3 VENN DIAGRAMS

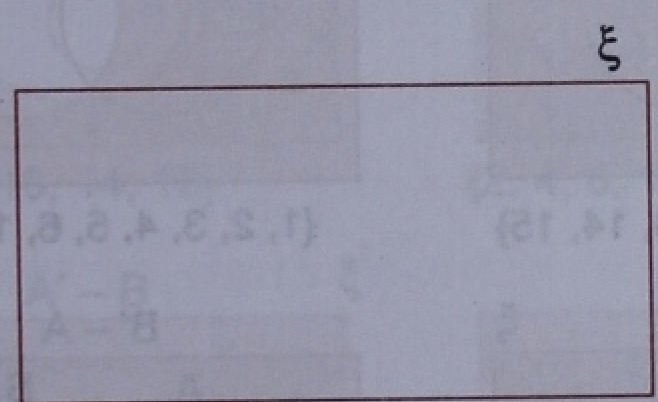
- Venn Diagrams and Set Operations
- Properties of Cardinal Numbers
- Operations on Three Sets
- Associative Laws
- Distributive Laws
- De Morgan's Laws

Introduction

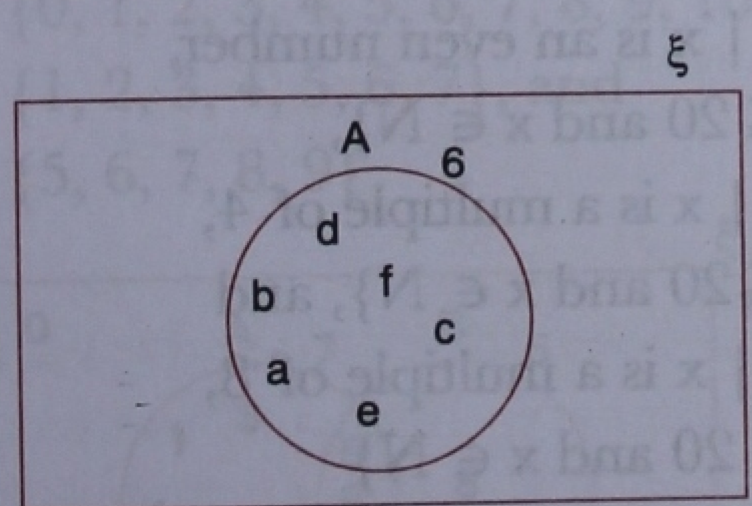
John Venn (1834–1923), an English mathematician, introduced the concept of pictorially representing sets by elements bounded within a closed figure. This is why these geometrical figures are known as Venn diagrams.

Some of the basic facts about Venn diagrams are:

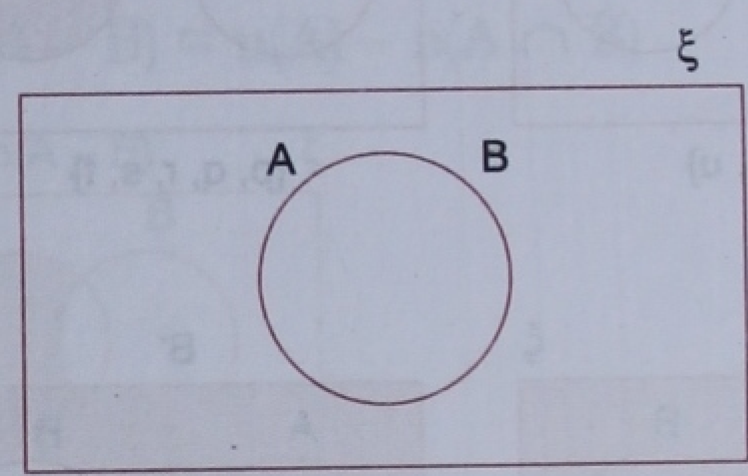
1. The universal set is represented by a rectangle. All the elements of the sets under consideration lie in the interior of this rectangle.



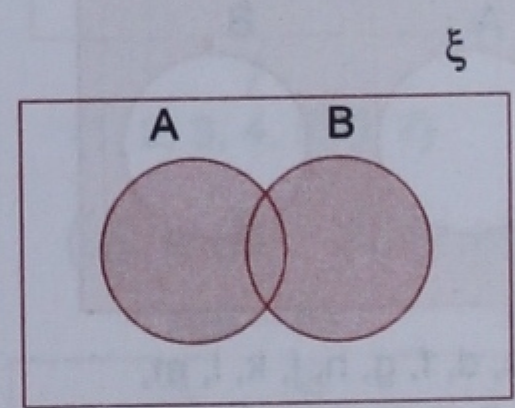
2. A set is represented by a circle or an ellipse. All the elements of the set lie in the interior of this circle and are written inside the circle. The name and cardinal number of the set are written on the boundary of the circle.



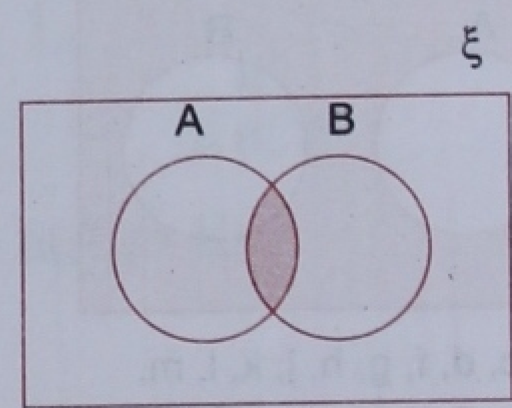
3. If two circles of equal radii describe two sets, this does not necessarily mean that the sets are equal. The boundary of a circle only describes the limits of the set, just as curly brackets do, in set notation. Equal sets are represented by the same circle, as shown below.



4. To highlight the relation between sets, or to describe the result of an operation on sets, particular areas are shaded.



$A \cup B$

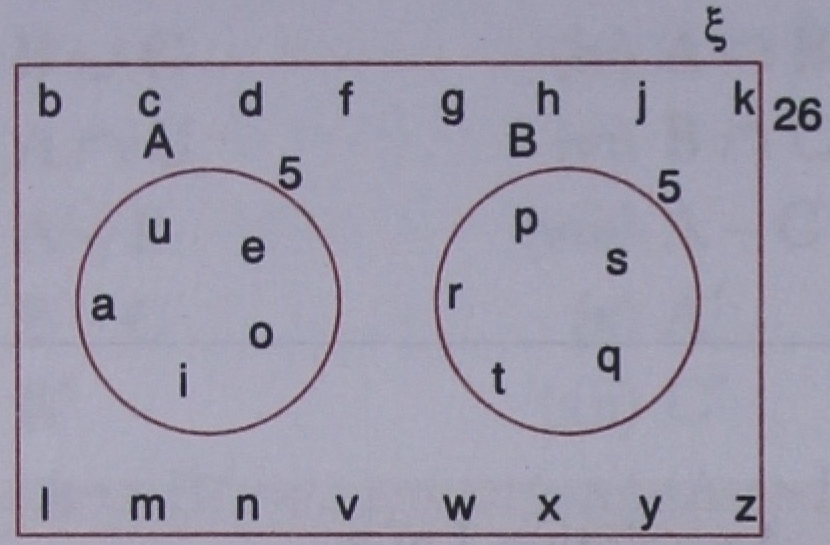


$A \cap B$

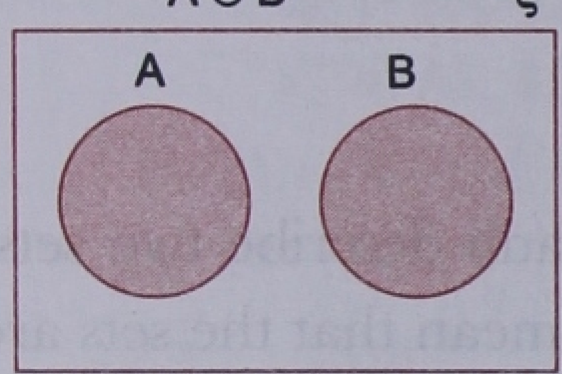
Venn Diagrams and Set Operations

Disjoint Sets

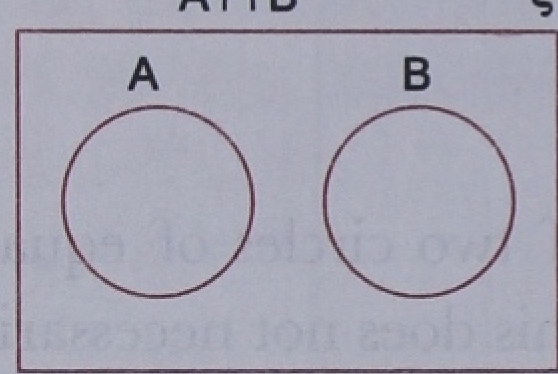
Let $\xi = \{x \mid x \text{ is a letter of the English alphabet}\}$,
$A = \{a, e, i, o, u\}$, and $B = \{p, q, r, s, t\}$



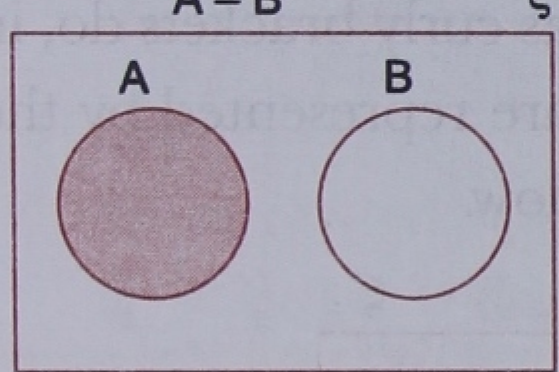
Now observe the following operations represented by Venn diagrams:

A ∪ B


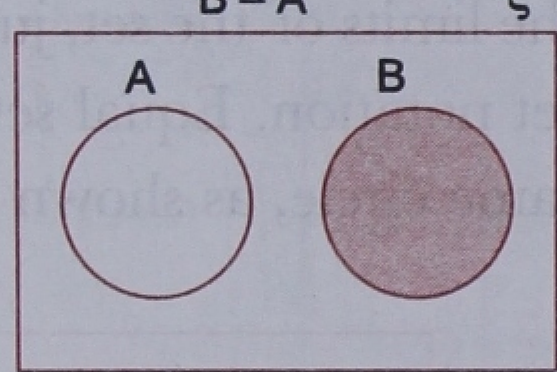
{a, e, i, o, u, p, q, r, s, t}

A ∩ B


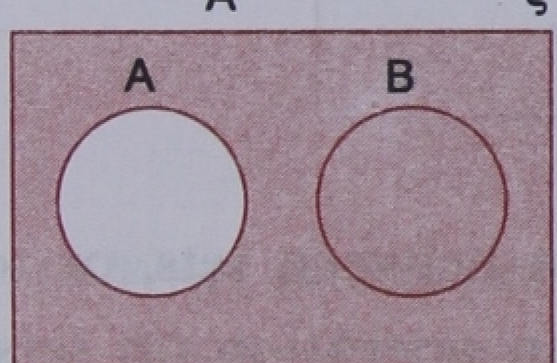
{ } or ϕ

A − B


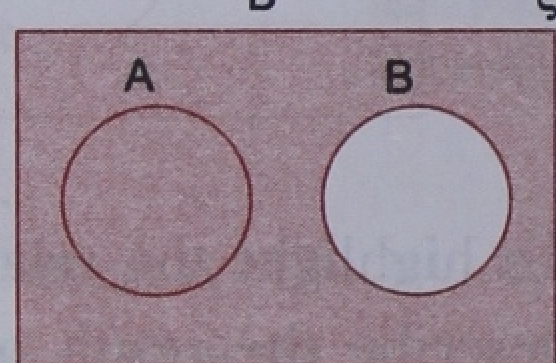
{a, e, i, o, u}

B − A


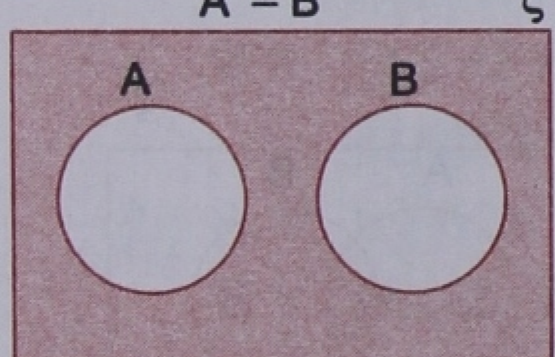
{p, q, r, s, t}

A′


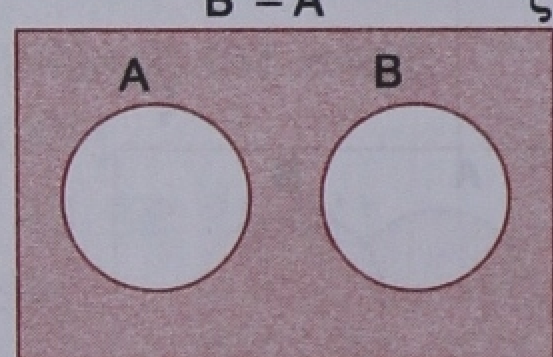
{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}

B′


{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, u, v, w, x, y, z}

A′ − B


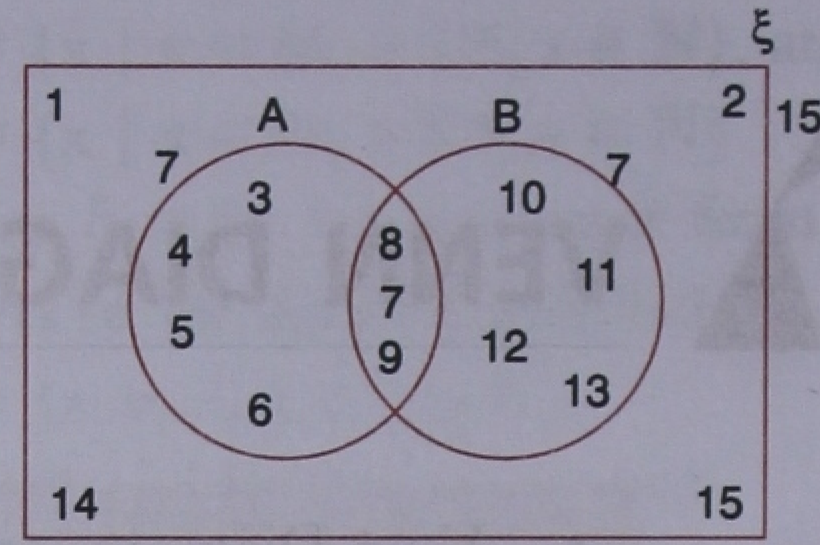
{b, c, d, f, g, h, j, k, l, m, n, v, w, x, y, z}

B′ − A


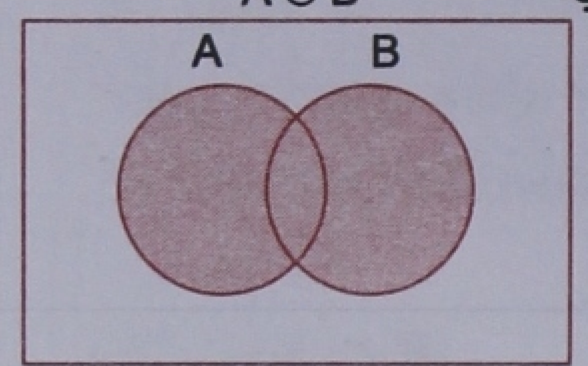
{b, c, d, f, g, h, j, k, l, m, n, v, w, x, y, z}

Intersecting Sets

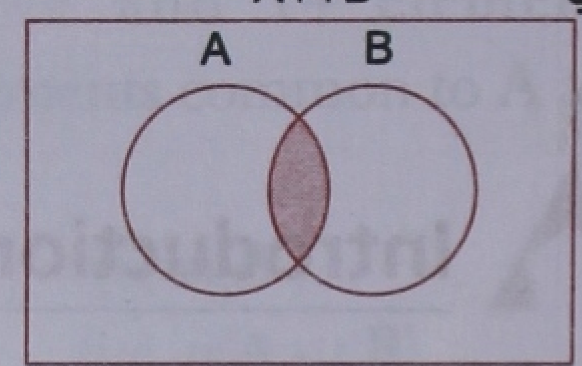
Let ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15},
A = {3, 4, 5, 6, 7, 8, 9}, and
B = {7, 8, 9, 10, 11, 12, 13}



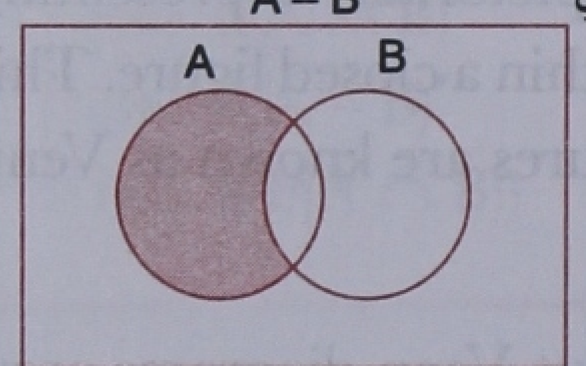
Now observe the following operations represented by Venn diagrams:

A ∪ B


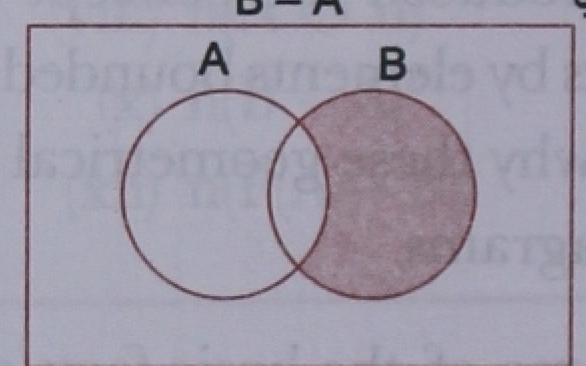
{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

A ∩ B


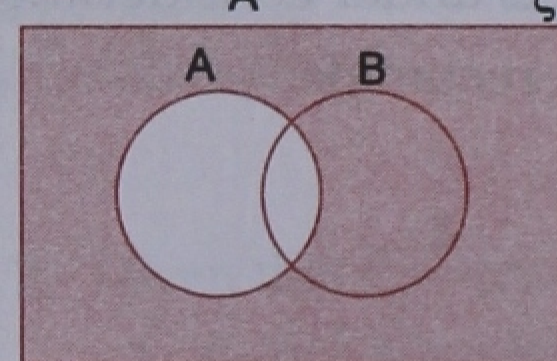
{7, 8, 9}

A − B


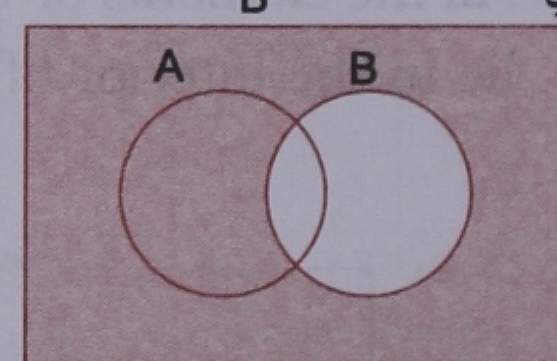
{3, 4, 5, 6}

B − A


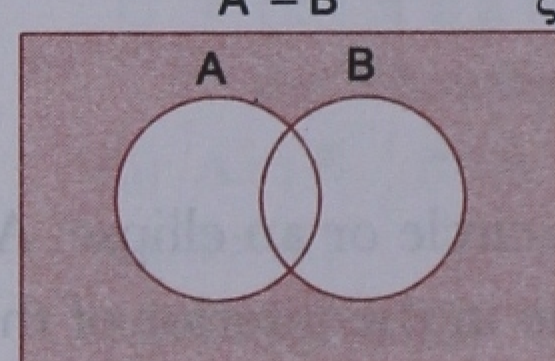
{10, 11, 12, 13}

A′


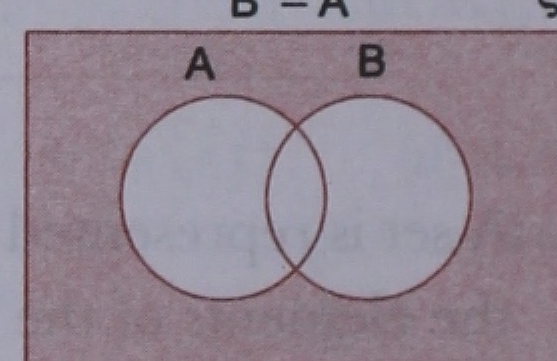
{1, 2, 10, 11, 12, 13, 14, 15}

B′


{1, 2, 3, 4, 5, 6, 14, 15}

A′ − B


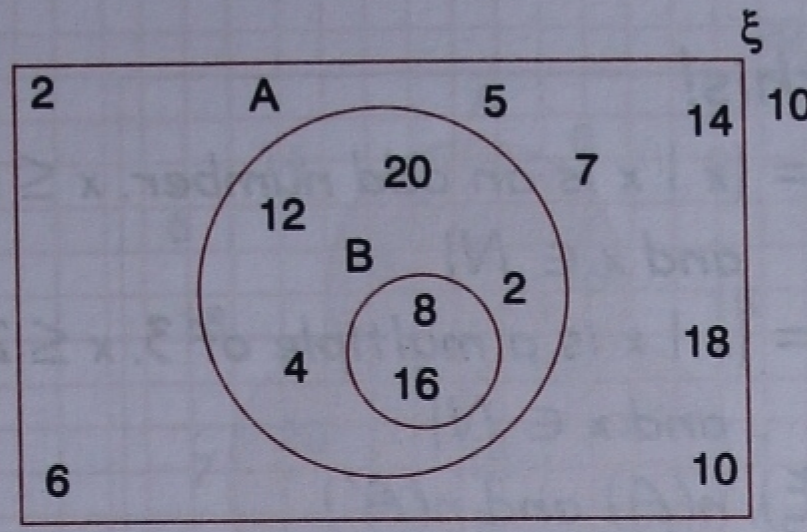
{1, 2, 14, 15}

B′ − A


{1, 2, 14, 15}

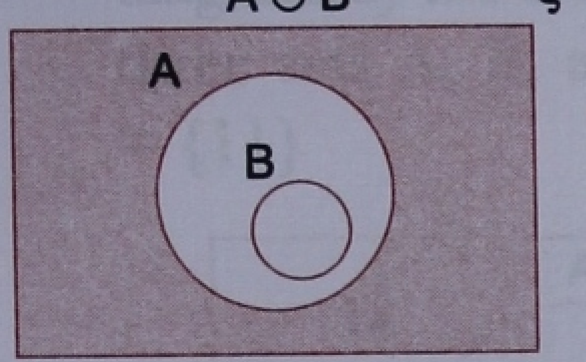
Subset

Let ξ = {x | x is an even number, x ≤ 20 and x ∈ N},
A = {x | x is a multiple of 4, x ≤ 20 and x ∈ N}, and
B = {x | x is a multiple of 8, x ≤ 20 and x ∈ N}



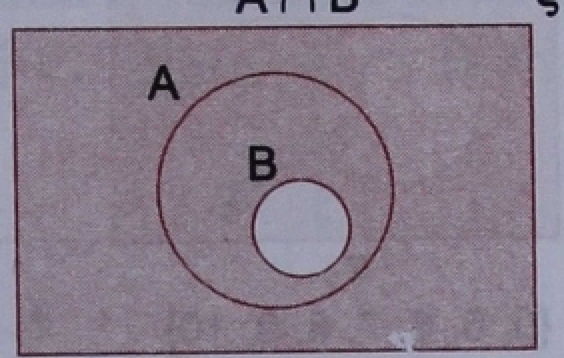
Now observe the following operations represented by Venn diagrams:

A ∪ B



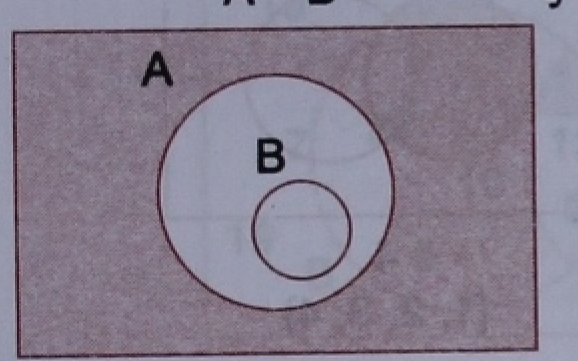
{4, 8, 12, 16, 20}

A ∩ B



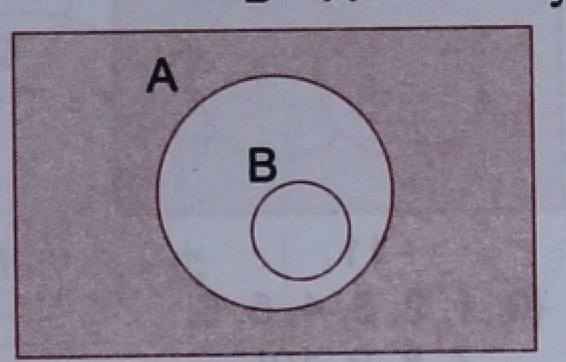
{8, 16}

A – B



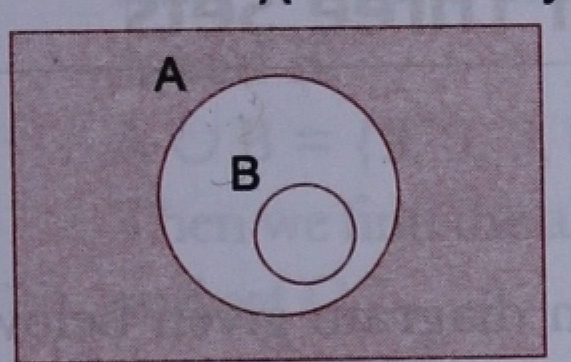
{4, 12, 20}

B – A



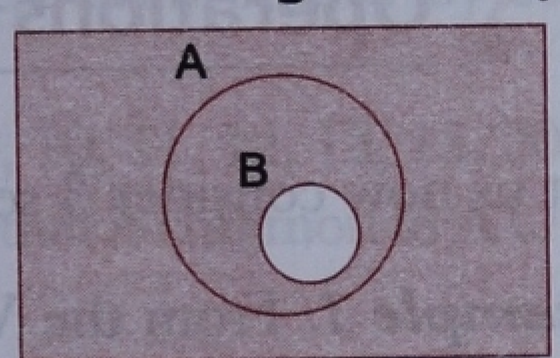
{ } or ϕ

A′



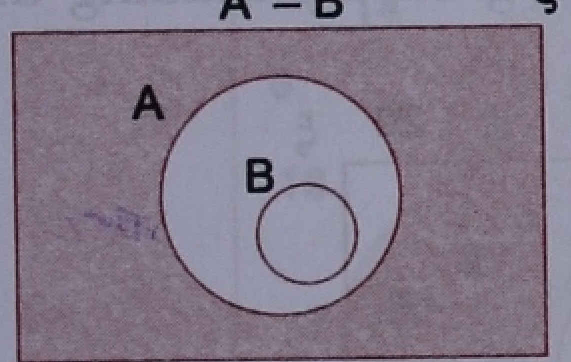
{2, 6, 10, 14, 18}

B′



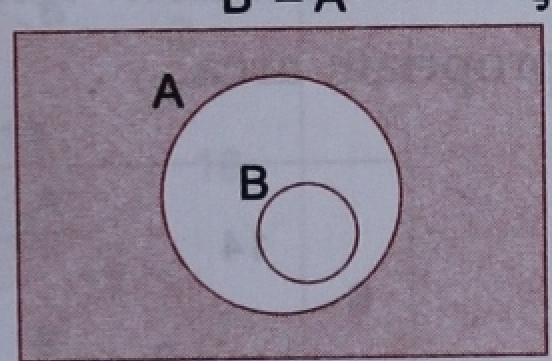
{2, 4, 6, 10, 12, 14, 18, 20}

A′ – B



{2, 6, 10, 14, 18}

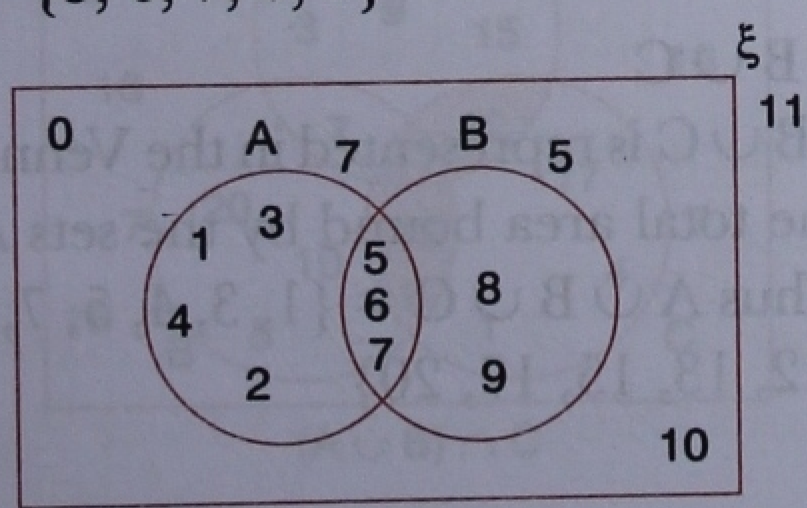
B′ – A



{2, 6, 10, 14, 18}

Properties of Cardinal Numbers

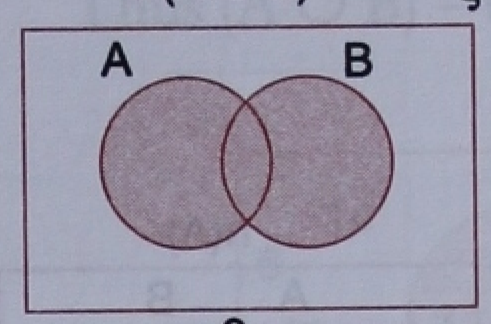
Let ξ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
A = {1, 2, 3, 4, 5, 6, 7}, and
B = {5, 6, 7, 8, 9}



We shall now verify the properties of cardinal numbers by constructing Venn diagrams for the given sets for different operations.

1. n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

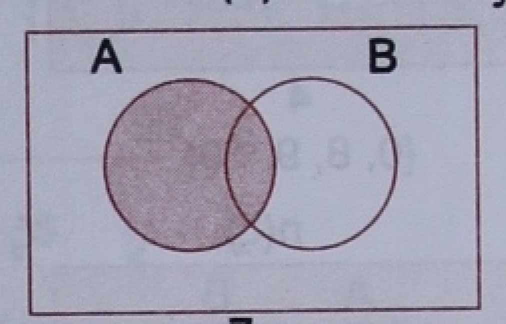
n(A ∪ B)



9
{1, 2, 3, 4, 5, 6, 7, 8, 9}

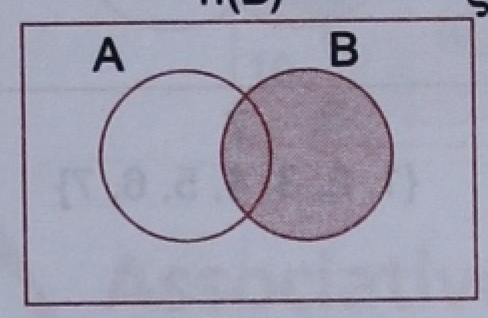
=

n(A)



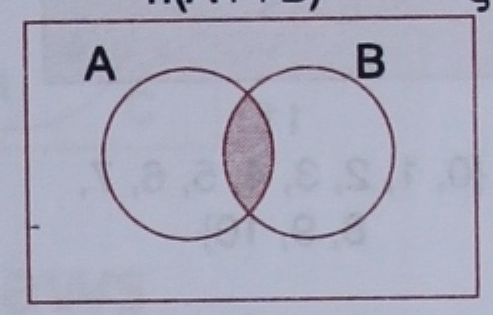
7
{1, 2, 3, 4, 5, 6, 7}

+ n(B)



5
{5, 6, 7, 8, 9}

– n(A ∩ B)

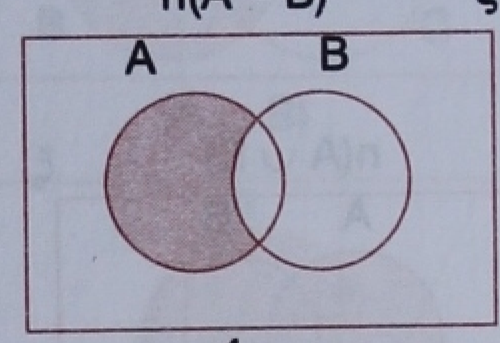


3
{5, 6, 7}

The last area n(A ∩ B) has been taken twice in the union of A and B. Thus, n(A ∩ B) is subtracted once.

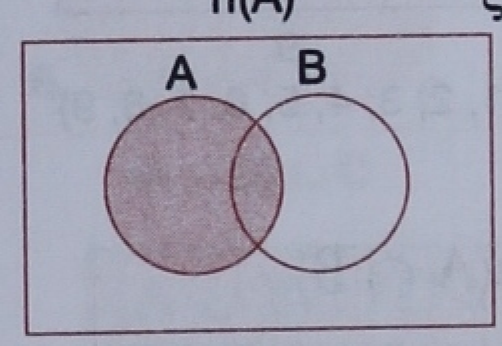
2. (i) n(A – B) = n(A) – n(A ∩ B)

n(A – B)



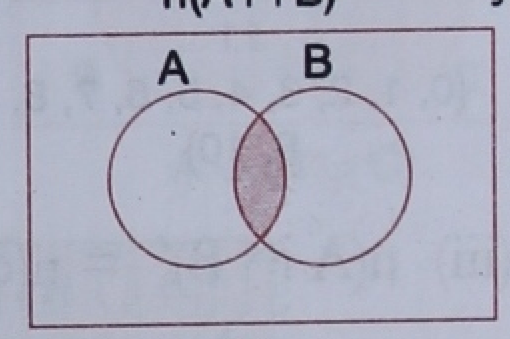
4
{1, 2, 3, 4}

= n(A)



7
{1, 2, 3, 4, 5, 6, 7}

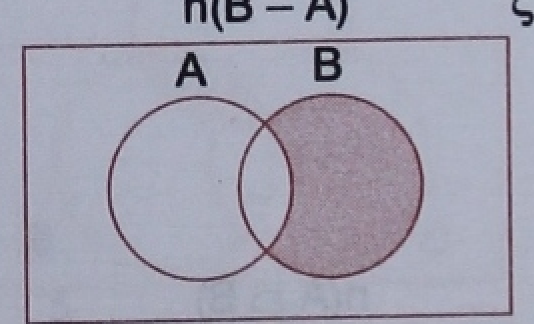
– n(A ∩ B)



3
{5, 6, 7}

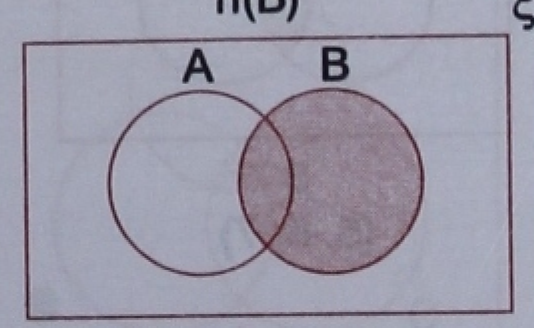
(ii) n (B – A) = n(B) – n(A ∩ B)

n(B – A)



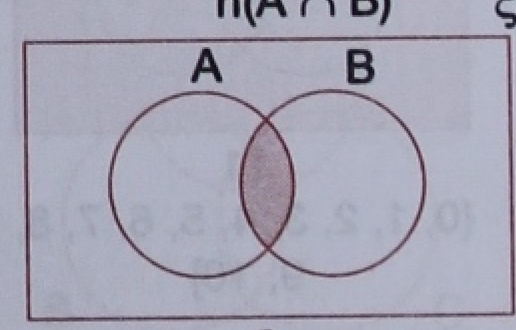
2
{8, 9}

= n(B)



5
{5, 6, 7, 8, 9}

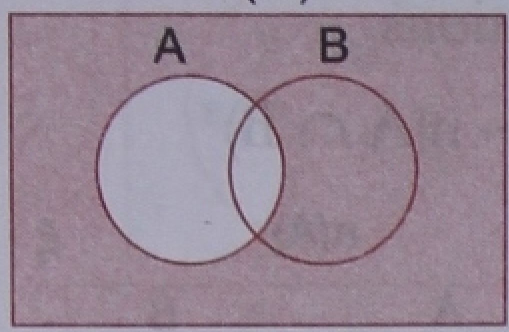
– n(A ∩ B)



3
{5, 6, 7}

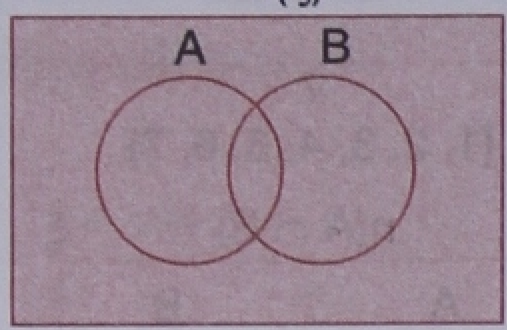
3. (i) $n(A') = n(\xi) - n(A)$

n(A′) ξ



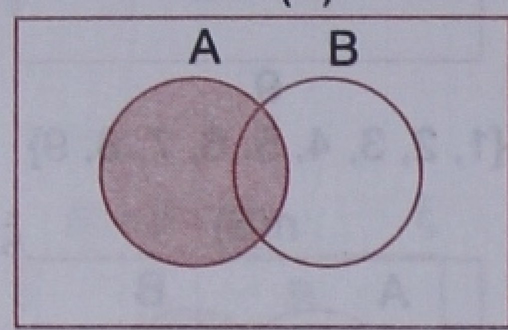
4
{0, 8, 9, 10}

n(ξ) ξ

= 

11
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

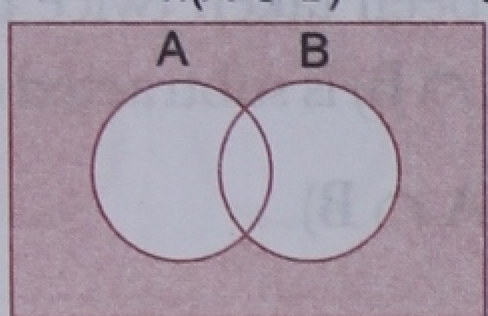
− n(A) ξ



7
{1, 2, 3, 4, 5, 6, 7}

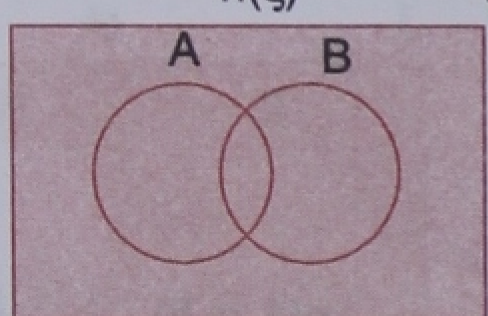
(ii) $n(A \cup B)' = n(\xi) - n(A \cup B)$

n(A ∪ B)′ ξ



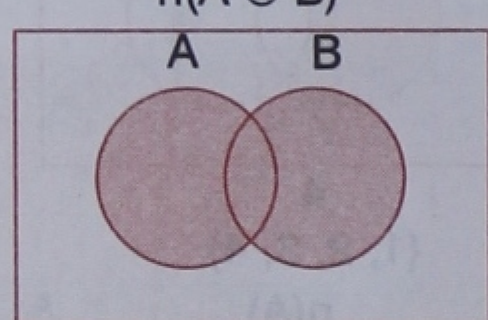
2
{0, 10}

n(ξ) ξ

= 

11
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

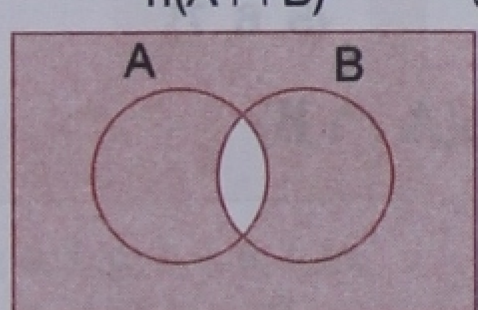
− n(A ∪ B) ξ



9
{1, 2, 3, 4, 5, 6, 7, 8, 9}

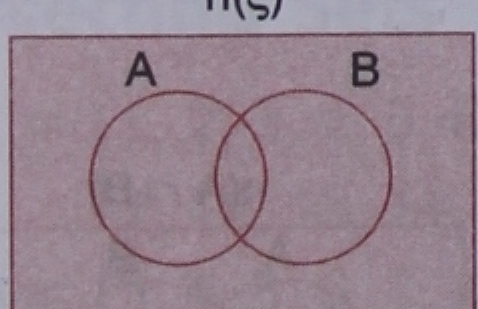
(iii) $n(A \cap B)' = n(\xi) - n(A \cap B)$

n(A ∩ B)′ ξ



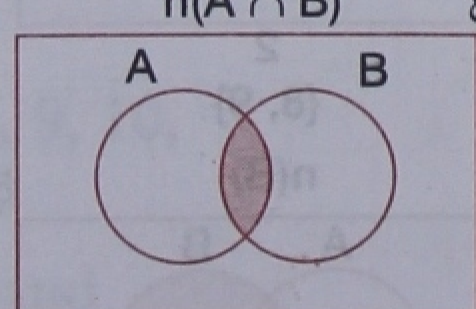
8
{0, 1, 2, 3, 4, 8, 9, 10}

n(ξ) ξ

= 

11
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

− n(A ∩ B) ξ



3
{5, 6, 7}

Try this!

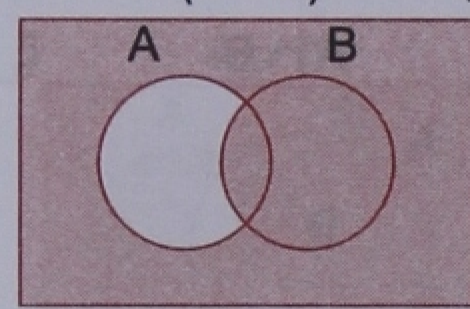
Let $\xi = \{x \mid x$ is an odd number, $x \leq 25$ and $x \in N\}$

$A = \{x \mid x$ is a multiple of 3, $x \leq 25$ and $x \in N\}$

find $n(\xi)$, $n(A)$ and $n(A')$

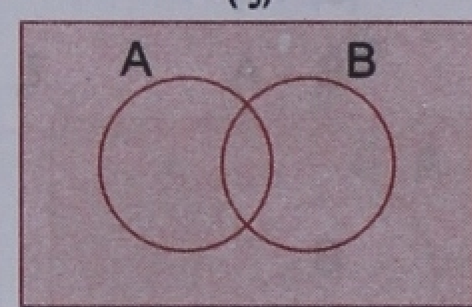
(iv) $n(A - B)' = n(\xi) - n(A - B)$

n(A − B)′ ξ



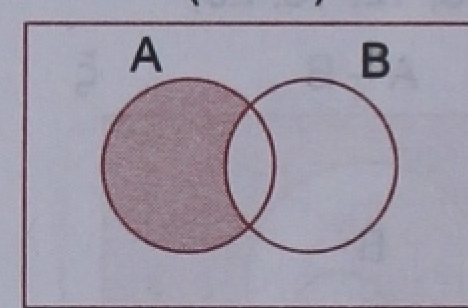
7
{0, 5, 6, 7, 8, 9, 10}

n(ξ) ξ

= 

11
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

− n(A − B) ξ

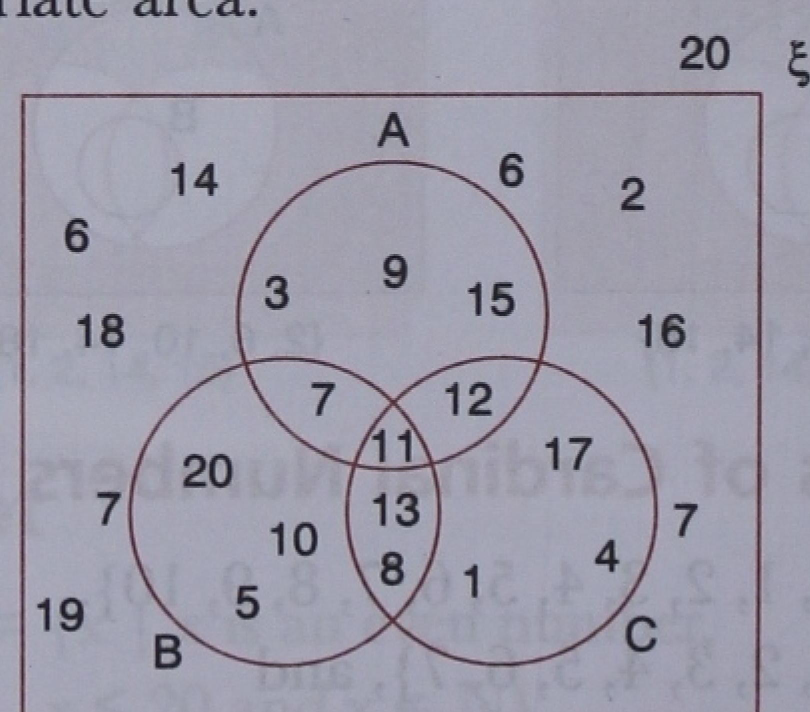


4
{1, 2, 3, 4}

Operations on Three Sets

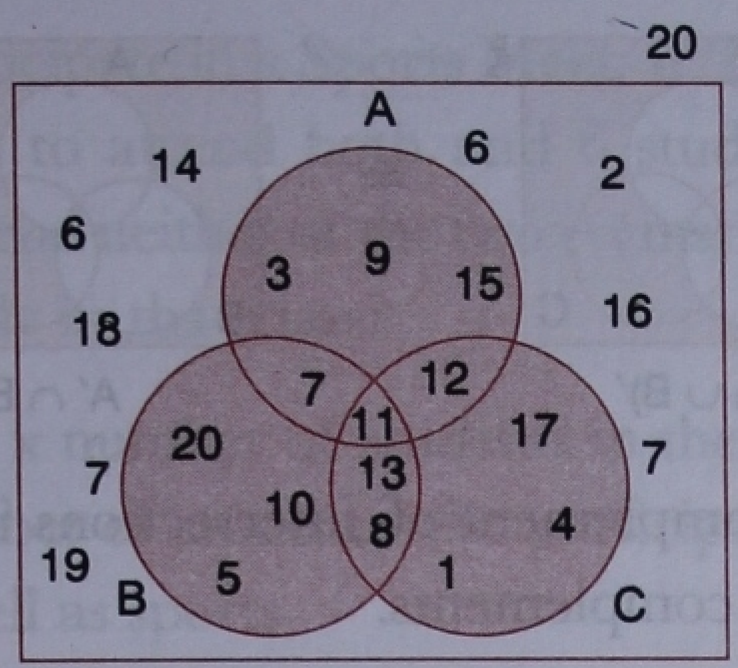
Let us now consider 3 sets.

Example 1: From the Venn diagram given below, find the results of the following operations and show the result in a new Venn diagram by shading the appropriate area.



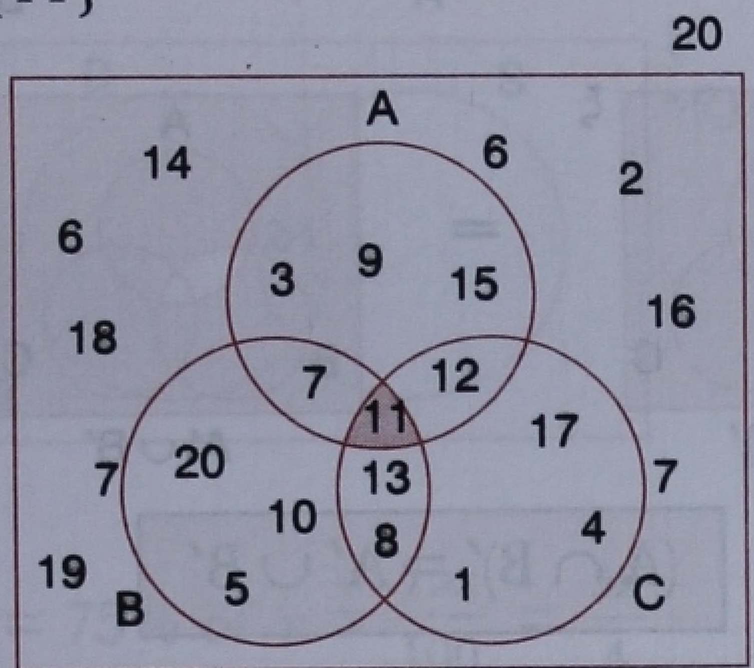
(i) $A \cup B \cup C$

$A \cup B \cup C$ is represented in the Venn diagram by the total area bound by the sets A, B and C. Thus $A \cup B \cup C = \{1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 17, 20\}$



(ii) A ∩ B ∩ C

A ∩ B ∩ C is represented in the Venn diagram by the area that is common to all the three sets A, B, and C. Thus A ∩ B ∩ C = {11}

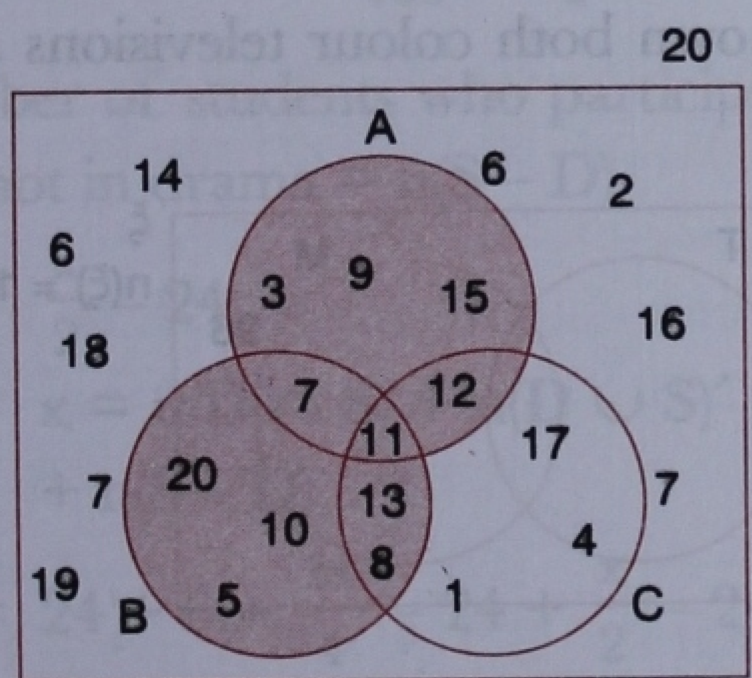


(iii) (A ∪ B) ∩ C

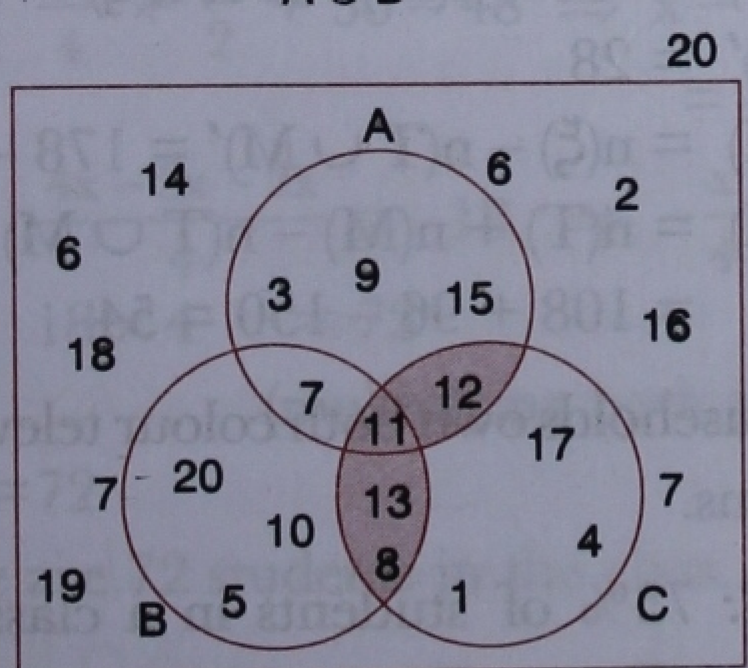
First we find the total area bound by sets A and B.

A ∪ B = {3, 5, 7, 8, 9, 10, 11, 12, 13, 15, 20}.

Then we find the area that is common to A ∪ B and set C.



A ∪ B

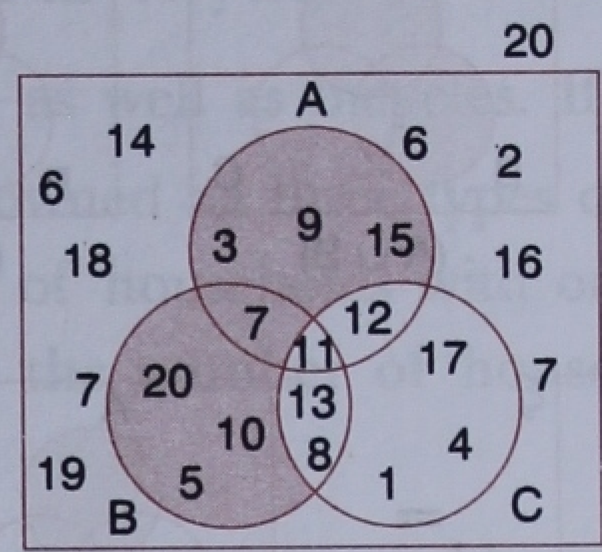


(A ∪ B) ∩ C

Thus (A ∪ B) ∩ C = {8, 11, 12, 13}

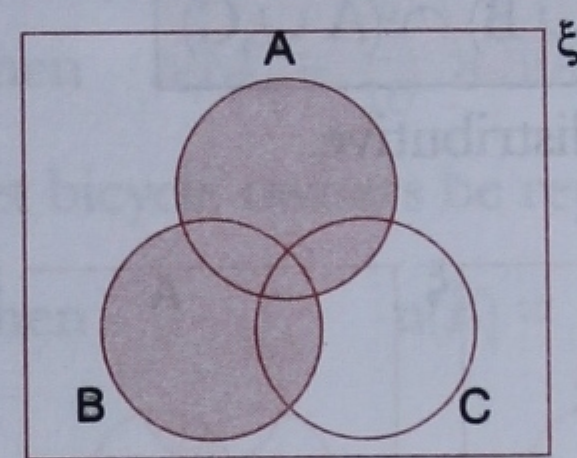
(iv) (A ∪ B) – C

We have identified the area that represents (A ∪ B) in the above example. From this area, we exclude the area that is common with set C. Thus (A ∪ B) – C = {3, 5, 7, 9, 10, 15, 20}

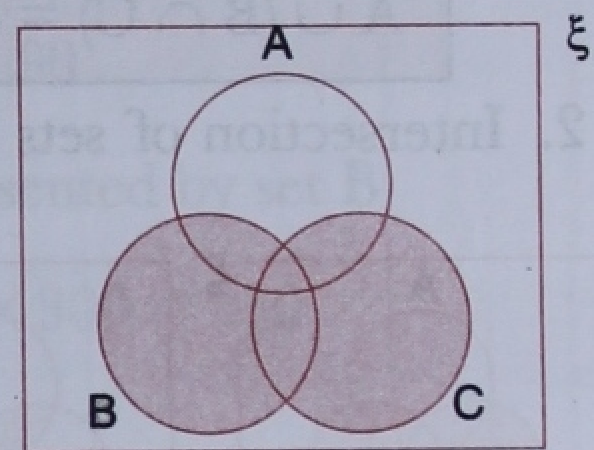


Associative Laws

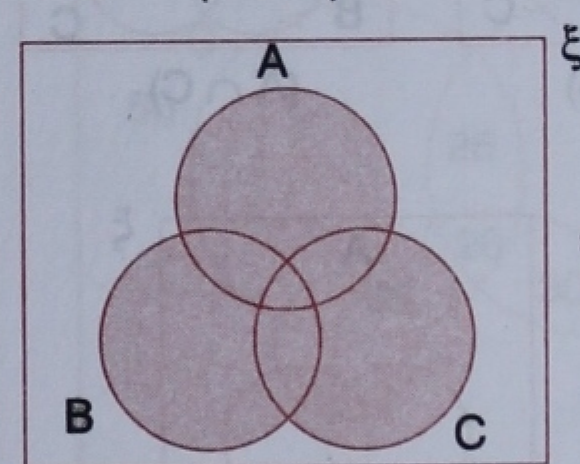
1. Union of sets is associative.



(A ∪ B)

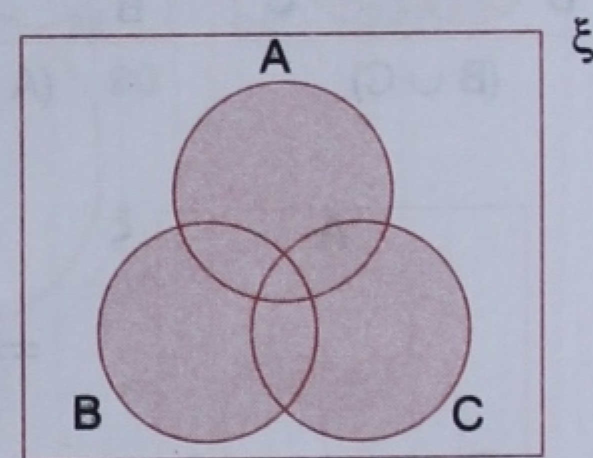


(B ∪ C)



(A ∪ B) ∪ C

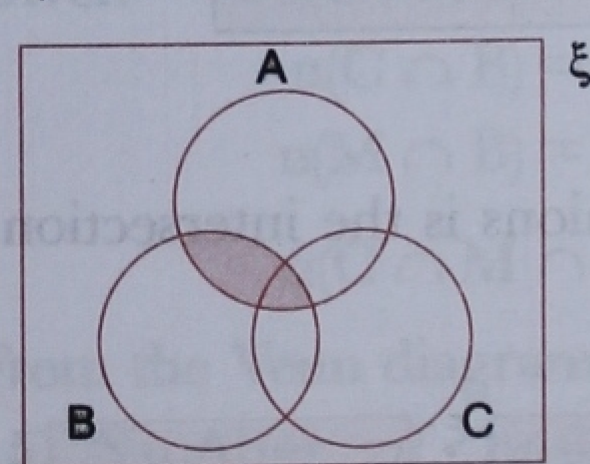
=



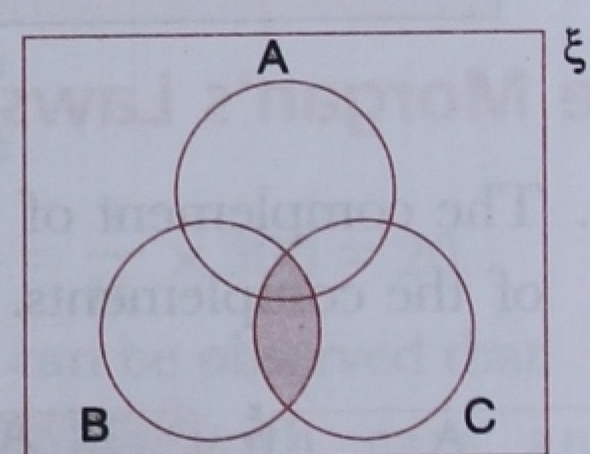
A ∪ (B ∪ C)

(A ∪ B) ∪ C = A ∪ (B ∪ C)

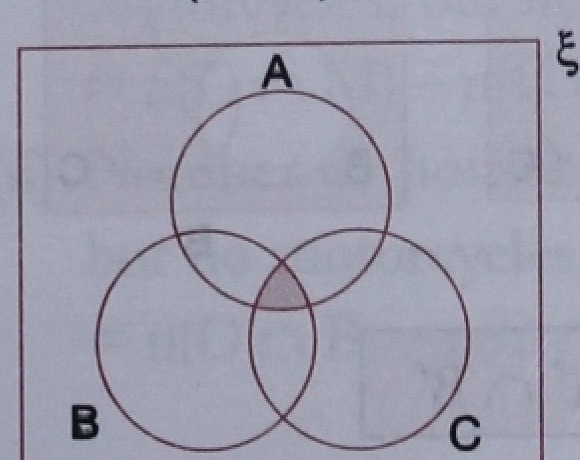
2. Intersection of sets is associative.



(A ∩ B)

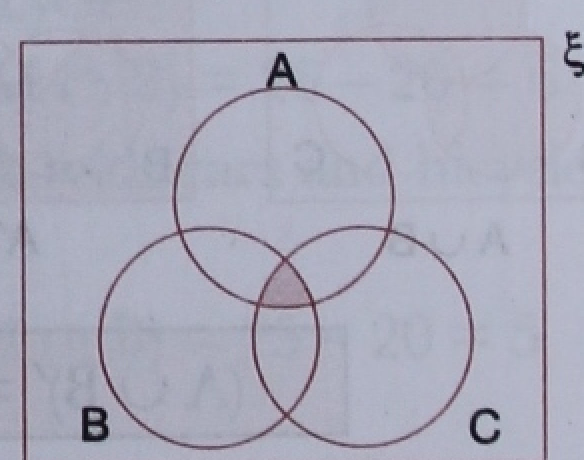


(B ∩ C)



(A ∩ B) ∩ C

=

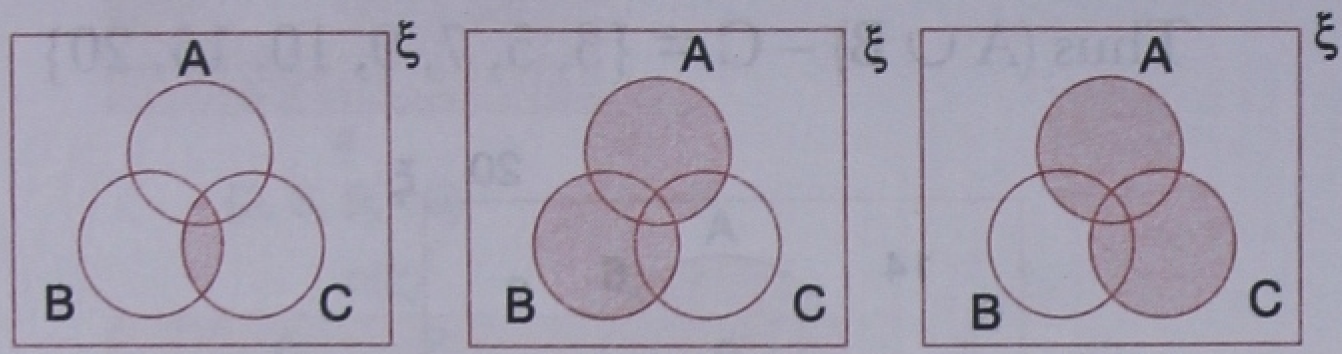


A ∩ (B ∩ C)

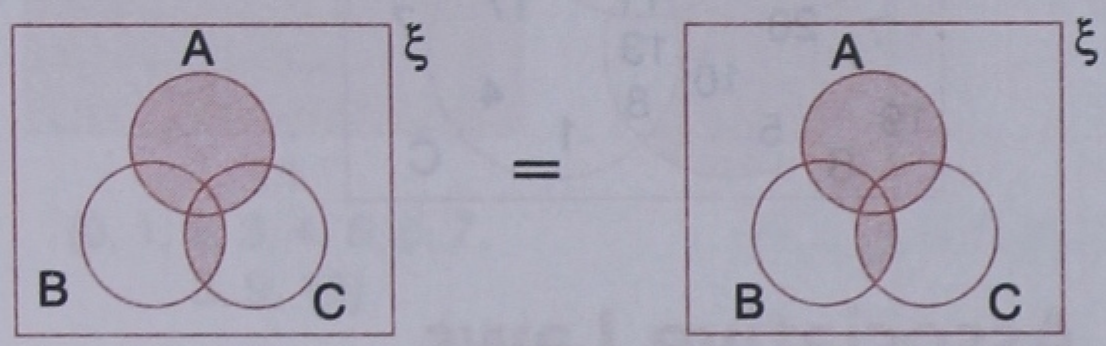
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive Laws

1. Union of sets is distributive.



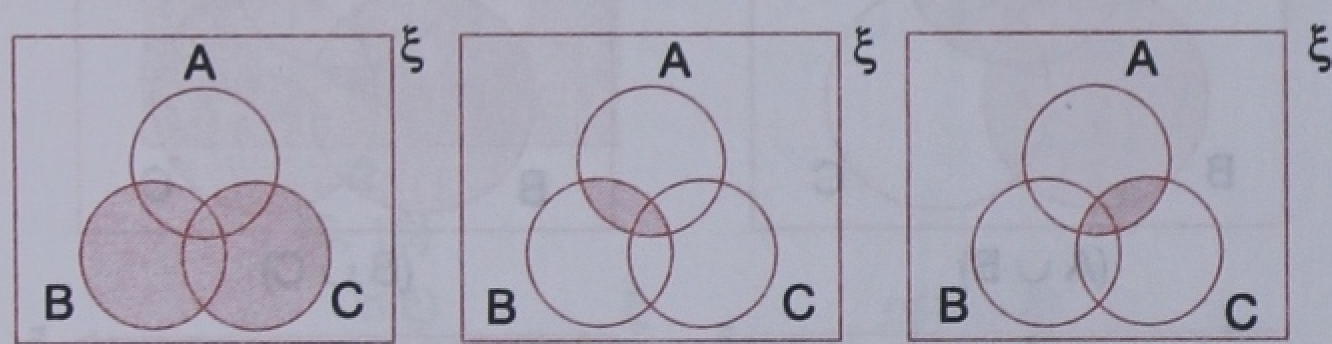
(B ∩ C) (A ∪ B) (A ∪ C)



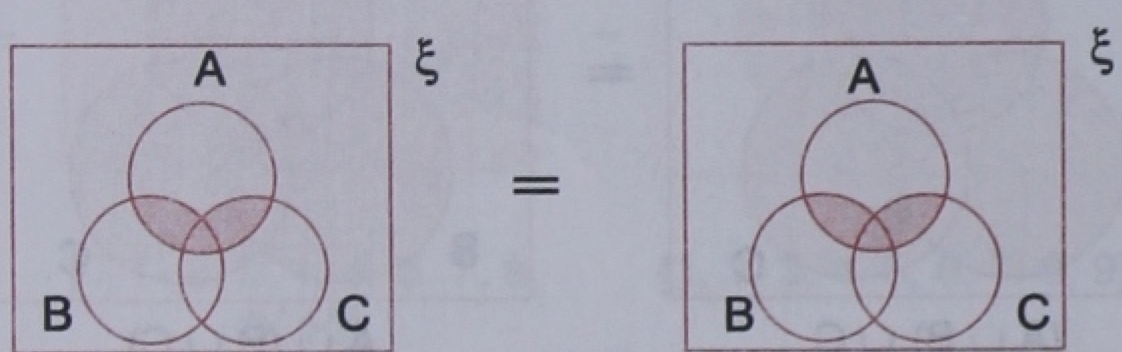
A ∪ (B ∩ C) (A ∪ B) ∩ (A ∪ C)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

2. Intersection of sets is distributive.



(B ∪ C) (A ∩ B) (A ∩ C)

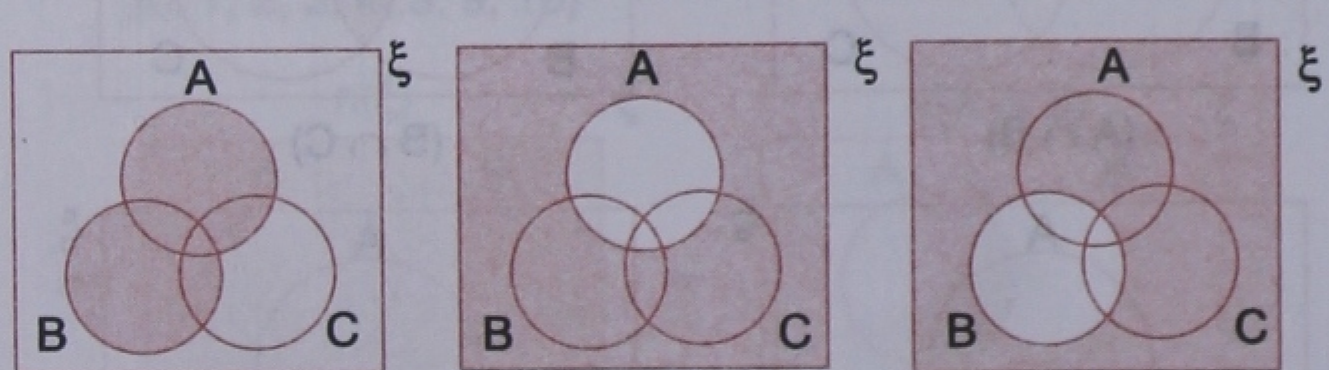


A ∩ (B ∪ C) (A ∩ B) ∪ (A ∩ C)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

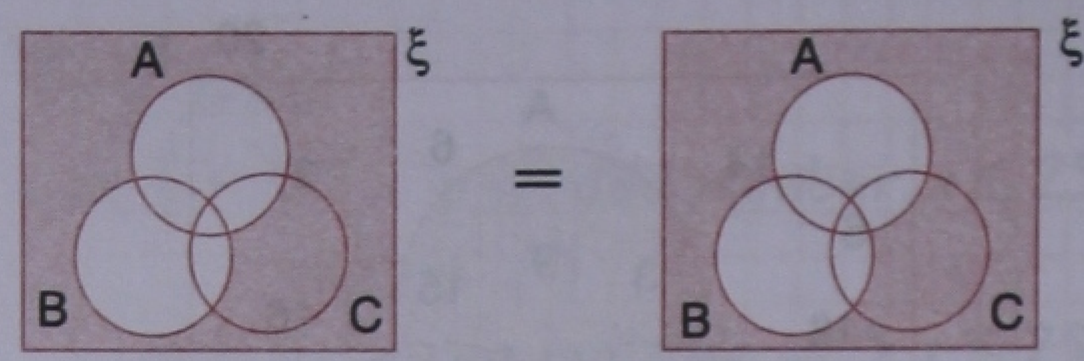
De Morgan's Laws

1. The complement of unions is the intersection of the complements.



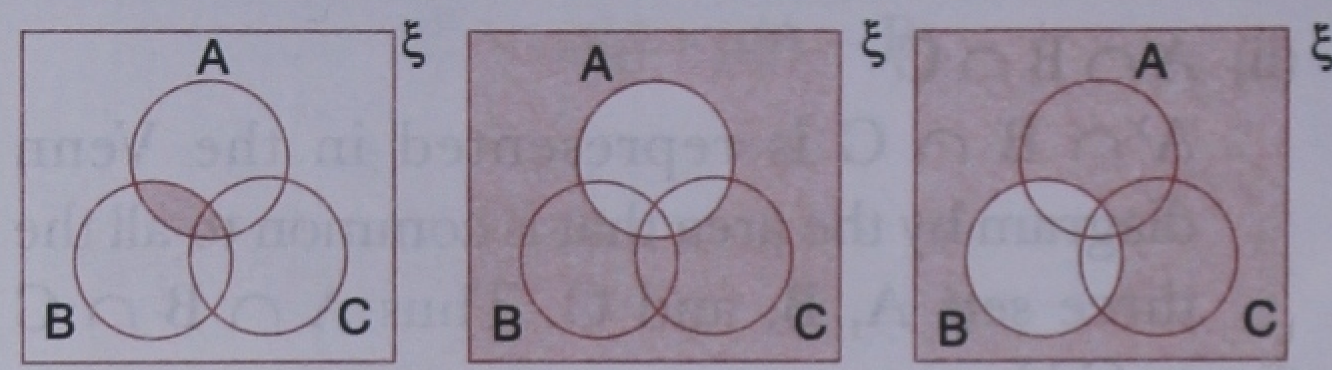
A ∪ B A′ B′

$$(A \cup B)' = A' \cap B'$$

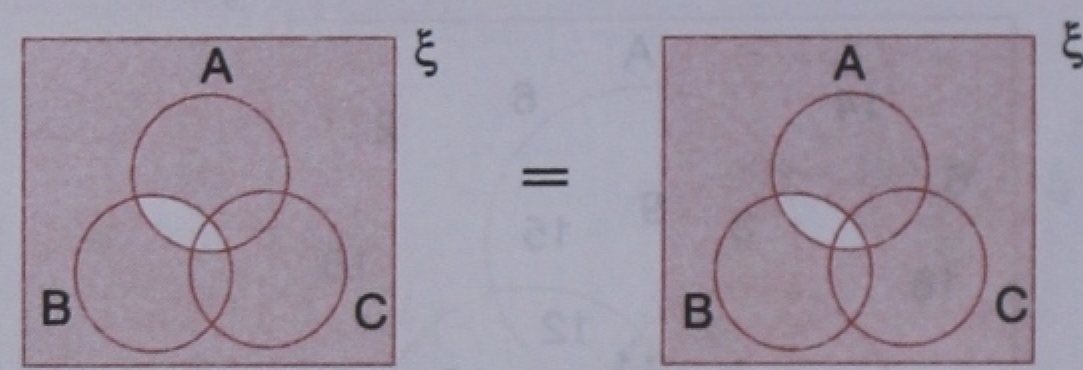


(A ∪ B)′ A′ ∩ B′

2. The complement of intersections is the union of the complements.



A ∩ B A′ B′

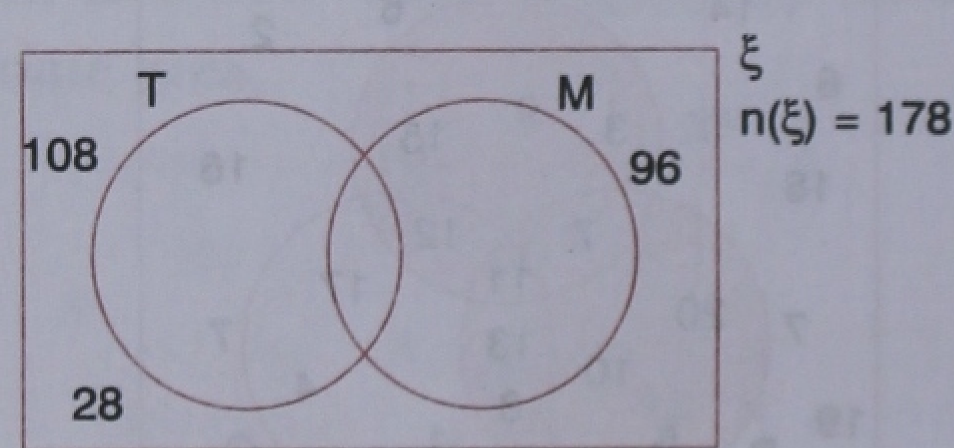


(A ∩ B)′ A′ ∪ B′

$$(A \cap B)' = A' \cup B'$$

Word Problems

Example 2: In a colony of 178 houses, 108 households own colour televisions and 96 households own music systems. If 28 households own neither a colour TV nor a music system, find how many households own both colour televisions and music systems.



Given $n(T) = 108$, $n(M) = 96$, $n(\xi) = 178$ and

$$n(T \cup M)' = 28$$

$$n(T \cup M) = n(\xi) - n(T \cup M)' = 178 - 28 = 150$$

$$n(T \cap M) = n(T) + n(M) - n(T \cup M)$$

$$= 108 + 96 - 150 = 54$$

Thus, 54 households own both colour televisions and music systems.

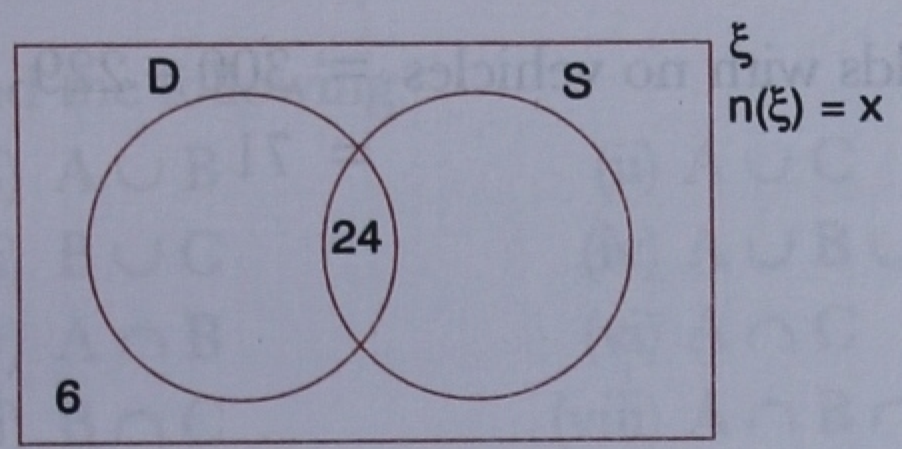
Example 3: 75% of students in a class chose to participate in a Drama Fest and 50% of the students chose to participate in a Sports Meet. If 24 students have chosen to attend both and 6 students have chosen to attend neither of the two events, how many students study in that class?

Let there be x number of students in the class.
Given: Number of students who participate in both drama as well as sports

$$= n(D \cap S) = 24$$

Number of students who participate neither in drama nor in sports

$$= n(D \cup S)' = 6$$



Now $D = 75\% \text{ of } x = \frac{75x}{100} = \frac{3x}{4}$

Thus, number of students who participate only in drama but not in sports = n(D – S)

$$= \frac{3x}{4} - 24$$

Now $S = 50\% \text{ of } x = \frac{50x}{100} = \frac{x}{2}$

Thus, number of students who participate only in sports but not in drama = n(S – D)

$$= \frac{x}{2} - 24$$

$$n(\xi) = x = n(D \cap S) + n(D \cup S)' + n(D - S) + n(S - D)$$

$$= 24 + 6 + \frac{3x}{4} - 24 + \frac{x}{2} - 24$$

$$\Rightarrow \quad x = \frac{3x}{4} + \frac{x}{2} + 30 - 48 \Rightarrow x - \frac{3x}{4} - \frac{x}{2} = -18$$

$$\Rightarrow \quad \frac{4x - 3x - 2x}{4} = -18 \Rightarrow -\frac{x}{4} = -18$$

$$\Rightarrow \quad x = 18 \times 4 = 72$$

(multiplying both sides by –4)

Thus, n(ξ) = 72
Thus, there are 72 students in the class.

Example 4: In a residential complex with 300 households, $\frac{1}{6}$ owned cars, $\frac{3}{10}$ owned motorcycles, and $\frac{1}{2}$ owned bicycles. 26 households owned cars as well as motorcycles while 25 households owned cars as well as bicycles. 30 households owned motorcycles as well as bicycles. If $\frac{1}{15}$ of all the households owned all three types of vehicles, find the number of households with only one type of vehicle and the number of households with no vehicles.

Let car owners be represented by set C

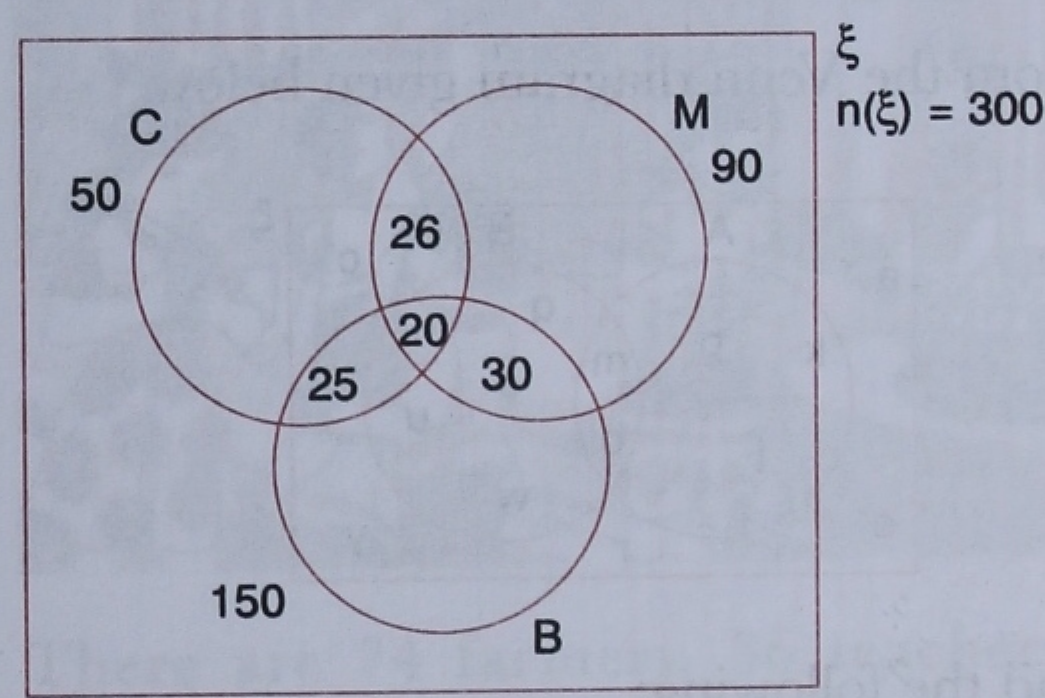
Then $n(C) = \frac{1}{6} \times 300 = 50$

Let motorcycle owners be represented by set M

Then $n(M) = \frac{3}{10} \times 300 = 90$

Let bicycle owners be represented by set B

Then $n(B) = \frac{1}{2} \times 300 = 150$



Given $n(C \cap M) = 26$
$n(C \cap B) = 25$
$n(M \cap B) = 30$
$n(C \cap M \cap B) = \frac{1}{15} \times 300 = 20$

From the Venn diagram it can be observed that

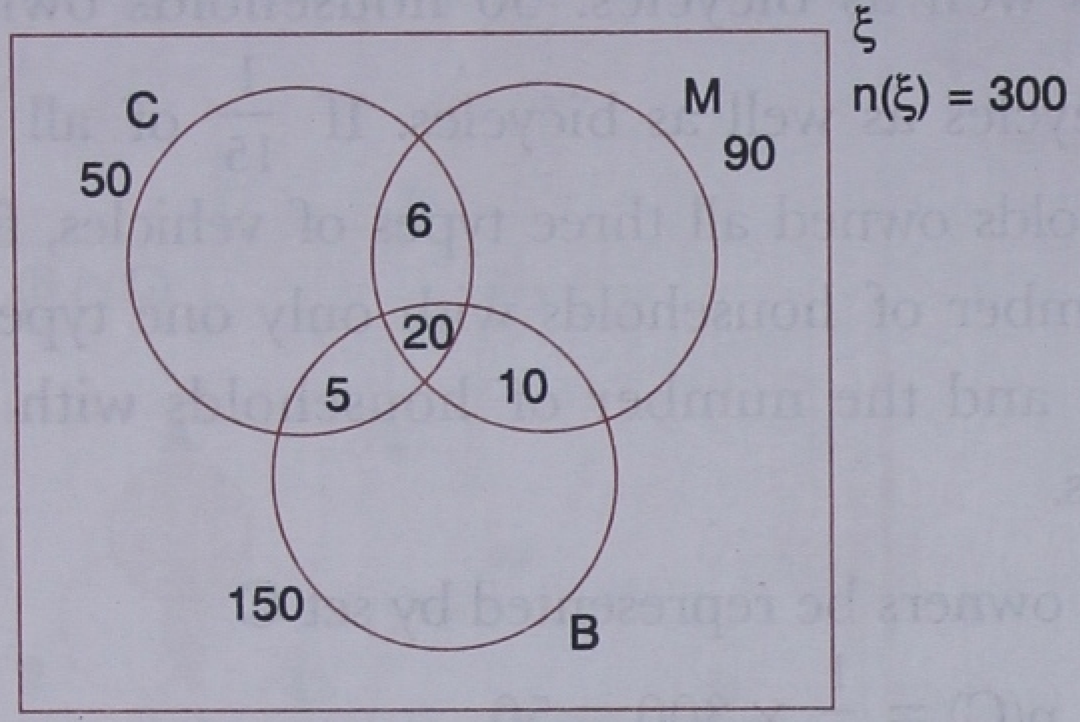
(i) Number of households with cars and motorcycles, but no bicycles
$= n(C \cap M) - n(C \cap M \cap B) = 26 - 20 = 6$

(ii) Number of households with cars and bicycles, but no motorcycles
$= n(C \cap B) - n(C \cap M \cap B) = 25 - 20 = 5$

(iii) Number of households with bicycles and motorcycles, but no cars

$= n(M \cap B) - n(C \cap M \cap B) = 30 - 20 = 10$

Let us draw a fresh Venn diagram showing the cardinal numbers calculated above.



Thus, number of households with only cars $= 50 - (6 + 20 + 5)$

$= 50 - 31 = 19$

Number of households with only motorcycles $= 90 - (6 + 20 + 10)$

$= 90 - 36 = 54$

Number of households with only bicycles

$= 150 - (10 + 20 + 5)$

$= 150 - 35 = 115$

Thus, number of households with only one vehicle $= 19 + 54 + 115$

$= 188$

Total vehicle onwers $= 188 + 6 + 5 + 10 + 20$

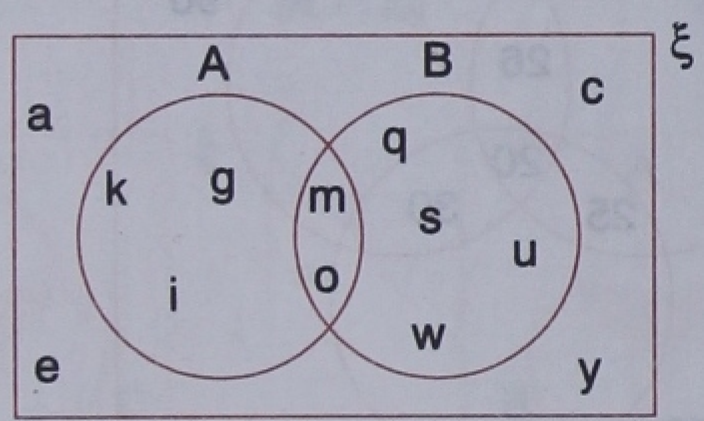
$= 229$

Households with no vehicles $= 300 - 229$

$= 71$

Exercise 3.1

1. From the Venn diagram given below,



find the following:

(i) $A \cup B$ (ii) $A \cap B$
(iii) A' (iv) B'
(v) $(A \cup B)'$ (vi) $(A \cap B)'$
(vii) $A - B$ (viii) $B - A$
(ix) $A' \cap B$ (x) $A \cap B'$

2. Given that

$\xi = \{x \mid x \leq 20 \text{ and } x \in N\}$,

$A = \{x \mid x \text{ is a multiple of } 2, x \leq 20 \text{ and } x \in N\}$, and

$B = \{x \mid x \text{ is a multiple of } 3, x \leq 20 \text{ and } x \in N\}$,

draw Venn diagrams and find the following:

(i) $A \cup B$ (ii) $A \cap B$
(iii) A' (iv) B'
(v) $(A \cup B)'$ (vi) $(A \cap B)'$
(vii) $A - B$ (viii) $B - A$
(ix) $A' \cap B$ (x) $A \cap B'$

3. Given that

$\xi = \{50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60\}$,

$A = \{51, 53, 55, 57, 59\}$, and

$B = \{52, 54, 56, 58\}$,

draw Venn diagrams and find the following:

(i) $A \cup B$ (ii) A'
(iii) B' (iv) $A' - B$
(v) $B' - A$

Show that $A - B = A$ and $B - A = B$.

4. Given that

$\xi = \{x \mid x \text{ is a multiple of } 2, 20 \leq x \leq 40 \text{ and } x \in N\}$,

$A = \{x \mid x \text{ is a multiple of } 4, 20 \leq x \leq 40 \text{ and } x \in N\}$, and

$B = \{x \mid x \text{ is a multiple of } 8, 20 \leq x \leq 40 \text{ and } x \in N\}$,

draw Venn diagrams and find the following:

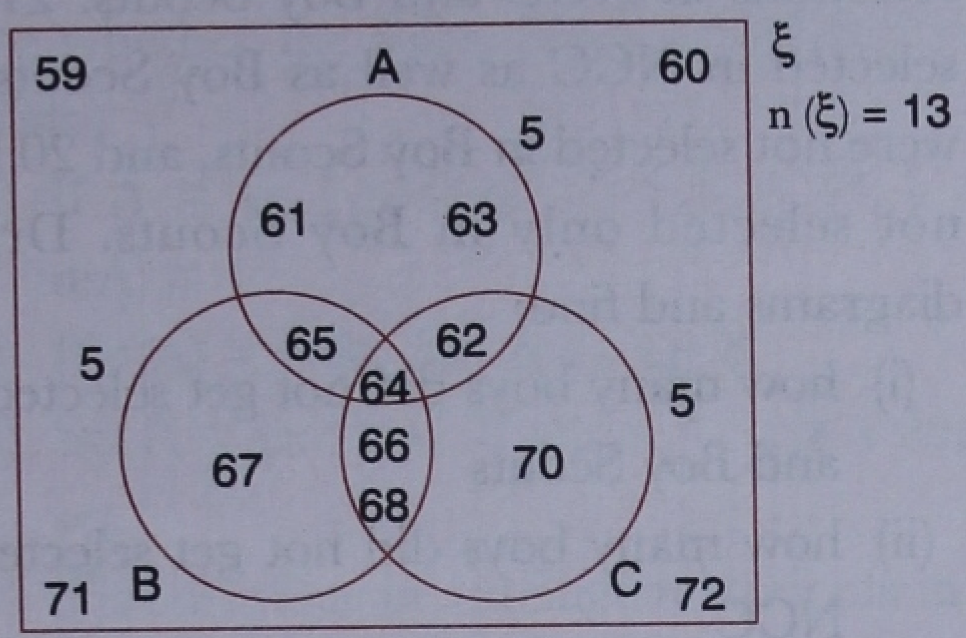
(i) $A \cup B$ (ii) $A \cap B$

(iii) A′ (iv) B′

(v) A – B

Show that B′ – A = A′.

5. From the Venn diagram given below,



find the following:

(i) A ∪ B (ii) A ∪ C

(iii) B ∪ C (iv) A ∪ B ∪ C

(v) A ∩ B (vi) A ∩ C

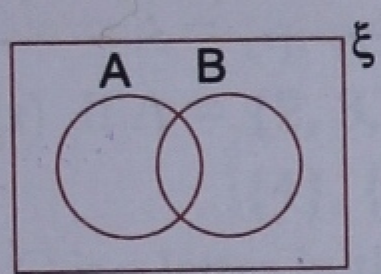
(vii) B ∩ C (viii) A ∩ B ∩ C

(ix) (A – B) ∪ C (x) (A – C) ∩ B

(xi) (A ∪ B ∪ C)′ (xii) (A ∩ C ∩ B)′

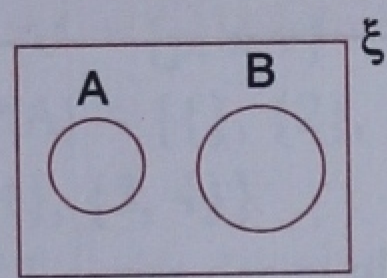
6. Shade the given Venn diagrams to describe the following sets:

(i)



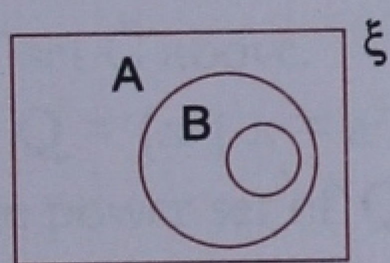
(A ∩ B)′

(ii)



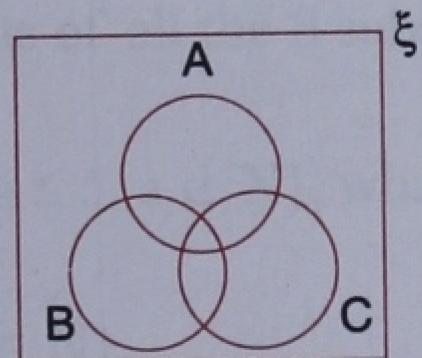
B′

(iii)



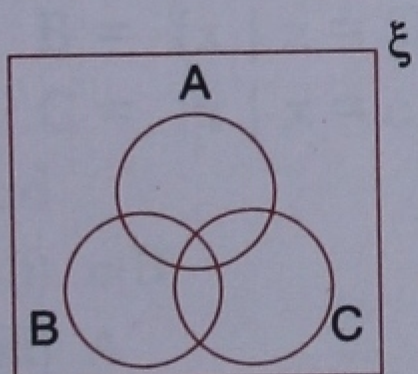
A – B

(iv)



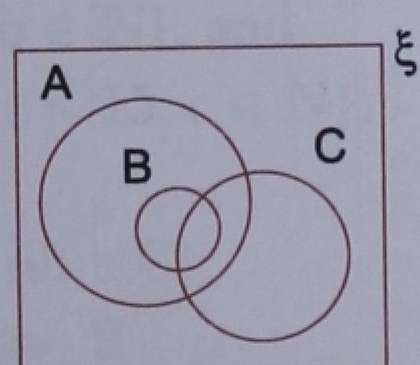
(A – B) ∩ C

(v)



A ∪ (B ∩ C)

(vi)



(A ∩ C) – B

7. In a shopping mall, 36 employees are wearing ties and 25 employees are wearing caps. If there are 55 employees in all, find the number of employees who are wearing a tie as well as a cap if

(i) all the employees are wearing either a cap or a tie

(ii) 5 employees are wearing neither a cap nor a tie

8. $\frac{5}{7}$ of all students in a hostel read an English newspaper while $\frac{3}{5}$ of all students read a Hindi newspaper every day. If 82 students read newspapers in both languages, while 16 students do not read a newspaper at all, find how many students stay in the hostel.

9. In a birthday party, out of 53 children, 8 children have neither an ice cream nor a cold drink. If 30 children have ice cream and 23 children have cold drinks, find:

(i) how many children have only ice cream

(ii) how many children have only cold drinks

(iii) how many children have both cold drinks as well as ice creams

10. There are 74 farmers, 36 teachers, and 40 graduates in a village. 12 farmers are teachers as well and 14 farmers are graduates too. If there are 16 graduate teachers in the village and 4 graduate teachers do farming as well, find:

(i) the number of farmers who are neither graduates nor teachers

(ii) the number of teachers who are not graduates

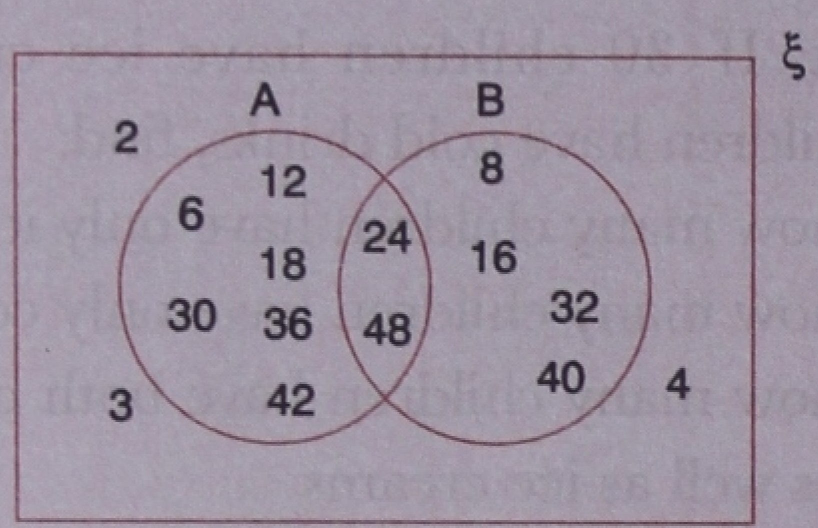
(iii) the number of non-graduate teachers who are not farmers

(iv) the number of graduates who are neither farmers nor teachers

(v) the number of graduate teachers who are not farmers

Revision Exercise

1. Represent the following sets in Venn diagrams.
   $\xi = \{x \mid x = n, n < 40, n \in N\}$
   $A = \{x \mid x = 6n, n < 6, n \in N\}$
   $B = \{x \mid x = 3n, n < 13, n \in N\}$
2. If $x = \{x \mid$ is a prime number, $x < 29\}$ and. $A = \{x \mid x$ is a prime factor of $210\}$, represent A in venn diagram and find A'.
3. The following Venn diagram shows intersecting sets A and B. Write the sets for the following in roster form.



(a) $A \cup B$ (b) $A \cap B$ (c) $A - B$ (d) $B - A$
(e) A' (f) B' (g) $(A \cup B)'$ (h) $(A \cap B)'$
(i) $(A - B)'$ (j) $(B - A)'$

4. 95 boys of a school appeared for a physical test for selections in NCC and Boy Scouts. 21 boys got selected in NCC as well as Boy Scouts, 44 boys were not selected in Boy Scouts, and 20 boys were not selected only in Boy Scouts. Draw Venn diagrams and find:
   (i) how many boys did not get selected in NCC and Boy Scouts
   (ii) how many boys did not get selected only in NCC
   (iii) how many boys got selected in NCC
   (iv) how many boys got selected in Boy Scouts
   (v) how many boys got selected in NCC but not Boy Scouts

   (Hint : Solving the question will be easier if you begin with a Venn diagram for boys who did not get selected.)