = \sqrt{3}$ and $AC = 2$



Now in right ΔABC,

$AC^2 = AB^2 + BC^2$

(Pythagoras Theorem)

$\Rightarrow (2)^2 = AB^2 + (\sqrt{3})^2$

$\Rightarrow 4 = AB^2 + 3$

$\Rightarrow AB^2 = 4 - 3 = 1 = (1)^2$

$\therefore AB = 1$

Now $\operatorname{cosec}\theta = \frac{\text{Hyp.}}{\text{Perp.}} = \frac{2}{\sqrt{3}}$

$$\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{1}{\sqrt{3}}$$

$\therefore$ cosec $\theta + \cot\theta$

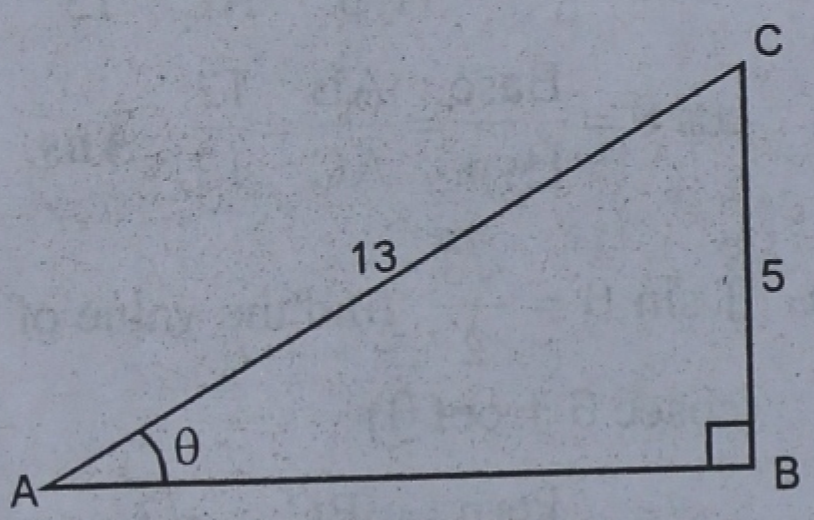
$$= \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$ **Ans.**

Q. 11. If 13 sin θ = 5, find the value of $\frac{(5\sin\theta - 2\cos\theta)}{\tan\theta}$.

Sol. $13\sin\theta = 5 \Rightarrow \sin\theta = \frac{5}{13}$

But $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{5}{13}$

$\therefore$ BC = 5, AC = 13



Now in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = AB^2 + (5)^2$

$\Rightarrow 169 = AB^2 + 25$

$\Rightarrow AB^2 = 169 - 25 = 144$

$\Rightarrow AB^2 = (12)^2 \therefore AB = 12$

Now $\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{12}{13}$

$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB} = \frac{5}{12}$

$$\therefore \frac{5\sin\theta - 2\cos\theta}{\tan\theta} = \frac{5\times\frac{5}{13} - 2\times\frac{12}{13}}{\frac{5}{12}}$$

$$= \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}} = \frac{\frac{1}{13}}{\frac{5}{12}} = \frac{1}{13}\times\frac{12}{5} = \frac{12}{65}$$ **Ans.**

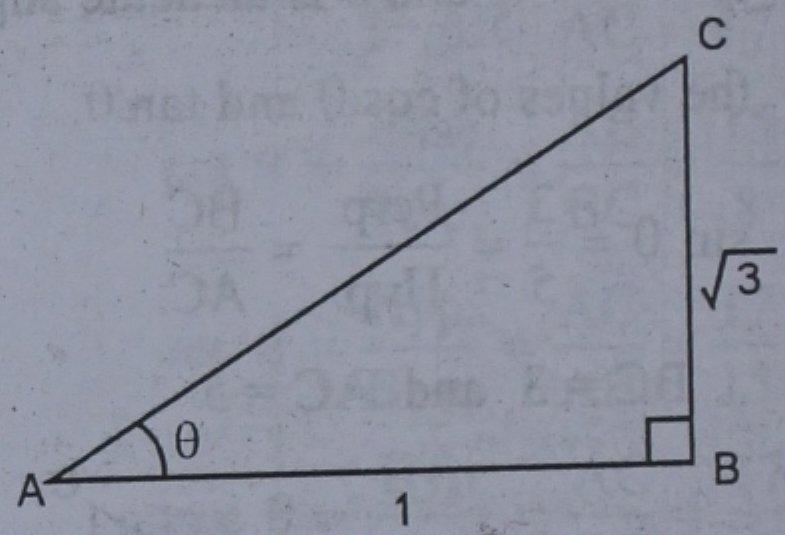
Q. 12. If $\cot\theta = \frac{1}{\sqrt{3}}$, show that

$$\left[\frac{1-\cos^2\theta}{2-\sin^2\theta}\right] = \frac{3}{5}.$$

Sol. $\cot\theta = \frac{1}{\sqrt{3}}$

But $\cot\theta = \frac{\text{Base}}{\text{Perp.}} = \frac{1}{\sqrt{3}} = \frac{AB}{BC}$

$\therefore$ AB = 1 and BC = $\sqrt{3}$



Now in right ΔABC,

$AC^2 = AB^2 + BC^2$

(Pythagoras Theorem)

$= (1)^2 = (\sqrt{3})^2$

$= 1 + 3 = 4 = (2)^2$

$\therefore$ AC = 2

Now $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

and $\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{1}{2}$

$$\therefore \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{4}+\frac{4}{5} = \frac{3}{5}$$

Which is given.

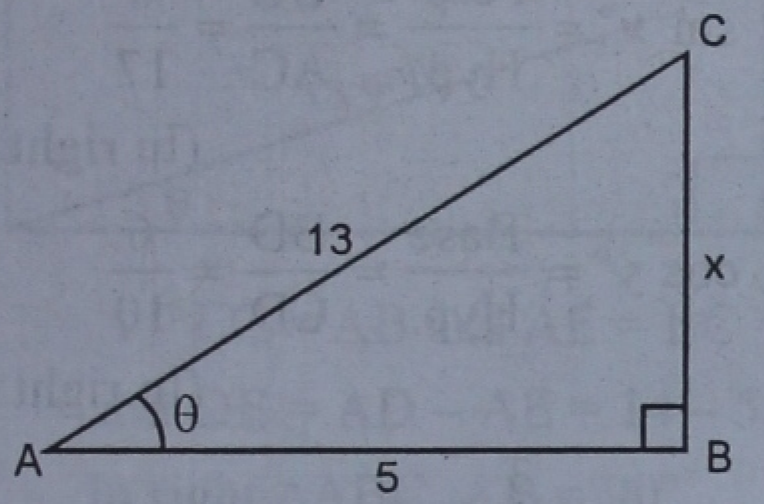
Hence proved.

Q. 13. If $\sec\theta = \frac{13}{5}$, show that

$$\left(\frac{2\sin\theta - 3\tan\theta}{4\sin\theta - 9\cos\theta}\right) = 3.$$

Sol. $\sec\theta = \frac{13}{5} = \frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{AB}$

$\therefore \quad AC = 13, AB = 5$



Now in right ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (13)^2 = (5)^2 + (BC)^2$$

$$\Rightarrow \quad 169 = 25 + BC^2$$

$$\Rightarrow BC^2 = 169 - 25 = 144 = (12)^2$$

$$\therefore \quad BC = 12$$

Now, $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{12}{13}$

$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AB}{AC} = \frac{5}{13}$

Now L.H.S. $= \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$$= \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13}\times\frac{13}{3} = 3$$

= R.H.S.

Hence proved.

Q. 14. If $3\tan\theta = 4$, show that

$$\left(\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}\right) = 3.$$

Sol. $3\tan\theta = 4 \qquad \therefore \tan\theta = \frac{4}{3}$

Now L.H.S. $= \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$

$$= \frac{3\frac{\sin\theta}{\cos\theta} + 2\frac{\cos\theta}{\cos\theta}}{3\frac{\sin\theta}{\cos\theta} - 2\frac{\cos\theta}{\cos\theta}}$$

(Dividing by $\cos\theta$)

$$= \frac{3\tan\theta + 2}{3\tan\theta - 2}$$

Substituting the value of $3\tan\theta = 4$

$$= \frac{4+2}{4-2} = \frac{6}{2} = 3 = \text{R.H.S.}$$

Q. 15. If $\cot\theta = \frac{q}{p}$, show that

$$\left(\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}\right) = \left(\frac{p^2 - q^2}{p^2 + q^2}\right).$$

Sol. $\cot\theta = \frac{q}{p}$

L.H.S. $= \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

$$= \frac{p\frac{\sin\theta}{\sin\theta} - q\frac{\cos\theta}{\sin\theta}}{p\frac{\sin\theta}{\sin\theta} + q\frac{\cos\theta}{\sin\theta}}$$

(Dividing by $\sin\theta$)

$$= \frac{p - q\cot\theta}{p + q\cot\theta}$$

$$= \frac{p - q\times\frac{q}{p}}{p + q\times\frac{q}{p}} = \frac{p - \frac{q^2}{p}}{p + \frac{q^2}{p}} \quad \left(\because \cos + \theta = \frac{p}{q}\right)$$

$$= \frac{p^2 - q^2}{p}\times\frac{p}{p^2 + q^2} = \frac{p^2 - q^2}{p^2 + q^2}$$

= R.H.S.

Q. 16. If $4 \cot \theta = 3$, show that

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{1}{7}.$$

Sol. $4 \cot \theta = 3 \Rightarrow \cot \theta = \frac{3}{4}$

$$\text{L.H.S.} = \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\frac{\sin \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta}} \quad \text{(Dividing by } \sin \theta)$$

$$= \frac{1 - \cot \theta}{1 + \cot \theta}$$

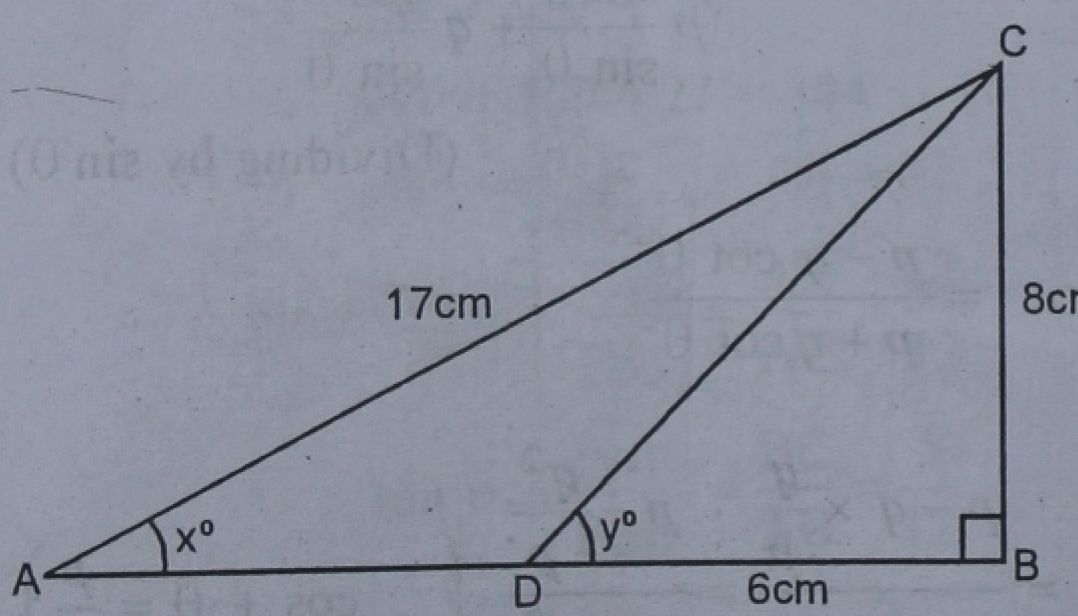
$$= \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}} = \frac{\frac{4-3}{4}}{\frac{4+3}{4}} \quad \left(\because \cot + \theta = \frac{3}{4}\right)$$

$$= \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{4} \times \frac{4}{7} = \frac{1}{7} = \text{R.H.S.}$$

Q. 17. Use the adjoining figure and write the values of :

(*i*) $\sin x^\circ$ (*ii*) $\cos y^\circ$

(*iii*) $3 \tan x^\circ - 2 \sin y^\circ + 4 \cos y^\circ$



Sol. In right ΔABC, $\angle B = 90^\circ$

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (17)^2 = AB^2 + (8)^2$$

$$\Rightarrow 289 = AB^2 + 64$$

$$\Rightarrow AB^2 = 289 - 64 = 225 = (15)^2$$

$$\therefore AB = 15 \text{ cm}$$

and in right ΔBCD, $\angle B = 90^\circ$

$$CD^2 = BD^2 + BC^2$$

(Pythagoras Theorem)

$$= (6)^2 + (8)^2$$

$$= 36 + 64 = 100 = (10)^2$$

$$\therefore CD = 10 \text{ cm}$$

(*i*) $\sin x^\circ = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC} = \frac{8}{17}$

(In right ΔABC)

(*ii*) $\cos y^\circ = \frac{\text{Base}}{\text{Hyp.}} = \frac{BD}{CD} = \frac{6}{10}$

(In right ΔBCD)

$$= \frac{3}{5}$$

(*iii*) $\tan x^\circ = \frac{BC}{AB} = \frac{8}{15},$

$$\sin y^\circ = \frac{BC}{CD} = \frac{8}{10} = \frac{4}{5}$$

$$\therefore 3 \tan x^\circ - 2 \sin y^\circ + 4 \cos y^\circ$$

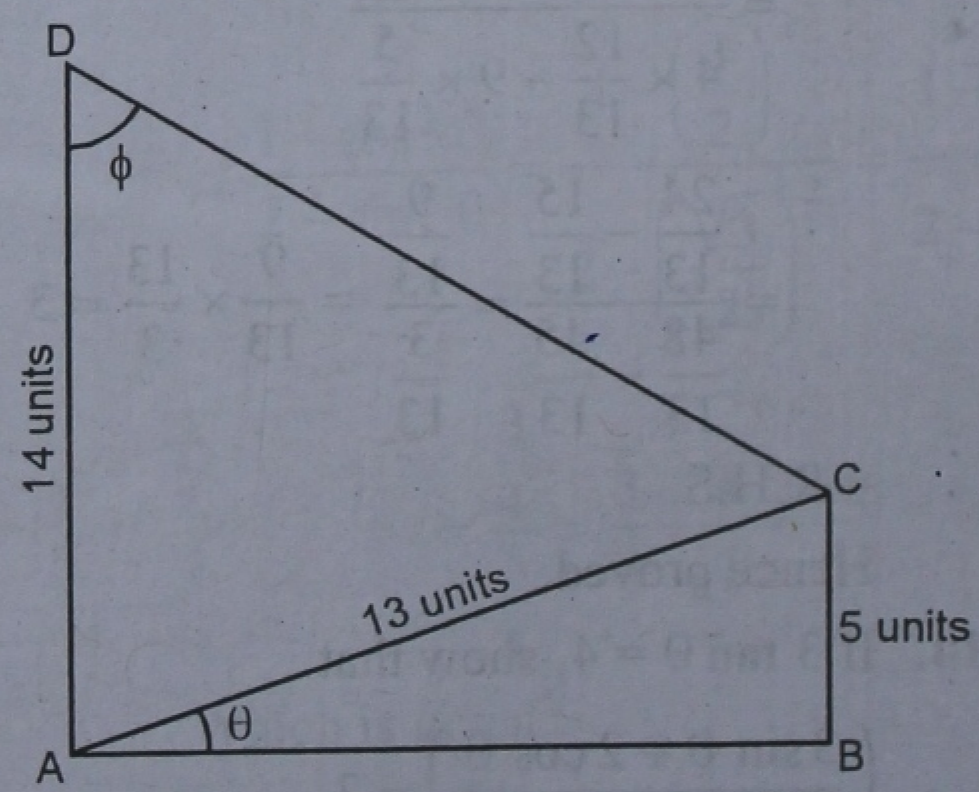
$$= 3 \times \frac{8}{15} - 2 \times \frac{4}{5} + 4 \times \frac{3}{5}$$

$$= \frac{8}{5} - \frac{8}{5} + \frac{12}{5} = \frac{12}{5} = 2\frac{2}{5} \quad \textbf{Ans.}$$

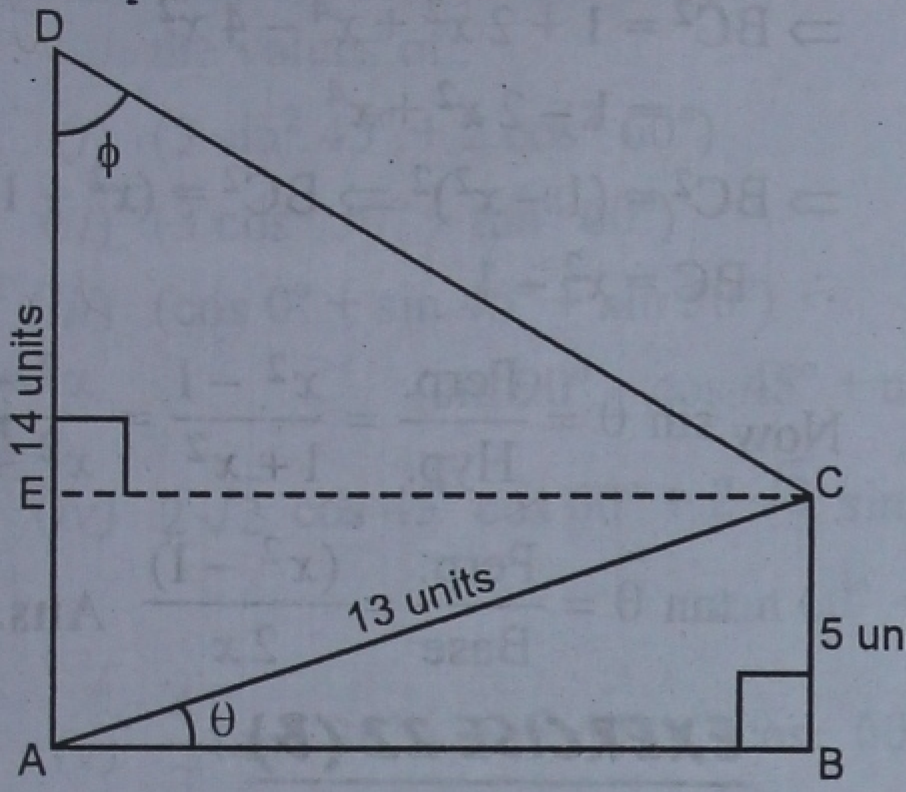
Q. 18. Using the adjoining figure, calculate the values of :

(*i*) $\cos \theta$ (*ii*) $\tan \phi$

(*iii*) $\operatorname{cosec} \phi$



Sol. Draw CE ⊥ AD



∴ EC = AB and AE = BC = 5 units

and DE = AD – AE = 14 – 5 = 9 units

In right ΔABC, ∠B = 90°

$AC^2 = AB^2 + BC^2$

$\Rightarrow (13)^2 = AB^2 + (5)^2$

$\Rightarrow 169 = AB^2 + 25$

$\Rightarrow AB^2 = 169 - 25 = 144 = (12)^2$

∴ AB = 12 units

Hence EC = 12 units

Now in right ΔDEC,

$DC^2 = DE^2 + EC^2$

(Pythagoras Theorem)

$= (9)^2 + (12)^2 = 81 + 144$

$= 225 = (15)^2$

∴ DC = 15 units

Now (*i*) $\cos\theta = \frac{AB}{AC} = \frac{12}{13}$

(*ii*) $\tan\phi = \frac{CE}{ED} = \frac{12}{9} = \frac{4}{3}$

(*iii*) $\text{cosec}\,\phi = \frac{CD}{CE} = \frac{15}{12} = \frac{5}{4}$ **Ans.**

Q. 19. If $(\tan\theta + \cot\theta) = 5$, find the value of $(\tan^2\theta + \cot^2\theta)$.

Sol. $\tan\theta + \cot\theta = 5$

Squaring both sides,

$(\tan\theta + \cot\theta)^2 = (5)^2$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

$\{(a+b)^2 = a^2 + b^2 + 2ab\}$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta \times \frac{1}{\tan\theta} = 25$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 25$

$\Rightarrow \tan^2\theta + \cot^2\theta = 25 - 2 = 23$ **Ans.**

Q. 20. If $(\cos\theta + \sec\theta) = \frac{5}{2}$, find the value of $(\cos^2\theta + \sec^2\theta)$.

Sol. $(\cos\theta + \sec\theta) = \frac{5}{2}$

Squaring both sides,

$(\cos\theta + \sec\theta)^2 = \left(\frac{5}{2}\right)^2$

$\cos^2\theta + \sec^2 + 2\cos\theta\sec\theta = \frac{25}{4}$

$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$

$\Rightarrow \cos^2\theta + \sec^2\theta + 2\cos\theta + \frac{1}{\cos\theta} = \frac{25}{4}$

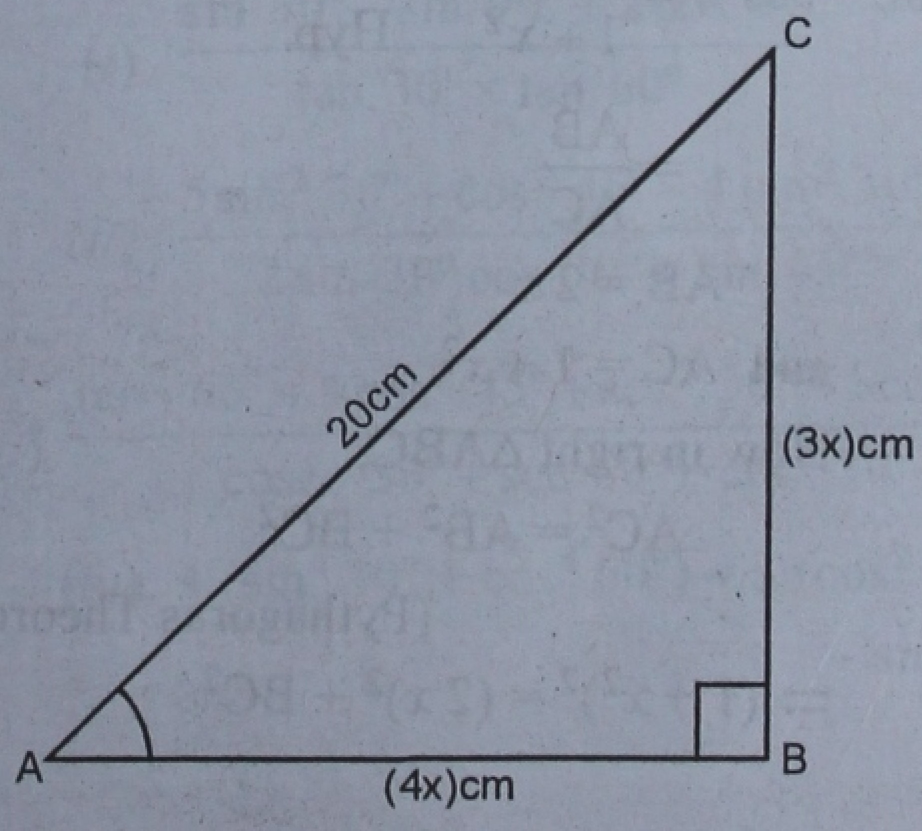
$\Rightarrow \cos^2\theta + \sec^2\theta + 2 = \frac{25}{4}$

$\Rightarrow \cos^2\theta + \sec^2\theta = \frac{25}{4} - 2$

$= \frac{25-8}{4} = \frac{17}{4}$

$\Rightarrow \cos^2 + \sec^2\theta = \frac{17}{4}$ **Ans.**

Q. 21. In the given figure, ΔABC is right angled at B. If AC = 20 cm and $\tan A = \frac{3}{4}$, find the lengths of AB and BC.



Sol. In right ΔABC, $\angle B = 90^\circ$

$$AC = 20 \text{ cm}, \tan A = \frac{3}{4}$$

But $\tan A = \frac{BC}{AB} = \frac{3}{4} = \frac{3x}{4x}$

$\therefore BC = 3x$ cm and $AB = 4x$ **cm**

Now $AC^2 = AB^2 + BC^2$

$\Rightarrow (20)^2 = (4x)^2 + (3x)^2$

$\Rightarrow 400 = 16x^2 + 9x^2 = 25x^2$

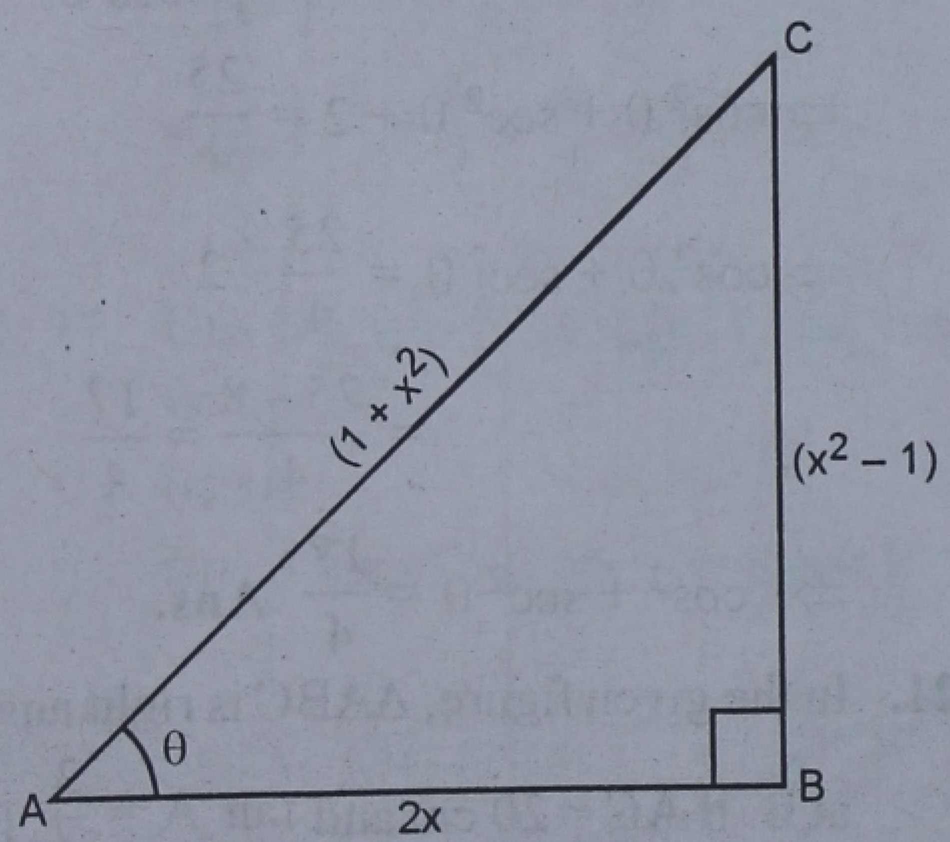
$\therefore x^2 = \frac{400}{25} = 16 = (4)^2$

$\therefore x = 4$

Hence $AB = 4x = 4 \times 4 = 16$ cm

and $BC = 3x = 3 \times 4 = 12$ cm **Ans.**

Q. 22. If $\cos\theta = \frac{2x}{1+x^2}$, find the values of $\sin\theta$ and $\tan\theta$ in terms of x.



Sol. $\cos\theta = \frac{2x}{1+x^2} = \frac{\text{Base}}{\text{Hyp.}}$

$$= \frac{AB}{AC}$$

$\therefore AB = 2x$

and $AC = 1 + x^2$

Now in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (1 + x^2)^2 = (2x)^2 + BC^2$

$\Rightarrow 1 + 2x^2 + x^4 = 4x^2 + BC^2$

$\Rightarrow BC^2 = 1 + 2x^2 + x^4 - 4x^2$

$= 1 - 2x^2 + x^4$

$\Rightarrow BC^2 = (1 - x^2)^2 \Rightarrow BC^2 = (x^2 - 1)^2$

$\therefore BC = x^2 - 1$

Now $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{x^2 - 1}{1 + x^2} = \frac{x^2 - 1}{x^2 + 1}$

$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{(x^2 - 1)}{2x}$ **Ans.**

EXERCISE 22 (B)

Q. 1. Without using trigonometric tables, find the values of :

(*i*) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(*ii*) $\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

(*iii*) $\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$

(*iv*) $\cos 90^\circ + \cos^2 45^\circ \sin 30^\circ \tan 45^\circ$

Sol. (*i*) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ **Ans.**

(*ii*) $\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$ **Ans.**

(*iii*) $\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1 + \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$ **Ans.**

(*iv*) $\cos 90^\circ + \cos^2 45^\circ \sin 30^\circ \tan 45^\circ$

$= 0 + \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} \times 1$

$= 0 + \frac{1}{2} \times \frac{1}{2} \times 1 = 0 + \frac{1}{4} = \frac{1}{4}$ **Ans.**

Q. 2. Without using trigonometric tables, find the values of :

(*i*) $(3 \sin^2 45^\circ + 2 \cos^2 60^\circ)$

(*ii*) $(3 \cos^2 30^\circ + \tan^2 60^\circ)$

(*iii*) $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

(*iv*) $2\sqrt{2} \cos 45^\circ \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \tan 60^\circ - \cos 0^\circ$

(*v*) $\frac{4}{3}\tan^2 30^\circ + \sin^2 60^\circ - 3\cos^2 60^\circ + \frac{3}{4}\tan^2 60^\circ - 2\tan^2 45^\circ$

(*vi*) $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$

Sol. (*i*) $3 \sin^2 45^\circ + 2 \sec^2 60^\circ$

$$= 3\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{1}{2}\right)^2 = 3 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$= \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2 \text{ **Ans.**}$$

(*ii*) $3 \cos^2 30^\circ + \tan^2 60^\circ$

$$= 3\left(\frac{\sqrt{3}}{2}\right)^2 + (\sqrt{3})^2 = 3 \times \frac{3}{4} + 3$$

$$= \frac{9}{4} + \frac{3}{1} = \frac{9+12}{4} = \frac{21}{4} = 5\frac{1}{4} \text{ **Ans.**}$$

(*iii*) $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$\{(a+b)(a-b) = a^2 - b^2\}$

$$= \frac{9}{4} - \frac{1}{2} = \frac{9-2}{4} = \frac{7}{4} = 1\frac{3}{4} \text{ **Ans.**}$$

(*iv*) $2\sqrt{2} \cos 45^\circ \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \tan 60^\circ - \cos 0^\circ$

$$= 2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{1}{2} \times \sqrt{3} - 1$$

$$= 1 + 3 - 1 = 4 - 1 = 3 \text{ **Ans.**}$$

(*v*) $\frac{4}{3}\tan^2 30^\circ + \sin^2 60^\circ - 3\cos^2 60^\circ + \frac{3}{4}\tan^2 60^\circ - 2\tan^2 45^\circ$

$$= \frac{4}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - 3\left(\frac{1}{2}\right)^2 + \frac{3}{4}(\sqrt{3})^2 - 2(1)^2$$

$$= \frac{4}{3} \times \frac{1}{3} + \frac{3}{4} - 3 \times \frac{1}{4} + \frac{3}{4} \times 3 - 2 \times 1$$

$$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2 = \frac{4}{9} + \frac{9}{4} - \frac{2}{1}$$

$$= \frac{16 + 81 - 72}{36} = \frac{25}{36} \text{ **Ans.**}$$

(*vi*) $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}{(\sqrt{3})^2} = \frac{\frac{1}{2} + \frac{1}{2}}{3} = \frac{1}{3} \text{ **Ans.**}$$

Q. 3. Without using trigonometric tables, find the values of :

(*i*) $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \times \tan 60^\circ}$

(*ii*) $\frac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ \cos 30^\circ + \tan 45^\circ}$

(*iii*) $\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + \sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

(*iv*) $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$.

Sol. (*i*) $\dfrac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \times \tan 60^\circ}$

$$= \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{\frac{1}{2} - 1 + 2}{1}$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \textbf{ Ans.}$$

(*ii*) $\dfrac{5\sin^2 30^\circ + \cos^2 45^\circ - 4\tan^2 30^\circ}{2\sin 30^\circ \cos 30^\circ + \tan 45^\circ}$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 4\left(\frac{1}{\sqrt{3}}\right)^2}{2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + 1}$$

$$= \frac{5 \times \frac{1}{4} + \frac{1}{2} - 4 \times \frac{1}{3}}{2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + 1} = \frac{\frac{5}{4} + \frac{1}{2} - \frac{4}{3}}{\frac{\sqrt{3}}{2} + 1}$$

$$= \frac{\frac{15 + 6 - 16}{12}}{\frac{\sqrt{3} + 2}{2}} = \frac{5}{12} \times \frac{2}{2 + \sqrt{3}}$$

$$= \frac{5}{6(2 + \sqrt{3})} \textbf{ Ans.}$$

(*iii*) $\dfrac{\tan^2 60^\circ + 4\cos^2 45^\circ + \sec^2 30^\circ + 5\cos^2 90^\circ}{\text{cosec } 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

$$= \frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 5 \times 0}{2 + 2 - (\sqrt{3})^2}$$

$$= \frac{3 + 4 \times \frac{1}{2} + \frac{4}{3} + 0}{4 - 3} = \frac{3 + 2 + \frac{4}{3}}{1}$$

$$= 5 + \frac{4}{3}$$

$$= \frac{15 + 4}{3} = \frac{19}{3} = 6\frac{1}{3} \textbf{ Ans.}$$

(*iv*) $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left(\frac{1}{2} - 1\right) = 4 \times \frac{1}{8} - 3\left(-\frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \textbf{ Ans.}$$

Q. 4. Verify each of the following :

(*i*) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0.$

(*ii*) $\cos 60^\circ = (1 - 2\sin^2 30^\circ).$
$= (2\cos^2 30^\circ - 1)$

(*iii*) $\tan 30^\circ = \left(\dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}\right).$

Sol. (*i*) L.H.S. $= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$= 0 =$ R.H.S.

(*ii*) L. H. S. $= \cos 60^\circ = \dfrac{1}{2}$

$$1 - 2\sin^2 30^\circ = 1 - 2 \times \left(\frac{1}{2}\right)^2$$

$$= 1 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$2\cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

It is clear from above that

$\cos 60^\circ = 1 - 2\sin^2 30^\circ = 2\cos^2 30^\circ - 1$

(*iii*) L. H. S. $= \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

R. H. S. $= \dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$$

Hence, L.H.S. = R.H.S.

Q. 5. Verify each of the following :

(*i*) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$.

(*ii*) $2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$.

(*iii*) $2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$.

Sol. (*i*) L.H.S. = $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

R.H.S. = $\sin 30^\circ = \frac{1}{2}$

Hence L.H.S. = R.H.S.

(*ii*) L.H.S. = $2 \sin 30^\circ \cos 30^\circ$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

R.H.S. = $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, L.H.S. = R.H.S.

(*iii*) L.H.S. = $2 \sin 45^\circ \cos 45^\circ$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$$

R.H.S. = $\sin 90^\circ = 1$

Hence, L.H.S. = R.H.S.

Q. 6. If A = 45°, verify that

(*i*) $\sin 2A = 2 \sin A \cos A$

(*ii*) $\cos 2A = (2\cos^2 A - 1) = (1 - 2\sin^2 A)$

Sol. (*i*) L.H.S. = $\sin 2A = \sin (2 \times 45^\circ)$

$$= \sin 90^\circ = 1$$

R.H.S. = $2 \sin A \cos A$

$$= 2 \times \sin 45^\circ \cos 45^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$$

Hence, L.H.S. = R.H.S.

(*ii*) $\cos 2A = \cos (2 \times 45^\circ) = \cos 90^\circ = 0$

and $2\cos^2 A - 1 = 2\cos^2 45^\circ - 1$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1$$

$$= 1 - 1 = 0$$

Again $1 - 2\sin^2 A = 1 - 2\sin^2 45^\circ$

$$= 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2 = 1 - 2 \times \frac{1}{2}$$

$$= 1 - 1 = 0$$

Hence, $\cos 2A = (2\cos^2 A - 1)$

$$= (1 - 2\sin^2 A)$$

Q. 7. If A = 30°, prove that

(*i*) $\sin 2A = \frac{2 \tan A}{(1 + \tan^2 A)}$

(*ii*) $\cos 2A = \left(\frac{1 - \tan^2 A}{1 + \tan^2 A}\right)$.

Sol. (*i*) L.H.S. = $\sin 2A = \sin (2 \times 30^\circ)$

$$= \sin 60^\circ = \frac{\sqrt{3}}{2}$$

R.H.S. = $\frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

Hence, L.H.S. = R.H.S.

(*ii*) L.H.S. = $\cos 2A = \cos (2 \times 30^\circ)$

$$= \cos 60^\circ = \frac{1}{2}$$

$$\text{R.H.S.} = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}}$$

$$= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

Hence, L.H.S. = R.H.S.

Q. 8. If $A = B = 45^\circ$, show that :

(*i*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*ii*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Sol. We are given $A = B = 45^\circ$

(*i*) L.H.S. $= \sin(A - B) = \sin(45^\circ - 45^\circ)$

$= \sin 0^\circ = 0$

R.H.S. $= \sin A \cos B - \cos A \sin B$

$= \sin 45^\circ \cos 45^\circ - \cos 45^\circ \sin 45^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} - \frac{1}{2} = 0$$

$\therefore$ L.H.S. = R.H.S.

(*ii*) L.H.S. $= \cos(A + B) = \cos(45^\circ + 45^\circ)$

$= \cos 90^\circ = 0$

R.H.S. $= \cos A \cos B - \sin A \sin B$

$= \cos 45^\circ \cos 45^\circ - \sin 45^\circ \sin 45^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} - \frac{1}{2} = 0$$

$\therefore$ L.H.S. = R.H.S.

Q. 9. If $A = 60^\circ$ and $B = 30^\circ$, show that :

$(\sin A \cos B + \cos A \sin B)^2 + (\cos A \cos B - \sin A \sin B)^2 = 1$.

Sol. We are given $A = 60^\circ$ and $B = 30^\circ$

L.H.S. $= (\sin A \cos B + \cos A \sin B)^2 + (\cos A \cos B - \sin A \sin B)^2$

$= (\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ)^2 - (\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ)^2$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right)^2 - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right)^2$$

$$= \left(\frac{3}{4} + \frac{1}{4}\right)^2 - \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right)^2$$

$= (1)^2 - (0)^2 = 1^2 = 1 =$ R.H.S.

Hence proved.

Q. 10. If $A = 60^\circ$ and $B = 30^\circ$, prove that :

(*i*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

(*ii*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

(*iii*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

(*iv*) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

Sol. We are given $A = 60^\circ$ and $B = 30^\circ$

(*i*) L.H.S. $= \sin(A + B) = \sin(60^\circ + 30^\circ)$

$= \sin 90^\circ = 1$

R.H.S. $= \sin A \cos B + \cos A \sin B$

$= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

$\therefore$ L.H.S. = R.H.S.

(*ii*) L.H.S. $= \cos(A + B) = \cos(60^\circ + 30^\circ)$

$= \cos 90^\circ = 0$

R.H.S. $= \cos A \cos B - \sin A \sin B$

$= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

$\therefore$ L.H.S. = R.H.S.

(*iii*) L.H.S. $= \cos(A - B)$

$= \cos(60^\circ - 30^\circ)$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2}$$

R.H.S. $= \cos A \cos B + \sin A \sin B$

$= \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

∴ L.H.S. = R.H.S.

(*iv*) L.H.S. = tan (A – B)

$$= \tan(60^\circ - 30^\circ)$$

$$= \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{R.H.S.} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$= \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{\sqrt{3} \times 2} = \frac{1}{\sqrt{3}}$$

∴ L.H.S. = R.H.S.

Q. 11. Evaluate : $\frac{\cos 3A + 2\cos 4A}{\sin 3A + 2\sin 4A}$, when A = 15°.

Sol. A = 15°

$$\frac{\cos 3A + 2\cos 4A}{\sin 3A + 2\sin 4A}$$

$$= \frac{\cos(3 \times 15^\circ) + 2\cos(4 \times 15^\circ)}{\sin(3 \times 15^\circ) + 2\sin(4 \times 15^\circ)}$$

$$= \frac{\cos 45^\circ + 2\cos 60^\circ}{\sin 45^\circ + 2\sin 60^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}} + 2 \times \frac{1}{2}}{\frac{1}{\sqrt{2}} + 2 \times \frac{\sqrt{3}}{2}} = \frac{\frac{1}{\sqrt{2}} + 1}{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{1}}$$

$$= \frac{\frac{1+\sqrt{2}}{\sqrt{2}}}{\frac{1+\sqrt{6}}{\sqrt{2}}} = \frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{1+\sqrt{6}}$$

$$= \frac{1+\sqrt{2}}{1+\sqrt{6}} \quad \textbf{Ans.}$$

Q. 12. Evaluate : $\frac{3\sin 3A + 2\cos(2A + 5^\circ)}{2\cos 3A - \sin(2A - 10^\circ)}$, when A = 20°.

Sol. $\frac{3\sin 3A + 2\cos(2A + 5^\circ)}{2\cos 3A - \sin(2A - 10^\circ)}$

$$= \frac{3\sin(3 \times 20^\circ) + 2\cos(2 \times 20^\circ + 5)}{2\cos(3 \times 20^\circ) - \sin(2 \times 20^\circ - 10^\circ)}$$

[∵ A = 20°]

$$= \frac{3\sin 60^\circ + 2\cos(40^\circ + 5^\circ)}{2\cos 60^\circ - \sin(40^\circ - 10^\circ)}$$

$$= \frac{3\sin 60^\circ + 2\cos 45^\circ}{2\cos 60^\circ - \sin 30^\circ}$$

$$= \frac{\frac{3 \times \sqrt{3}}{2} + 2 \times \frac{1}{\sqrt{2}}}{2 \times \frac{1}{2} - \frac{1}{2}}$$

$$= \frac{\frac{3\sqrt{3}}{2} + \frac{2}{\sqrt{2}}}{1 - \frac{1}{2}} = \frac{\frac{3\sqrt{6}+4}{2\sqrt{2}}}{\frac{1}{2}}$$

$$= \frac{2(3\sqrt{6}+4)}{2\sqrt{2}} = \frac{(3\sqrt{6}+4)2}{2\sqrt{2}}$$

$$= \frac{3\sqrt{6}+4}{\sqrt{2}} = \frac{(3\sqrt{6}+4) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{3\sqrt{6} \times \sqrt{2} + 4\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{12} + 4\sqrt{2}}{2}$$

$$= \frac{3 \times \sqrt{4 \times 3} + 4\sqrt{2}}{2}$$

$$= \frac{3 \times 2\sqrt{3} + 4\sqrt{2}}{2}$$

$$= \frac{2(3\sqrt{3} + 2\sqrt{2})}{2} = \frac{3\sqrt{3} + 2\sqrt{2}}{1}$$

$$= 3\sqrt{3} + 2\sqrt{2}$$ **Ans.**

Q. 13. Show that $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$.

Sol. L.H.S. $= 4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4 \times \frac{2}{16} - 3 \times \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 = \text{R.H.S.}$$

Q. 14. Without using tables, verify that :

(*i*) $\cos 60^\circ = (\cos^2 30^\circ - \sin^2 30^\circ)$.

(*ii*) $\sin 60^\circ = \frac{2\tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{\sqrt{3}}{2}$.

(*iii*) $\cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1}{2}$.

Sol. (*i*) L.H.S. $= \cos 60^\circ = \frac{1}{2}$

R.H.S. $= \cos^2 30^\circ - \sin^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ L.H.S. = R.H.S.

(*ii*) $\sin 60^\circ = \frac{\sqrt{3}}{2}$

and
$$\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{2 \times \sqrt{3} \times \sqrt{3}}$$

$$= \frac{3\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{2}$$

Hence proved.

(*iii*) L.H.S. $= \cos 60^\circ = \frac{1}{2}$

and
$$\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

Hence proved.

Q. 15. If $0^\circ \le x \le 90^\circ$, state the numerical value of x for which $\sin x^\circ = \cos x^\circ$.

Sol. $\because \sin x^\circ = \cos x^\circ$

$$\Rightarrow \frac{\sin x^\circ}{\cos x^\circ} = \frac{\cos x^\circ}{\cos x^\circ}$$

(Dividing by $\cos x^\circ$)

$\Rightarrow \tan x^\circ = 1 = \tan 45^\circ$

$\therefore \quad x^\circ = 45^\circ$

$\therefore \quad x = 45$ **Ans.**

EXERCISE 22 (C)

Q. 1. Without using trigonometric tables, evaluate the following :

(*i*) $\frac{\sin 10^\circ}{\cos 80^\circ}$ (*ii*) $\frac{\cos 12^\circ}{\sin 78^\circ}$.

(*iii*) $\frac{\tan 25^\circ}{\cot 65^\circ}$

Sol. (*i*) $\dfrac{\sin 10^\circ}{\cos 80^\circ} = \dfrac{\sin (90^\circ - 80^\circ)}{\cos 80^\circ}$

$= \dfrac{\cos 80^\circ}{\cos 80^\circ} = 1$

(*ii*) $\dfrac{\cos 12^\circ}{\sin 78^\circ} = \dfrac{\cos (90^\circ - 78^\circ)}{\sin 78^\circ}$

$= \dfrac{\sin 78^\circ}{\sin 78^\circ} = 1$

(*iii*) $\dfrac{\tan 25^\circ}{\cot 65^\circ} = \dfrac{\tan (90^\circ - 65^\circ)}{\cot 65^\circ}$

$= \dfrac{\cot 65^\circ}{\cot 65^\circ} = 1$

Q. 2. Without using trigonometric tables, evaluate the following :

(*i*) $\left(\dfrac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\dfrac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

(*ii*) $\left(\dfrac{\cot 40^\circ}{\tan 50^\circ}\right) - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$

Sol. (*i*) $\left(\dfrac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\dfrac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

$= \left[\dfrac{\sin (90^\circ - 41^\circ)}{\cos 41^\circ}\right]^2 + \left[\dfrac{\cos (90^\circ - 49^\circ)}{\sin 49^\circ}\right]^2$

$= \left(\dfrac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\dfrac{\sin 49^\circ}{\sin 49^\circ}\right)$

$= (1)^2 + (1)^2 = 1 + 1 = 2$

(*ii*) $\dfrac{\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$

$= \dfrac{\cot (90^\circ - 50^\circ)}{\tan 50^\circ} - \dfrac{1}{2}\dfrac{\cos (90^\circ - 55^\circ)}{\sin 55^\circ}$

$= \dfrac{\tan 50^\circ}{\tan 50^\circ} - \dfrac{1}{2}\dfrac{\sin 55^\circ}{\sin 55^\circ} = 1 - \dfrac{1}{2} \times 1$

$= 1 - \dfrac{1}{2} = \dfrac{1}{2}$

Q. 3. Without using trigonometric tables, prove that :

(*i*) $\cos 72^\circ - \sin 18^\circ = 0$

(*ii*) $\sin^2 56^\circ - \cos^2 34^\circ = 0$

(*iii*) $\sin^2 48^\circ + \sin^2 42^\circ = 1$

(*iv*) $\cos^2 75^\circ + \cos^2 15^\circ = 1$

Sol. (*i*) L.H.S. $= \cos 72^\circ - \sin 18^\circ$

$= \cos (90^\circ - 18^\circ) - \sin 18^\circ$

$= \sin 18^\circ - \sin 18^\circ = 0 =$ R.H.S.

(*ii*) L.H.S. $= \sin^2 56^\circ - \cos^2 34^\circ$

$= \sin^2 (90^\circ - 34^\circ) - \cos^2 34^\circ$

$= \cos^2 34^\circ - \cos^2 34^\circ = 0 =$ R.H.S.

(*iii*) L.H.S. $= \sin^2 48^\circ + \sin^2 42^\circ$

$= \sin^2 (90^\circ - 42^\circ) + \sin^2 42^\circ$

$= \cos^2 42^\circ + \sin^2 42^\circ = 1$

$(\because \sin^2 \theta + \cos^2 \theta = 1)$

= R.H.S.

(*iv*) L.H.S. $= \cos^2 75^\circ + \cos^2 15^\circ$

$= \cos^2 (90^\circ - 15^\circ) + \cos^2 15^\circ$

$= \sin^2 15^\circ + \cos^2 15^\circ = 1 =$ R.H.S.

Q. 4. Without using trigonometric tables, prove that :

(*i*) $\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ = 1$

(*ii*) $\cos 64^\circ \cos 26^\circ - \sin 64^\circ \sin 26^\circ = 0$

Sol. (*i*) $\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ$

$= \sin 20^\circ \cos (90^\circ - 20^\circ)$

$+ \cos 20^\circ \sin (90^\circ - 20^\circ)$

$= \sin 20^\circ \sin 20^\circ + \cos 20^\circ \cos 20^\circ$

$= \sin^2 20^\circ + \cos^2 20^\circ = 1 =$ R.H.S.

$(\because \sin^2 \theta + \cos^2 \theta = 1)$

(*ii*) $\cos 64^\circ \cos 26^\circ - \sin 64^\circ \sin 26^\circ$

$= \cos (90^\circ - 26^\circ) \cos 26^\circ$

$- \sin (90^\circ - 26^\circ) \sin 26^\circ$

$= \sin 26^\circ \cos 26^\circ - \cos 26^\circ \sin 26^\circ$

$= 0 =$ R.H.S.

Q. 5. Without using the trigonometric tables, prove that :

(i) $\dfrac{\cos 81^\circ}{\sin 9^\circ} + \dfrac{\cos 14^\circ}{\sin 76^\circ} = 2$

(ii) $\left(\dfrac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ = 0$

(iii) $\sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ = 0$

Sol. (i) L.H.S. $= \dfrac{\cos 81^\circ}{\sin 9^\circ} + \dfrac{\cos 14^\circ}{\sin 76^\circ}$

$= \dfrac{\cos(90^\circ - 9^\circ)}{\sin 9^\circ} + \dfrac{\cos(90^\circ - 76^\circ)}{\sin 76^\circ}$

$= \dfrac{\sin 9^\circ}{\sin 9^\circ} + \dfrac{\sin 76^\circ}{\sin 76^\circ} = 1 + 1 = 2 =$ R.H.S.

(ii) L.H.S. $= \left(\dfrac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$

$= \left[\dfrac{\sin(90^\circ - 43^\circ)}{\cos 43^\circ}\right]^2 + \left[\dfrac{\cos(90^\circ - 47^\circ)}{\sin 47^\circ}\right]^2 - 4\left(\dfrac{1}{\sqrt{2}}\right)^2 \quad \left[\because \cos 45^\circ = \dfrac{1}{\sqrt{2}}\right]$

$= \left(\dfrac{\cos 43^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\sin 47^\circ}{\sin 47^\circ}\right)^2 - 4 \times \dfrac{1}{2}$

$= (1)^2 + (1)^2 - 2$

$= 2 - 2 = 0 =$ R.H.S.

(iii) L.H.S. $= \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ$

$= \sin^2 20^\circ + \sin^2(90^\circ - 20^\circ) - \tan^2 45^\circ$

$= \sin^2 20^\circ + \cos^2 20^\circ - \tan^2 45^\circ$

$= 1 - 1 = 0$

$\{\because \sin^2\theta + \cos^2\theta = 1$ and $\tan 45^\circ = 1\}$

= R.H.S.

Q. 6. Prove that

(i) $\dfrac{\cos\theta}{\sin(90^\circ - \theta)} + \dfrac{\sin\theta}{\cos(90^\circ - \theta)} = 2$

(ii) $\sin\theta\cos(90^\circ - \theta) + \cos\theta\sin(90^\circ - \theta) = 1.$

Sol. (i) L.H.S.

$= \dfrac{\cos\theta}{\sin(90^\circ - \theta)} + \dfrac{\sin\theta}{\cos(90^\circ - \theta)}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\sin\theta} = 1 + 1 = 2 =$ R.H.S.

(ii) $\sin\theta\cos(90^\circ - \theta) + \cos\theta\sin(90^\circ - \theta)$

$= \sin\theta.\sin\theta + \cos\theta.\cos\theta$

$= \sin^2\theta + \cos^2\theta = 1 =$ R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$

Q. 7. Prove that : $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) = 0.$

Sol. L.H.S. $= \sin(50^\circ + \theta) - \cos(40^\circ - \theta)$

$= \sin[90^\circ - (40^\circ - \theta)] - \cos(40^\circ - \theta)$

$= \cos(40^\circ - \theta) - \cos(40^\circ - \theta)$

$= 0 =$ R.H.S.

Q. 8. Prove that :

$\dfrac{\cos 35^\circ}{\sin 55^\circ} + \dfrac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \operatorname{cosec} 62^\circ = 1.$

Sol. L.H.S. $= \dfrac{\cos 35^\circ}{\sin 55^\circ} + \dfrac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \operatorname{cosec} 62^\circ$

$= \dfrac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} + \dfrac{\sin(90^\circ - 79^\circ)}{\cos 79^\circ} - \cos(90^\circ - 62^\circ)\operatorname{cosec} 62^\circ$

$= \dfrac{\sin 55^\circ}{\sin 55^\circ} + \dfrac{\cos 79^\circ}{\cos 79^\circ} - \sin 62^\circ \operatorname{cosec} 62^\circ$

$= 1 + 1 - 1 \quad (\because \sin\theta\operatorname{cosec}\theta = 1)$

$= 2 - 1 = 1 =$ R.H.S.

Q. 9. Prove that : $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ = 1.$

Sol. L.H.S. $= \tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

$= \tan(90^\circ - 55^\circ)\tan(90^\circ - 50^\circ)\tan 45^\circ \tan 50^\circ \tan 55^\circ$

$= \cot 55^\circ \cot 50^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

$= \cot 55^\circ . \tan 55^\circ \cot 50^\circ \tan 50^\circ \tan 45^\circ$

$= 1 \times 1 \times 1$

$\{\because \tan\theta \cot\theta = 1 \text{ and } \tan 45^\circ = 1\}$

$= 1 = \text{R.H.S.}$

Q. 10. If A, B, C are the interior angles of a triangle, prove that :

$$\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}.$$

Sol. $\because$ A, B and C are the angles of a triangle ABC

$\therefore \quad \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = 90^\circ$

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90^\circ - \frac{A}{2}$

$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$

Now $\text{L.H.S.} = \tan\left(\frac{B+C}{2}\right)$

$= \tan\left(90^\circ - \frac{A}{2}\right)$

$= \cot\frac{A}{2} = \text{R.H.S.}$

EXERCISE 22 (D)

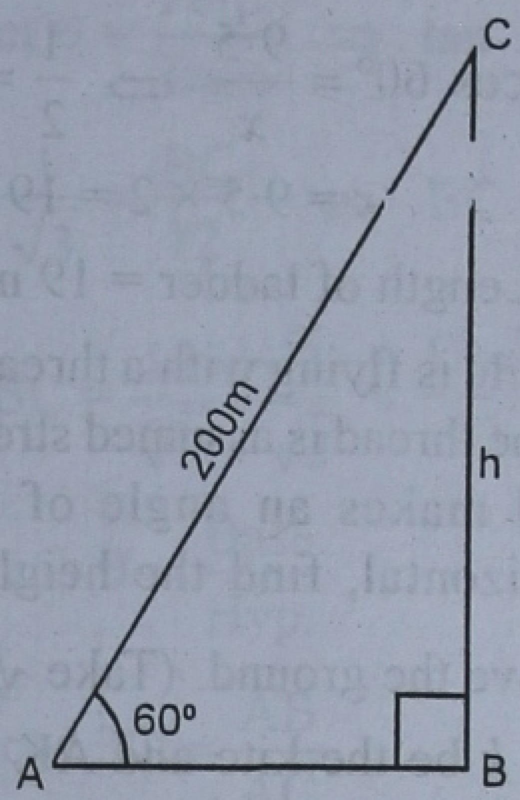
Q. 1. A balloon is connected to a meteorological station by a cable of length 200 metres, inclined at 60° to the horizontal. Determine the height of the balloon from the ground, assuming that there is no slack in the string. (Take $\sqrt{3} = 1.73$)

Sol. Let B is the balloon and AB is the string which inclined an angle of 60° with the ground.

then AB = 200 m, $\angle A = 60^\circ$

and BC is the height of balloon from the ground and let BC = h metres

$$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AB}$$



$\Rightarrow \sin 60^\circ = \frac{h}{200} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{200}$

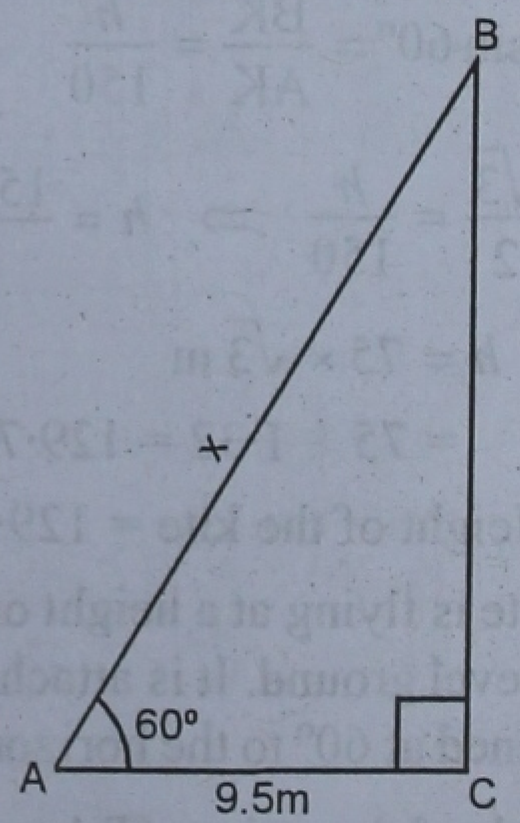
$\Rightarrow h = \frac{200 \times \sqrt{3}}{2}$

$= \frac{200 \times 1.73}{2} = 173 \text{ m}$

Hence, height = 173 m **Ans.**

Q. 2. A ladder leaning against a wall, makes an angle of 60° with the horizontal and the foot of the ladder is 9·5 metres away from the wall. Find the length of the ladder.

Sol. Let AB be the ladder which makes an angle of 60° with AC and BC is the height of wall. Then



AC = 9·5 m

and AB = x m (say)

$\therefore \cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AC}{AB}$

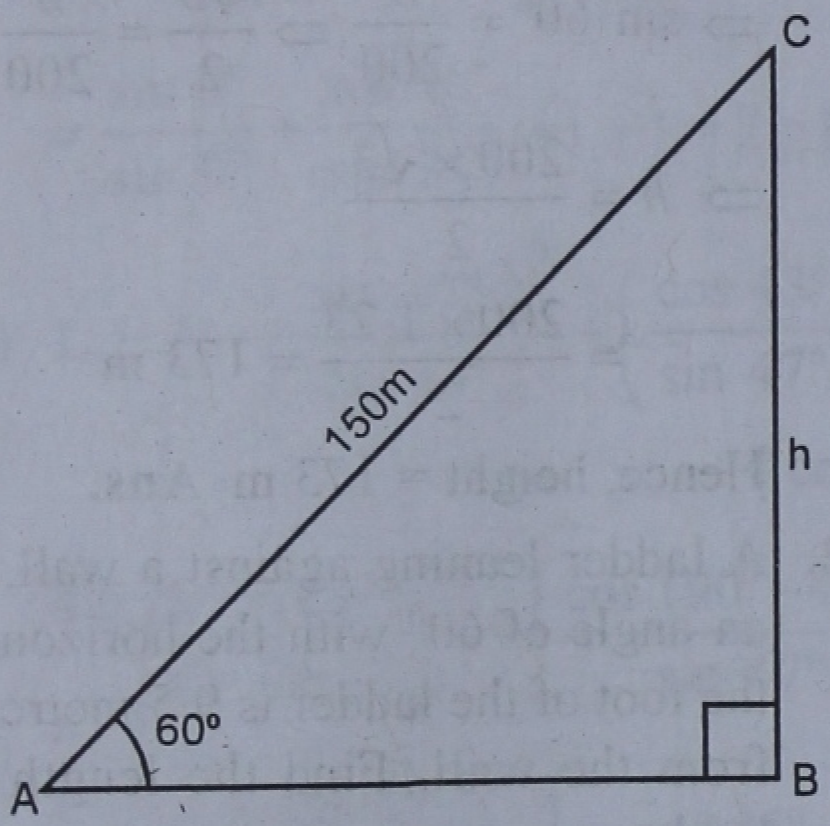
$$\Rightarrow \cos 60^\circ = \frac{9\cdot5}{x} \quad \Rightarrow \frac{1}{2} = \frac{9\cdot5}{x}$$

$$\Rightarrow \quad x = 9\cdot5 \times 2 = 19$$

$\therefore$ Length of ladder = 19 m **Ans.**

Q. 3. A kite is flying with a thread 150 m long. If the thread is assumed stretched straight and makes an angle of 60° with the horizontal, find the height of the kite above the ground. (Take $\sqrt{3} = 1\cdot73$)

Sol. Let k be the kite and AK is the thread. BK is the height of the kite, then



AK = 150 m, BK = h (say)

$$\therefore \quad \sin\theta = \frac{\text{Perp.}}{\text{Hyp.}}$$

$$\Rightarrow \sin 60^\circ = \frac{BK}{AK} = \frac{h}{150}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{150} \quad \Rightarrow \quad h = \frac{150 \times \sqrt{3}}{2}$$

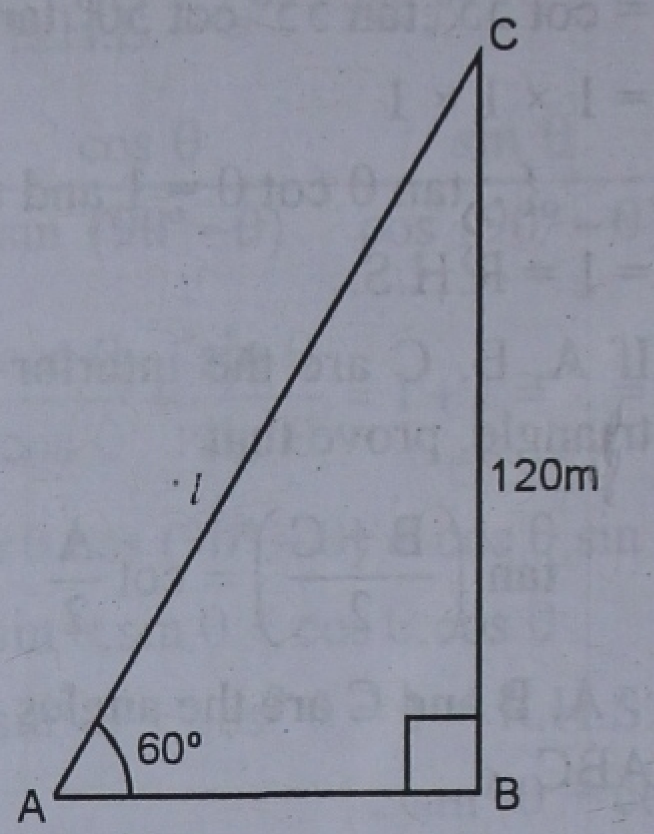
$$\Rightarrow \quad h = 75 \times \sqrt{3} \text{ m}$$

$$= 75 \times 1\cdot32 = 129\cdot75 \text{ m}$$

$\therefore$ Height of the kite = 129·75 m **Ans.**

Q. 4. A kite is flying at a height of 120 m from the level ground. It is attached to a string inclined at 60° to the horizontal. Find the length of the string. (Take $\sqrt{3} = 1\cdot73$)

Sol. Let C be the kite and AC be the string which inclined an angle of 60° with the horizontal and BC is the height. Then



Let AC = l, BC = 120 m

$$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} \Rightarrow \sin 60^\circ = \frac{BC}{AC} = \frac{120}{l}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{120}{l}$$

$$\Rightarrow \quad l = \frac{120 \times 2}{\sqrt{3}} = \frac{120 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{ m}$$

$$= \frac{120 \times 2 \times \sqrt{3}}{3}$$

$$= 40 \times 2 \times \sqrt{3} \text{ m}$$

$$= 80\,(1\cdot73) \text{ m}$$

$$= 138\cdot40$$

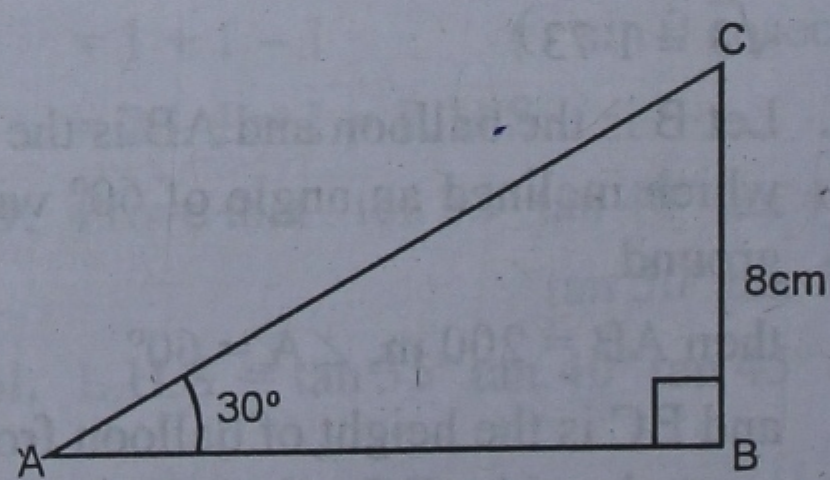
$$= 138\cdot4 \text{ m}$$

$\therefore$ Length of string = 138·4 m. **Ans.**

Q. 5. In a ΔABC, right angled at B, if $\angle A = 30^\circ$ and BC = 8 cm, find the remaining angles and sides.

Sol. In right angled ΔABC

$$\angle B = 90^\circ, \angle A = 30^\circ$$



$$\therefore \quad \angle C = 180^\circ - (\angle B + \angle A)$$

$$= 180^\circ - (90^\circ + 30^\circ)$$

$$= 180^\circ - 120^\circ = 60^\circ$$

$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{BC}{AC}$

$\Rightarrow \sin 30^\circ = \frac{8}{AC} \Rightarrow \frac{1}{2} = \frac{8}{AC}$

$\Rightarrow \quad AC = 16 \text{ cm}$

Again $\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{BC}{AB}$

$\Rightarrow \tan 30^\circ = \frac{8}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{AB}$

Hence, $AB = 8\sqrt{3}$ cm,

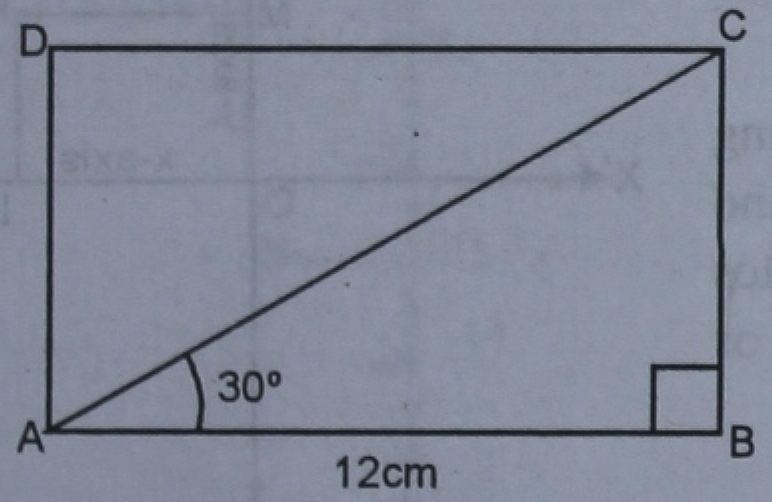
$AC = 16$ cm **Ans.**

Q. 6. In a rectangle ABCD, AB = 12 cm and $\angle BAC = 30^\circ$. Calculate the lengths of side BC and diagonal AC.

Sol. In rect. ABCD,

$AB = 12$ cm

$\angle BAC = 30^\circ, \angle B = 90^\circ$



$\tan\theta = \frac{\text{Perp.}}{\text{Base}} \Rightarrow \tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{12} \Rightarrow BC = \frac{12}{\sqrt{3}}$

$\Rightarrow BC = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ cm

$\cos\theta = \frac{\text{Base}}{\text{Hyp.}}$

$\Rightarrow \cos 30^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{AC}$

$\Rightarrow \sqrt{3}\, AC = 12 \times 2$

$AC = \frac{24}{\sqrt{3}} = \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{24\sqrt{3}}{3} = 8\sqrt{3}$ cm

Hence, $BC = 4\sqrt{3}$ cm

and $AC = 8\sqrt{3}$ cm **Ans.**