UNIT 3 – ALGEBRA

4

Expansions

POINTS TO REMEMBER

1. (*i*) $(a+b)^2 = a^2 + b^2 + 2\,ab$ (*ii*) $(a-b)^2 = a^2 + b^2 - 2\,ab$

(*iii*) $(a+b)^2 + (a-b)^2 = 2\,(a^2 + b^2)$ (*iv*) $(a+b)^2 - (a-b)^2 = 4\,ab$

(*v*) $(a+b)\,(a-b) = (a^2 - b^2)$ (*vi*) $(a+b)^2 = (a-b)^2 + 4\,ab$

(*vii*) $(a-b)^2 = (a+b)^2 - 4\,ab$.

2. (*i*) $\left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$ (*ii*) $\left(a-\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

(*iii*) $\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right) = \left(a^2 - \frac{1}{a^2}\right)$ (*iv*) $\left(a+\frac{1}{a}\right)^2 + \left(a-\frac{1}{a}\right)^2 = 2\left(a^2 + \frac{1}{a^2}\right)$

(*v*) $\left(a+\frac{1}{a}\right)^2 - \left(a-\frac{1}{a}\right)^2 = 4$ (*vi*) $\left(a+\frac{1}{a}\right)^2 = \left(a-\frac{1}{a}\right)^2 + 4$

(*vii*) $\left(a-\frac{1}{a}\right)^2 = \left(a+\frac{1}{a}\right)^2 - 4$.

3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2\,(ab + bc + ca)$.

4. (*i*) $(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

(*ii*) $(x+a)\,(x-b) = x^2 + (a-b)\,x - ab$

(*iii*) $(x-a)\,(x+b) = x^2 - (a-b)\,x - ab$

(*iv*) $(x-a)\,(x-b) = x^2 - (a+b)\,x + ab$.

Note $(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

$= x^2 +$ (Algebraic sum of 2nd terms) $x +$ (Product of second terms).

5. $(a+b)^3 = a^3 + b^3 + 3\,ab\,(a+b) = a^3 + 3\,a^2\,b + 3\,ab^2 + b^3$ and $\left(a+\frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a+\frac{1}{a}\right)$.

6. $(a-b)^3 = a^3 - b^3 - 3\,ab\,(a-b) = a^3 - 3\,a^2\,b + 3\,ab^2 - b^3$ and $\left(a-\frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a-\frac{1}{a}\right)$.

7. If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3\,abc$.

EXERCISE 4 (A)

Using the standard formulae, expand each of the following (Q. No. 1 to 13) :

Q. 1. (*i*) $(4a+9)^2$

(*ii*) $(3x+10y)^2$

(*iii*) $(\sqrt{2}\,m+\sqrt{3}\,n)^2$.

Sol. (*i*) $(4a+9)^2 = (4a)^2 + (9)^2 + 2\times 4a\times 9$

$= 16a^2 + 81 + 72a$

(*ii*) $(3x+10y)^2$

$= (3x)^2 + (10y)^2 + 2\times 3x\times 10y$

$= 9x^2 + 100y^2 + 60xy$

(*iii*) $(\sqrt{2}\,m+\sqrt{3}\,n)^2 = (\sqrt{2}\,m)^2 + (\sqrt{3}\,n)^2 + 2\times\sqrt{2}\,m\times\sqrt{3}\,n$

$= 2m^2 + 3n^2 + 2\sqrt{6}\,mn$ **Ans.**

Q. 2. (*i*) $(2a^2+3b)^2$

(*ii*) $(3x^2y+z)^2$

(*iii*) $\left(2x+\frac{1}{3x}\right)^2$.

Sol. (*i*) $(2a^2+3b)^2$

$= (2a^2)^2 + (3b)^2 + 2\times 2a^2\times 3b$

$= 4a^4 + 9b^2 + 12a^2b$

(*ii*) $(3x^2y+z)^2$

$= (3x^2y)^2 + (z)^2 + 2\times 3x^2y\times z$

$= 9x^4y^2 + z^2 + 6x^2yz$

(*iii*) $\left(2x+\frac{1}{3x}\right)^2$

$= (2x)^2 + \left(\frac{1}{3x}\right)^2 + 2\times 2x\times\frac{1}{3x}$

$= 4x^2 + \frac{1}{9x^2} + \frac{4}{3}$ **Ans.**

Q. 3. (*i*) $\left(\frac{2}{5}x+\frac{5}{6}y\right)^2$ (*ii*) $\left(\frac{x}{3}+\frac{6}{x}\right)^2$

(*iii*) $\left(6+\frac{5}{x}\right)^2$.

Sol. (*i*) $\left(\frac{2}{5}x+\frac{5}{6}y\right)^2 = \left(\frac{2}{5}x\right)^2 + \left(\frac{5}{6}y\right)^2 + 2\times\frac{2}{5}x\times\frac{5}{6}y$

$= \frac{4}{25}x^2 + \frac{25}{36}y^2 + \frac{2}{3}xy$

(*ii*) $\left(\frac{x}{3}+\frac{6}{x}\right)^2 = \left(\frac{x}{3}\right)^2 + \left(\frac{6}{x}\right)^2 + 2\times\frac{x}{3}\times\frac{6}{x}$

$= \frac{x^2}{9} + \frac{36}{x^2} + 4$

(*iii*) $\left(6+\frac{5}{x}\right)^2 = (6)^2 + \left(\frac{5}{x}\right)^2 + 2\times 6\times\frac{5}{x}$

$= 36 + \frac{25}{x^2} + \frac{60}{x}$ **Ans.**

Q. 4. (*i*) $(5x-3y)^2$

(*ii*) $(3a-7b)^2$

(*iii*) $\left(\frac{1}{2}x-\frac{3}{2}y\right)^2$.

Sol. (*i*) $(5x-3y)^2$

$= (5x)^2 + (3y)^2 - 2\times 5x\times 3y$

$= 25x^2 + 9y^2 - 30xy$

(*ii*) $(3a-7b)^2$

$= (3a)^2 + (7b)^2 - 2\times 3a\times 7b$

$= 9a^2 + 49b^2 - 42ab$

(*iii*) $\left(\frac{1}{2}x-\frac{3}{2}y\right)^2 = \left(\frac{1}{2}x\right)^2 + \left(\frac{3}{2}y\right)^2 - 2\times\frac{1}{2}x\times\frac{3}{2}y$

$= \frac{1}{4}x^2 + \frac{9}{4}y^2 - \frac{3}{2}xy$ **Ans.**

Q. 5. (*i*) $\left(a^2-\frac{b}{2}\right)^2$ (*ii*) $\left(\frac{3a}{2b}-\frac{2b}{3a}\right)^2$

(*iii*) $\left(5x-\frac{2}{3x}\right)^2$.

Sol. (*i*) $\left(a^2-\frac{b}{2}\right)^2=(a^2)^2+\left(\frac{b}{2}\right)^2$

$$-2\times a^2\times\frac{b}{2}$$

$$=a^4+\frac{b^2}{4}-a^2b$$

(*ii*) $\left(\frac{3a}{2b}-\frac{2b}{3a}\right)^2=\left(\frac{3a}{2b}\right)^2+\left(\frac{2b}{3a}\right)^2$

$$-2\times\frac{3a}{2b}\times\frac{2b}{3a}$$

$$=\frac{9a^2}{4b^2}+\frac{4b^2}{9a^2}-2$$

(*iii*) $\left(5x-\frac{2}{3x}\right)^2=(5x)^2+\left(\frac{2}{3x}\right)^2$

$$-2\times 5x\times\frac{2}{3x}$$

$$=25x^2+\frac{4}{9x^2}-\frac{20}{3}$$ **Ans.**

Q. 6. (*i*) $(a+2b+3c)^2$

(*ii*) $(3x+5y-2z)^2$

(*iii*) $(2x-3y+7z)^2$

Sol. (*i*) $(a+2b+3c)^2$

$=(a)^2+(2b)^2+(3c)^2+2\times a\times 2b$

$+2\times 2b\times 3c+2\times 3c\times a$

$=a^2+4b^2+9c^2+4ab+12bc+6ca$

(*ii*) $(3x+5y-2z)^2=(3x)^2+(5y)^2$

$+(2z)^2+2\times 3x\times 5y-2\times 5y\times 2z$

$-2\times 2z\times 3x$

$=9x^2+25y^2+4z^2+30xy-20yz$

$-12zx$

(*iii*) $(2x-3y+7z)^2=(2x)^2+(3y)^2$

$+(7z)^2-2\times 2x\times 3y-2\times 3y\times 7z$

$+2\times 7z\times 2x$

$=4x^2+9y^2+49z^2-12xy$

$-42yz+28zx$ **Ans.**

Q. 7. (*i*) $(6-2y+4z)^2$

(*ii*) $(4x-3y+z)^2$

(*iii*) $(7-2x-3y)^2$

Sol. (*i*) $(6-2y+4z)^2=(6)^2+(2y)^2$

$+(4z)^2-2\times 6\times 2y-2\times 2y\times 4z$

$+2\times 4z\times 6$

$=36+4y^2+16z^2-24y-16yz+48z$

(*ii*) $(4x-3y+z)^2=(4x)^2+(3y)^2+(z)^2$

$-2\times 4x\times 3y-2\times 3y\times z$

$+2\times z\times 4x$

$=16x^2+9y^2+z^2-24xy-6yz+8zx$

(*iii*) $(7-2x-3y)^2=(7)^2+(2x)^2+(3y)^2$

$-2\times 7\times 2x+2\times 2x\times 3y$

$-2\times 3y\times 7$

$=49+4x^2+9y^2-28x+12xy-42y$

Ans.

Q. 8. (*i*) $\left(\frac{a}{2}+\frac{b}{3}+\frac{c}{4}\right)^2$

(*ii*) $\left(\frac{2x}{3}+\frac{3}{2y}-2\right)^2$

(*iii*) $\left(2x+\frac{3}{x}-1\right)^2$.

Sol. (*i*) $\left(\frac{a}{2}+\frac{b}{3}+\frac{c}{4}\right)^2$

$=\left(\frac{a}{2}\right)^2+\left(\frac{b}{3}\right)^2+\left(\frac{c}{4}\right)^2$

$+2\times\frac{a}{2}\times\frac{b}{3}+2\times\frac{b}{3}\times\frac{c}{4}+2\times\frac{c}{4}\times\frac{a}{2}$

$=\frac{a^2}{4}+\frac{b^2}{9}+\frac{c^2}{16}+\frac{1}{3}ab+\frac{1}{6}bc+\frac{1}{4}ca$

(*ii*) $\left(\frac{2x}{3}+\frac{3}{2y}-2\right)^2$

$=\left(\frac{2x}{3}\right)^2+\left(\frac{3}{2y}\right)^2+(2)^2$

$+2\times\frac{2}{3}x\times\frac{3}{2y}-2\times\frac{3}{2y}\times 2-2\times 2\times\frac{2x}{3}$

$$= \frac{4}{9}x^2 + \frac{9}{4y^2} + 4 + 2.\frac{x}{y} - \frac{6}{y} - \frac{8}{3}x$$

$$\Rightarrow \frac{4x^2}{9} + \frac{9}{4y^2} + 4 + 2.\frac{x}{y} - \frac{6}{y} - \frac{8x}{3}$$

(*iii*) $\left(2x + \frac{3}{x} - 1\right)^2 = (2x)^2 + \left(\frac{3}{x}\right)^2 + (1)^2$

$$+2 \times 2x \times \frac{3}{x} - 2 \times \frac{3}{x} \times 1 - 2 \times 1 \times 2x$$

$$= 4x^2 + \frac{9}{x^2} + 1 + 12 - \frac{6}{x} - 4x$$

$$= 4x^2 + \frac{9}{x^2} + 13 - \frac{6}{x} - 4x \text{ Ans.}$$

Q. 9. (*i*) $(x + 7)(x + 4)$

(*ii*) $(a + 13)(a - 8)$

(*iii*) $(y - 6)(y - 4)$.

Sol. (*i*) $(x + 7)(x + 4)$

$= (x)^2 + (7 + 4)x + 7 \times 4$

$= x^2 + 11x + 28$

(*ii*) $(a + 13)(a - 8)$

$= a^2 + (13 - 8)a + 13 \times (-8)$

$= a^2 + 5a - 104$

(*iii*) $(y - 6)(y - 4)$

$= y^2 + (-6 - 4)y + (-6)(-4)$

$= y^2 - 10y + 24$ **Ans.**

Q. 10. (*i*) $(9 + 2x)(9 - 3x)$

(*ii*) $(5x - 4y).(5x + 3y)$

(*iii*) $(3 - 7a)(3 + 4a)$.

Sol. (*i*) $(9 + 2x)(9 - 3x)$

$= (9)^2 + (2x - 3x) \times 9 + 2x \times (-3x)$

$= 81 - 9x - 6x^2$

(*ii*) $(5x - 4y)(5x + 3y)$

$= (5x)^2 + (-4y + 3y) \times 5x$

$+ (-4y)(3y)$

$= 25x^2 - 5xy - 12y^2$

(*iii*) $(3 - 7a)(3 + 4a)$

$= (3)^2 + (-7a + 4a) \times 3 + (-7a)(4a)$

$= 9 - 9a - 28a^2$. **Ans.**

Q. 11. (*i*) $(3a + 2b)(3a - 2b)$

(*ii*) $\left(5x + \frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$

(*iii*) $\left(2x^2 + \frac{3}{x^2}\right)\left(2x^2 - \frac{3}{x^2}\right)$.

Sol. (*i*) $(3a + 2b)(3a - 2b)$

$= (3a)^2 - (2b)^2 = 9a^2 - 4b^2$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

(*ii*) $\left(5x + \frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$

$= (5x)^2 - \left(\frac{1}{5x}\right)^2$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$= 25x^2 - \frac{1}{25x^2}$ **Ans.**

(*iii*) $\left(2x^2 + \frac{3}{x^2}\right)\left(2x^2 - \frac{3}{x^2}\right)$

$= (2x^2)^2 - \left(\frac{3}{x^2}\right)^2$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$= 4x^4 - \frac{9}{x^4}$ **Ans.**

Q. 12. (*i*) $(2 - x)(2 + x)(4 + x^2)$

(*ii*) $(x + y)(x - y)(x^2 + y^2)$.

Sol. (*i*) $(2 - x)(2 + x)(4 + x^2)$

$= \{(2)^2 - (x)^2\}(4 + x^2)$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$= (4 - x^2)(4 + x^2)$

$= (4)^2 - (x^2)^2 = 16 - x^4$ **Ans.**

(*ii*) $(x + y)(x - y)(x^2 + y^2)$

$= \{(x)^2 - (y)^2\}(x^2 + y^2)$

$= (x^2 - y^2)(x^2 + y^2)$

$= (x^2)^2 - (y^2)^2 = x^4 - y^4$ **Ans.**

Q. 13. (*i*) $(x - 2)(x - 3)(x + 4)$

(*ii*) $(x - 5)(2x - 1)(2x + 3)$

Sol. (*i*) $(x-2)(x-3)(x+4)$

$= \{x^2 + (-2-3)x + (-2)(-3)\}(x+4)$

$= (x^2 - 5x + 6)(x+4)$

$= x^3 - 5x^2 + 6x + 4x^2 - 20x + 24$

$= x^3 - x^2 - 14x + 24$ **Ans.**

(*ii*) $(x-5)(2x-1)(2x+3)$

$= (x-5)\{(2x)^2 + (-1+3)2x + (-1)(3)\}$

$= (x-5)\{4x^2 + 4x - 3\}$

$= 4x^3 + 4x^2 - 3x - 20x^2 - 20x + 15$

$= 4x^3 - 16x^2 - 23x + 15$ **Ans.**

Q. 14. Simplify :

(*i*) $(a+b)^2 + (a-b)^2$

(*ii*) $(a+b)^2 - (a-b)^2$

(*iii*) $\left(x+\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^2$

(*iv*) $\left(x+\frac{1}{x}\right)^2 - \left(x-\frac{1}{x}\right)^2$

(*v*) $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2 - \left(\frac{2b}{a}-\frac{a}{2b}\right)^2$

(*vi*) $\left(3x-\frac{1}{3x}\right)^2 - \left(3x+\frac{1}{3x}\right)\left(3x-\frac{1}{3x}\right)$

(*vii*) $(5a+3b)^2 - (5a-3b)^2 - 60ab$

(*viii*) $(3x+1)^2 - (3x+2)(3x-1)$.

Sol. (*i*) $(a+b)^2 + (a-b)^2$

$= a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$

$= 2a^2 + 2b^2 = 2(a^2 + b^2)$ **Ans.**

(*ii*) $(a+b)^2 - (a-b)^2$

$= (a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$

$= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$

$= 4ab$ **Ans.**

(*iii*) $\left(x+\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^2$

$= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^2 + \frac{1}{x^2} - 2\right)$

$= x^2 + \frac{1}{x^2} + 2 + x^2 + \frac{1}{x^2} - 2$

$= 2x^2 + \frac{2}{x^2} = 2\left(x^2 + \frac{1}{x^2}\right)$ **Ans.**

(*iv*) $\left(x+\frac{1}{x}\right)^2 - \left(x-\frac{1}{x}\right)^2$

$= \left(x^2 + \frac{1}{x^2} + 2\right) - \left(x^2 + \frac{1}{x^2} - 2\right)$

$= x^2 + \frac{1}{x^2} + 2 - x^2 - \frac{1}{x^2} + 2 = 4$ **Ans.**

(*v*) $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2 - \left(\frac{2b}{a}-\frac{a}{2b}\right)^2$

$= \left\{\left(\frac{a}{2b}\right)^2 + \left(\frac{2b}{a}\right)^2 + 2 \times \frac{a}{2b} \times \frac{2b}{a}\right\}$

$- \left\{\left(\frac{2b}{a}\right)^2 + \left(\frac{a}{2b}\right)^2 - 2 \times \frac{2b}{a} \times \frac{a}{2b}\right\}$

$= \left(\frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2\right)$

$- \left(\frac{4b^2}{a^2} + \frac{a^2}{4b^2} - 2\right)$

$= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2 - \frac{4b^2}{a^2} - \frac{a^2}{4b^2} + 2$

$= 4$ **Ans.**

(*vi*) $\left(3x-\frac{1}{3x}\right)^2 - \left(3x+\frac{1}{3x}\right)\left(3x-\frac{1}{3x}\right)$

$\left(3x-\frac{1}{3x}\right)^2 - \left\{(3x)^2 - \left(\frac{1}{3x}\right)^2\right\}$

$\{\because (a+b)(a-b) = a^2 - b^2\}$

$= \left\{(3x)^2 + \left(\frac{1}{3x}\right)^2 - 2 \times 3x \times \frac{1}{3x}\right\}$

$- \left\{9x^2 - \frac{1}{9x^2}\right\}$

$$= \left(9x^2 + \frac{1}{9x^2} - 2\right) - \left(9x^2 - \frac{1}{9x^2}\right)$$

$$= 9x^2 + \frac{1}{9x^2} - 2 - 9x^2 + \frac{1}{9x^2}$$

$$= \frac{2}{9}x^2 - 2 = 2\left(\frac{1}{9}x^2 - 1\right)$$ **Ans.**

(*vii*) $(5a + 3b)^2 - (5a - 3b)^2 - 60ab$

$= \{(5a)^2 + (3b)^2 + 2 \times 5a \times 3b\}$

$- \{(5a)^2 + (3b)^2 - 2 \times 5a \times 3b\} - 60ab$

$= (25a^2 + 9b^2 + 30ab)$

$- (25a^2 + 9b^2 - 30ab) - 60ab$

$= 25a^2 + 9b^2 + 30ab - 25a^2 - 9b^2$

$+ 30ab - 60ab$

$= 0$ **Ans.**

(*viii*) $(3x + 1)^2 - (3x + 2)(3x - 1)$

$= \{(3x)^2 + (1)^2 + 2 \times 3x \times 1\}$

$- \{(3x)^2 + (2 - 1)3x + 2 \times (-1)\}$

$= (9x^2 + 1 + 6x) - (9x^2 + 3x - 2)$

$= 9x^2 + 1 + 6x - 9x^2 - 3x + 2$

$= 3x + 3 = 3(x + 1)$ **Ans.**

Q. 15. (*i*) If $a + b = 7$ and $ab = 10$, find the value of $(a - b)$.

(*ii*) If $x - y = 5$ and $xy = 24$, find the value of $(x + y)$.

Sol. (*i*) $(a - b)^2 = (a + b)^2 - 4ab$

$= (7)^2 - 4 \times 10 = 49 - 40$

$= 9 = (\pm 3)^2$

$\therefore \quad a - b = \pm 3$ **Ans.**

(*ii*) $(x + y)^2 = (x - y)^2 + 4xy$

$= (5)^2 + 4 \times 24 = 25 + 96 = 121$

$= (\pm 11)^2$

$\therefore x + y = \pm 11$ **Ans.**

Q. 16. If $(3a + 4b) = 16$ and $ab = 4$, find the value of $(9a^2 + 16b^2)$.

Sol. $(3a + 4b) = 16$

Squaring both sides

$(3a)^2 + (4b)^2 + 2 \times 3a \times 4b = (16)^2$

$\Rightarrow 9a^2 + 16b^2 + 24ab = 256$

$\Rightarrow 9a^2 + 16b^2 + 24 \times 4 = 256$

$\Rightarrow 9a^2 + 16b^2 + 96 = 256$

$\Rightarrow 9a^2 + 16b^2 = 256 - 96 = 160$

Hence $9a^2 + 16b^2 = 160$ **Ans.**

Q. 17. If $(a + b) = 2$ and $(a - b) = 10$, find the values of :

(*i*) $(a^2 + b^2)$ (*ii*) ab.

Sol. $a + b = 2$ and $a - b = 10$

(*i*) $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

$= (2)^2 + (10)^2 = 4 + 100 = 104$

$\therefore a^2 + b^2 = \frac{104}{2} = 52$ **Ans.**

(*ii*) $4ab = (a + b)^2 - (a - b)^2$

$= (2)^2 - (10)^2 = 4 - 100 = -96$

$\therefore ab = \frac{-96}{4} = -24$ **Ans.**

Q. 18. If $(a - b) = 0{\cdot}9$ and $ab = 0{\cdot}36$, find the values of (*i*) $(a + b)$ (*ii*) $(a^2 - b^2)$.

Sol. $a - b = 0{\cdot}9$ and $ab = 0{\cdot}36$

Now

(*i*) $(a + b)^2 = (a - b)^2 + 4ab$

$= (0{\cdot}9)^2 + 4 \times 0{\cdot}36$

$= 0{\cdot}81 + 1{\cdot}44 = 2{\cdot}25 = (\pm 1{\cdot}5)^2$

$\therefore a + b = \pm 1{\cdot}5$

(*ii*) $a^2 - b^2 = (a + b)(a - b)$

$= \pm 1{\cdot}5 \times 0{\cdot}9 = \pm 1{\cdot}35$ **Ans.**

Q. 19. If $\left(x + \frac{1}{x}\right) = 5$, find the values of

(*i*) $\left(x^2 + \frac{1}{x^2}\right)$ (*ii*) $\left(x^4 + \frac{1}{x^4}\right)$.

Sol. $x + \frac{1}{x} = 5$

Squaring both sides,

(*i*) $\left(x + \frac{1}{x}\right)^2 = (5)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 25 - 2 = 23$

$\therefore \quad x^2 + \frac{1}{x^2} = 23$ **Ans.**

Again squaring both sides,

(*ii*) $\left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$

$\Rightarrow (x^2)^2 + \frac{(1)^2}{(x^2)^2} + 2 \times x \times \frac{1}{x} = 529$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 529$

$\Rightarrow x^4 + \frac{1}{x^4} = 529 - 2 = 527$ **Ans.**

Q. 20. If $\left(x - \frac{1}{x}\right) = 4$, find the values of

(*i*) $\left(x^2 + \frac{1}{x^2}\right)$ (*ii*) $\left(x^4 + \frac{1}{x^4}\right)$.

Sol. (*i*) $x - \frac{1}{x} = 4 \Rightarrow \left(x - \frac{1}{x}\right)^2 = 4^2$

Squaring both sides,

$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 16$

$x^2 + \frac{1}{x^2} - 2 = 16$

$x^2 + \frac{1}{x^2} = 16 + 2 = 18$

$\therefore x^2 + \frac{1}{x^2} = 18$

(*ii*) Again squaring both sides,

$\left(x^2 + \frac{1}{x^2}\right)^2 = (18)^2$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 324$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 324$

$\Rightarrow x^4 + \frac{1}{x^4} = 324 - 2 = 322$

$\therefore \quad x^4 + \frac{1}{x^4} = 322$. **Ans.**

Q. 21. If $x - 2 = \frac{1}{3x}$ find the values of

(*i*) $\left(x^2 + \frac{1}{9x^2}\right)$ (*ii*) $\left(x^4 + \frac{1}{81x^4}\right)$.

Sol. $x - 2 = \frac{1}{3x} \quad \Rightarrow \quad x - \frac{1}{3x} = 2'$

Squaring both sides,

(*i*) $\left(x - \frac{1}{3x}\right)^2 = (2)^2$

$\Rightarrow x^2 + \frac{1}{9x^2} - 2 \times x \times \frac{1}{3x} = 4$

$\Rightarrow x^2 + \frac{1}{9x^2} - \frac{2}{3} = 4$

$\Rightarrow x^2 + \frac{1}{9x^2} = 4 + \frac{2}{3} = \frac{14}{3}$

$\therefore \quad x^2 + \frac{1}{9x^2} = \frac{14}{3}$

(*ii*) Again squaring both sides,

$\left(x^2 + \frac{1}{9x^2}\right)^2 = \left(\frac{14}{3}\right)^2$

$\Rightarrow (x^2)^2 + \left[\frac{1}{9x^2}\right]^2 + 2 \times x^2 \times \frac{1}{9x^2} = \frac{196}{9}$

$\Rightarrow x^4 + \frac{1}{81x^4} + \frac{2}{9} = \frac{196}{9}$

$\Rightarrow x^4 + \frac{1}{81x^4} = \frac{196}{9} - \frac{2}{9} = \frac{194}{9}$ **Ans.**

Q. 22. If $\left(x + \frac{1}{x}\right) = 6$, find the values of

(*i*) $\left(x - \frac{1}{x}\right)$ (*ii*) $\left(x^2 - \frac{1}{x^2}\right)$.

Sol. $x + \frac{1}{x} = 6$

(*i*) $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$

$= (6)^2 - 4 = 36 - 4 = 32$

$\therefore \quad x - \frac{1}{x} = \pm\sqrt{32} = \pm\sqrt{16 \times 2}$

$= \pm 4\sqrt{2}$

(*ii*) $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 6 \times (\pm 4\sqrt{2}) = \pm 24\sqrt{2}$ **Ans.**

Q. 23. If $\left(x - \frac{1}{x}\right) = 8$, find the values of

(*i*) $\left(x + \frac{1}{x}\right)$ (*ii*) $\left(x^2 - \frac{1}{x^2}\right)$.

Sol. $x - \frac{1}{x} = 8$

(*i*) $\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4$

$= (8)^2 + 4 = 64 + 4 = 68$

$= 4 \times 17$

$\therefore\ x + \frac{1}{x} = \pm\sqrt{4 \times 17} = \pm 2\sqrt{17}$.

(*ii*) $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= \pm 2\sqrt{17} \times 8 = \pm 16\sqrt{17}$ **Ans.**

Q. 24. If $\left(x^2 + \frac{1}{x^2}\right) = 7$, find the values of

(*i*) $\left(x + \frac{1}{x}\right)$ (*ii*) $\left(x - \frac{1}{x}\right)$

(*iii*) $\left(2x^2 - \frac{2}{x^2}\right)$.

Sol. (*i*) $x^2 + \frac{1}{x^2} = 7$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$= 7 + 2 = 9 = (\pm 3)^2$

$\therefore\ x + \frac{1}{x} = \pm 3$

(*ii*) $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$= 7 - 2 = 5 = (\pm\sqrt{5})^2$

$\therefore\ x - \frac{1}{x} = \pm\sqrt{5}$ **Ans.**

(*iii*) $2x^2 - \frac{2}{x^2} = 2\left(x^2 - \frac{1}{x^2}\right)$

$= 2\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 2\,(\pm 3)\ (\pm\sqrt{5}) = \pm 6\sqrt{5}$ **Ans.**

Q. 25. If $\left(x^2 + \frac{1}{25x^2}\right) = 9\frac{2}{5}$, find the value of $\left(x - \frac{1}{5x}\right)$.

Sol. $x^2 + \frac{1}{25x^2} = 9\frac{2}{5} = \frac{47}{5}$

$\left(x - \frac{1}{5x}\right)^2 = x^2 + \frac{1}{25x^2} - 2 \times x \times \frac{1}{5x}$

$= x^2 + \frac{1}{25x^2} - \frac{2}{5}$

$= \frac{47}{5} - \frac{2}{5} = \frac{45}{5} = 9 = (\pm 3)^2$

$\therefore\ x - \frac{1}{5x} = \pm 3$ **Ans.**

Q. 26. If $(a^2 - 4a - 1) = 0$ and $a \neq 0$, find the value of :

(*i*) $\left(a - \frac{1}{a}\right)$ (*ii*) $\left(a + \frac{1}{a}\right)$

(*iii*) $\left(a^2 - \frac{1}{a^2}\right)$ (*iv*) $\left(a^2 + \frac{1}{a^2}\right)$.

Sol. $a^2 - 4a - 1 = 0$

Dividing by a $(\because a \neq 0)$

$\therefore\ a - 4 - \frac{1}{a} = 0$

(*i*) $a - \frac{1}{a} - 4 = 0 \Rightarrow a - \frac{1}{a} = 4$

$\therefore\ a - \frac{1}{a} = 4$

(*ii*) $\left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4$

$= (4)^2 + 4 = 16 + 4 = 20 = 4 \times 5$

$$\therefore \quad a+\frac{1}{a}=\pm\sqrt{4\times5}=\pm2\sqrt{5}$$

$$(iii)\quad a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$$

$$=\pm2\sqrt{5}\times4=\pm8\sqrt{5}$$

$$(iv)\quad a^2+\frac{1}{a^2}=\left(a-\frac{1}{a}\right)^2+2$$

$$=(4)^2+2=16+2=18 \quad \textbf{Ans.}$$

Q. 27. If $a=\dfrac{1}{(a-5)}$ where $a\neq5$ and $a\neq0$, find the values of :

$(i)\left(a-\dfrac{1}{a}\right)$ $\qquad$ $(ii)\left(a+\dfrac{1}{a}\right)$

$(iii)\left(a^2-\dfrac{1}{a^2}\right)$ $\qquad$ $(iv)\left(a^2+\dfrac{1}{a^2}\right)$.

Sol. $a=\dfrac{1}{a-5} \quad\Rightarrow\quad a-5=\dfrac{1}{a}$

$$\Rightarrow\ a-\frac{1}{a}=5$$

$$(i)\quad \therefore\ a-\frac{1}{a}=5$$

$$(ii)\quad \left(a+\frac{1}{a}\right)^2=\left(a-\frac{1}{a}\right)^2+4=(5)^2+4$$

$$=25+4=29$$

$$\therefore\quad a+\frac{1}{a}=\pm\sqrt{29}$$

$$(iii)\quad a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$$

$$=\pm\sqrt{29}\times5=\pm5\sqrt{29}$$

$$(iv)\quad a^2+\frac{1}{a^2}=\left(a-\frac{1}{a}\right)^2+2$$

$$=(5)^2+2=25+2\ \Rightarrow\ 27$$

$$\Rightarrow\ \therefore\ a^2+\frac{1}{a^2}=\sqrt{27}=\sqrt{9\times3}$$

$$=3\sqrt{3} \quad \textbf{Ans.}$$

Q. 28. Using $(a+b)^2=(a^2+b^2+2ab)$ evaluate :

$(i)\ (137)^2$ $\qquad$ $(ii)\ (1008)^2$

$(iii)\ (11{\cdot}6)^2$

Sol. $(i)\ (137)^2=(130+7)^2$

$$=(130)^2+(7)^2+2\times130\times7$$

$$=16900+49+1820=18769$$

$(ii)\ (1008)^2=(1000+8)^2$

$$=(1000)^2+(8)^2+2\times1000\times8$$

$$=1000000+64+16000$$

$$=1016064$$

$(iii)\ (11{\cdot}6)^2=(11+0{\cdot}6)^2$

$$=(11)^2+(0{\cdot}6)^2+2\times11\times0{\cdot}6$$

$$=121+0{\cdot}36+13{\cdot}2$$

$$=134{\cdot}56 \quad \textbf{Ans.}$$

Q. 29. Using $(a-b)^2=(a^2+b^2-2ab)$, evaluate :

$(i)\ (97)^2$ $\qquad$ $(ii)\ (992)^2$

$(iii)\ (9{\cdot}98)^2$

Sol. $(i)\ (97)^2=(100-3)^2$

$$=(100)^2+(3)^2-2\times100\times3$$

$$=10000+9-600=10009-600$$

$$=9409$$

$(ii)\ (992)^2=(1000-8)^2$

$$=(1000)^2+(8)^2-2\times1000\times8$$

$$=1000000+64-16000$$

$$=1000064-16000=984064$$

$(iii)\ (9{\cdot}98)^2=(10-0{\cdot}02)^2$

$$=(10)^2+(0{\cdot}02)^2-2\times10\times0{\cdot}02$$

$$=100+0{\cdot}0004-0{\cdot}4$$

$$=99{\cdot}6004 \quad \textbf{Ans.}$$

Q. 30. Fill in the blanks to make the given expression a perfect square :

$(i)\ 16\,a^2+9\,b^2+$.....

$(ii)\ 25\,a^2+16\,b^2-$ .....

$(iii)\ 4\,a^2+20\,ab+$ .....

$(iv)\ 9\,a^2-24\,ab+$ .....

Sol. $(i)\ 16\,a^2+9\,b^2+$.....

$= (4a)^2 + (3b)^2 + 2 \times 4a \times 3b$

$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$

$= 16a^2 + 9b^2 + \mathbf{24\,ab}$

(*ii*) $25a^2 + 16b^2 - .....$

$= (5a)^2 + (4b)^2 - 2 \times 5a \times 4b$

$= 25a^2 + 16b^2 - \mathbf{40\,ab}$

$\{\because (a-b)^2 = a^2 + b^2 - 2ab\}$

(*iii*) $4a^2 + 20ab + ...$

$= (2a)^2 + 2 \times 2a \times 5b + (5b)^2$

$= 4a^2 + 20ab + \mathbf{25\,b^2}$

(*iv*) $9a^2 - 24ab + ...$

$= (3a)^2 - 2 \times 3a \times 4b + (4b)^2$

$= 9a^2 - 24ab + \mathbf{16\,b^2}$ **Ans.**

Q. 31. If $(a+b+c) = 14$ and $(a^2+b^2+c^2) = 74$, find the value of $(ab+bc+ca)$.

Sol. $a+b+c = 14$

Squaring both sides,

$(a+b+c)^2 = (14)^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 196$

$\Rightarrow 2 + 2(ab+bc+ca) = 196$

$\Rightarrow 74(ab+bc+ca) = 196$

$\Rightarrow 2(ab+bc+ca) = 196 - 74$

$\Rightarrow 2(ab+bc+ca) = 122$

$\therefore \quad ab+bc+ca = \dfrac{122}{2} = 61$ **Ans.**

Q. 32. If $(a+b+c) = 15$ and $(ab+bc+ca) = 74$, find the value of $(a^2+b^2+c^2)$.

Sol. $a+b+c = 15$

Squaring both sides,

$(a+b+c)^2 = (15)^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 225$

$\Rightarrow a^2+b^2+c^2+2 \times 74 = 225$

$\Rightarrow \quad a^2+b^2+c^2+148 = 225$

$\therefore a^2+b^2+c^2 = 225 - 148 = 77$ **Ans.**

Q. 33. If $(a^2+b^2+c^2) = 50$ and $(ab+bc+ca) = 47$, find the value of $(a+b+c)$.

Sol. We know that

$(a+b+c)^2 = a^2+b^2+c^2$

$+ 2(ab+bc+ca)$

$= 50 + 2 \times 47 = 50 + 94$

$= 144 = (\pm 12)^2$

$\therefore a+b+c = \pm 12$ **Ans.**

Q. 34. If $(a^2+b^2+c^2) = 89$ and $(ab-bc-ca) = 16$, find the value of $(a+b-c)$.

Sol. We know that

$(a+b-c)^2 = a^2+b^2+c^2$

$+ 2(ab-bc-ca)$

$= 89 + 2 \times 16 = 89 + 32$

$= 121 = (\pm 11)^2$

$\therefore a+b-c = \pm 11$ **Ans.**

EXERCISE 4 (B)

Q. 1. Expand :

(*i*) $(3a+5b)^3$ (*ii*) $(2p-3q)^3$

(*iii*) $\left(2x + \dfrac{1}{3x}\right)^3$ (*iv*) $(3ab-2c)^3$

(*v*) $\left(3a - \dfrac{1}{a}\right)^3$ (*vi*) $\left(\dfrac{1}{2}x - \dfrac{2}{3}y\right)^3$

Sol. (*i*) $(3a+5b)^3 = (3a)^3 + 3 \times (3a)^2 (5b)$

$+ 3(3a)(5b)^2 + (5b)^3$

$= 27a^3 + 135a^2b + 225ab^2 + 125b^3$

Ans.

(*ii*) $(2p-3q)^3 = (2p)^3 - 3(2p)^2(3q)$

$+ 3(2p)(3q)^2 - (3q)^3$

$= 8p^3 - 36p^2q + 54pq^2 - 27q^3$ **Ans.**

(*iii*) $\left(2x + \dfrac{1}{3x}\right)^3 = (2x)^3 + \left(\dfrac{1}{3x}\right)^3$

$+ 3 \times 2x \times \dfrac{1}{3x}\left(2x + \dfrac{1}{3x}\right)$

$= 8x^3 + \dfrac{1}{27x^3} + 2\left(2x + \dfrac{1}{3x}\right)$

$= 8x^3 + \dfrac{1}{27x^3} + 4x + \dfrac{2}{3x}$

$= 8x^3 + 4x + \dfrac{2}{3x} + \dfrac{1}{27x^3}$ **Ans.**

(*iv*) $(3ab-2c)^3 = (3ab)^3 - (2c)^3$

$- 3 \times 3ab \times 2c(3ab-2c)$

$= 27\,a^3\,b^3 - 8\,c^3 - 18\,abc\,(3\,ab - 2\,c)$

$= 27\,a^3\,b^3 - 8\,c^3 - 54\,a^2\,b^2\,c + 36\,abc^2$

$= 27\,a^3\,b^3 - 54\,a^2\,b^2\,c + 36\,abc^2 - 8\,c^3$

Ans.

(v) $\left(3a - \frac{1}{a}\right)^3 = (3a)^3 - \left(\frac{1}{a}\right)^3$

$$-3 \times 3a \times \frac{1}{a}\left(3a - \frac{1}{a}\right)$$

$= 27a^3 - \frac{1}{a^3} - 9\left(3a - \frac{1}{a}\right)$

$= 27a^3 - \frac{1}{a^3} - 27a + \frac{9}{a}$

$= 27a^3 - 27a + \frac{9}{a} - \frac{1}{a^3}$ **Ans.**

(vi) $\left(\frac{1}{2}x - \frac{2}{3}y\right)^3 = \left(\frac{1}{2}x\right)^3 - \left(\frac{2}{3}y\right)^3$

$$-3 \times \frac{1}{2}x \times \frac{2}{3}y\left(\frac{1}{2}x - \frac{2}{3}y\right)$$

$= \frac{1}{8}x^3 - \frac{8}{27}y^3 - xy\left(\frac{1}{2}x - \frac{2}{3}y\right)$

$= \frac{1}{8}x^3 - \frac{8}{27}y^3 - \frac{1}{2}x^2\,y + \frac{2}{3}xy^2$

$= \frac{1}{8}x^3 - \frac{1}{2}x^2\,y + \frac{2}{3}xy^2 - \frac{8}{27}y^3$ **Ans.**

Q. 2. If $(4\,a + 3\,b) = 10$ and $ab = 2$, find the value of $(64\,a^3 + 27\,b^3)$.

Sol. $(4\,a + 3\,b) = 10$

Cubing both sides,

$(4\,a + 3\,b)^3 = (10)^3$

$\Rightarrow (4\,a)^3 + (3\,b)^3 + 3 \times 4\,a \times 3\,b\,(4\,a + 3\,b) = 1000$

$\Rightarrow 64\,a^3 + 27\,b^3 + 36\,ab\,(4\,a + 3\,b) = 1000$

$\Rightarrow 64\,a^3 + 27\,b^3 + 36 \times 2 \times 10 = 1000$

$\Rightarrow 64\,a^3 + 27\,b^3 + 720 = 1000$

$\therefore 64\,a^3 + 27\,b^3 = 1000 - 720 = 280$ **Ans.**

Q. 3. If $(3\,x - 2\,y) = 5$ and $xy = 6$, find the value of $(27\,x^3 - 8\,y^3)$.

Sol. $3\,x - 2\,y = 5$

Cubing both sides,

$(3\,x - 2\,y)^3 = (5)^3$

$\Rightarrow (3\,x)^3 - (2\,y)^3 - 3 \times 3\,x \times 2\,y\,(3\,x - 2\,y) = 125$

$\Rightarrow 27\,x^3 - 8\,y^3 - 18\,xy\,(3\,x - 2\,y) = 125$

$\Rightarrow 27\,x^3 - 8\,y^3 - 18 \times 6 \times 5 = 125$

$\Rightarrow 27\,x^3 - 8\,y^3 - 540 = 125$

$\therefore 27\,x^3 - 8\,y^3 = 125 + 540 = 665$ **Ans.**

Q. 4. If $(a + 3\,b) = 6$, show that $a^3 + 27\,b^3 + 54\,ab = 216$.

Sol. $a + 3\,b = 6$

Cubing both sides,

$(a + 3\,b)^3 = (6)^3$

$\Rightarrow (a)^3 + (3\,b)^3 + 3 \times a \times 3\,b\,(a + 3\,b) = 216$

$\Rightarrow a^3 + 27\,b^3 + 9\,ab\,(a + 3\,b) = 216$

$\Rightarrow a^3 + 27\,b^3 + 9\,ab \times 6 = 216$

$\Rightarrow a^3 + 27\,b^3 + 54\,ab = 216$

Hence proved.

Q. 5. If $a + 2\,b + 3\,c = 0$, show that $a^3 + 8\,b^3 + 27\,c^3 = 18\,abc$.

Sol. $a + 2\,b + 3\,c = 0$

$a + 2\,b = -3\,c$

Cubing both sides,

$(a + 2\,b)^3 = (-3\,c)^3$

$\Rightarrow (a)^3 + (2\,b)^3 + 3 \times a \times 2\,b\,(a + 2\,b) = -27\,c^3$

$\Rightarrow a^3 + 8\,b^3 + 6\,ab \times (-3\,c) = -27\,c^3$

$\Rightarrow a^3 + 8\,b^3 - 18\,abc = -27\,c^3$

$\Rightarrow a^3 + 8\,b^3 + 27\,c^3 = 18\,abc$

Hence proved.

Q. 6. If $\left(x + \frac{1}{x}\right) = 3$,

find the value of $\left(x^3 + \frac{1}{x^3}\right)$.

Sol. $x + \frac{1}{x} = 3$

Cubing both sides,

$$\left(x+\frac{1}{x}\right)^3 = (3)^3$$

$$\Rightarrow x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=27$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\times 3=27$$

$$\Rightarrow x^3+\frac{1}{x^3}+9=27$$

$$\Rightarrow x^3+\frac{1}{x^3}=27-9=18$$

Hence $x^3+\frac{1}{x^3}=18$ **Ans.**

Q. 7. If $\left(x-\frac{1}{x}\right)=5$, find the value of $\left(x^3-\frac{1}{x^3}\right)$.

Sol. $x-\frac{1}{x}=5$

Cubing both sides,

$$\left(x-\frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\times x\times\frac{1}{x}\left(x-\frac{1}{x}\right)=125$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\times 5=125$$

$$\Rightarrow x^3-\frac{1}{x^3}-15=125$$

$$\Rightarrow x^3-\frac{1}{x^3}=125+15=140$$

Hence $x^3-\frac{1}{x^3}=140$ **Ans.**

Q. 8. If $\left(x-\frac{2}{x}\right)=6$, find the value of $\left(x^3-\frac{8}{x^3}\right)$.

Sol. $x-\frac{2}{x}=6$

Cubing both sides,

$$\left(x-\frac{2}{x}\right)^3 = (6)^3$$

$$\Rightarrow x^3-\frac{8}{x^3}-3\times x\times\frac{2}{x}\left(x-\frac{2}{x}\right)=216$$

$$\Rightarrow x^3-\frac{8}{x^3}-6\times 6=216$$

$$\Rightarrow x^3-\frac{8}{x^3}-36=216$$

$$\Rightarrow x^3-\frac{8}{x^3}=216+36=252$$

$$\therefore x^3-\frac{8}{x^3}=252$$ **Ans.**

Q. 9. If $\left(x+\frac{1}{x}\right)=4$, find the values of :

(*i*) $\left(x^3+\frac{1}{x^3}\right)$ (*ii*) $\left(x-\frac{1}{x}\right)$

(*iii*) $\left(x^3-\frac{1}{x^3}\right)$.

Sol. $x+\frac{1}{x}=4$

Cubing both sides,

(*i*) $$\left(x+\frac{1}{x}\right)^3 = (4)^3$$

$$= x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=64$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\times 4=64$$

$$\Rightarrow x^3+\frac{1}{x^3}+12=64$$

$$\Rightarrow x^3+\frac{1}{x^3}=64-12=52$$

Hence $x^3+\frac{1}{x^3}=52$.

(*ii*) $$\left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4$$

$$=(4)^2-4=16-4=12$$

$$\Rightarrow x - \frac{1}{x} = \pm\sqrt{12} = \pm\sqrt{4 \times 3}$$

$$\Rightarrow x - \frac{1}{x} = \pm 2\sqrt{3}$$

(*iii*) $\left(x - \frac{1}{x}\right)^3 = (\pm 2\sqrt{3})^3$

$$x^3 - \frac{1}{x^3} - 3x \times \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$$= \pm 8 \times 3\sqrt{3} = \pm 24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} - 3 \times (\pm 2\sqrt{3}) = \pm 24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} - \pm 6\sqrt{3} = \pm 24\sqrt{3}$$

$$\therefore x^3 - \frac{1}{x^3} = \pm 24\sqrt{3} \pm 6\sqrt{3}$$

$$= \pm 30\sqrt{3}$$ **Ans.**

Q. 10. If $\left(a^2 + \frac{1}{a^2}\right) = 23$, find the values of :

(*i*) $\left(a + \frac{1}{a}\right)$ (*ii*) $\left(a^3 + \frac{1}{a^3}\right)$.

Sol. $a^2 + \frac{1}{a^2} = 23$

(*i*) $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$$= 23 + 2 = 25$$

$$\therefore a + \frac{1}{a} = \pm\sqrt{25} = \pm 5$$

(*ii*) Cubing both sides,

$$\left(a + \frac{1}{a}\right)^3 = (\pm 5)^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\ xa \times \frac{1}{a}\left(a + \frac{1}{a}\right) = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times (\pm 5) = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} \pm 15 = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} = (\pm 125) - (\pm 15) = \pm 110.$$

Q. 11. If $\left(a - \frac{1}{a}\right) = \sqrt{5}$, find the values of :

(*i*) $\left(a + \frac{1}{a}\right)$ (*ii*) $\left(a^3 + \frac{1}{a^3}\right)$.

Sol. $a - \frac{1}{a} = \sqrt{5}$

(*i*) $\left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4$

$$= (\sqrt{5})^2 + 4 = 5 + 4 = 9$$

$$\therefore a + \frac{1}{a} = \pm\sqrt{9} = \pm 3$$

Cubing both sides,

(*ii*) $\left(a + \frac{1}{a}\right)^3 = (\pm 3)^3$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = \pm 27$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\ (\pm 3) = \pm 27$$

$$\Rightarrow a^3 + \frac{1}{a^3} \pm 9 = \pm 27$$

$$\therefore a^3 + \frac{1}{a^3} = \pm 27 - (\pm 9) = \pm 18$$

Q. 12. If $\left(a^2 + \frac{1}{a^2}\right) = 27$, find the values of

(*i*) $\left(a - \frac{1}{a}\right)$ (*ii*) $\left(a^3 - \frac{1}{a^3}\right)$.

Sol. (*i*) $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = 27 - 2 = 25$$

$$\therefore a - \frac{1}{a} = \pm\sqrt{25} = \pm 5$$

Cubing both sides,

(*ii*) $\left(a - \frac{1}{a}\right)^3 = (\pm 5)^3$

$$\Rightarrow \quad a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right) = \pm 125$$

$$\Rightarrow \quad a^3 - \frac{1}{a^3} - 3 \times (\pm 5) = \pm 125$$

$$\Rightarrow \quad a^3 - \frac{1}{a^3} - (\pm 15) = \pm 125$$

$$\Rightarrow \quad a^3 - \frac{1}{a^3} = \pm 125 + (\pm 15)$$

$$\therefore \quad a^3 - \frac{1}{a^3} = \pm 140$$ **Ans.**

Q. 13. If $\left(x^2 + \frac{1}{25x^2}\right) = 8\frac{3}{5}$, find the values of

(*i*) $\left(x + \frac{1}{5x}\right)$ (*ii*) $\left(x^3 + \frac{1}{125x^3}\right)$.

Sol. (*i*) $\left(x + \frac{1}{5x}\right)^2$

$$= x^2 + \frac{1}{25x^2} + 2 \times x \times \frac{1}{5x}$$

$$= x^2 + \frac{1}{25x^2} + \frac{2}{5}$$

$$= \frac{43}{5} + \frac{2}{5} = \frac{45}{5} = 9$$

$$\therefore \quad x + \frac{1}{5x} = \pm\sqrt{9} = \pm 3$$

(*ii*) Cubing both sides,

$$\left(x + \frac{1}{5x}\right)^3 = (\pm 3)^3$$

$$(x^3) + \left(\frac{1}{5x}\right)^3 + 3 \times x \times \frac{1}{5x}\left(x + \frac{1}{5x}\right) = \pm 27$$

$$\Rightarrow \quad x^3 + \frac{1}{125x^3} + \frac{3}{5} \times (\pm 3) = \pm 27$$

$$\Rightarrow \quad x^3 + \frac{1}{125x^3} \pm \frac{9}{5} = \pm 27$$

$$\Rightarrow x^3 + \frac{1}{125}x^3 = \pm 27 - \left(\pm\frac{9}{5}\right)$$

$$\Rightarrow x^3 + \frac{1}{125}x^3 = \pm\frac{126}{5} = \pm 25\frac{1}{5}$$ **Ans.**

Q. 14. If $\left(x + \frac{1}{x}\right)^2 = 3$, show that

$$\left(x^3 + \frac{1}{x^3}\right) = 0.$$

Sol. $\left(x + \frac{1}{x}\right)^2 = 3 \quad \therefore \quad x + \frac{1}{x} = \pm\sqrt{3}$

Cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = (\pm\sqrt{3})^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = \pm 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times (\pm\sqrt{3}) = \pm 3\sqrt{3}$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + (\pm 3\sqrt{3}) = \pm 3\sqrt{3}$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = \pm 3\sqrt{3} - (\pm 3\sqrt{3}) = 0$$

Hence $x^3 + \frac{1}{x^3} = 0$.

Q. 15. If $\frac{a}{b} = \frac{b}{c}$, prove that

$(a + b + c)(a - b + c) = (a^2 + b^2 + c^2)$.

Sol. Let $\frac{a}{b} = \frac{b}{c} = k$, then

$b = ck$ and $a = bk = ck \times k = ck^2$

L.H.S. $= (a + b + c)(a - b + c)$

$= (ck^2 + ck + c)(ck^2 - ck + c)$

$= c(k^2 + k + 1)\, c(k^2 - k + 1)$

$= c^2 (k^2 + k + 1)(k^2 - k + 1)$

$= c^2 (k^4 + k^2 + 1)$

R.H.S. $= a^2 + b^2 + c^2$

$= (ck^2)^2 + (ck)^2 + c^2$

$= c^2 k^4 + c^2 k^2 + c^2$

$= c^2 (k^4 + k^2 + 1)$

$\therefore$ L.H.S. = R.H.S.

Second Method

$\because \frac{a}{b} = \frac{b}{c}, \quad \therefore b^2 = ac$

Now L.H.S. $= (a + b + c)(a - b + c)$

$= [(a + c) + b)]\ [(a + c) - (b)]$

$= (a + c)^2 - b^2$

$= (a + c)^2 - ac \qquad (\because b^2 = ac)$

$= a^2 + c^2 + 2ac - ac = a^2 + c^2 + ac$

$= a^2 + c^2 + b^2 = a^2 + b^2 + c^2 =$ R.H.S.

EXERCISE 4 (C)

Write down each of the following products :

Q. 1. $(x + 6)(x + 2)$

Sol. $(x + 6)(x + 2) = x^2 + (6 + 2)x + 6 \times 2$

$= x^2 + 8x + 12$ **Ans.**

Q. 2. $(x + 8)(x - 3)$

Sol. $(x + 8)(x - 3) = x^2 + (8 - 3)x + 8 \times (-3)$

$= x^2 + 5x - 24$ **Ans.**

Q. 3. $(x - 5)(x - 7)$

Sol. $(x - 5)(x - 7)$

$= x^2 + (-5 - 7)x + (-5)(-7)$

$= x^2 - 12x + 35$ **Ans.**

Q. 4. $(2 - x)(4 - x)$

Sol. $\{-(x - 2)\} \times \{-(x - 4)\}$

$= (x - 2)(x - 4)$

$= x^2 + (-2 - 4)x + (-2)(-4)$

$= x^2 - 6x + 8 \Rightarrow 8 - 6x + x^2$ **Ans.**

Q. 5. $(y - 7)(y + 4)$

Sol. $(y - 7)(y + 4)$

$= y^2 + (-7 + 4)y + (-7)(4)$

$= y^2 - 3y - 28$ **Ans.**

Q. 6. $(ab + 3)(ab - 2)$

Sol. $(ab + 3)(ab - 2)$

$= (ab)^2 + (3 - 2)ab + 3 \times (-2)$

$= a^2 b^2 + ab - 6$ **Ans.**

Q. 7. $(5 - xy)(2 + xy)$

Sol. $(5 - xy)(2 + xy)$

$= -(xy - 5)(xy + 2)$

$= -\{(xy)^2 + (-5 + 2)xy + (-5) \times 2\}$

$= -[x^2 y^2 - 3xy - 10]$

$= 10 + 3xy - x^2 y^2$ **Ans.**

Q. 8. $(x^2 + 1)(x^2 + 2)$

Sol. Let $x^2 = y$, then

$(y + 1)(y + 2) = y^2 + (1 + 2)y + 1 \times 2$

$= y^2 + 3y + 2$

$= (x^2)^2 + 3x^2 + 2$

$= x^4 + 3x^2 + 2$ **Ans.**

Q. 9. $(3 - x^2)(5 + x^2)$

Sol. $(3 - x^2)(5 + x^2) = -(x^2 - 3)(x^2 + 5)$

$= -\{(x^2)^2 + (-3 + 5)x^2 + (-3) \times 5\}$

$= -\{x^4 + 2x^2 - 15\}$

$= -x^4 - 2x^2 + 15$

$= 15 - 2x^2 - x^4$ **Ans.**

Q. 10. $(6 - x)(x + 5)$

Sol. $(6 - x)(x + 5) = -(x - 6)(x + 5)$

$= -\{x^2 + (-6 + 5)x + (-6) \times 5\}$

$= -\{x^2 - x - 30\} = -x^2 + x + 30$

$= 30 + x - x^2 = x - x^2 + 30$ **Ans.**

Q. 11. $(2x + 3)(3x + 5)$

Sol. $(2x + 3)(3x + 5)$

$= 2x(3x + 5) + 3(3x + 5)$

$= 6x^2 + 10x + 9x + 15$

$= 6x^2 + 19x + 15$ **Ans.**

Q. 12. $(7m - 2)(4m + 3)$

Sol. $(7m - 2)(4m + 3)$

$= 7m(4m + 3) - 2(4m + 3)$

$= 28m^2 + 21m - 8m - 6$

$= 28m^2 + 13m - 6$ **Ans.**

Q. 13. $(2y - 3)(3y - 5)$

Sol. $(2y - 3)(3y - 5)$

$= 2y(3y - 5) - 3(3y - 5)$

$= 6y^2 - 10y - 9y + 15$

$= 6y^2 - 19y + 15$ **Ans.**

Q. 14. $(3a^2 - b^2)(2a^2 + 5b^2)$

Sol. $(3a^2 - b^2)(2a^2 + 5b^2)$

$= 3a^2(2a^2 + 5b^2) - b^2(2a^2 + 5b^2)$

$= 6a^4 + 15a^2 b^2 - 2a^2 b^2 - 5b^4$

$= 6a^4 + 13a^2 b^2 - 5b^4$ **Ans.**