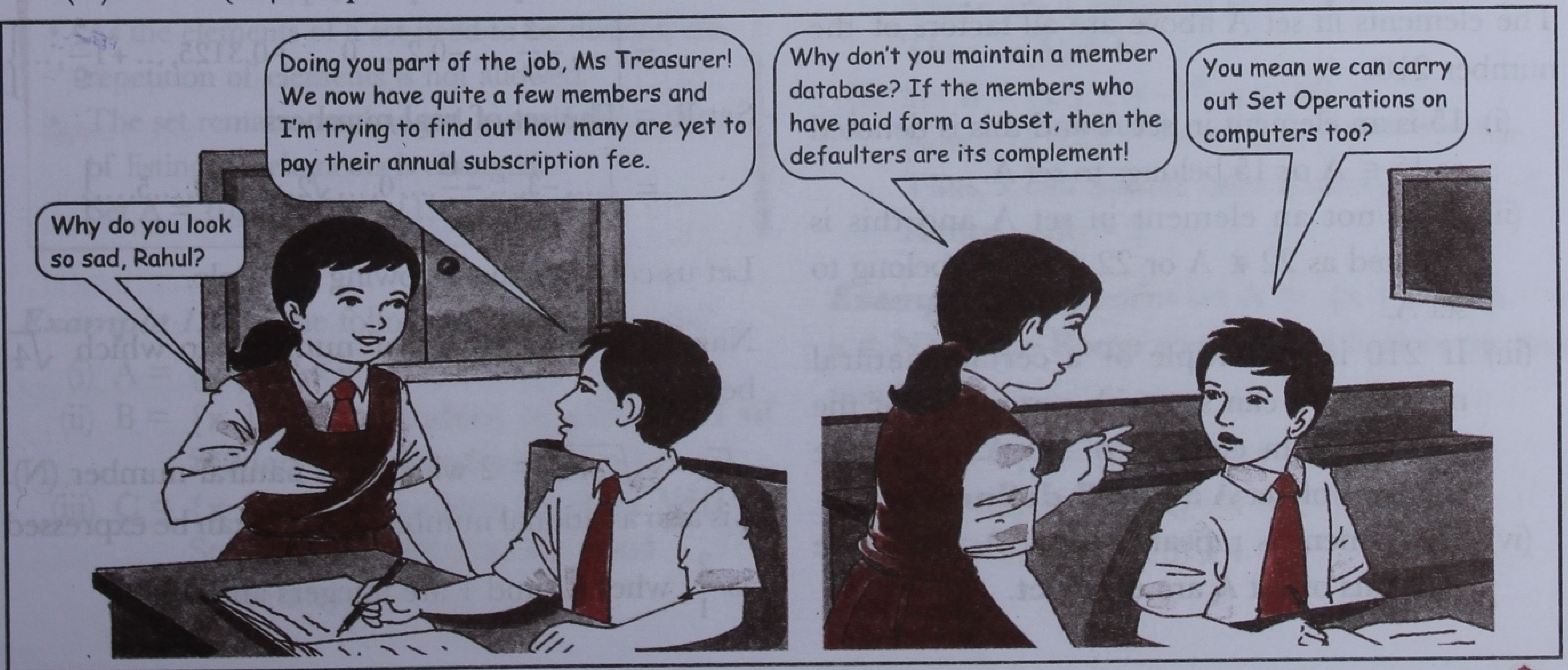
UNIT
ONE

- Sets
- Operations on Sets
- Venn Diagrams

Sets

Let's Recap

1. State the type of the following sets, based on their cardinal numbers.
   (i) A = {x | x is a consonant in the word AREA}
   (ii) B = $\{x \mid x = \frac{3a}{2}, a \in W\}$
   (iii) C = $\{x \mid x = 3x - 12, x \in W\}$
   (iv) D = $\{x \mid x = 3y + 4, -30 < y < 30\}$
   (v) E = {x | x is a prime number}
   (vi) F = $\{x \mid x$ is an odd number, $\frac{x}{2} = 0\}$
   (vii) G = $\{x \mid x$ is a composite number, $x < 1000, x \in Z\}$
2. Write all the possible subsets of the following sets:
   (i) A = $\{-5, -4, -3\}$
   (ii) B = {x | x is a letter in the word GNOME}
   (iii) C = {x | x is a prime factor of 210}
   (iv) D = $\{x \mid x$ is a multiple of 6, $x \le 24, x \in N\}$
3. If $\xi = \{x \mid x$ is an odd number, $30 < x < 50\}$, then write the complements of the following subsets:
   (i) A = {31, 35, 39, 43, 47}
   (ii) B = $\{x \mid x = 3a, 10 < a < 16, a$ is an odd number$\}$
   (iii) C = $\{x \mid x$ a prime number, $30 < x < 50\}$



1 SETS

- Roster Method
- Set-Builder Method
- Cardinal Number of a Set
- Finite and Infinite Sets
- Singleton and Null Sets
- Equivalent and Equal Sets
- Disjoint and Joint Sets
- Subset, Superset and Universal Set
- Proper Subset
- Power Set
- Complement of a Set
- Replacement Set

Introduction

Let us first recall what was learnt about sets and the different types of sets in our previous classes to reinforce our understanding of sets.

Definition of a Set

A set is a well-defined collection of distinct objects. The objects in a set, known as its elements, are enclosed within a pair of braces and the set is denoted by a capital letter of the English alphabet.

A = {1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210}

The elements in set A above are all factors of the number 210.

(i) 15 is an element in set A and this is denoted as 15 ∈ A or 15 belongs to set A.

(ii) 22 is not an element in set A and this is denoted as 22 ∉ A or 22 does not belong to set A.

(iii) If 210 is a multiple of a certain natural number, we can say with certainty that the number is an element of set A. Thus, the elements of set A are **well-defined**.

(iv) No element is repeated in set A. Thus, the elements of set A are **distinct**.

Sets of Numbers

We have learnt about different types of numbers in number systems in our previous classes. Although there are infinite numbers of each type, each number of a particular type of numbers is distinct and well-defined.

Set N = The set of natural numbers
$= \{1, 2, 3, 4, 5, 6, \dots\}$

Set W = The set of whole numbers
$= \{0, 1, 2, 3, 4, 5, \dots\}$

Set Z = The set of integers
$= \{\dots, -3, -2, -1, 0, +1, +2, +3, \dots\}$

Set Q = The set of rational numbers of the type $\frac{p}{q}$, where p and q ∈ Z, q ≠ 0

$$= \left\{\dots, -\frac{3}{7}, \dots -0.\dot{2}, \dots 0, \dots +0.3125, \dots +1\frac{2}{9}, \dots\right\}$$

Set R = The set of real numbers

$$= \left\{\dots, -3, \dots -\frac{6}{7}, \dots 0, \dots \sqrt{2}, \dots 3.78, \dots 5, \dots\right\}$$

Let us consider the following example.

Name the smallest set of numbers to which $\sqrt{4}$ belongs to.

$\sqrt{4} = \sqrt{2 \times 2} = 2$ which is a natural number (N). 2 is also a rational number (Q) as it can be expressed as $\frac{2}{1}$, where 2 and 1 are integers and $1 \neq 0$.

Being a rational number, 2 is also a real number (R). As $Q \subset R$, $Z \subset Q$, and $N \subset Z$, the smallest set that $\sqrt{4}$ belong to is the set of natural numbers (N).

Set Representation

Roster Method

In the **Roster** method, also known as the **listing** or the **tabular** method, the elements of the set are listed individually, enclosed within braces.

A = {n, v, r, m, t},

B = {8, 16, 24, 32, 40, …}, and

C = {66, 77, 88, 99}

are examples of set representation by the Roster method.

Set-Builder Method

In the **set-builder** method, also known as the **Rule** method, the elements of the set are described by a statement summarising their common property, which is enclosed within braces.

A = {x | x is a consonant in the word ENVIRONMENT},

$B = \{x \mid x = 8a,\ a \in N\}$, and

$C = \{x \mid x = 11a,\ 6 \le a \le 9,\ a \in N\}$

are examples that represent the same sets represented by the Roster method earlier, now represented by the set-builder method.

Remember

- As the elements of a set need to be distinct, the repetition of elements is not allowed.
- The set remains unchanged, even if the order of listing the elements is changed.
  Set A = {0, 1, 2, 3} = {1, 0, 3, 2}

Example 1: Are the following collections sets?

(i) $A = \{x \mid 12 < x < 13, x \in Q\}$

(ii) B = {x | x is a student in class VIII of St Mary's School who can swim}

(iii) C = {x | x is a student in class VIII of St Mary's School who is a good swimmer}

(iv) $D = \{x \mid 12 < x < 13, x \in N\}$

(i) Although infinite rational numbers lie between 12 and 13 on the number line, each number is distinct and the collection is well-defined. So, A is a set.

(ii) All students of the particular class who can swim will belong to this collection. Thus the collection is well-defined. As each of these students are different from one another, each element is distinct. Thus B is a set.

(iii) Whether a swimmer is 'good' or not is a matter of personal opinion. Thus it cannot be stated with certainty if a particular student will be in this collection or not. Thus collection C, being not well-defined, is not a set.

(iv) As no natural number lies between 12 and 13, there would be no element in collection D. As this fact defines collection D and as there are no elements in D, there is no chance of repetition of elements. Hence D, although empty, is a set.

Cardinal Number of a Set

The cardinal number of a set is the number of distinct elements in it.

The cardinal number of set A is denoted by n(A).

Example 2: Write the cardinal numbers of sets A and B.

(i) Set A = {0.1, 0.01, 0.001, 0.0001}
There are 4 elements in set A.
Thus, n(A) = 4

(ii) Set $B = \{x \mid x = -1^n, n \in N\}$
Now $(-1)^{\text{even number}} = +1$ and $(-1)^{\text{odd number}} = -1$.
Thus, x can assume only 2 values, +1 or –1.
Hence, n(B) = 2

Example 3: Represent set $A = \{x \mid x = 3a - 4, a \in N\}$ by the Roster method and find its cardinal number.

When $a = 1, x = (3 \times 1) - 4 = -1$

When $a = 2, x = (3 \times 2) - 4 = 2$

When $a = 3, x = (3 \times 3) - 4 = 5$

Thus, beginning from –1, the value of x increases by 3.

Thus A = {–1, 2, 5, 8, 11, 14, …}

As there are infinite elements in set A, n(A) = ∞.

Try this!

Write the cardinal number of the following set.

set A = {1, 3, 5, 7, 9, 11}

Exercise 1.1

1. Which of the following collections are sets?
   (i) {–1, +1}
   (ii) {x | x is either a vowel or a consonant in the English alphabet}
   (iii) {x | x is a pleasant hill station in Himachal Pradesh}
   (iv) {x | x is a Hindi letter in the English alphabet}
   (v) {–15, –10, –5, 0, +5, +10, +15, …}
   (vi) {… depth, loss, –3, 0, +3, profit, height, 63%, …}
   (vii) {…, –20, +20, –10, +10, 0, 0, +10, –10, +20, –20, …}
   (viii) {x | x is the quotient when 0 is divided by a natural number}
   (ix) {x | x is the quotient when a natural number is divided by 0}
   (x) {x | x is the name of a popular British rock star}
2. List the elements of the following sets:
   (i) A = {x | x is a letter in the word FOLLOWING}
   (ii) B = {x | x is a consonant in the word FOLLOWING}
   (iii) C = {x | x is a vowel in the word FOLLOWING}
   (iv) D = {x | x is a day of the week that ends with the letter Y}
   (v) E = {x | x = 4a, a < 10, a ∈ N}
   (vi) F = $\{x \mid x = \frac{35}{a}, a \in N \text{ and } x \in N\}$
   (vii) G = {x | x = 6a – 5, 1 < a < 6, a ∈ N}
   (viii) H = {x | x = 2a, 2 < a < 7, a ∈ N}
   (ix) I = $\{x \mid x = a^2 - 1, -3 < a < +3, a \in Z\}$
   (x) J = $\{x \mid x = \sqrt{64}, x \in Z\}$
3. To which of the smallest set of numbers, among N, W, Z, Q, and R, does each of the following numbers belong?
   (i) –8 (ii) +8
   (iii) –8.8 (iv) +8.8
   (v) 0 (vi) $\frac{3}{4}$
   (vii) $\sqrt{4}$ (viii) $\sqrt{3}$
   (ix) $\sqrt[3]{8}$ (x) $\sqrt[3]{2}$
4. Represent the following sets by the Roster method:
   (i) A = {x | x is a letter in the word DICTIONARY}
   (ii) B = {x | x is a vowel in the word DICTIONARY}
   (iii) C = {x | x = 7a, a < 7, a ∈ N}
   (iv) D = $\{x \mid x = \frac{105}{a}, a \in N \text{ and } x \in N\}$
   (v) E = {x | x = 2a + 17, 15 < a < 20, a ∈ N}
   (vi) F = $\{x \mid x = a^3, a < 5, a \in N\}$
   (vii) G = $\{x \mid x = a^3, -3 \le a \le +3, a \in Z\}$
   (viii) H = {x | x = 2x – 27}
   (ix) I = {x | x = 3.6a – 1.1, a < 5, a ∈ N}
   (x) J = {x | x is a bounding line segment in rectangle PQRS}
5. Represent the following sets by the set-builder method:
   (i) A = {q, r, s, t, u, v, w, x, y, z}
   (ii) B = {o, m, a, n, d, y}
   (iii) C = {January, June, July}
   (iv) D = {11, 22, 33, 44, 55, …}
   (v) E = {19, 38, 57, 76, 95, 114, 133}
   (vi) F = $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}$

(vii) G = {0.2, 0.04, 0.008, 0.0016}
(viii) H = {5, 25, 125, 625, 3125}
(ix) I = {1, 330, 2, 165, 3, 110, 5, 66, 6, 55, 10, 33, 11, 30, 15, 22}
(x) J = {∠ABC, ∠BCA, ∠CAB}

6. Write the cardinal number of each of the following sets:
(i) A = {–17, –13, –11, –7, –5, –3, –2, 2, 5, 7, 11, 13, 17}
(ii) B = {…,–8, –6, –4, –2, 0, 2, 4, 6, 8, …}
(iii) C = {x | x is a letter in the word ENVIRONMENT}
(iv) D = {x | x is a diagonal in rhombus ABCD}
(v) E = {x | x is a median in triangle PQR}
(vi) F = {x | x is a diametre in a circle with centre O}
(vii) G = {x | x is an axis of symmetry through an oval}
(viii) H = {x | x is an altitude in right angled triangle ABC}
(ix) I = {x | x is an axis of symmetry through equilateral triangle PQR}
(x) J = {x | x = a^2, $-1 \le a \le +1$, a ∈ Z}

Types of Sets

Sets are classified according to their cardinal numbers or based on their relationship with another set.

1. A = {x | x is a point on line segment AB}
The number of points on a line segment AB cannot be counted, hence set A is an **infinite set**.
B = {x | x is an axis of symmetry through square ABCD}
A square can have only four axes of symmetry about which it is symmetrical–two axes that divide it into two congruent rectangles, and two axes along its diagonals that divide it into two congruent triangles. The elements of set B can be counted, hence it is a **finite set**.
2. C = {x | x is a composite factor of 17, x ∈ N}
A prime number has only two factors, 1 and the number itself. Since none of the two numbers are composite factors, set C with no elements in it, is known as an **empty set**.

> **Remember**
> An empty set is denoted by {} or by the Greek letter ϕ, pronounced as 'phi'. An empty set is also known as a **null set** or a **void set**.

3. D = {x | x = $(-1)^n$, n ∈ N and n is an even number}
–1, multiplied by itself an even number of times is always equal to +1. As set D has only one element in it, it is known as a **singleton set.**
4. E = {x | x is a prime factor of 256, x ∈ N}
F = {x | x is a prime factor of 729, x ∈ N}
Set E has only one element, viz. 2, and set F too has only one element, 3. Thus, set E ↔ set F if n(E) = n(F). If both sets have the same number of elements, they are known as **equivalent sets**.
5. G = {x | x is a prime factor of 42, x ∈ N}
H = {x | x is a prime factor of 1764, x ∈ N}
Set G has three elements, viz. 2, 3, and 7, and set H too has the same three elements. So, set G = set H.
When two sets have the same elements, they are known as **equal sets**.
6. I = {x | x is a prime factor of 63}
J = {x | x is a prime factor of 55}
K = {x | x is a prime factor of 98}
Set I and set J which have no elements in common are known as **disjoint sets**. Set I and set K which have 7 as the common element, are known as **overlapping**, **joint**, or **intersecting sets**.

7. $L = \{x \mid x \in N\}$
$M = \{x \mid x = 2a, a \in N\}$
$P = \{x \mid x = 8a, a \in N\}$
Set P, consisting of all positive multiples of 8, is a part of set M which is the set of all even numbers. All the elements of set P belong to set M. Set P is known to be a **subset** of set M and set M is known to be a **superset** of set P. Considering sets M and P together, we find that all the elements in both the sets belong to set L, which is the set of all natural numbers. Set L is known as the **universal set** of sets M and P.

> The universal set is represented by the Greek letter ξ, pronounced as 'si'.

The relation between sets L, M, and P is represented by
$P \subset M$ (P is a subset of M), $P \subset \xi$, $M \subset \xi$
$M \supset P$ (M is a superset of P), $\xi \supset P$, $\xi \supset M$
Consider set $O = \{x \mid x = 8a + 8, a \in W\}$
We find that all elements in set O are the same as the elements in set P. Thus, by definition, set O is a subset of set P. Hence, for clarity, a subset is denoted by the symbol $\subseteq$ (is a subset of or equal to). Hence $O \subseteq P$, $P \subseteq M$, $M \subseteq \xi$, $P \subseteq \xi$, $O \subseteq \xi$ and $P \supseteq O$, $M \supseteq P$, $\xi \supseteq M$, $\xi \supseteq P$, $\xi \supseteq O$.
Furthermore, $O \subseteq O$, $P \subseteq P$, $M \subseteq M$, and $\xi \subseteq \xi$.

> **Remember**
> - A set is a subset of itself. $A \subseteq A$.
> - An empty set is a subset of every set.
> - If A = B, then $A \subseteq B$ and $B \subseteq A$.
> - If $A \subseteq B$ and $B \subseteq A$, then A = B.

Example 4: Write all the possible subsets of set $A = \{x \mid x = 3a + 4, a < 6, a \in N\}$.

In Roster form, set A = {7, 10, 13, 16, 19}
All possible subsets of set A are:
{ }, {7, 10, 13, 16, 19}, {7}, {10}, {13}, {16}, {19}, {7, 10}, {7, 13}, {7, 16}, {7, 19}, {10, 13}, {10, 16}, {10, 19}, {13, 16}, {13, 19}, {16, 19}, {7, 10, 13}, {7, 10, 16}, {7, 10, 19}, {7, 13, 16}, {7, 13, 19}, {7, 16, 19}, {10, 13, 16}, {10, 13, 19}, {10, 16, 19}, {13, 16, 19}, {7, 10, 13, 16}, {7, 10, 16, 19}, {7, 13, 16, 19}, {10, 13, 16, 19}, {7, 10, 13, 19}

Thus set A, which has 5 elements, can have 32 possible subsets.

Now $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

> **Remember**
> If a set has *n* elements, it can have 2^n possible subsets, where *n* is its cardinal number.

8. If the cardinal number of a subset is less than the cardinal number of its superset, it is known as a **proper subset**.

> **Remember**
> As a proper subset is not equal to its superset, it is denoted by the symbol $\subset$. If a set has n elements, it can have $2^n - 1$ possible proper subsets.

Example 5: Write all the possible proper subsets of set $A = \{x \mid x = a^2 - 1, a < 5, a \in N\}$.

In Roster form, set A = {0, 3, 8, 15}
As set A has four elements in it, it can have $2^4 = 16$ possible subsets, out of which only {0, 3, 8, 15} will not be a *proper subset* of set A. Thus the number of proper subsets possible $= 2^n - 1$ or $2^4 - 1 = 16 - 1 = 15$.

All possible proper subsets of set A = {}, {0}, {3}, {8}, {15}, {0, 3}, {0, 8}, {0, 15}, {3, 8} {3, 15}, {8, 15}, {0, 3, 8}, {0, 3, 15}, {0, 8, 15}, {3, 8, 15}.

9. A set containing all the possible subsets of set A is known as the **power set** of A.
The elements in a power set are all sets by themselves.

> **Remember**
> As set A can have 2^n possible subsets, where n is its cardinal number, the power set of A has 2^n elements in all.
> Thus $n(P(A)) = 2^n$

Example 6: Write the power set of set $A = \{x \mid x = a^3 + 5, a < 4, a \in N\}$.

In Roster form, set $A = \{6, 13, 32\}$

$P(A) = \{ \{\}, \{6, 13, 32\}, \{6\}, \{13\}, \{32\}, \{6, 13\}, \{6, 32\}, \{13, 32\}\}$

where $n(P(A)) = 2^3 = 8$

10. $\xi = \{x \mid x \text{ is an odd natural number}, x \leq 21\}$

$A = \{x \mid x = 2a + 1, a < 7, a \in N\}$

Written in Roster form,

$\xi = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$

and $A = \{3, 5, 7, 9, 11, 13\}$

All the elements of set A are a part of all the elements of its universal set, $A \subset \xi$. All the elements in the universal set that are not elements of set A, form a set that is known as the **complement of set A**, which is denoted by A′.

Thus $A' = \{1, 15, 17, 19, 21\}$

Remember

As the difference of the universal set ξ and a set is its complement ($\xi - A = A'$),

(i) $\xi' = \xi - \xi = \{ \} = \phi$, or the complement of the universal set is an empty set.

(ii) $\phi' = \xi - \{ \} = \xi$, or the complement of an empty set is the universal set.

11. If the solution to a problem involving sets is to be taken from a given universal set, then the universal set is known as the **replacement set**.

Example 7: Given $\xi = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, find $A = \{x \mid x = 2a + 2, a \geq 6 \text{ and } a \in N\}$ and $B = \{x \mid x = 2a, 1 < a < 15 \text{ and } a \in N\}$.

Now $A = \{14, 16, 18, 20, 22, 24, 26, \ldots\}$ and $B = \{4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$. But since $\xi = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$, the elements 22, 24, 26, … cannot belong to set A. Thus $A = \{14, 16, 18, 20\}$ and similarly $B = \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

Exercise 1.2

1. Which of the following are finite and infinite sets?
   (i) $A = \{\ldots, -2, -1, 0, +1, +2, \ldots\}$
   (ii) $B = \{-20, -10, 0, +10, +20\}$
   (iii) $C = \{\ldots, -33, -22, -11, 0\}$
   (iv) $D = \{x \mid x = 3a - 5, a < 15, a \in N\}$
   (v) $E = \{x \mid x < 3a - 5, a < 15, a \in Z\}$
2. Identify the empty sets and the singleton sets from the following sets:
   (i) A = {x | x is a point in a circle}
   (ii) B = {x | x is a curved line segment in isosceles ΔABC}
   (iii) C = {x | x is a right angle in rectangle ABCD}
   (iv) D = {x | x is an obtuse angle in scalene ΔABC}
   (v) E = {x | x is a right angle in acute angled ΔABC}
3. Which of the following pairs are equivalent sets?
   (i) $A = \{a, b, c, d\}$
   $B = \{w, x, y, z\}$
   (ii) C = {x | x is a side in equilateral ΔABC}
   $D = \{x \mid x = 21a, a < 4, a \in N\}$
   (iii) $E = \{x \mid x = 13a + 4, a < 4, a \in N\}$
   $F = \{x \mid x = \frac{4}{a}, a \in N \text{ and } x \in N\}$
   (iv) $G = \{x \mid x = a^2, 3 < a < 10, a \in N\}$
   $H = \{x \mid x = a^3, 3 < a < 10, a \in N\}$
   (v) $I = \{x \mid x = \frac{4a}{5} + 7, a < 200, a \in N\}$
   $J = \{x \mid x = 27a + 38, 100 < a < 200, a \in N\}$
4. Which of the following pairs are equal sets?
   (i) $A = \{23, 24, 25, 26, 27\}$
   $B = \{27, 26, 25, 24, 23\}$
   (ii) $C = \{x \mid x \text{ is a prime number}, 7 < x < 23\}$
   $D = \{x \mid x \text{ is an odd number}, 9 < x < 23\}$

(iii) $E = \{x \mid x = 4a, a < 10, a \in N\}$

$F = \{x \mid x = 2a + 2, a < 10, a \in N\}$

(iv) $G = \{x \mid x = a + 1, a < 5, a \in N\}$

$H = \{x \mid x$ is a prime factor of $900\}$

(v) $I = \{x \mid x = a^2 + 1, 1 < a < 4, a \in N\}$

$J = \{x \mid x = 5a, a \le 2, a \in N\}$

5. Write an equivalent set and an equal set for each of the following sets:
   (i) $A = \{x \mid x$ is a letter in the word IRIS$\}$
   (ii) $B = \{x \mid x = 7a, 3 < a < 9, a \in N\}$
   (iii) $C = \{x \mid x = \frac{24}{a}, a \in N \text{ and } x \in N\}$
   (iv) $D = \{x \mid x$ is a point of intersection of a pair of parallel lines$\}$
   (v) $E = \{x \mid x$ is a point of intersection of a pair of intersecting lines$\}$
6. Identify the overlapping and the disjoint sets from the following pairs of sets:
   (i) $A = \{1, 2, 3, 4, 5\}$
   $B = \{a, b, c, d, e\}$
   (ii) $C = \{x \mid x = 72a, a < 10, a \in N\}$
   $D = \{x \mid x = \frac{72}{a}, a \in N \text{ and } x \in N\}$
   (iii) $E = \{x \mid x = 5a, a \in N\}$
   $F = \{x \mid x = 6a, a \in N\}$
   (iv) $G = \{x \mid x = 11a, a \in N\}$
   $H = \{x \mid x = 13a, a \in N\}$
   (v) $I = \{x \mid x = -3a, a \in N\}$
   $J = \{x \mid x = +3a, a \in N\}$
7. Write the common elements between the following pairs of overlapping sets:
   (i) $A = \{a, b, c, d, e, f\}$
   $B = \{e, f, g, h, i, j\}$
   (ii) $C = \{x \mid x = 27a, a < 10, a \in N\}$
   $D = \{x \mid x = \frac{27}{a}, a \in N \text{ and } x \in N\}$
   (iii) $E = \{x \mid x = 6a, 5 \le a \le 9, a \in N\}$
   $F = \{x \mid x = 12a, 3 \le a \le 5, a \in N\}$
   (iv) $G = \{x \mid x$ is a letter in the word ALGEBRA$\}$
   $H = \{x \mid x$ is a letter in the word GEOMETRY$\}$
   (v) $I = \{x \mid x = a^2, a < 5, a \in N\}$
   $J = \{x \mid x = 4a, a < 5, a \in N\}$
8. Write all the possible subsets of the following sets:
   (i) $A = \{7, 11\}$
   (ii) $B = \{x \mid x$ is a vowel in the word ELATION$\}$
   (iii) $C = \{x \mid x = \frac{12}{a}, a < 12, a \in N, \text{ and } x \in N\}$
   (iv) $D = \{x \mid x = 12a, a < 4, a \in N\}$
   (v) $E = \{x \mid x = a^2 - 1, a < 6, a \in N\}$
9. Write all the possible proper subsets of the following sets:
   (i) $A = \{x, y, z\}$
   (ii) $B = \{x \mid x$ is a consonant in the word ALGEBRA$\}$
   (iii) $C = \{x \mid x = 5a + 3, 7 < a < 11, a \in N\}$
   (iv) $D = \{x \mid x = 7a, a < 6, a \in N\}$
   (v) $E = \{x \mid x = a^3 + 1, a < 4, a \in N\}$
10. How many proper subsets can the following sets have?
    (i) $A = \{a, b, c, d, e, f\}$
    (ii) $B = \{6, 16, 26, 36, 46, 56, 66\}$
    (iii) $C = \{x \mid x = 3a, a < 5, a \in N\}$
    (iv) $D = \{x \mid x = 63a, 10 < a < 20, a \in N\}$
    (v) $E = \{x \mid x = 3a^3 + 27, 18 < a < 26, a \in N\}$
11. Write the power sets of the following sets:
    (i) $A = \{25, 35, 45\}$
    (ii) $B = \{x \mid x = a + 11, a < 5, a \in N\}$
    (iii) $C = \{x \mid x = (-1)^a, a \in N\}$
    (iv) $D = \{x \mid x = 12a, a \le 5, a \in N\}$
    (v) $E = \{x \mid x = \frac{a\ \ 2}{2}, a \le 3, a \in N\}$
12. Find the cardinal numbers of the power sets of the following sets:
    (i) $A = \{p, q, r, s, t\}$
    (ii) $B = \{x \mid x = (+1)^a, a \in N\}$
    (iii) $C = \{x \mid x = 5a, -3 < a < +3, a \in Z\}$
    (iv) $D = \{x \mid x = \frac{a}{2}, -3 < a < +3, a \in N\}$
    (v) $E = \{x \mid x = \frac{30}{a}, a \in N \text{ and } x \in N\}$
13. Fill in the boxes with $=, \leftrightarrow, \supset, \subset$ symbols to describe the relation between the following sets:
    (i) A ☐ B, given

A = {a, c, e, g, i}
B = {x | x is a letter from the first 10 letters of the English alphabet}

(ii) C ☐ D, given
C = {x | x is a letter in the word SYSTEM}
D = {x | x is a letter in the word MISTY}

(iii) E ☐ F, given
$E = \{x \mid x = 7a, a \leq 3, a \in N\}$
$F = \{x \mid x = \frac{42}{a}, 6 < a < 42, a \text{ is a factor of } 6, a \neq 1\}$

(iv) G ☐ H, given
$G = \{x \mid x = 3a, a \in N\}$
$H = \{x \mid x = 12a, a \in N\}$

(v) I ☐ J, given
$I = \{x \mid x \in N\}$
$J = \{x \mid x \in Z\}$

14. Write a universal set for each of the following:
(i) A = {x | x is a point on line segment AB with mid-point O}
B = {x | x is a point on ray $\overrightarrow{OA}$ }
C = {x | x is a point on ray $\overrightarrow{OB}$ }
(ii) D = {x | x is a letter in the word EXCESS}
E = {x | x is a letter in the word AXIS}
(iii) $F = \{x \mid x = 4a, a \in N\}$
$G = \{x \mid x = 6a, a \in N\}$
(iv) $H = \{x \mid x = a^4, a \in N\}$
$I = \{x \mid x = a^6, a \in N\}$
(v) $J = \{x \mid x = \frac{12}{a}, a \in N \text{ and } x \in N\}$
$K = \{x \mid x = \frac{24}{a}, a \in N \text{ and } x \in N\}$

15. Which of the following are not proper subsets of set $X = \{x \mid x = 2a, a \leq 10, a \in N\}$?
(i) A = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
(ii) B = {4, 8, 12, 16, 20}
(iii) C = {2, 4, 6, 8, 10, 12, 14, 16, 18}
(iv) $D = \{x \mid x = 6a, a \leq 3, a \in N\}$
(v) $E = \{x \mid x = 4a, a \leq 6, a \in N\}$
(vi) $F = \{x \mid x = 2 + 2b, b \leq 9, b \in N\}$
(vii) $G = \{x \mid x = c + 1, c \text{ is an odd number}, c \leq 19, c \in N\}$

16. Given $\xi = \{x \mid x = 3a, a \leq 10, a \in N\}$, write the complement of each of the following subsets:
(i) $A = \{x \mid x = 6a, a < 4, a \in N\}$
(ii) $B = \{x \mid x = 9a, a < 3, a \in N\}$
(iii) $C = \{x \mid x = 12a, a < 2, a \in N\}$
(iv) $D = \{x \mid x = \frac{27}{a}, a < 27, a \in N, \text{ and } x \in N\}$

17. Write the proper subsets of the following sets:
(i) A = {a, e, i, o}
(ii) B = {–2, –1, 1}
(iii) C = {X, Y, Z}

18. Given ξ = {x | x is a letter in the word MATHEMATICIAN}, write the complement of each of the following subsets:
(i) A = {x | x is a letter in the word MATCH}
(ii) B = {x | x is a letter in the word INMATE}
(iii) C = {x | x is a vowel in the word LAMINATE}
(iv) D = {x | x is a consonant in the word ANTHEM}
(v) E = {x | x is a consonant in the word THEMATIC}

Challenge

1. Given the universal set of all quadrilaterals possible on a plane, which of the following sets is/are a superset of the set of all squares?
(i) The set of rectangles
(ii) The set of parallelograms
(iii) The set of rhombi
2. Given the universal set of all natural numbers, which of the following sets is/are a subset of set $A = \{x \mid x = 35a, a \in N\}$?
(i) $B = \{x \mid x = 5a, a \in N\}$
(ii) $C = \{x \mid x = 70a, a \in N\}$
(iii) $D = \{x \mid x = 7a, a \in N\}$
(iv) $E = \{x \mid x = 175a, a \in N\}$
3. Given replacement set $\xi = \{x \mid x \in N, x \leq 25\}$, find
(i) $P = \{x \mid x = 2p + 1, p \in N\}$
(ii) $Q = \{x \mid x = 3q + 1, q \in N\}$
(iii) $R = \{x \mid x = r^2, r \in N\}$

Revision Exercise

1. Represent the following sets by the listing method.
   (a) A = {x | x is one of the first 5 prime numbers}
   (b) B = {x | x = 6 n –1, n ≤ 5 and n ∈ N }
   (c) C = {x | x = 6 n – 1, n < 5 and n ∈ W}
   (d) D = {x | x = n^2, 3 < n < 7 and n ∈ N}
   (e) E = {x | x = 1.2p, p ∈ N}
2. Represent the following sets by the rule method.
   (a) A = {5, 6, 7, 8, 9,}
   (b) B = {11, 22, 33, 44, 55, 66, 77, 88, 99}
   (c) C = {violet, indigo, blue, green, yellow, orange, red}
   (d) D = {0, 1, 4, 9, 16, 25, ...}
   (e) E = $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}$
3. Write F for finite and I for infinite against the following sets.
   (a) A = {0}
   (b) B = {..., –2, –1, 0, +1, +2, ...}
   (c) C = {x = $(-1)^n$, n ∈ N}
   (d) D = {x | x = 3 > f > 2, f is a fractional number}
   (e) E = {x | x = –1 < n < +1, n ∈ W}
4. Write S for singleton and E for empty against the following sets.
   (a) A = {x | x = $(+1)^n$, n ∈ W}
   (b) B = {x | x = $(-1)^{2n}$, n ∈ W}
   (c) C = {x | x = 11 < n < 13, n is a prime natural number}
   (d) D = {x | x is factor of 26, x ≠ 1 and x ≠ 13}
   (e) E = {x | x is a factor of 23, x ≠ 1 and x ≠ 23}
5. Write an equivalent set for the following sets.
   (a) A = {tungsten, uranium, zinc}
   (b) B = {x | x = 3q – 1, 15 < q < 25, q ∈ N}
   (c) C = {x | x = $(n)^0$, n ∈ N}
6. Write an equal set for each of the following sets.
   (a) A = {x | x is a letter in the word AMEN}
   (b) B = {x | x is a factor of 6, x ∈ N}
   (c) C = {x | x = 3y, y ≤ 4 and y ∈ W}
7. Write all the possible subsets of the following sets.
   (a) A = {–16, –9, –4, 0}
   (b) B = {x | x is a factor of 6, x ∈ N}
   (c) C = {x | x = 3y, 3y ≤ 9 and y ∈ N}