# 14 Polygons

## POINTS TO REMEMBER

1. **Polygon :** A closed plane figure bounded by three or more line segments is called a polygon. These line segments are called its sides and by joining two non-consecutive vertices of a polygon is called its diagonal and the point of intersection of two consecutive sides of a polygon is called a vertex.
2. **Convex Polygon :** If each angle of a polygons is less than 180°, then it is called a convex polygon.
3. **Concave Polygon :** If at least one angle of a polygon is a reflex angle *i.e.* more than 180°, then it is called a concave polygon.
4. **Regular Polygon :** If all sides of a polygon are equal and all angles are equal, then it is called a regular polygon.
5. **Theorems :**

(*i*) The sum of all the interior angles of a convex polygon of $n$ sides is $(2n - 4)$ right angles.

(*ii*) The sum of all the exterior angles of a convex polygon is 4 right angles.

6. **Some more results :**

**(i) ForAll Convex Polygons :**

(*i*) Sum of all interior angles of a polygon of $n$ sides = $(2n - 4)$ right angles.

(*ii*) Sum of all exterior angles of a polygon of $n$ sides = 4 right angles.

(*iii*) At each vertex of a polygon, we have :

Interior Angle + Exterior Angle = 180°.

**(ii) For Regular Polygons :**

(*i*) Each interior angle of a regular polygon of $n$ sides $= \dfrac{(2n-4)}{n} rt. \angle s = \left[\dfrac{(2n-4)\times 90}{n}\right]^{\circ}$.

(*ii*) Each exterior angle of a regular polygon of $n$ sides $= \left(\dfrac{360}{n}\right)^{\circ}$.

(*iii*) If each exterior angle of a regular polygon is $x°$, then number of its sides $= \left(\dfrac{360}{x}\right)$.

**Note :** Greater is the number of sides in a regular polygon, greater is the value of its interior angle and smaller is the value of its each exterior angle.

**(iii) Number of diagonals in a polygon of $n$ sides** $= \left[\dfrac{n(n-1)}{2} - n\right]$.

## EXERCISE 14 (A)

**Q. 1.** Write in degrees the sum of all interior angles of a :

(*i*) Hexagon (*ii*) Septagon

(*iii*) Nonagon (*iv*) 15-gon

**Sol.** (*i*) Sum of interior angles of a hexagon

$= (2n - 4)$ right angles

$= (2 \times 6 - 4) \times 90°$

$= (12 - 4) \times 90° = 8 \times 90°$

$= 720°$ **Ans.**

(*ii*) Sum of interior angles of a septagon

$= (2n - 4)$ right angles

$= (2 \times 7 - 4) \times 90°$

$= (14 - 4) \times 90° = 10 \times 90°$

$= 900°$ **Ans.**

(*iii*) Sum of interior angles of nonagon

$= (2n - 4)$ right angles

$= (2 \times 9 - 4) \times 90° = (18 - 4) \times 90°$

$= 14 \times 90° = 1260°$.

(*iv*) Sum of interior angles of a 15-gon

$= (2n - 4)$ right angles

$= (2 \times 15 - 4) \times 90° = (30 - 4) \times 90°$

$= 26 \times 90° = 2340°$ **Ans.**

**Q. 2.** Find the measure, in degrees, of each interior angle of a regular :

(*i*) Pentagon (*ii*) Octagon

(*iii*) Decagon (*iv*) 16-gon

**Sol.** We know that each interior angle of a regular polygon of $n$ sides

$= \frac{(2n-4)}{n}$ right angles.

(*i*) Each interior angle of pentagon

$= \frac{(2n-4)}{n}$ right angles

$= \frac{2 \times 5 - 4}{5} \times 90°$

$= \frac{10-4}{5} \times 90° = \frac{6}{5} \times 90° = 108°$ **Ans.**

(*ii*) Each interior angle of octagion

$= \frac{2n-4}{n}$ *rt.* angles.

$= \frac{2 \times 8 - 4}{8} \times 90°$

$= \frac{16-4}{8} \times 90° = \frac{12}{8} \times 90°$

$= 135°$ **Ans.**

(*iii*) Each interior angle of decagon

$= \frac{2n-4}{n}$ *rt.* angles.

$= \frac{2 \times 10 - 4}{10} \times 90°$

$= \frac{20-4}{10} \times 90° = \frac{16}{10} \times 90°$

$= 144°$ **Ans.**

(*iv*) Each interior angle of 16-gon

$= \frac{2n-4}{n}$ *rt.* angles.

$= \frac{2 \times 16 - 4}{16} \times 98 = \frac{32-4}{16} \times 90°$

$= \frac{28}{16} \times 90° = \frac{315}{2} = 157{\cdot}5°$ **Ans.**

**Q. 3.** Find the measure, in degrees, of each exterior angle of a regular polygon containing :

(*i*) 6 sides (*ii*) 8 sides

(*iii*) 15 sides (*iv*) 20 sides

**Sol.** We know that each exterior angle of a polygon of $n$ sides $= \frac{360°}{n}$.

(*i*) Each exterior angle of 6 sided polygon

$= \frac{360°}{6} = 60°$

(*ii*) Each exterior angle of 8 sided polygon

$= \frac{360°}{8} = 45°$

(*iii*) Each exterior angle of 15 sided polygon

$= \frac{360°}{15} = 24°$

(*iv*) Each exterior angle of 20 sided polygon

$= \frac{360°}{20} = 18°$ **Ans.**

**Q. 4.** Find the number of sides of a polygon, the sum of whose interior angles is :

(*i*) 24 right angles (*ii*) 1620°

(*iii*) 2880°

**Sol.** We know that sum of interior angles of a regular polygon of $n$ sides = $(2n-4)$ right angles.

(*i*) Sum = 24 right angles

$\therefore (2n-4) = 24 \Rightarrow 2n = 24 + 4 = 28$

$\therefore \quad n = \frac{28}{2} = 14$

Hence polygon has 14 sides.

(*ii*) Sum = 1620°

$\therefore (2n-4)$ right angles = 1620°

$\Rightarrow (2n-4) \times 90° = 1620°$

$\Rightarrow 2n - 4 = \frac{1620°}{90°}$

$\Rightarrow 2n - 4 = 18° \Rightarrow 2n = 18 + 4 = 22$

$\therefore \quad n = \frac{22}{2} = 11$

Hence polygon has 11 sides.

(*iii*) Sum = 2888°

$\Rightarrow (2n-4)$ right angles = 2880°

$\Rightarrow 2n - 4 = \frac{2880°}{90°}$

$\Rightarrow 2n - 4 = 32$

$\Rightarrow \quad 2n = 32 + 4 = 36$

$\therefore \quad x = \frac{36}{2} = 18$

Hence polygon has 18 sides. **Ans.**

**Q. 5.** Find the number of sides in a regular polygon, if each of its exterior angles is :

(*i*) 72° (*ii*) 24°

(*iii*) (22·5)° (*iv*) 15°

**Sol.** We know that each exterior angle of a regular polygon of $n$ sides $= \frac{360°}{n}$

(*i*) Exterior angle = 72°

$\therefore \frac{360°}{n} = 72° \Rightarrow n = \frac{360°}{72°} = 5$

$\therefore$ No. of sides of polygon = 5 **Ans.**

(*ii*) Each exterior angle = 24°

$\therefore \frac{360°}{n} = 24 \Rightarrow n = \frac{360°}{24°} = 15$

$\therefore$ No. of sides of the regular polygon = 15 **Ans.**

(*iii*) Each exterior angle = (22·5)°

$\therefore \frac{360°}{n} = 22{\cdot}5°$

$\Rightarrow \quad n = \frac{360°}{22{\cdot}5°} = \frac{360 \times 10}{225} = 16$

$\therefore$ No. of sides of the regular polygon = 16. **Ans.**

(*iv*) Each exterior angle = 15°

$\therefore \quad \frac{360°}{n} = 15° \Rightarrow n = \frac{360°}{15°} = 24$

Hence no. of sides of the regular polygon = 24 **Ans.**

**Q. 6.** Find the number of sides in a regular polygon, if each of its interior angles is :

(*i*) 120° (*ii*) 150°

(*iii*) 160° (*iv*) 165°

**Sol.** We known that each interior angle of a regular polygon of $n$ sides $= \frac{2n-4}{n}$ right angles.

(*i*) Each interior angle = 120°

$\therefore = \frac{2n-4}{n}$ *rt.* Angle = 120°

$\Rightarrow \quad \frac{2n-4}{n} \times 90° = 120°$

$\Rightarrow \frac{2n-4}{n} = \frac{120°}{90°}$

$\Rightarrow \frac{2n-4}{n} = \frac{4}{3} \Rightarrow 6n - 12 = 4n$

$\Rightarrow 6n - 4n = 12 \Rightarrow 2n = 12$

$\therefore n = 6$

Hence number of sides = 6. **Ans.**

(*ii*) Each interior angle = 150°

$\therefore \frac{2n-4}{n}$ right angle = 150°

$\Rightarrow \frac{2n-4}{n} \times 90° = 150°$

$\Rightarrow \frac{2n-4}{n} = \frac{150°}{90} = \frac{5}{3}$

$\Rightarrow 6n - 12 = 5n \Rightarrow 6n - 5n = 12$

$\Rightarrow n = 12$

Hence no. of sides = 12 **Ans.**

(*iii*) Each interior angle = 160°

$\therefore \frac{2n-4}{n}$ right angles = 160°

$\Rightarrow \frac{2n-4}{n} \times 90° = 160°$

$\Rightarrow \frac{2n-4}{n} = \frac{160°}{90} = \frac{16}{9}$

$18n - 36 = 16n \Rightarrow 18n - 16n = 36$

$\Rightarrow 2n = 36 \Rightarrow n = \frac{36}{2} = 18.$

Hence no. of sides = 18 **Ans.**

(*iv*) Each interior angle = 165°

$\therefore \frac{2n-4}{n}$ right angles = 165°

$\Rightarrow \frac{2n-4}{n} \times 90° = 165°$

$\Rightarrow \frac{2n-4}{n} = \frac{165°}{90°} = \frac{11}{6}$

$\Rightarrow 12n - 24 = 11n$

$\Rightarrow 12n - 11n = 24$

$\Rightarrow n = 24$

Hence no. of sides = 24 **Ans.**

**Q. 7.** Is it possible to describe a polygon, the sum of whose interior angles is :

(*i*) 320° (*ii*) 540°

(*iii*) 11 right angles (*iv*) 14 right angles ?

**Sol.** We know that sum of interior angles of a regular polygon of *n* sides = $(2n-4)$ right angles.

(*i*) Sum of interior angles = 320°

$\therefore (2n-4)$ right angles = 320°

$\Rightarrow (2n-4) \times 90° = 320°$

$\Rightarrow 2n - 4 = \frac{320°}{90°} = \frac{32}{9}$

$\Rightarrow 2n = \frac{32}{9} + 4 = \frac{32+36}{9} = \frac{68}{9}$

$n = \frac{68}{9 \times 2} = \frac{34}{9}$

Which is in fraction.

Hence it is not possible to describe a polygon.

(*ii*) Sum of interior angles = 540°

$\therefore (2n-4)$ right angles = 540°

$\Rightarrow (2n-4) \times 90° = 540°$

$\Rightarrow 2n - 4 = \frac{540°}{90°} = 6$

$\Rightarrow 2n = 6 + 4 = 10 \Rightarrow n = \frac{10}{2} = 5$

Yes, it is possible to describe a polygon.

(*iii*) Sum of interior angles = 11 right angles

$\therefore (2n-4)$ right angles = 11 right angles

$\Rightarrow 2n - 4 = 11 \Rightarrow 2n = 11 + 4 = 15$

$\Rightarrow n = \frac{15}{2}$

Which is in fraction.

Hence it is not possible to describe a polygon.

(*iv*) Sum of interior angles = 14 right angles

$\therefore (2n-4)$ right angles = 14 right angles

$\Rightarrow 2n - 4 = 14$

$\Rightarrow 2n = 14 + 4 = 18$

$\Rightarrow \quad n = \frac{18}{2} = 9$

Hence it is possible to describe a polygon.

**Ans.**

**Q. 8.** Is it possible to have a regular polygon, each of whose exterior angle is :

(i) 32° (ii) 18°

(iii) $\frac{1}{8}$ of a right angle (iv) 80° ?

**Sol.** We know that exterior angle of a regular polygon of $n$ sides $= \frac{360°}{n}$

(i) Exterior angle = 32°

$\therefore \frac{360°}{n} = 32° \Rightarrow n = \frac{360°}{32°} = \frac{45}{4}$

which is in fraction.

∴ It is not possible to have a regular polygon.

(ii) Exterior angle = 180°

$\therefore \frac{360°}{n} = 18° \Rightarrow n = \frac{360°}{18°} = 20$

Hence it is possible to have a regular polygon.

(iii) Exterior angle $= \frac{1}{8}$ of right angle

$= \frac{1}{8} \times 90° = \frac{45°}{4}$

$\therefore \frac{360°}{n} = \frac{45°}{4} \Rightarrow n = \frac{360° \times 4}{45} = 32$

∴ It is possible to have a regular polygon.

(iv) Exterior angle = 80°

$\therefore \frac{360°}{n} = 80° \Rightarrow \frac{360°}{80°} = \frac{9}{2}$

which is in fraction.

∴ It is not possible to have a regular polygon. **Ans.**

**Q. 9.** Is it possible to have a regular polygon, each of whose interior angles is :

(i) 120° (ii) 105°

(iii) 175° (iv) 130° ?

**Sol.** We know that each interior angle of a regular polygon of $n$ sides $= \frac{2n-4}{n}$ right angles.

(i) Interior angle = 120°

$\therefore \frac{2n-4}{n}$ right angles = 120°

$\Rightarrow \frac{2n-4}{n} \times 90° = 120°$

$\Rightarrow \frac{2n-4}{n} = \frac{120°}{90°} = \frac{4}{3}$

$\Rightarrow 6n - 12 = 4n \Rightarrow 6n - 4n = 12$

$\Rightarrow 2n = 12 \quad \Rightarrow n = 6$

It is possible to have a regular polygon.

(ii) Interior angle = 105°

$\therefore \frac{2n-4}{n}$ right angle = 105°

$\Rightarrow \frac{2n-4}{n} \times 90° = 105°$

$\Rightarrow \frac{2n-4}{n} = \frac{105°}{90°}$

$\Rightarrow \frac{2n-4}{n} = \frac{7}{6} \Rightarrow 12n - 24 = 7n$

$\Rightarrow 12n - 7n = 24 \Rightarrow 5n = 24$

$\Rightarrow \quad n = \frac{24}{5}$

which is in fraction.

Hence it is not possible to have a regular polygon.

(iii) Interior angle = 175°

$\therefore \frac{2n-4}{n}$ right angle = 175°

$\Rightarrow \frac{2n-4}{n} \times 90° = 175°$

$\Rightarrow \frac{2n-4}{n} = \frac{175°}{90°}$

$\Rightarrow \frac{2n-4}{n} = \frac{35}{18} \Rightarrow 36n - 72 = 35n$

$\Rightarrow 36n - 35n = 72 \Rightarrow n = 72$

∴ It is possible to have a regular polygon.

(*iv*) Interior angle = 130°

$\therefore \frac{2n-4}{n}$ right angle $= 130°$

$\Rightarrow \frac{2n-4}{n} \times 90° = 130° \Rightarrow \frac{2n-4}{n} = \frac{130°}{90}$

$\Rightarrow \frac{2n-4}{n} = \frac{13}{9} \Rightarrow 18n - 36 = 13n$

$\Rightarrow 18n - 13n = 36 \Rightarrow 5n = 36$

$n = \frac{36}{5}$

which is in fraction.

$\therefore$ It is not possible to have a regular polygon. **Ans.**

**Q. 10.** The angles of a quadrilateral are in the ratio 6 : 3 : 2 : 4. Find the angles.

**Sol.** Sum of four angles of a quadrilateral ABCD = 360°

then $\angle A + \angle B + \angle C + \angle D = 360°$

and $\angle A : \angle B : \angle C : \angle D = 6 : 3 : 2 : 4$

Let $\angle A = 6x$, then $\angle B = 3x$,

$\angle C = 2x$ and $\angle D = 4x$

then $6x + 3x + 2x + 4x = 360°$

$\Rightarrow 15x = 360° \Rightarrow x = \frac{360°}{15} = 24°$

$\therefore \angle A = 6x = 6 \times 24° = 144°$

$\angle B = 3x = 3 \times 24° = 72°$

$\angle C = 2x = 2 \times 24° = 48°$

$\angle D = 4x = 4 \times 24° = 96°$ **Ans.**

**Q. 11.** The angles of a pentagon are in the ratio 3 : 4 : 5 : 2 : 4. Find the angles.

**Sol.** Sum of five angles of a pentagon ABCDE

= (2*n* – 4) right angles

= (2 × 5 – 4) × 90° = (10 – 4) × 90°

= 6 × 90° = 540°

The ratio between the angles say $\angle A$, $\angle B$, $\angle C$, $\angle D$, $\angle E$

= 3 : 4 : 5 : 2 : 4

Let $\angle A = 3x$, then $\angle B = 4x$, $\angle C = 5x$, $\angle D = 2x$ and $\angle E = 4x$

$\therefore 3x + 4x + 5x + 2x + 4x = 540°$

$\Rightarrow 18x = 540° \Rightarrow x = \frac{540°}{18} = 30°$

$\therefore \angle A = 3x = 3 \times 30° = 90°$

$\angle B = 4x = 4 \times 30° = 120°$

$\angle C = 5x = 5 \times 30° = 150°$

$\angle D = 2x = 2 \times 30° = 60°$

$\angle E = 4x = 4 \times 30° = 120°$ **Ans.**

**Q. 12.** The angles of a pentagon are $(3x + 5)°$, $(x + 16)°$, $(2x + 9)°$, $(3x - 8)°$ and $(4x - 15)°$ respectively. Find the value of $x$ and hence find the measures of all the angles of the pentagon.

**Sol.** Let angles of pentagon ABCDE are $(3x+5)°$, $(x+16)°$, $(2x+9)°$, $(3x-8)°$ and $(4x-15)°$.

But the sum of these five angles

= (2*n* – 4) right angle

= (2 × 5 – 4) × 90°

= (10 – 4) × 90° = 6 × 90° = 540°

$\therefore 3x + 5 + x + 16 + 2x + 9 + 3x - 8 + 4x - 15 = 540°$

$\Rightarrow 13x + 30 - 23 = 540°$

$\Rightarrow 13x + 7 = 540°$

$\Rightarrow 13x = 540° - 7 = 533°$

$x = \frac{533}{13} = 41$

$\therefore$ First angle $= 3x + 5 = 3 \times 41 + 5 = 123 + 5 = 128°$

Second angle $= x + 16 = 41 + 16 = 57°$

Third angle $= 2x + 9 = 2 \times 41 + 9 = 82 + 9 = 91°$

Fourth angle $= 3x - 8 = 3 \times 41 - 8 = 123 - 8 = 115°$

Fifth angle $= 4x - 15 = 4 \times 41 - 15 = 164 - 15 = 149°$ **Ans.**

**Q. 13.** The angles of a hexagon are $2x°$, $(2x + 25)°$, $3(x - 15)°$, $(3x - 20)°$, $2(x + 5)°$ and $3(x - 5)°$ respectively. Find the value of $x$ and hence find the measures of all the angles of the hexagon.

**Sol.** Angles a hexagon are $2x°$, $(2x + 25)°$, $3(x - 15)°$, $(3x - 20)°$, $2(x + 5)°$ and $3(x - 5)°$.

But sum of angles of a hexagon

$= (2n - 4)$ right angles

$= (2 \times 6 - 4) \times 90°$

$= (12 - 4) \times 90° = 8 \times 90°$

$= 720°$

$\therefore 2x + 2x + 25 + 3(x - 15) + 3x - 20 + 2(x + 5) + 3(x - 5) = 720°$

$\Rightarrow 2x + 2x + 25 + 3x - 45 + 3x - 20 + 2x + 10 + 3x - 15 = 720°$

$\Rightarrow 15x + 35° - 80° = 720$

$\Rightarrow 15x - 45° = 720°$

$\Rightarrow 15x = 720° + 45° \Rightarrow 15x = 765°$

$\Rightarrow x = \frac{765}{15} = 51°$

$\therefore$ First angle $= 2x = 2 \times 51° = 102°$

Second angle $= 2x + 25 = 2 \times 51° + 25° = 102 + 25 = 127°$

Third angle $= 3(x - 15) = 3(51° - 15°) = 3 \times 36° = 108°$

Fourth angle $= 3x - 20 = 3 \times 51° - 20 = 153 - 20 = 133°$

Fifth angle $= 2(x + 5) = 2(51 + 5) = 2 \times 56 = 112°$

Sixth angle $= 3(x - 5) = 3(51 - 5) = 3 \times 46 = 138°$

Hence angles are 102°, 127°, 108°, 133°, 112° and 138°. **Ans.**

**Q. 14.** Three of the exterior angles of a hexagon as 40°, 52° and 85° respectively and each of the remaining exterior angles is $x°$. Calculate the value of $x$.

**Sol.** Sum of exterior angles of a hexagon

$= 360°$

Three angles are 40°, 52° and 85° and three angles are $x°$ each.

$\therefore 40° + 52° + 85° + x° + x° + x° = 360°$

$\Rightarrow 177° + 3x° = 360°$

$\Rightarrow 3x° = 360° - 177° = 183°$

$\therefore x = \frac{183}{3} = 61°$

Hence $x = 61°$ **Ans.**

**Q. 15.** One angle of an octagon is 100° and the other angles are all equal. Find the measure of each of the equal angles.

**Sol.** One angles of an octagon = 100°

Let each of the other 3 angles $= x°$

But sum of interior angles of an octagon

$= (2x - 4)$ right angles

$= (2 \times 8 - 4) \times 90° = (16 - 4) \times 90°$

$= 12 \times 90° = 1080°$

$\therefore 100 + 7x = 1080 \Rightarrow 7x = 1080 - 100$

$\Rightarrow 7x = 980° \Rightarrow x = \frac{980}{7} = 140°$

Hence each angle of the remaining angles $= 140°$ **Ans.**

**Q. 16.** The interior angle of a regular polygon is double the exterior angle. Find the number of sides in the polygon.

**Sol.** Let no. of sides of a regular polygon $= x$

But sum of interior angle and exterior angle = 180°

Let each exterior angle $= x°$

then interior angle $= 2x$

$\therefore x + 2x = 180° \Rightarrow 3x = 180°$

$x = \frac{180°}{3} = 60°$.

Now $x \times$ exterior angle = 360°

$x \times 60° = 360° \Rightarrow x = \frac{360°}{60°} = 6$

$\therefore$ No. of sides of the regular polygon $= 6$.

**Q. 17.** The ratio of each interior angle to each exterior angle of a regular polygon is 7 : 2. Find the number of sides in the polygon.

**Sol.** Let number of sides of regular polygon $= 3$

and ratio of interior angle with exterior angle $= 7 : 2$

Let each interior angle $= 7x$

and each exterior angle $= 2x$

$\therefore \quad 7x + 2x = 180^\circ$

$\Rightarrow \quad 9x = 180^\circ$

$$x = \frac{180^\circ}{9} = 20^\circ$$

$\therefore$ Each exterior angles $= 20x^\circ = 2 \times 20^\circ = 40^\circ$

But sum of exterior angles of a regular polygon of $x$ sides $= 360^\circ$

$\Rightarrow \quad x \times 40^\circ = 360^\circ$

$$\Rightarrow \quad x = \frac{360^\circ}{40^\circ} = 9$$

$\therefore$ No. of sides of a regular polygon $= 9$.

**Q. 18.** The sum of the interior angles of a polygon is 6 times the sum of its exterior angles. Find the number of sides in the polygon.

**Sol.** Sum of the exterior angles of a regular polygon of $x$ sides $= 360^\circ$

$\therefore$ Sum of its interior angles

$= 360^\circ \times 6 = 2160^\circ$

But sum of interior angles of the polygon

$= (2x - 4)$ right angles

$\therefore (2x - 4) \times 90^\circ = 2160^\circ$

$$\Rightarrow \quad 2x - 4 = \frac{2160^\circ}{90^\circ}$$

$\Rightarrow \quad 2x - 4 = 24$

$\Rightarrow \quad 2x = 24 + 4 = 28$

$$\therefore \quad x = \frac{28}{2} = 14$$

Hence number of sides $= 14$ **Ans.**

**Q. 19.** Two angles of a convex polygon are right angles and each of the other angles is $120^\circ$. Find the number of sides of the polygon.

**Sol.** $\because$ Two angles of a convex polygon $= 90^\circ$ each

$\therefore$ Exterior angles will be $180^\circ - 90^\circ = 90^\circ$ each

Each of other interior angles is $120^\circ$

$\therefore$ Each of exterior angles will be $180^\circ - 120^\circ = 60^\circ$

But, the sum of its exterior angles $= 360^\circ$

Let no. of sides $= n$

Then $90^\circ + 90^\circ + (n - 2) \times 60^\circ = 360^\circ$

$180^\circ + (n - 2)\, 60^\circ = 360^\circ$

$60^\circ\,(n - 2) = 360^\circ - 180^\circ = 180^\circ$

$$n - 2 = \frac{180^\circ}{60^\circ} = 3$$

$\therefore \quad n = 3 + 2 = 5$

Hence, number of sides $= 5$ **Ans.**

**Q. 20.** The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides is each polygon.

**Sol.** The ratio between the sides of two regular polygon $= 3 : 4$

Let number of sides in the first polygon $= 3x$

and number of sides in the second $= 4x$

Sum of interior angles of the first polygon

$= (2 \times 3x - 4)$ right angles

$= (6x - 4)$ right angles

and sum of interior angles of the second polygon

$= (2 \times 4x - 4)$ right angles

$= (8x - 4)$ right angles.

$\therefore (6x - 4)$ right angles $: (8x - 4)$ right angles $= 2 : 3$

$\Rightarrow \quad (6x - 4) : (8x - 4) = 2 : 3$

$$\Rightarrow \quad \frac{6x - 4}{8x - 4} = \frac{2}{3}$$

$\Rightarrow \quad 18x - 12 = 16x - 8$

$\Rightarrow \quad 18x - 16x = -8 + 12$

$\Rightarrow \quad 2x = 4$

$\therefore \quad x = 2$

Hence number of sides in the first polygon

$= 3x = 3 \times 2 = 6$

and no. of sides of the second polygon

$= 4x = 4 \times 2 = 8$ **Ans.**

**Q. 21.** The number of sides of two regular polygons are in the ratio 4 : 5 and their interior angles are in the ratio 15 : 16. Find the number of sides in each polygon.

**Sol.** Ratio between the sides of two regular polygon = 4 : 5

Let no. of sides of first polygon = $4x$

and no. of sides the second polygon = $5x$

$\therefore$ Interior angle of the first polygon

$$= \frac{2 \times 4x - 4}{4x} \text{ right angles}$$

$$= \frac{8x-4}{4x} = \frac{2x-1}{x} \text{ right angles}$$

and interior angle of second polygn

$$= \frac{2 \times 5x - 4}{5x} \text{ right angle}$$

$$= \frac{10x-4}{5x} \text{ right angle}$$

$$\therefore \quad \frac{2x-1}{x} : \frac{10x-4}{5x} = 15:16$$

$$\Rightarrow \quad \frac{2x-1}{x} \times \frac{5x}{10x-4} = \frac{15}{16}$$

$$\Rightarrow \quad \frac{5(2x-1)}{10x-4} = \frac{15}{16}$$

$$\Rightarrow \quad \frac{10x-5}{10x-4} = \frac{15}{16}$$

$$\Rightarrow \quad 160x - 80 = 150x - 60$$

$$\Rightarrow \quad 160x - 150x = -60 + 80$$

$$\Rightarrow \quad 10x = 20$$

$$x = \frac{20}{10} = 2$$

$\therefore$ No. of sides of the first polygon = $4x$

$= 4 \times 2 = 8$

and no. of sides of the second polygon

$= 5 \times 2 = 10$ **Ans.**

**Q. 22.** How many diagonals are there in a

(*i*) Pentagon (*ii*) Hexagon

(*iii*) Octagon ?

**Sol.** No. of diagonals of a polygon of $n$ sides

$$= \frac{1}{2}n(n-1) - n$$

(*i*) $\therefore$ No.of diagonals in a pentagon

$$= \frac{1}{2}n(n-1) - n$$

$$= \frac{1}{2} \times 5(5-1) - 5 \quad \text{(Here } n = 5)$$

$$= \frac{1}{2} \times 5 \times 4 - 5$$

$$= 10 - 5 = 5$$

(*ii*) No. of diagonals in a hexagon

$$= \frac{1}{2} \times 6(6-1) - 6 \quad \text{(Here } n = 6)$$

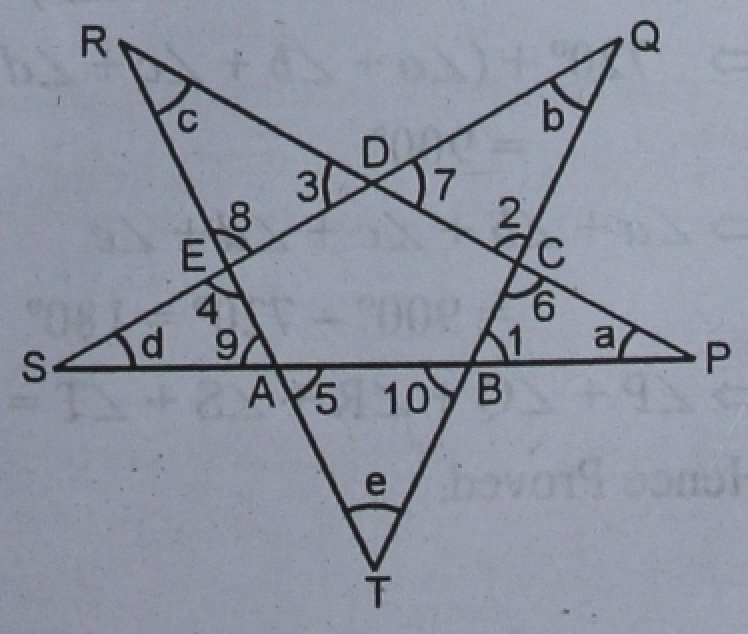
$$= \frac{1}{2} \times 6 \times 5 - 6 = 15 - 6 = 9$$

(*iii*) No. of diagonals in an octagon

$$= \frac{1}{2} \times 8(8-1) - 8 \quad \text{(Here } x = 8)$$

$$= \frac{1}{2} \times 8 \times 7 - 8 = 28 - 8 = 20 \text{ Ans.}$$

**Q. 23.** The alternate sides of any pentagon are produced to meet, so as to form a star-shaped figure, shown in the figure. Prove that the sum of measures of the angles at the vertices of the star is 180°.



**Given :** The alternate sides of a pentagon ABCDE are produce to meet at P, Q, R, S and T so as to form a star shaped figure.

**To Prove :** $\angle P + \angle Q + \angle R + \angle S + \angle T = 180^\circ$ ...(*i*)

or $\angle a + \angle b + \angle c + \angle d + \angle e = 180^\circ$

**Proof :** $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360^\circ$ ...(*i*)

(Sum of ext. angles of a polygon)

Similarly $\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 360^\circ$ ...(*ii*)

But in $\Delta BCP = \angle 1 + \angle b + \angle a = 180^\circ$ ...(*iii*)

(Sum of angles of a triangle)

Similarly in $\Delta CDQ$, $\angle 2 + \angle 7 + \angle b = 180^\circ$ ...(*iv*)

In $\Delta$ DER,

$= \angle 3 + \angle 8 + \angle C = 180^\circ$ ...(*v*)

In $\Delta$ EAS,

$= \angle 4 + \angle 9 + \angle d = 180^\circ$ ...(*vi*)

and in $\Delta ABT = \angle 5 + \angle 10 + \angle e = 180$ ...(*vii*)

Adding (*iii*) to (*vii*)

$\angle 1 + \angle 6 + \angle a + \angle 2 + \angle 7 + \angle b + \angle 3 + \angle 8 + \angle c + \angle 4 + \angle 9 + d + \angle 5 + \angle 10 + \angle e$

$= 180^\circ + 180^\circ + 180^\circ + 180^\circ + 180^\circ$

$\Rightarrow (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5) + (\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10) + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$

$\Rightarrow 360^\circ + 360^\circ + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$

$\Rightarrow 720^\circ + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$

$\Rightarrow \angle a + \angle b + \angle c + \angle d + \angle e = 900^\circ - 720^\circ = 180^\circ$

$\Rightarrow \angle P + \angle Q + \angle R + \angle S + \angle T = 180^\circ$

Hence Proved.

## EXERCISE 14 (B)

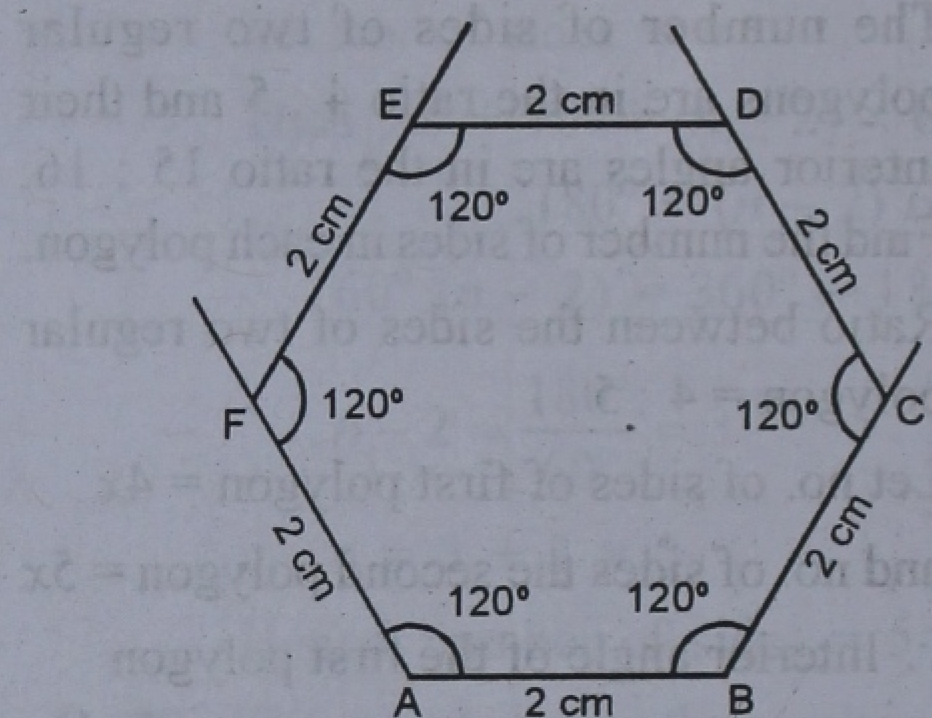
**Q. 1.** Construct a regular hexagon of side 2 cm, using ruler and compasses only.

**Sol.** We know that each angle of a regular hexagon = 120°.

**Steps of Construction :**

(*i*) Draw AB = 2 cm.

(*ii*) At A and B, draw rays making an angle of 120° each.




(*iii*) Cut off AF = BC = 2 cm.

(*iv*) Again at F and C, draw rays making an angle of 120° each.

(*v*) Cut off FE = CD = 2 cm.

(*vi*) Join ED.

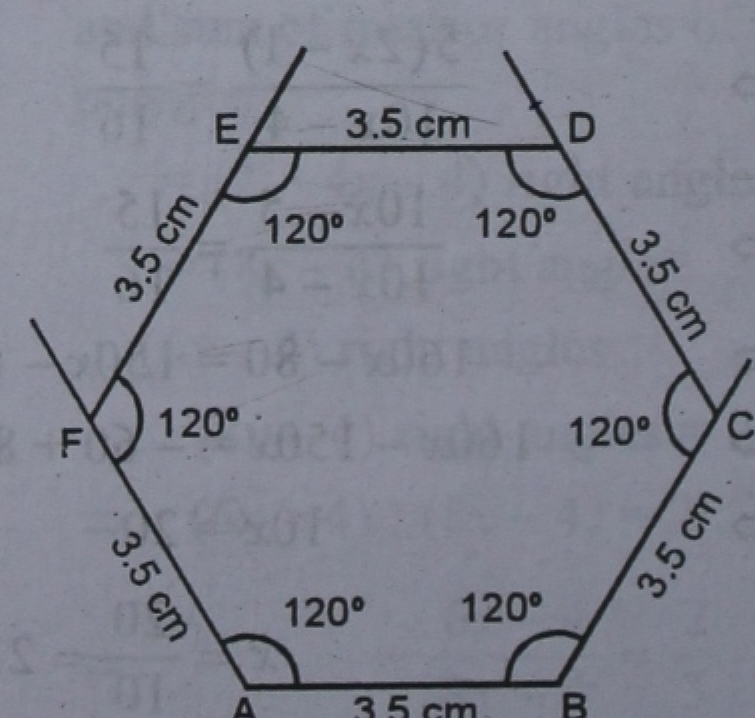
Then, ABCDEF is the required hexagon.

**Q. 2.** Construct a regular hexagon of side 3·5 cm, using ruler and compasses only.

**Sol.** We know that each angle of a regular hexagon = 120°

**Steps of Construction :**

(*i*) Draw AB = 3·5 cm.



(*ii*) At A and B, draw rays making an angle of 120° each.

(*iii*) Cut off AF = BC = 3·5 cm

(*iv*) At F and C, draw rays making an angle of 120° each.

(*v*) Cut off FE = CD = 3·5 cm.

(*vi*) Join ED.

Then, ABCDEF is the required to regular hexagon.

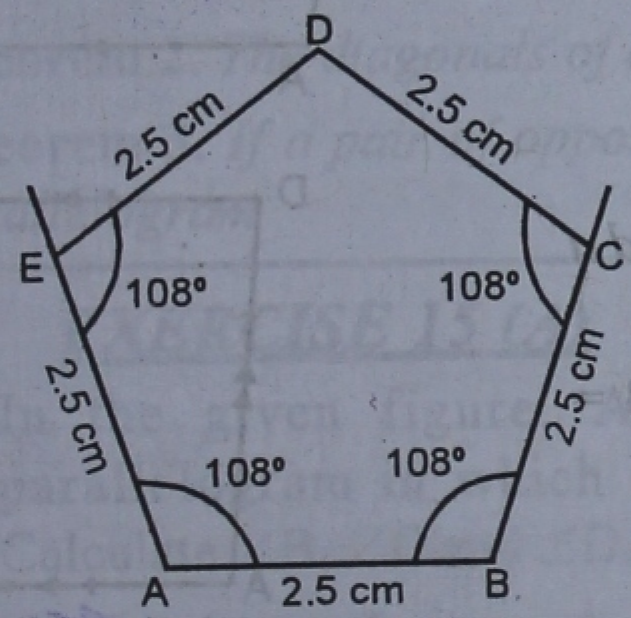
**Q. 3.** Construct a regular pentagon of side 2·5 cm. Using ruler, compasses and protractor.

**Sol.** We know that each angle of a regular pentagon = 108°.

**Steps of Construction :**

(*i*) Draw a line segment AB = 2·5 cm.

(*ii*) At A and B, draw rays making an angle of 108° each.




(*iii*) Cut off AE = BC = 2·5 cm.

(*iv*) At E and C, draw rays making an angle of 108° each meeting each other at D.

Then, ABCDE is the required regular pentagon.

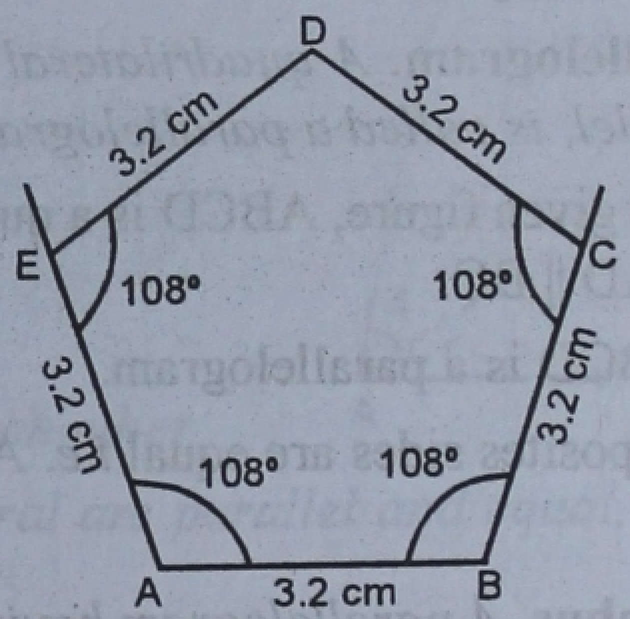
**Q. 4.** Construct a regular pentagon of side 3·2 cm, using ruler, compasses and protractor.

**Sol.** We know that each angle of a regular pentagon = 108°

**Steps of Construction :**

(*i*) Draw a line segment AB = 3·2 cm.

(*ii*) At A and B, draw rays making an angle of 108°.




(*iii*) Cut off AE = BC = 3·2 cm.

(*iv*) At E and C, draw rays making an angle of 108° each meeting each other at D.

Then, ABCDE is the required pentagon.