2
Operations and Venn Diagrams

Operations on Sets

When we carry out the operations of addition, multiplication, etc., on two numbers, we get new numbers. Similarly, when we carry out the operations of union and intersection on two sets, we get new sets. Just as we can find the difference between two numbers, we can also find the difference between two sets.

Union of two sets

The union of the sets A and B is the set of all the elements that belong to either A or B or both. It is denoted by $A \cup B$ (read as "A union B").

We can write, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Examples (i) Let $A = \{2, 4, 6\}$ and $B = \{6, 8, 10\}$.
Then $A \cup B = \{2, 4, 6\} \cup \{6, 8, 10\} = \{2, 4, 6, 8, 10\}$.

(ii) Let $P = \{a, b, c\}$ and $Q = \{x, y, z\}$.
Then $P \cup Q = \{a, b, c, x, y, z\}$.

(iii) Let $A = \{5, 6, 8\}$.
Then $A \cup A = \{5, 6, 8\} \cup \{5, 6, 8\} = \{5, 6, 8\} = A$.
Also, $A \cup \phi = \{5, 6, 8\} \cup \phi = \{5, 6, 8\} = A$.

(iv) Let $A = \{a, b, c\}$ and $B = \{a, b, c, d\}$.
Here, $A \subseteq B$ then $A \cup B = \{a, b, c, d\} = B$.

Note The results we have arrived at in (iii) and (iv) are always true. In other words, they are laws of operations on sets.

Intersection of two sets

The intersection of the sets A and B is the set of all the elements which belong to both A and B. It is denoted by $A \cap B$ (read as "A intersection B").

We can write, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

If A and B do not have any element in common then $A \cap B$ = a null set = ϕ.

Examples (i) Let $A = \{2, 4, 6, 8\}$ and $B = \{4, 8, 12\}$.
Then $A \cap B = \{2, 4, 6, 8\} \cap \{4, 8, 12\} = \{4, 8\}$.

(ii) Let $P = \{1, 3, 5, 7, 9, 11, 13\}$ and $Q = \{2, 4, 6, 8, 10, 12, 14\}$.
Then $P \cap Q = \{1, 3, 5, 7, 9, 11, 13\} \cap \{2, 4, 6, 8, 10, 12, 14\}$
$= \phi$, because there is no element that is common to both P and Q.

(iii) Let $A = \{a, b, c\}$.

Then $A \cap A = \{a, b, c\} \cap \{a, b, c\} = \{a, b, c\} = A$.

Also, $A \cap \phi = \{a, b, c\} \cap \phi = \phi$.

(iv) Let $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 1, 2\}$.

Here, $B \subseteq A$ then $A \cap B = \{-2, -1, 0, 1, 2\} \cap \{0, 1, 2\} = \{0, 1, 2\} = B$.

Note (iii) and (iv) are also laws of operations on sets.

Disjoint sets

Two sets A and B are called disjoint sets if they have no element in common, that is, $A \cap B = \phi$.

Example Let $A = \{x : x \text{ is a positive integer}\}$ and $B = \{x : x \text{ is a negative integer}\}$.

Then $A = \{1, 2, 3, 4, \ldots\}$ and $B = \{-1, -2, -3, -4, \ldots\}$.

Then $A \cap B = \{1, 2, 3, 4, \ldots\} \cap \{-1, -2, -3, -4, \ldots\} = \phi$. So A and B are disjoint sets.

Overlapping sets

Two sets A and B are called overlapping sets if they have at least one element in common, that is $A \cap B \neq \phi$.

Example Let $A = \{x : x \text{ is prime and } x < 10\}$ and $B = \{\text{first two natural numbers}\}$.

Then $A = \{2, 3, 5, 7\}$ and $B = \{1, 2\}$.

So, $A \cap B = \{2, 3, 5, 7\} \cap \{1, 2\} = \{2\}$.

$\therefore$ $A \cap B \neq \phi$. So, A and B are overlapping sets.

Difference of two sets

Let A and B be any two sets. Then the difference of A and B is the set of elements which belong to A but not to B. This is denoted by $A - B$.

We can write $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.

Example Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$.

Then $A - B = \{1, 3\}$ and $B - A = \{6\}$.

Complement of a set

Given the universal set U, the complement (or complementary set) of a set A is the set of those elements of U which are not elements of A. It is denoted by A' (or A^C or $\overline{A}$). Symbolically, $A' = \{x : x \in U \text{ and } x \notin A\}$. In other words, A' is the difference of the universal set U and A.

$$A' = U - A$$

Examples (i) Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 3, 5\}$.

Then $A' = U - A = \{1, 2, 3, 4, 5\} - \{1, 3, 5\} = \{2, 4\}$.

(ii) Let $U = \{x : x \in N \text{ and } x \leq 10\}$ and $A = \{x : x \in W \text{ and } 4 \leq x \leq 6\}$.

Then, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{4, 5, 6\}$.

Thus, $A' = U - A = \{1, 2, 3, 7, 8, 9, 10\}$.

Solved Examples

EXAMPLE 1 **Let A = {letters of the word CRICKET} and B = {letters of the word HOCKEY}. Find (i) $A \cup B$, (ii) $B \cup A$, (iii) $A \cap B$, (iv) $B \cap A$, (v) $A - B$, (vi) $B - A$.**

Solution In the tabular form: $A = \{C, R, I, K, E, T\}$, $B = \{H, O, C, K, E, Y\}$

(i) $A \cup B = \{C, R, I, K, E, T\} \cup \{H, O, C, K, E, Y\} = \{C, R, I, K, E, T, H, O, Y\}$.

(ii) $B \cup A = \{H, O, C, K, E, Y\} \cup \{C, R, I, K, E, T\} = \{H, O, C, K, E, Y, R, I, T\}$.

(iii) $A \cap B = \{C, R, I, K, E, T\} \cap \{H, O, C, K, E, Y\} = \{C, K, E\}$.

(iv) $B \cap A = \{H, O, C, K, E, Y\} \cap \{C, R, I, K, E, T\} = \{C, K, E\}$.

(v) $A - B = \{C, R, I, K, E, T\} - \{H, O, C, K, E, Y\} = \{R, I, T\}$.

(vi) $B - A = \{H, O, C, K, E, Y\} - \{C, R, I, K, E, T\} = \{H, O, Y\}$.

Note From (i) and (ii) it should be clear that $A \cup B = B \cup A$. Similarly, from (iii) and (iv), $A \cap B = B \cap A$. These two are laws of operations on sets.

EXAMPLE 2 **If $U = \{x : x \in W \text{ and } 6 \leq x \leq 11\}$, $A = \{6, 8, 9\}$, $B = \{7, 8, 11\}$ and $C = \{6\}$, find the following sets.**

(i) A' (ii) B' (iii) C' (iv) $A - B$ (v) $(B \cup C)'$ (vi) $(A \cap C)'$

(vii) $A - (B \cup C)$ (viii) $A - (B \cap C)$

Solution Here, $U = \{x : x \in W \text{ and } 6 \leq x \leq 11\} = \{6, 7, 8, 9, 10, 11\}$

$A = \{6, 8, 9\}$, $B = \{7, 8, 11\}$ and $C = \{6\}$.

(i) $A' = U - A = \{6, 7, 8, 9, 10, 11\} - \{6, 8, 9\} = \{7, 10, 11\}$.

(ii) $B' = U - B = \{6, 7, 8, 9, 10, 11\} - \{7, 8, 11\} = \{6, 9, 10\}$.

(iii) $C' = U - C = \{6, 7, 8, 9, 10, 11\} - \{6\} = \{7, 8, 9, 10, 11\}$.

(iv) $A - B = \{6, 8, 9\} - \{7, 8, 11\} = \{6, 9\}$.

(v) $B \cup C = \{7, 8, 11\} \cup \{6\} = \{6, 7, 8, 11\}$.

$\therefore \quad (B \cup C)' = U - (B \cup C) = \{6, 7, 8, 9, 10, 11\} - \{6, 7, 8, 11\} = \{9, 10\}$.

(vi) $A \cap C = \{6, 8, 9\} \cap \{6\} = \{6\}$.

$\therefore \quad (A \cap C)' = U - (A \cap C) = \{6, 7, 8, 9, 10, 11\} - \{6\} = \{7, 8, 9, 10, 11\}$.

(vii) $A - (B \cup C) = \{6, 8, 9\} - \{6, 7, 8, 11\} = \{9\}$.

(viii) $B \cap C = \{7, 8, 11\} \cap \{6\} = \phi$. So, $A - (B \cap C) = \{6, 8, 9\} - \phi = \{6, 8, 9\}$.

EXAMPLE 3 **Let $U = \{x : x \in N \text{ and } 2 \leq x \leq 9\}$, $A = \{x : x \text{ is an even number}\}$, $B = \{x : x \text{ is a multiple of } 3\}$ and $C = \{x : x \text{ is a multiple of } 4\}$. Verify the following.**

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$ (iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution Here, $U = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{3, 6, 9\}$ and $C = \{4, 8\}$.

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{3, 6, 9\} = \{2, 3, 4, 6, 8, 9\}$. So, $(A \cup B)' = \{5, 7\}$.

$A' = U - A = \{2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\} = \{3, 5, 7, 9\}$.

$B' = U - B = \{2, 3, 4, 5, 6, 7, 8, 9\} - \{3, 6, 9\} = \{2, 4, 5, 7, 8\}$.

$\therefore \quad A' \cap B' = \{3, 5, 7, 9\} \cap \{2, 4, 5, 7, 8\} = \{5, 7\}$.

$$\therefore \quad (A \cup B)' = A' \cap B'$$

(ii) $A \cap B = \{6\}$, so $(A \cap B)' = \{2, 3, 4, 5, 7, 8, 9\}$.

$A' \cup B' = \{3, 5, 7, 9\} \cup \{2, 4, 5, 7, 8\} = \{2, 3, 4, 5, 7, 8, 9\}$.

$$\therefore \quad (A \cap B)' = A' \cup B'$$

(iii) $B \cap C = \phi$, so $A \cup (B \cap C) = A \cup \phi = A = \{2, 4, 6, 8\}$.

$(A \cup B) \cap (A \cup C) = \{2, 3, 4, 6, 8, 9\} \cap \{2, 4, 6, 8\} = \{2, 4, 6, 8\}$.

$$\therefore \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(iv) $A \cap (B \cup C) = \{2, 4, 6, 8\} \cap \{3, 4, 6, 8, 9\} = \{4, 6, 8\}$.

$(A \cap B) \cup (A \cap C) = \{6\} \cup \{4, 8\} = \{4, 6, 8\}$.

$$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Note (i), (ii), (iii) and (iv) are laws of operations on sets.

Remember These

1. **Symbol** — **Meaning**

Symbol	Meaning
$A \cup B$	The union of the sets A and B
$A \cap B$	The intersection of the sets A and B
$A - B$	The difference of the sets A and B
A' or A^C	The complementary set of the set A

2. Two sets A and B are
   (i) disjoint if $A \cap B = \phi$ (ii) overlapping if $A \cap B \neq \phi$
3. To find $A - B$, list all the members of A which are not members of B.
4. To find A', list all the members of the universal set U which are not members of A.
   For any set A, $A \subseteq A$ and $\phi \subseteq A$.
5. Laws of operations on sets
   (i) $A \cup A = A,\ A \cap A = A,\ A \cup \phi = A,\ A \cap \phi = \phi$ [Idempotent laws]
   (ii) $A \cup B = B \cup A,\ A \cap B = B \cap A$ [Commutative laws]
   (iii) $(A \cup B) \cup C = A \cup (B \cup C),\ (A \cap B) \cap C = A \cap (B \cap C)$ [Associative laws]
   (iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C),\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Distributive laws]
   (v) $(A \cup B)' = A' \cap B',\ (A \cap B)' = A' \cup B'$ [De Morgan's laws]
   (vi) $A \subseteq A \cup B,\ B \subseteq A \cup B,\ A \cap B \subseteq A,\ A \cap B \subseteq B$
   (vii) If $A \subseteq B$ then $A \cup B = B$, $A \cap B = A$ and $A - B = \phi$.

EXERCISE 2A

1. If $A = \{3, 4, 5\}$ and $B = \{5, 6, 7, 8\}$ then find (i) $A \cup B$, (ii) $A \cap B$, (iii) $A - B$, (iv) $B - A$ and verify the following.
   (a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$ (c) $A \cup A = A$ (d) $A \cup \phi = A$
   (e) $A \cap A = A$ (f) $A \cap \phi = \phi$
2. Let $U = \{5, 6, 7, 8, 9, 10\}$, $P = \{5, 6, 7, 9\}$, $Q = \{5, 6, 10\}$ and $R = \{5, 9\}$. Find:
   (i) P' (ii) Q' (iii) R' (iv) $P - Q$
   (v) $Q \cup R$ (vi) $P \cap R$ (vii) $P - (Q \cup R)$ (viii) $P - (Q \cap R)$
3. If A = {letters of the word RICE}, B = {letters of the word NICE}, find the following.
   (i) $A \cup B$ (ii) $B \cup A$ (iii) $A \cap B$ (iv) $B \cap A$
   (v) $A - B$ (vi) $B - A$
   Also verify that
   (a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$ (c) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
4. Let the universal set $U = \{x : x \in W \text{ and } x < 9\}$, A = {factors of 6} and B = {factors of 8}. Find the following sets.
   (i) A' (ii) B' (iii) $A' \cap B'$ (iv) $A' \cup B'$
   Also verify the following.
   (a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$ (c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, $B = \{1, 3, 5, 7\}$, verify the following.
   (i) $A' = B$ (ii) $B' = A$ (iii) $A' \cup B' = U$ (iv) $A \cup B = U$

ANSWERS

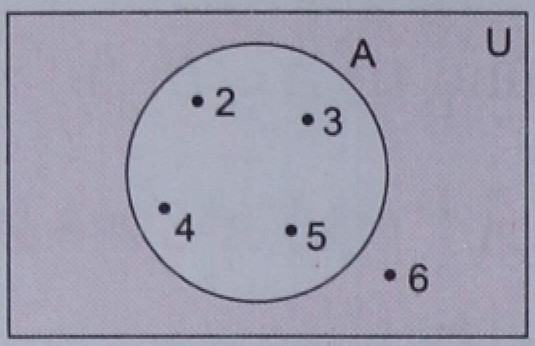
1. (i) {3, 4, 5, 6, 7, 8} (ii) {5} (iii) {3, 4} (iv) {6, 7, 8}
2. (i) {8, 10} (ii) {7, 8, 9} (iii) {6, 7, 8, 10} (iv) {7, 9} (v) {5, 6, 9, 10} (vi) {5, 9} (vii) {7} (viii) {6, 7, 9}
3. (i) {R, I, C, E, N} (ii) {R, I, C, E, N} (iii) {I, C, E} (iv) {I, C, E} (v) {R} (vi) {N}
4. (i) {0, 4, 5, 7, 8} (ii) {0, 3, 5, 6, 7} (iii) {0, 5, 7} (iv) {0, 3, 4, 5, 6, 7, 8}

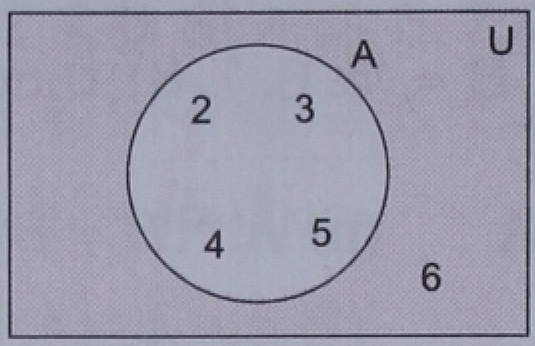
Venn Diagrams

A Venn diagram is a closed figure used to denote a set. Points within such a closed figure represent the elements of the set.

The universal set U is usually represented by a rectangle. All other sets are represented by closed curves (usually circles) within the rectangle. All the elements of a set are indicated by points inside the circle representing the set. The elements can also be written inside the circle without marking the points. An object that is not a member of a set is placed outside the circle representing the set.

Example The set $A = \{2, 3, 4, 5\}$ can be represented by either of the two diagrams shown. Here, $6 \notin A$.



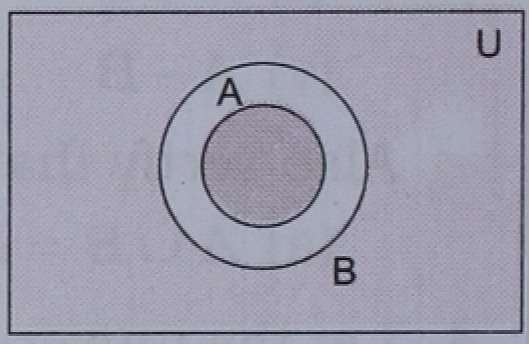


Relationships between sets

Venn diagrams can be used to represent the relationships between sets.

Subset

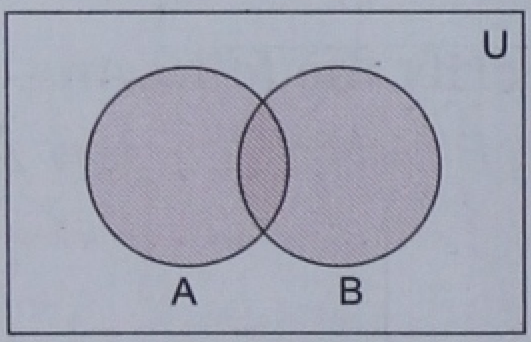
If $A \subseteq B$, the set A is represented by a circle within the circle representing the set B.



Union of sets

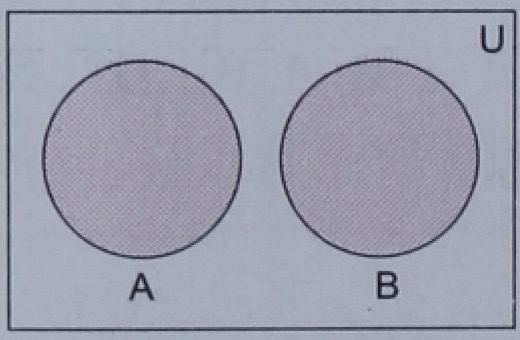
The set $A \cup B$ may be represented by the shaded portion of the following Venn diagrams under the three situations—(i) A and B are overlapping, (ii) A and B are disjoint and (iii) $A \subseteq B$.

(i)



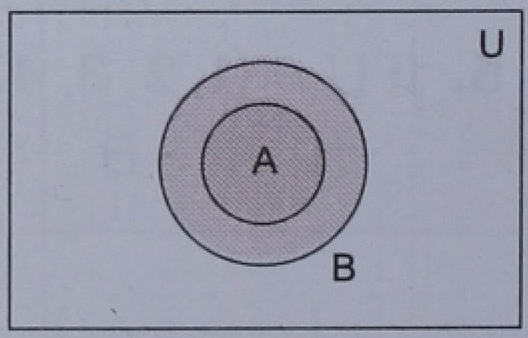
A and *B* are overlapping sets.

(ii)



A and *B* are disjoint sets.

(iii)

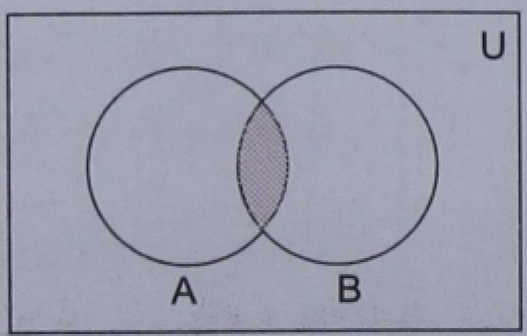


$A \subseteq B$

Intersection of sets

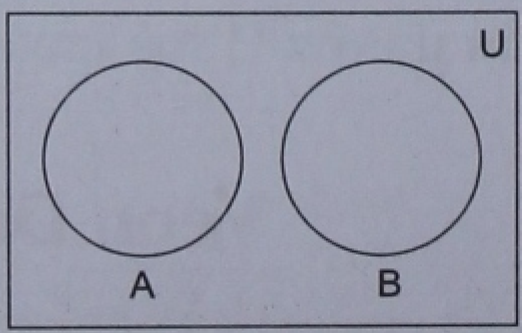
The shaded portion of the following Venn diagrams represent the set $A \cap B$ when (i) A and B are overlapping sets, (ii) A and B are disjoint sets and (iii) $A \subseteq B$. No portion of the diagram is shaded when A and B are disjoint sets because in this case $A \cap B = \phi$.

(i)



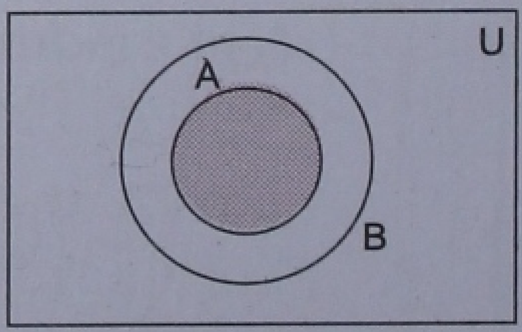
A and *B* are overlapping sets.

(ii)



A and *B* are disjoint sets.

(iii)

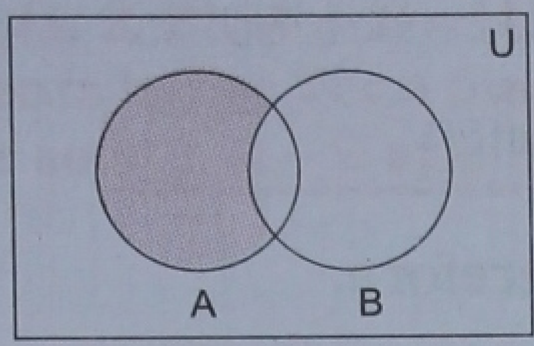


$A \subseteq B$

Difference of two sets

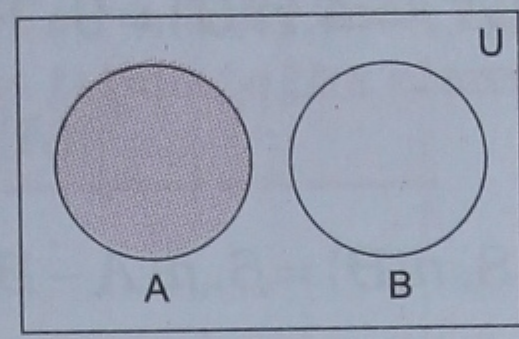
The shaded portion of the following Venn diagrams represent the set $A - B$ when (i) A and B are overlapping sets, (ii) A and B are disjoint sets and (iii) $A \subseteq B$. No portion of the third diagram is shaded because $A - B = \phi$.

(i)



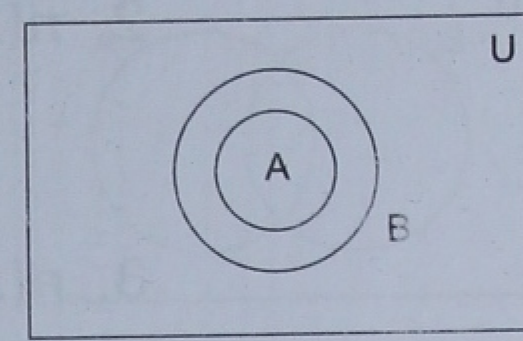
A and *B* are overlapping sets.

(ii)



A and *B* are disjoint sets.

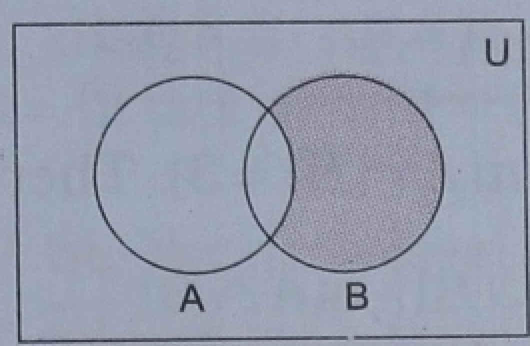
(iii)



$A \subseteq B$

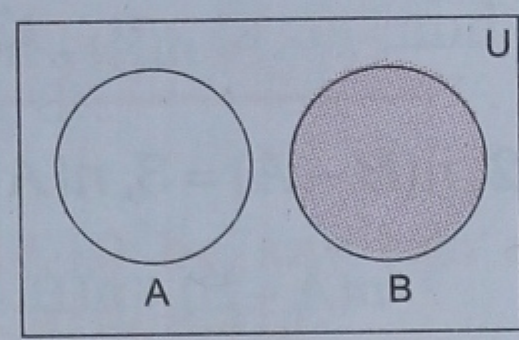
The shaded portion of the following diagrams show the set $B - A$ in three cases.

(i)



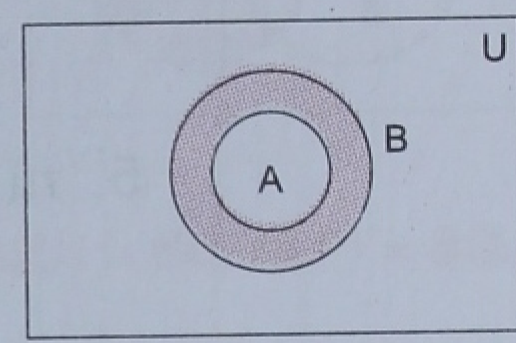
A and *B* are overlapping sets.

(ii)



A and *B* are disjoint sets.

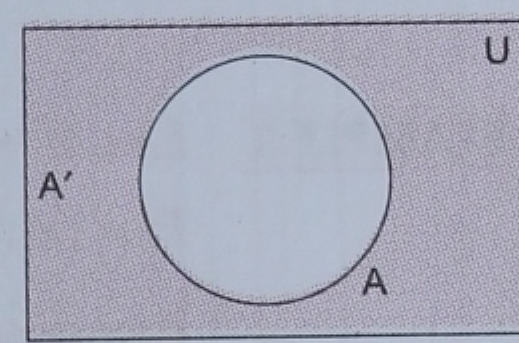
(iii)



$A \subseteq B$

Complement of a set

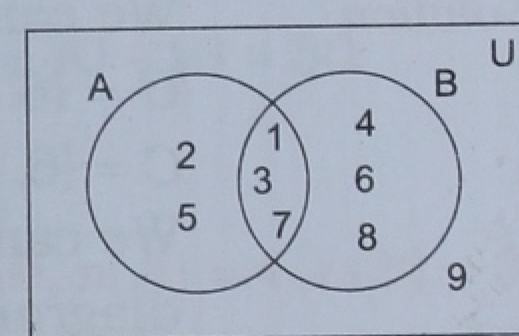
The shaded portion of the diagram represents the set A', which is the complement of the set A.



A': shaded portion

Interpreting a Venn diagram

In the Venn diagram shown alongside, the rectangle representing the universal set U contains the elements 2, 5, 1, 3, 7, 4, 6, 8, 9.



$\therefore \quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $n(U) = 9$.

The closed region in the rectangle representing the set A contains the elements 2, 5, 1, 3, 7, while that representing the set B contains 1, 3, 7, 4, 6, 8. Thus,

$A = \{1, 2, 3, 5, 7\}$ and $n(A) = 5$.

$B = \{1, 3, 7, 4, 6, 8\} = \{1, 3, 4, 6, 7, 8\}$ and $n(B) = 6$;

$A \cup B = \{2, 5, 1, 3, 7, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $n(A \cup B) = 8$;

$A \cap B = \{1, 3, 7\}$ and $n(A \cap B) = 3$;

$A - B = \{2, 5\}$ and $n(A - B) = 2$; $\quad B - A = \{4, 6, 8\}$ and $n(B - A) = 3$;

$A' = \{4, 6, 8, 9\}$ and $n(A') = 4$; $\quad B' = \{2, 5, 9\}$ and $n(B') = 3$;

$(A \cup B)' = \{9\}$ and $n(A \cup B)' = 1$; $(A \cap B)' = \{2, 5, 4, 6, 8, 9\}$ and $n(A \cap B)' = 6$.

From these, we can arrive at certain general relations between the cardinal numbers of sets, which are known as cardinal properties of sets.

1. $n(A) + n(B) = 5 + 6 = 11$, $n(A \cup B) = 8$, $n(A \cap B) = 3$. Therefore,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

You know that $n(A \cap B) = 0$ when A and B are disjoint sets. Therefore, for disjoint sets,

$$n(A \cup B) = n(A) + n(B)$$

2. $n(A) = 5, n(A') = 4, n(U) = 9$. Therefore,

$$n(A) + n(A') = n(U)$$

3. $n(A \cup B) = 8, n(B) = 6, n(A - B) = 2$. Therefore,

$$n(A - B) + n(B) = n(A \cup B)$$

4. $n(A \cap B) = 3, n(A) = 5, n(A - B) = 2$. Therefore,

$$n(A) - n(A - B) = n(A \cap B)$$

5. $n(A - B) = 2, n(B - A) = 3, n(A \cup B) = 8, n(A \cap B) = 3)$. Therefore,

$$n(A - B) + n(B - A) = n(A \cup B) - n(A \cap B)$$

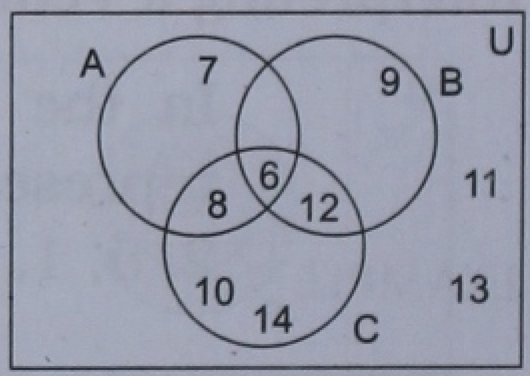
Solved Examples

EXAMPLE 1 **Let $U = \{x : x \in W \text{ and } 6 \le x < 15\}$, $A = \{x : x \text{ is a positive integer and } x \le 8\}$, $B = \{x : x \text{ is a multiple of } 3\}$ and $C = \{\text{even integers}\}$.**

Represent the given sets by a Venn diagram and find the following.

(i) $A \cap B$ (ii) $B \cap C$ (iii) $C \cap A$ (iv) A' (v) B' (vi) C' (vii) $(A \cup B)'$ (viii) $(B \cup C)'$

Solution We can represent the given sets in the tabular form as $U = \{6, 7, 8, 9, \ldots, 14\}$, $A = \{6, 7, 8\}$, $B = \{6, 9, 12\}$ and $C = \{6, 8, 10, 12, 14\}$.



We can arrive at the following from the adjoining Venn diagram which represents the given sets.

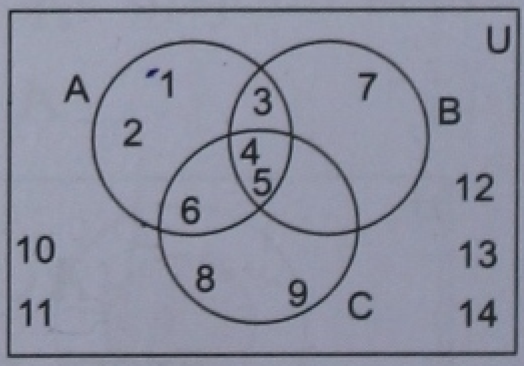
(i) $A \cap B = \{6\}$ (ii) $B \cap C = \{6, 12\}$

(iii) $C \cap A = \{6, 8\}$ (iv) $A' = \{9, 10, 11, 12, 13, 14\}$

(v) $B' = \{7, 8, 10, 11, 13, 14\}$ (vi) $C' = \{7, 9, 11, 13\}$

(vii) $(A \cup B)' = \{10, 11, 13, 14\}$ (viii) $(B \cup C)' = \{7, 11, 13\}$

EXAMPLE 2 **Find the following sets from the Venn diagram.**



(i) U (ii) A (iii) B (iv) C (v) A'

(vi) B' (vii) C' (viii) $A \cap B$ (ix) $B \cap C$ (x) $A \cap C$

(xi) $A - B$ (xii) $B - C$ (xiii) $A - C$ (xiv) $(A \cup B)'$

Solution

(i) $U = \{1, 2, 3, 4, \ldots, 14\}$ (ii) $A = \{1, 2, 3, 4, 5, 6\}$

(iii) $B = \{3, 4, 5, 7\}$ (iv) $C = \{4, 5, 6, 8, 9\}$

(v) $A' = \{7, 8, 9, \ldots, 14\}$ (vi) $B' = \{1, 2, 6, 8, 9, 10, 11, 12, 13, 14\}$

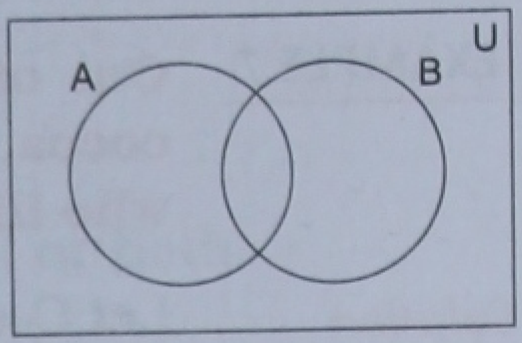
(vii) $C' = \{1, 2, 3, 7, 10, 11, 12, 13, 14\}$ (viii) $A \cap B = \{3, 4, 5\}$

(ix) $B \cap C = \{4, 5\}$ (x) $A \cap C = \{4, 5, 6\}$

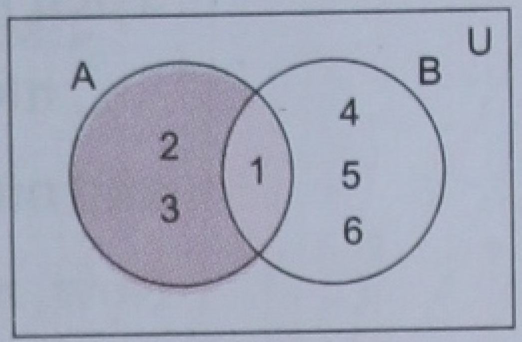
(xi) $A - B = \{1, 2, 6\}$ (xii) $B - C = \{3, 7\}$

(xiii) $A - C = \{1, 2, 3\}$ (xiv) $(A \cup B)' = \{8, 9, 10, 11, 12, 13, 14\}$

EXAMPLE 3 **If $A = \{1, 2, 3\}$, $A \cap B = \{1\}$ and $B - A = \{4, 5, 6\}$ then fill the elements in the Venn diagram. Also tabulate the elements of the sets B, $A - B$ and $A \cup B$.**



Solution Let us first fill the elements of $A \cap B$ and $B - A$. Then let us fill the elements of A that are not contained in $A \cap B$. These elements are 2 and 3. Now, from the Venn diagram. $B = \{1, 4, 5, 6\}$, $A - B = \{2, 3\}$ and $A \cup B = \{2, 3, 1, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$.



EXAMPLE 4 **Let U be the universal set and P, Q be any two sets. If $n(U) = 60$, $n(P) = 30$, $n(Q) = 20$ and $n(P \cap Q)' = 50$, find the following.**

(i) $n(P \cap Q)$ (ii) $n(P \cup Q)$ (iii) $n(P - Q)$ (iv) $n(Q - P)$

Solution (i) We know that $n(A) + n(A') = n(U)$

Replacing A by $P \cap Q$, $n(P \cap Q) + n(P \cap Q)' = n(U)$.

$\therefore \quad n(P \cap Q) + 50 = 60, \quad$ or $\quad n(P \cap Q) = 10$.

(ii) We know that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$.

$\therefore \quad n(P \cup Q) = 30 + 20 - 10 = 40$.

(iii) $\because \quad n(P - Q) + n(Q) = n(P \cup Q)$.

$\therefore \quad n(P - Q) = n(P \cup Q) - n(Q) = 40 - 20 = 20$.

(iv) $\because \quad n(P - Q) + n(Q - P) = n(P \cup Q) - n(P \cap Q)$, $20 + n(Q - P) = 40 - 10 = 30$.

$\therefore \quad n(Q - P) = 30 - 20 = 10$.

EXAMPLE 5 **Let U be the universal set and A and B any two sets. If $n(U) = 10$, $n(A) = 4$, $n(B) = 3$ and $n(A \cup B)' = 4$, find the following.**

(i) $n(A')$ (ii) $n(B')$ (iii) $n(A \cup B)$ (iv) $n(A \cap B)$ (v) $n(A - B)$ (vi) $n(B - A)$

Solution (i) $n(A) + n(A') = n(U)$. So, $n(A') = n(U) - n(A) = 10 - 4 = 6$.

(ii) $n(B) + n(B') = n(U)$. So, $n(B') = n(U) - n(B) = 10 - 3 = 7$.

(iii) $n(A) + n(A') = n(U)$. Replacing A by $A \cup B$,

$n(A \cup B) + n(A \cup B)' = n(U) \quad$ or $\quad n(A \cup B) + 4 = 10$.

$\therefore \quad n(A \cup B) = 10 - 4 = 6$.

(iv) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 4 + 3 - 6 = 1$.

(v) $n(A - B) + n(B) = n(A \cup B)$. So, $n(A - B) + 3 = 6$ or $n(A - B) = 3$.

(vi) $n(A - B) + n(B - A) = n(A \cup B) - n(A \cap B)$.

So, $3 + n(B - A) = 6 - 1$ or $n(B - A) = 2$.

EXAMPLE 6 **Let A and B be any two sets. If $n(A - B) = 20$, $n(A \cup B) = 60$, $n(A \cap B) = 10$, find.**

(i) $n(B - A)$ (ii) $n(A)$ (iii) $n(B)$

Solution (i) $n(A - B) + n(B - A) = n(A \cup B) - n(A \cap B)$.

$\therefore \quad 20 + n(B - A) = 60 - 10 = 50 \quad$ or $\quad n(B - A) = 50 - 20 = 30$.

(ii) $n(A) - n(A - B) = n(A \cap B)$. So, $n(A) - 20 = 10 \quad$ or $\quad n(A) = 30$.

(iii) $n(A - B) + n(B) = n(A \cup B)$. So, $20 + n(B) = 60 \quad$ or $\quad n(B) = 60 - 20 = 40$.

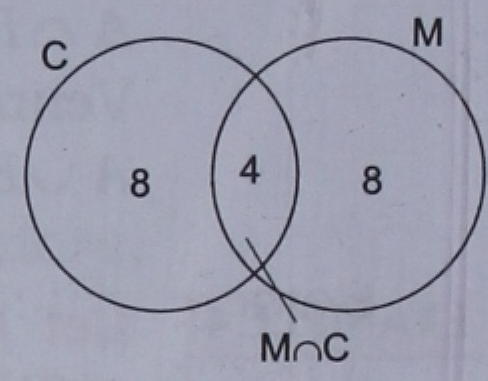
EXAMPLE 7 **Out of 20 students of a class who like either cocoa or milk or both, 12 like cocoa, while 4 like both. Draw a Venn diagram and find the number of students who like (i) milk (ii) only milk (iii) only cocoa.**

Solution Let C = {students who like cocoa}, M = {students who like milk}.

Then $n(C)$ = number of students who like cocoa = 12,

$n(C \cup M)$ = number of students who like either cocoa or milk or both = 20

and $n(C \cap M)$ = number of students who like both = 4.



Now, $n(C \cup M) = n(M) + n(C) - n(C \cap M)$

or $20 = n(M) + 12 - 4$ or $n(M) = 12$.

$\therefore$ the number of students who like milk = 12.

But the number of students who like both milk and cocoa = 4.

$\therefore$ the number of students who like only milk $= n(M) - n(C \cap M) = 12 - 4 = 8$.

Also, the number of students who like only cocoa $= n(C) - n(C \cap M) = 12 - 4 = 8$.

EXAMPLE 8 **Out of a class of 100 students, 70 watch cartoons, 80 watch sports and all watch either cartoons or sports or both. Find the number of students who watch (i) both cartoons and sports, (ii) only cartoons, (iii) only sports.**

Solution Let C = {students who watch cartoons}, S = {students who watch sports},

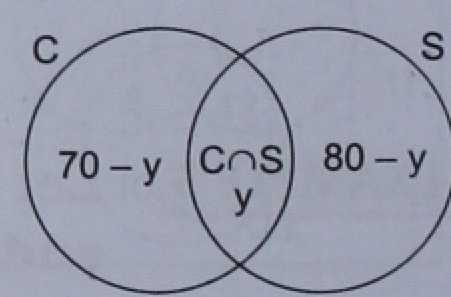
$C \cup S$ = {students who watch either cartoons or sports or both},

$C \cap S$ = {students who watch both}.

Given, $n(C) = 70$, $n(S) = 80$ and $n(C \cup S) = 100$.

Let $n(C \cap S)$ = the number of students who watch both = y.

(See Venn diagram.)



Then the number of students who watch only cartoons = $70 - y$

and the number of students who watch only sports = $80 - y$.

$\therefore$ the total number of students $= 100 = (70 - y) + y + (80 - y)$

or $100 = 150 - y$ or $y = 50$.

(i) So, the number of students who watch both = $y = 50$.

(ii) The number of students who watch only cartoons $= 70 - y = 70 - 50 = 20$.

(iii) The number of students who watch only sports $= 80 - y = 80 - 50 = 30$.

EXAMPLE 9 **Out of a group of 500 people, 240 watch cricket, 50 watch football and 15 watch both. Find the number of people who watch**

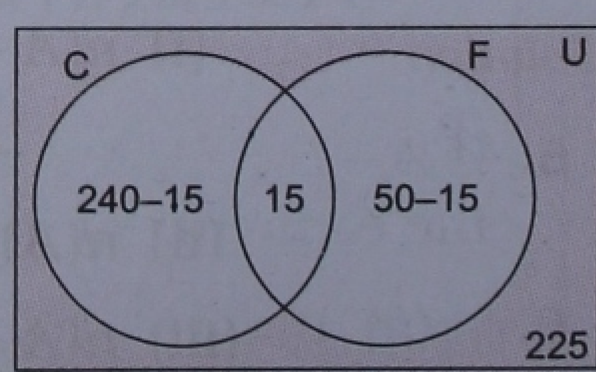
(i) only cricket **(ii) only football** **(iii) either cricket or football or both**

(iv) neither cricket nor football

Solution Let U = {the entire group}, C = {people who watch cricket},

F = {people who watch football}.

Given $n(C) = 240$, $n(F) = 50$, $n(C \cap F) == 15$.



(i) The number of people who watch only cricket

$= n(C) - n(C \cap F) = 240 - 15 = 225$.

(ii) The number of people who watch only football

$= n(F) - n(C \cap F) = 50 - 15 = 35.$

(iii) The number of people who watch either cricket or football or both

$= n(C \cup F) = n(C) + n(F) - n(C \cap F) = 240 + 50 - 15 = 275.$

(iv) The number of people who watch neither cricket nor football

$= n(U) - n(C \cup F) = 500 - 275 = 225.$

Remember These

Some cardinal properties of sets

(i) $n(A) + n(A') = n(U)$

(ii) $n(A \cup B = n(A) + n(B) - n(A \cap B)$

(iii) $n(A \cup B) = n(A - B) + n(B)$

(iv) $n(A) - n(A - B) = n(A \cap B)$

(v) $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$

EXERCISE 2B

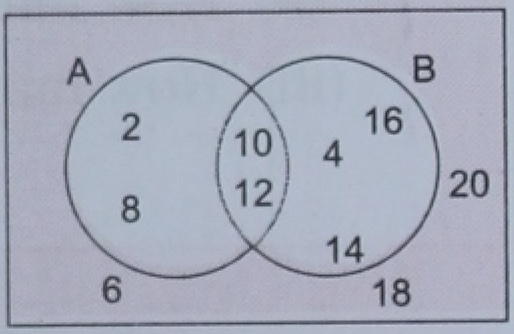
1. Let the universal set $U = \{0, 1, 2, 3, \ldots, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 3, 5, 8\}$, $C = \{2, 5, 6, 7\}$. Draw a Venn diagram to represent these sets. Also, find the following.

(i) $A \cap B$ (ii) $B \cap C$ (iii) $C \cap A$ (iv) $A \cup B$

(v) $B \cup C$ (vi) $C \cup A$ (vii) A' (viii) B'

(ix) C' (x) $A - B$ (xi) $B - C$ (xii) $(A \cup B)'$

2. Use the given Venn diagram to find the following sets.

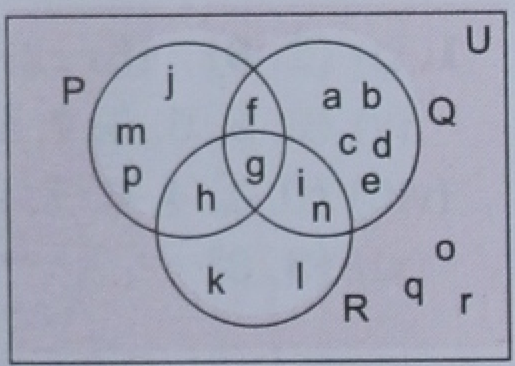


(i) U (ii) A (iii) B

(iv) $A \cup B$ (v) A' (vi) B'

(vii) $A \cap B$

3. Use the given Venn diagram to find the following.



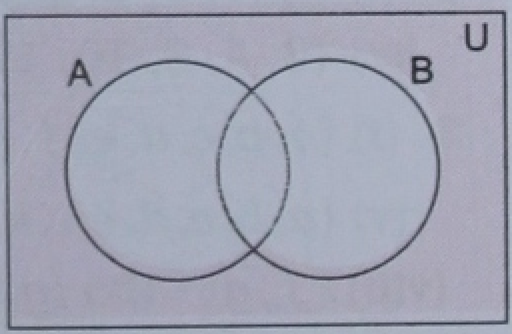
(i) U (ii) P (iii) Q

(iv) R (v) P' (vi) Q'

(vii) R' (viii) $P \cap Q$ (ix) $Q \cap R$

(x) $P - Q$ (xi) $Q - R$ (xii) $P - R$

4. If $A = \{a, b, c\}$, $B - A = \{d, e, f, g\}$ and $A \cap B = \{b, c\}$ then fill the elements of the given sets in the Venn diagram and tabulate the elements of the sets B, $A - B$ and $A \cup B$.



5. If $A - B = \{1, 3, 7, 8\}$, $B - A = \{2, 4, 6\}$ and $A \cap B = \{5\}$ then use a Venn diagram to display the elements of the given sets and tabulate the elements of the sets A, B and $A \cup B$.

6. (a) If $n(A) = 10$, $n(B) = 15$ and $n(A \cap B) = 7$ then find $n(A \cup B)$.

(b) If $n(P) = 50$, $n(Q) = 30$ and $n(P \cup Q) = 70$ then find $n(P \cap Q)$.

7. Let U be the universal set and A and B be any two sets. If $n(U) = 20$, $n(A) = 12$, $n(B) = 4$ and $n(A \cap B)' = 17$, find the following.

(i) $n(A \cap B)$ (ii) $n(A \cup B)$ (iii) $n(A - B)$ (iv) $n(B - A)$

8. Let U denote the universal set and A and B any two sets. If $n(U) = 25$, $n(A) = 17$, $n(B) = 7$ and $n(A \cup B)' = 2$, find the following.

(i) $n(A')$ (ii) $n(B')$ (iii) $n(A \cup B)$ (iv) $n(A \cap B)$

(v) $n(A - B)$ (vi) $n(B - A)$

9. If A and B be any two sets and $n(A - B) = 8$, $n(A \cup B) = 20$, $n(A \cap B) = 6$, find the following.

(i) $n(B - A)$ (ii) $n(A)$ (iii) $n(B)$

10. 50 students of a class like either oranges or grapes or both. 20 of them like oranges, while 10 like both. Draw a Venn diagram and find how many like (i) only oranges (ii) grapes (iii) only grapes?

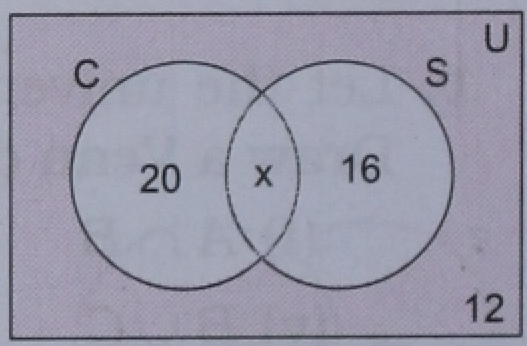
11. A class has 100 students. If 60 know English, 80 know Hindi and all the students of the class know either Hindi or English or both, find the number of students who know both the languages.

12. Out of a group of 1000 people, 560 have a TV, 250 have a computer and 130 have both. Find the number of people who have

(i) only a TV (ii) only a computer (iii) either a TV or a computer or both

(iv) neither a TV nor a computer

13. Let U = {students in a sports class},
C = {students who like cricket},
S = {students who like soccer}.



There are 60 students in the class. Use the Venn diagram to answer the following.

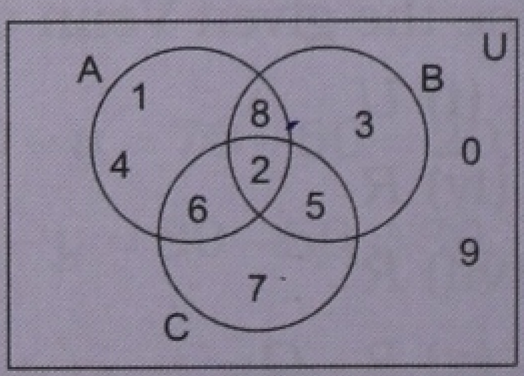
(i) What is the value of x?

(ii) How many students like cricket?

(iii) How many do not like soccer?

ANSWERS

1. (i) {2, 8} (ii) {2, 5} (iii) {2, 6} (iv) {1, 2, 3, 4, 5, 6, 8} (v) {2, 3, 5, 6, 7, 8} (vi) {1, 2, 4, 5, 6, 7, 8} (vii) {0, 3, 5, 7, 9} (viii) {0, 1, 4, 6, 7, 9} (ix) {0, 1, 3, 4, 8, 9} (x) {1, 4, 6} (xi) {3, 8} (xii) {0, 7, 9}



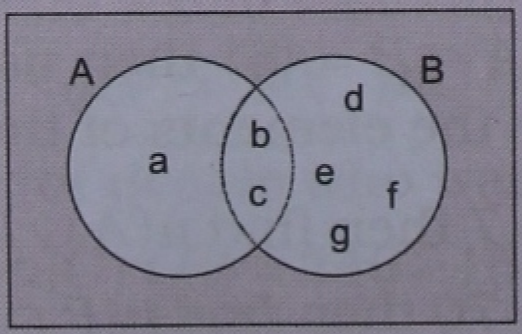
2. (i) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} (ii) {2, 8, 10, 12} (iii) {4, 10, 12, 14, 16} (iv) {2, 4, 8, 10, 12, 14, 16} (v) {4, 6, 14, 16, 18, 20} (vi) {2, 6, 8, 18, 20} (vii) {10, 12}

3. (i) $\{a, b, c, d, e, f, g, h, i, \ldots, r\}$ (ii) $\{f, g, h, j, m, p\}$ (iii) $\{a, b, c, d, e, f, g, i, n\}$ (iv) $\{g, h, i, k, l, n\}$ (v) $\{a, b, c, d, e, i, k, l, n, o, q, r\}$ (vi) $\{h, j, k, l, m, o, p, q, r\}$ (vii) $\{a, b, c, d, e, f, j, m, o, p, q, r\}$ (viii) $\{f, g\}$ (ix) $\{g, i, n\}$ (x) $\{h, j, m, p\}$ (xi) $\{a, b, c, d, e, f\}$ (xii) $\{f, j, m, p\}$

4. $B = \{b, c, d, e, f, g\}$

$A - B = \{a\}$

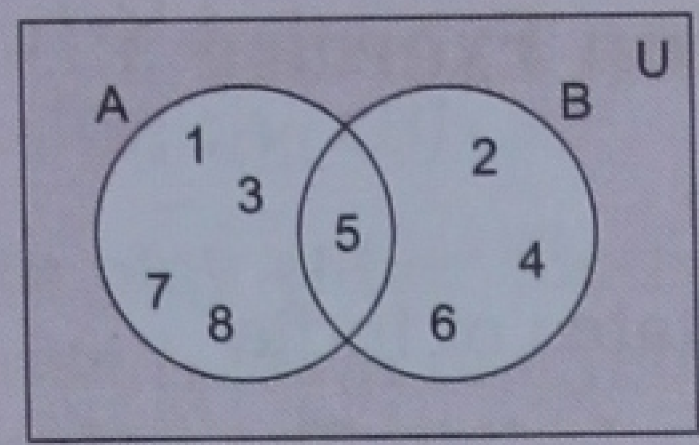
$A \cup B = \{a, b, c, d, e, f, g\}$



5. $A = \{1, 3, 5, 7, 8\}$

$B = \{2, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$



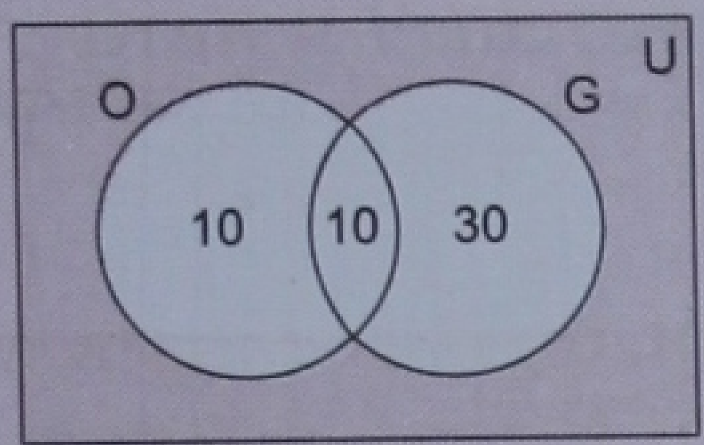
6. (a) 18 (b) 10

7. (i) 3 (ii) 13 (iii) 9 (iv) 1

8. (i) 8 (ii) 18 (iii) 23 (iv) 1 (v) 16 (vi) 6

9. (i) 6 (ii) 14 (iii) 12

10.



(i) 10 (ii) 40 (iii) 30

11. 40 students

12. (i) 430 (ii) 120 (iii) 680 (iv) 320

13. (i) 12 (ii) 32 (iii) 32

Revision Exercise 1

1. Which of the following collections are sets?

(i) The collection of the capitals of the states of India

(ii) The collection of the top singers of the Indian film industry

(iii) The collection of the natural numbers which are perfect cubes and are less than 100

(iv) The collection of good politicians of India

2. Write each of the following sets by the roster method.

(i) The set of the months of a year having 30 days

(ii) The set of the equal sides of an equilateral triangle ABC

(iii) The set of the planets of the solar system

(iv) The set of the months of a year that end with 'ber'

(v) The set of colours in traffic lights

(vi) The set of the oceans of the world

3. Express the following sets in the set-builder form.

(i) $\{4, 8, 12, 16, 20\}$ (ii) $\{-15, -10, -5, 0, 5, 10, 15, 20\}$

(iii) $\left\{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots\right\}$

4. If L = {letters of the word RATE}, M = {letters of the word TEAR}, N = {letters of the word RARE}, O = {letters of the word TREAT} then which of the following statements are true and which are false?

(i) $L \subset M$ (ii) $N \subset M$ (iii) $N \subset O$ (iv) $L = O$

(v) $L = N$ (vi) $M = O$

5. Let G = {letters of the word EUROPE}, H = {letters of the word NORWAY) and I = {letters of the word ENGLAND}. Find

(i) $G \cup H$ (ii) $H \cup I$ (iii) $G \cup I$ (iv) $G \cap H$

(v) $H \cap I$ (vi) $G \cap I$ (vii) $G - H$ (viii) $H - I$

(ix) $G - I$ (x) $G - (H \cap I)$ (xi) $H - (G \cap I)$ (xii) $I - (G \cap H)$

6. Let the universal set $U = \{3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{4, 5, 9, 10\}$, $B = \{3, 5, 9\}$ and $C = \{3, 10\}$. Find

(i) A' (ii) B' (iii) C' (iv) $A - B$

(v) $C - A$ (vi) $A \cap C$ (vii) $C - (A \cap B)$ (viii) $A - (B \cup C)$

7. Let the univesal set $U = \{x : x \in N \text{ and } x \leq 18\}$, A = {factors of 12}, B = {factors of 18}. Find

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$

Also, verify that (a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

8. Let the universal set $U = \{x : x \in W \text{ and } x \leq 10\}$, A = {prime numbers less than 10}, B = {even numbers less than 10} and C = {factors of 10}. Verify the following.

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

9. Let the universal set U = {letters of the English alphabet up to the letter 'm'}, $A = \{a, b, c\}$, $B = \{a, b, c, d, e, f\}$ and $C = \{a, e, f, g, h, i, j\}$. Represent the given sets by a Venn diagram and find the following.

(i) $A \cap B$ (ii) $B \cap C$ (iii) $C \cap A$ (iv) A'

(v) B' (vi) C' (vii) $(A \cap B)'$ (viii) $(B \cap C)'$

(ix) $(A \cup B) \cap C$ (x) $(A \cap B) \cup C$

10. In a class, 75 students play either football or cricket or both. Of these, 35 play football while 20 play both. Draw a Venn diagram and find how many play (i) only cricket (ii) only football?.

11. Out of a group of 56 students, 36 opted for mathematics, and 45 opted for biology. How many students opted for (i) both subjects, (ii) only mathematics and (iii) only biology? Draw a Venn diagram to represent the sets.

ANSWERS

1. (i), (iii)

2. (i) {April, June, September, November} (ii) {AB, BC, CA}
(iii) {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto}
(iv) {September, October, November, December} (v) {Red, Yellow, Green}
(vi) {Indian, Atlantic, Pacific, Arctic, Antarctic}

3. (i) $\{x : x = 4n, x \in N \text{ and } n \le 6\}$ (ii) $\{x \mid x = 5n, n \in I \text{ and } -3 \le n \le 4\}$ (iii) $\left\{x : x = \frac{n+1}{n}, x \in N\right\}$

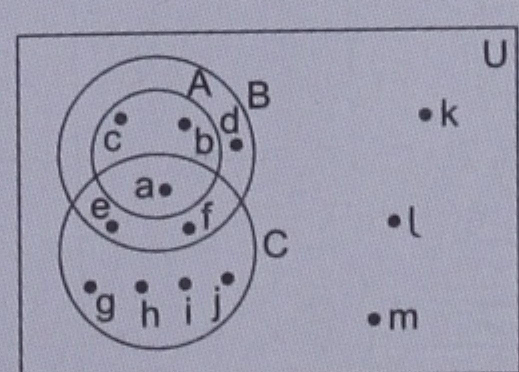
4. (i) False (ii) True (iii) True (iv) True (v) False (vi) True

5. (i) {$E, U, R, O, P, N, W, A, Y$} (ii) {$N, O, R, W, A, Y, E, G, L, D$} (iii) {$E, U, R, O, P, N, G, L, A, D$}
(iv) {R, O} (v) {N, A} (vi) {E} (vii) {E, U, P} (viii) {O, R, W, Y} (ix) {U, R, O, P} (x) {E, U, R, O, P}
(xi) {N, O, R, W, A, Y} (xii) {E, N, G, L, A, D}

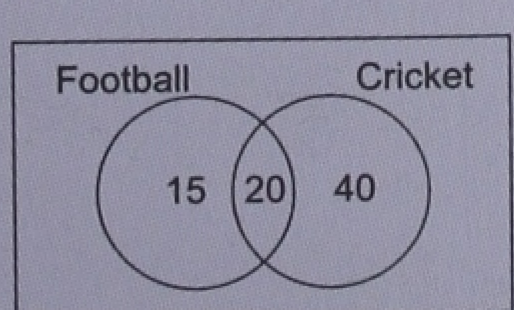
6. (i) {3, 6, 7, 8} (ii) {4, 6, 7, 8, 10} (iii) {4, 5, 6, 7, 8, 9} (iv) {4, 10} (v) {3} (vi) {10} (vii) {3, 10}
(viii) {4}

7. (i) {5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18} (ii) {4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17}
(iii) {4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} (iv) {5, 7, 8, 10, 11, 13, 14, 15, 16, 17}

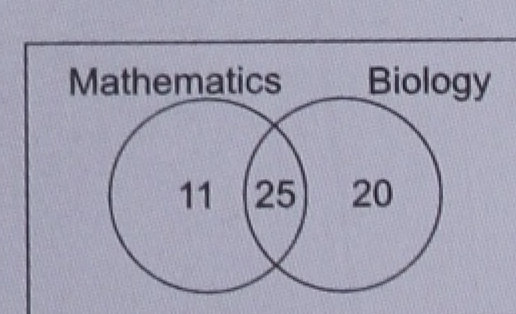
9. (i) $\{a, b, c\}$ (ii) $\{a, e, f\}$ (iii) $\{a\}$ (iv) $\{d, e, f, g, h, i, j, k, l, m\}$ (v) $\{g, h, i, j, k, l, m\}$ (vi) $\{b, c, d, k, l, m\}$
(vii) $\{d, e, f, g, h, i, j, k, l, m\}$ (viii) $\{b, c, d, g, h, i, j, k, l, m\}$ (ix) $\{a, e, f\}$ (x) $\{a, b, c, e, f, g, h, i, j\}$



10. (i) 40 (ii) 15



11. (i) 25 (ii) 11 (iii) 20



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