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Idea of Sets

Sets

You are already familiar with some basic concepts of sets. Let us review what you have learnt and then move on to new concepts.

A **set** is a collection of well-defined, distinct objects. The objects of a set are called **members** or **elements** of the set. We call a set a “well-defined collection of objects” because we can decide (with absolute certainty) whether a given object is a member of the set.

- Examples**
- (i) The set of all integers
 - (ii) The set of vowels of the English alphabet
 - (iii) The set of rivers of India
 - (iv) The set of students of class VIII of your school

We usually denote a set by a capital letter of the English alphabet, such as A, B, C, X, Y and Z and its elements, by small letters of the English alphabet, such as a, b, c, x and y .

If a is an element of the set X , we write $a \in X$ and read this as “ a belongs to the set X ”. If x is not an element of the set A , we write $x \notin A$ and read this as “ x does not belong to A ”.

Representation of sets

A set is usually represented in the **tabular form** or the **set-builder form**.

Roster method or tabular form

In this method, we represent a set by listing all its elements between braces.

- Examples**
- (i) The set A of the vowels of the English alphabet is represented as $A = \{a, e, i, o, u\}$.
 - (ii) The set X of the days of a week is represented as $X = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$.

The order in which we list the elements of a set within braces is immaterial. Thus, each of the following denotes the same set.

$$\{a, b, c\}, \{b, a, c\}, \{a, c, b\}, \{c, a, b\}, \{c, b, a\}, \{b, c, a\}$$

In listing the elements of a set in the tabular form, we do not repeat any element. Thus, if B is the set of digits in the number 15,312,142 then $B = \{1, 5, 3, 2, 4\}$.

Rule method or set-builder form

We can represent a set by stating a property which its elements satisfy. Thus, the set of natural numbers less than 10, is

$$A = \{x | x \text{ is a natural number, } x < 10\}$$

or $A = \{x : x \text{ is a natural number and } x < 10\}$.

We read this as "A is the set of all elements x such that x is a natural number and x is less than 10."

Examples (i) The set $X = \{2, 4, 6, 8\}$ can be written in the set-builder form as $X = \{x | x \text{ is an even natural number and } x \leq 8\}$.

(ii) The set of prime numbers that are less than 15 can be written in the set-builder form as $A = \{x | x \text{ is a prime number and } x < 15\}$.

The set A can be written in the tabular form as $A = \{2, 3, 5, 7, 11, 13\}$.

We sometimes represent a set by describing a property of its elements inside braces.

Examples $A = \{\text{days of a week}\}$; $B = \{\text{one-digit odd numbers}\}$

In the tabular form, $A = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$

$$B = \{1, 3, 5, 7, 9\}$$

Some special sets

(i) $N = \{1, 2, 3, 4, \dots\}$, (set of natural numbers)

(ii) $W = \{0, 1, 2, 3, \dots\}$, (set of whole numbers)

(iii) $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, (set of integers). This is also denoted as I .

Solved Examples

EXAMPLE 1 Write each of the following sets in the roster form.

(i) The set of the months in a year that end with y

(ii) The set of the months in a year that have 31 days

(iii) The set of odd numbers between 10 and 20

Solution

(i) $\{\text{January, February, May, July}\}$

(ii) $\{\text{January, March, May, July, August, October, December}\}$

(iii) $\{11, 13, 15, 17, 19\}$

EXAMPLE 2 Express each of the following sets in the set-builder form.

(i) The set of prime numbers between 20 and 30

(ii) The set of whole numbers which are divisible by 5 and are less than 35

(iii) The set of the factors of 25

- Solution** (i) $\{x : x \text{ is a prime number, } 20 < x < 30\}$ (ii) $\{x : x = 5n, n \in W \text{ and } n < 7\}$
 (iii) $\{x | x \text{ is a factor of } 25\}$

EXAMPLE 3 Write the following sets in the roster form.

(i) $\left\{x : x = \frac{n}{2n+1}, n \in N \text{ and } n < 4\right\}$ (ii) $\{x | 5x + 3 < 24, x \in W\}$

(iii) $\{x : x = 2r + 3, r \in I \text{ and } -2 < r \leq 3\}$

- Solution** (i) Given, $n \in N$ and $n < 4$. So, $n = 1, 2, 3$. Also, $x = \frac{n}{2n+1}$.

Substituting $n = 1, 2, 3$, we get $x = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ respectively.

\therefore the given set is $\left\{\frac{1}{3}, \frac{2}{5}, \frac{3}{7}\right\}$.

- (ii) Here, $5x + 3 < 24$. So, $5x < 21$ or $x < \frac{21}{5}$.

Since $x \in W$, we get $x = 0, 1, 2, 3, 4$. So, the given set is $\{0, 1, 2, 3, 4\}$.

- (iii) Given, $-2 < r \leq 3$ and $r \in I$. So, $r = -1, 0, 1, 2, 3$. Also, $x = 2r + 3$.

Substituting $r = -1, 0, 1, 2, 3$, we get $x = 1, 3, 5, 7, 9$ respectively.

Hence, the given set is $\{1, 3, 5, 7, 9\}$.

EXAMPLE 4 Express the following sets in the tabular and set-builder forms.

- (i) The set E of even natural numbers (ii) The set X of the factors of 36

- Solution** (i) $E = \{2, 4, 6, 8, \dots\}$ [tabular form]
 $= \{x : x = 2n, n \in N\}$ [set-builder form]
 (ii) $X = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ [tabular form]
 $= \{x : x \text{ is a factor of } 36\}$ [set-builder form]

EXAMPLE 5 Express the following sets in the set-builder form.

(i) $\left\{\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}\right\}$ (ii) $\{0, 2, 4, 6, 8, 10, 12\}$

(iii) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}\right\}$ (iv) $\{-40, -35, -30, \dots, 20\}$

- Solution** (i) $\left\{x : x = \frac{n}{n+1}, n \in N \text{ and } 4 \leq n \leq 9\right\}$ (ii) $\{x | x = 2n, n \in W \text{ and } n \leq 6\}$
 (iii) $\left\{x : x = \frac{1}{2^m}, m \in N \text{ and } m \leq 6\right\}$ (iv) $\{x | x = 5p, p \in I \text{ and } -8 \leq p \leq 4\}$

Remember These

1. A collection of distinct objects is a set if one can decide with absolute certainty whether a particular object is a member of the collection.
2. In the roster method (tabular form), the elements of a set are listed between braces $\{\}$.

3. In the rule method, the elements are described by a common property of the members.
4. No element of a set is repeated while writing a set by the roster method.

EXERCISE

1A

1. Which of the following collections are sets?
 - (i) The collection of the positive integers that are less than 6
 - (ii) The collection of big cities of India
 - (iii) The collection of rich people in India
 - (iv) The collection of the integers that are divisible by 3
2. If $A = \{1, 2, 3, 4, 5, 6\}$ then which of the following statements are true?
 - (i) $6 \in A$
 - (ii) $7 \notin A$
 - (iii) $2, 3$ and $5 \in A$
 - (iv) $\{1, 2, 3\} \in A$
 - (v) 6 and $8 \in A$
3. Write each of the following sets by the roster method.
 - (i) The set of the last three months of a year
 - (ii) The set of the angles of $\triangle ABC$
 - (iii) The set of even integers between 25 and 35
4. Express each of the following sets in the set-builder form.
 - (i) The set of even integers between 11 and 21
 - (ii) The set of whole numbers that are divisible by 6 and are less than 48
 - (iii) The set of the factors of 30
5. Express the following sets in the tabular and set-builder forms.
 - (i) The set of positive integers
 - (ii) The set of odd natural numbers
 - (iii) The set of the factors of 24
 - (iv) The set of the prime factors of 48
 - (v) The set of natural numbers which are perfect squares and are less than 50
6. Write the following sets in the tabular form.
 - (i) $\left\{x \mid x = \frac{n}{n+1}, n \in N \text{ and } n \leq 9\right\}$
 - (ii) $\{p : p = 3n + 1, n \in W \text{ and } 2 \leq n \leq 5\}$
 - (iii) $\{r : 7r + 5 < 36, r \in W\}$
 - (iv) $\{x : x = y + 3, 10 \leq y \leq 14 \text{ and } y \in N\}$
 - (v) $\{p : p = 7m, m \in I \text{ and } -1 \leq m \leq 3\}$
 - (vi) $\{x \mid x \in N \text{ and } x^3 < 30\}$
7. Express the following sets in the set-builder form.
 - (i) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$
 - (ii) $\{3, 6, 9, 12, 15\}$
 - (iii) $\{0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$
 - (iv) $\{1, 2, 13, 26\}$
 - (v) $\{-24, -12, 0, 12, 24, 36, 48, 60\}$

ANSWERS

1. (i) and (iv) are sets

2. (i), (ii) and (iii)

3. (i) {October, November, December} (ii) $\{\angle BAC, \angle ABC, \angle ACB\}$ (iii) {26, 28, 30, 32, 34}
4. (i) $\{x \mid x = 2n, n \in I \text{ and } 6 \leq n \leq 10\}$ (ii) $\{x : x = 6p, p \in W \text{ and } p < 8\}$ (iii) $\{x : x \text{ is a factor of } 30\}$
5. (i) $\{1, 2, 3, \dots\}; \{x \mid x \in I \text{ and } x > 0\}$ (ii) $\{1, 3, 5, \dots\}; \{x : x = 2n - 1 \text{ and } n \in N\}$
 (iii) $\{1, 2, 3, 4, 6, 8, 12, 24\}; \{x \mid x \text{ is a factor of } 24\}$ (iv) $\{2, 3\}; \{x \mid x \text{ is a prime factor of } 48\}$
 (v) $\{1, 4, 9, 16, 25, 36, 49\}; \{x : x = n^2, x \in N \text{ and } n \leq 7\}$
6. (i) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}\right\}$ (ii) {7, 10, 13, 16} (iii) {0, 1, 2, 3, 4} (iv) {13, 14, 15, 16, 17}
 (v) {-7, 0, 7, 14, 21} (vi) {1, 2, 3}
7. (i) $\left\{x : x = \frac{1}{r}, r \in N\right\}$ (ii) $\{x \mid x = 3n, n \in N \text{ and } n \leq 5\}$ (iii) $\{x : x = 4p, p \in W \text{ and } p < 11\}$
 (iv) $\{x : x \in N \text{ and } x \text{ is a factor of } 26\}$ (v) $\{x \mid x = 12n, n \in I \text{ and } -2 \leq n \leq 5\}$

Types of Sets

Finite set

If the number of elements of a set is finite, the set is called a **finite set**.

Examples (i) The set of all the days of a week

(ii) $X = \{x : x \text{ is a factor of } 6\}$

(iii) The set of even numbers between 3 and 9

Infinite set

A set that does not have a fixed number of elements is called an **infinite set**. Such a set has an uncountable number of elements.

Examples (i) The set of all positive even integers

(ii) The set of integers $I = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

(iii) The set of natural numbers that are greater than 6
 $= \{x : x \in N \text{ and } x > 6\}$

Empty set

A set which contains no elements is called an **empty set**. Such a set is denoted by $\{\}$ or ϕ . An empty set is also called a **null set** or a **void set**.

Examples (i) The set of women who are 6 m tall

(ii) The set of odd integers with 6 as a factor

(iii) The set of even prime numbers that are greater than 2

Note The number of elements in an empty set is 0. But $\{0\}$ is not an empty set because it contains the element 0.

Singleton set

A set that has only one element is called a **singleton set**.

Examples Each of the sets $\{0\}$, $\{12\}$ and $\{a\}$ is a singleton set.

Cardinal number of a finite set

The **cardinal number** of a finite set A is the number of distinct elements in the set A . It is denoted by $n(A)$.

Examples (i) $A = \{\text{months of a year}\}$

There are 12 months in a year.

$\therefore n(A) = 12$ or the cardinal number of the set $A = 12$.

(ii) If $A = \{x, y, z\}$ and $B = \{p, q, r, s\}$ then $n(A) = 3$ and $n(B) = 4$.

- Note**
- It is not possible to define the cardinal number of an infinite set.
 - The cardinal number of the empty set ϕ is zero, as ϕ has no element, or $n(\phi) = 0$.
 - The cardinal number of a singleton set is 1. For example, if $A = \{\text{the present Prime Minister of India}\}$ then $n(A) = 1$.

Universal set

The set of all the possible objects (or elements) under consideration for a particular discussion is called the **universal set**. It is denoted by U or ξ . The universal set may be different for different problems.

Example Let $A = \{\text{students of your class who play badminton}\}$
and $B = \{\text{students of your class who play cricket}\}$

If our study is regarding the sets A and B , we may take the set of all the students of your class as the universal set. We may also take the set of all the students of your school as the universal set. However, in a particular problem there will be only one universal set.

Equivalent sets

Two finite sets are called **equivalent sets** if they contain the same number of elements. In other words, sets A and B are equivalent sets if $n(A) = n(B)$. We express this in symbols as $A \sim B$ or $A \leftrightarrow B$.

Example The sets $A = \{2, 5, 8, 12\}$ and $B = \{4, 6, 9, 11\}$ are equivalent sets because $n(A) = n(B) = 4$. In symbols, $A \sim B$ or $A \leftrightarrow B$.

Subsets and Supersets

Subset

If A and B are two sets such that every element of A is an element of B , we say that **A is a subset of B** or A is included in B . We express this in symbols as $A \subseteq B$. If the set A is not a subset of the set B , we write $A \not\subseteq B$.

Example Let $A = \{4, 5\}$ and $B = \{3, 4, 5\}$. Then $4 \in A$ and $4 \in B$. Also, $5 \in A$ and $5 \in B$. Hence, every element of A is an element of B . So, $A \subseteq B$.

However, $3 \in B$ but $3 \notin A$. So, every element of B is not an element of A . So, B is not a subset of A , that is, $B \not\subseteq A$.

- Note**
- Each set is a subset of itself. Thus, for any set A , $A \subseteq A$.
 - The null set ϕ is a subset of every set.

EXAMPLE

- (i) Find all the subsets of the set $A = \{0, 1\}$.
 (ii) Write all the subsets of the set $P = \{1, 2, 3\}$.
 (iii) Find all the subsets of the set $X = \{1, 2, 3, 4\}$.

Solution

- (i) The subsets of A containing one element are $\{0\}, \{1\}$. Also, the null set ϕ and the set A itself are subsets of A .
 \therefore the subsets of the set A are $\phi, \{0\}, \{1\}, \{0, 1\}$.
- (ii) The subsets of P containing one element are $\{1\}, \{2\}, \{3\}$.
 The subsets of P containing two elements are $\{1, 2\}, \{1, 3\}, \{2, 3\}$.
 The null set ϕ and the set P itself are also subsets of the set P .
 So, the subsets of the set P are $\{\phi\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.
- (iii) The subsets of X containing one element are $\{1\}, \{2\}, \{3\}, \{4\}$.
 The subsets of X containing two elements are $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$.
 The subsets of X containing three elements are $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$.
 The null set ϕ and the set X itself are also subsets of X .
 \therefore the subsets of X are $\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}$.

In the examples we have considered:

- (i) The number of subsets of the set $A = 4 = 2^2 = 2^{\text{number of elements of } A}$.
 (ii) The number of subsets of the set $P = 8 = 2^3 = 2^{\text{number of elements of } P}$.
 (iii) The number of subsets of the set $X = 16 = 2^4 = 2^{\text{number of elements of } X}$.

We can generalise this for any finite set as follows.

If the number of elements of a finite set A is n , the number of subsets of $A = 2^n$.

Proper subset

If the sets A and B are such that every element of A is an element of B but B has at least one element which is not an element of A then A is called a **proper subset** of B . This is expressed in symbols as $A \subset B$.

- Examples** (i) If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ then every element of A is an element of B but the element '4' of B is not an element of A . So, $A \subset B$.
- (ii) If $P = \{\text{all the students of Delhi University}\}$
 and $Q = \{\text{all the students of India}\}$
 then it is clear that $P \subset Q$.

If A is a finite set containing n elements then the number of proper subsets of $A = 2^n - 1$

- Example** The proper subsets of the set $A = \{1, 2\}$ are $\phi, \{1\}$ and $\{2\}$.
 $\{1, 2\}$ is a subset of A but not a proper subset.
 \therefore the number of proper subsets of $A = 2^2 - 1 = 4 - 1 = 3$.

Superset

Let A be a subset of B , i.e., $A \subseteq B$. Then we say that B is a **superset** of A . We express this in symbols as $B \supseteq A$.

Example Let $A = \{4, 6, 8\}$ and $B = \{2, 4, 6, 8, 10\}$ then $B \supseteq A$.

Equal sets

Two sets A and B are **equal** if each element of A is an element of B and each element of B is an element of A . In other words, **the sets A and B are equal if A is a subset of B and B is a subset of A** , that is, $A \subseteq B$ and $B \subseteq A$. The equal sets A and B are denoted by $A = B$.

Examples (i) If $P = \{1, 2, 3\}$ and $Q = \{x : 2x - 1 < 6, x \in N\}$ then $P = Q$.

(ii) If $A = \{2, 4, 6\}$ and $B = \{6, 4, 2\}$ then $A = B$.

(iii) Let $P = \{\text{letters of the word ROOF}\}$ and $Q = \{\text{letters of the word FOR}\}$. Then $P = \{R, O, F\}$ and $Q = \{F, O, R\}$. Thus, every element of P is a member of Q , or $P \subseteq Q$. Also, every element of Q is a member of P , or $Q \subseteq P$. So, $P = Q$.

Equal sets are equivalent sets, but equivalent sets need not be equal sets.

Example If $P = \{x | x = 2n + 1, n \in N \text{ and } n < 8\}$ and $Q = \{\text{days in a week}\}$ then $n(P) = n(Q) = 7$. So, $P \leftrightarrow Q$.

However, Monday $\in Q$ but Monday $\notin P$. Therefore, $P \neq Q$.

Solved Examples

EXAMPLE 1 If $P = \{\text{letters of the word BLAME}\}$, $Q = \{\text{letters of the word MEAL}\}$ and $R = \{\text{letters of the word MALE}\}$ then identify the true and false statements.

(i) $P = Q$ (ii) $P \subseteq Q$ (iii) $Q \subseteq R$ (iv) $P \leftrightarrow Q$ (v) $Q \subseteq P$ (vi) $Q = R$

Solution

The given sets can be represented in the tabular form as follows.

$$P = \{B, L, A, M, E\}, Q = \{M, E, A, L\}, R = \{M, A, L, E\}$$

(i) False, because $B \in P$ but $B \notin Q$, so, the set P is not equal to the set Q

(ii) False, because the letter 'B' of the set P does not belong to Q

(iii) True, because every element of Q is an element of R

(iv) False, because $n(P) = 5, n(Q) = 4$, so $n(P) \neq n(Q)$

(v) True, because all the letters of the set Q are in P

(vi) True, because $Q \subseteq R$ and $R \subseteq Q$

EXAMPLE 2 If $A = \{x : x \in W \text{ and } 2x - 1 < 10\}$, $B = \{x : x = n^3, n \in N \text{ and } n < 3\}$, $C = \{x : x \in N \text{ and } 3x + 5 \leq 20\}$ then identify the true and false statements.

(i) $A \leftrightarrow C$ (ii) $A \subseteq B$ (iii) $B \subseteq C$ (iv) $C \subseteq A$

Solution

$$A = \{0, 1, 2, 3, 4, 5\}, B = \{1, 8\}, C = \{1, 2, 3, 4, 5\}$$

(i) False, because $n(A) = 6, n(C) = 5$, so $n(A) \neq n(C)$

- (ii) False, because $3 \in A$ but $3 \notin B$
- (iii) False, because the element '8' of B is not an element of C
- (iv) True, because all the elements of C are elements of A

EXAMPLE 3 If the universal set $U = \{1, 2, 3, 4, 5\}$ and $A = \{x | x \text{ is a prime factor of } 420\}$ then write the set A in the tabular form. Also, write all the subsets of A which are singleton sets.

Solution $420 = 2 \times 2 \times 105 = 2 \times 2 \times 3 \times 5 \times 7$. So, the prime factors of 420 are 2, 3, 5, 7.
But $7 \notin U$, so $A = \{2, 3, 5\}$.
Clearly, the singleton subsets of A are $\{2\}$, $\{3\}$, $\{5\}$.

Remember These

- The set A is
 - a finite set if $n(A)$ is finite
 - an infinite set if $n(A)$ is not finite
 - a singleton set if $n(A) = 1$
 - a null set if $n(A) = 0$
- U is the universal set. It may be finite or infinite. Every set of a problem is a subset of the universal set of the problem.
- Two sets A and B are equivalent if $n(A) = n(B)$.
- If every element of a set A is an element of the set B , the set A is called a subset of the set B . This is denoted by $A \subseteq B$. For any set A , $A \subseteq A$ and $\phi \subseteq A$.
- Two sets A and B are equal if $A \subseteq B$ and $B \subseteq A$.
- If A is a finite set containing n elements then the number of proper subsets of $A = 2^n - 1$.

EXERCISE

1B

- State whether each of the following sets is a finite set or an infinite set.
 - The set of multiples of 5
 - The set of positive integers greater than 20
 - The set of numbers which are factors of 32
 - $A = \{x : x = 2n, n \in N\}$
 - $B = \{x : x = 3n + 1, n \in N \text{ and } n \leq 20\}$
 - The set of positive integers which have the digit 2 in the units place
 - $A = \left\{ x : x = \frac{n-1}{n}, n \in N \right\}$
- Let the universal set $U = \{x : x \text{ is a multiple of } 3 \text{ and } 0 < x < 37\}$ and $A = \{x : x \text{ is a multiple of } 6\}$. Write A by the roster method.

3. Let the universal set $U = \{1, 2, 3, 4, \dots, 15\}$, $P = \{x: x = n^2, n \in N\}$ and $Q = \{x: x = 2^n, n \in W\}$. Write P and Q in the tabular form. Also, find the cardinal numbers of P and Q .
4. (i) Let $R = \{\text{letters of the word APPLE}\}$ and $S = \{\text{letters of the word PALE}\}$. Find the cardinality of the sets R and S . Are R and S equivalent sets? Are R and S equal sets?
 (ii) Let $B = \{\text{letters of the word WARD}\}$, and $C = \{\text{letters of the word DRAW}\}$. Is $B \leftrightarrow C$? Is $B = C$?
5. State whether each of the following statements is true or false.
 (i) $\{2, 5\} \subseteq \{2, 15, 3, 5\}$
 (ii) $\{1, 2, 8\} \subseteq \{6, 2, 3, 1, 8\}$
 (iii) $\{0, 3, 5\} \subseteq \{x: x \text{ is a natural number}\}$
 (iv) $\{x: x \text{ is an even number}\} \subseteq \{x: x \text{ is an integer}\}$
 (v) $\{x: x \text{ is a multiple of } 3\} \subseteq \{x: x \text{ is an integer}\}$
 (vi) $\{p, q, r\} \subseteq \{x: x \text{ is a small letter of the English alphabet}\}$
 (vii) $\{x: x \text{ is a prime number between } 10 \text{ and } 20\} \subseteq \{x: x \text{ is odd and } 12 \leq x < 25\}$
6. If $P = \{4, 5, 6, 7, 8, 9\}$, $Q = \{2, 3, 4\}$, $R = \{4, 5, 8\}$ then which of the following statements are true?
 (i) $P \subseteq Q$ (ii) $Q \subseteq R$ (iii) $R \subseteq P$ (iv) $P \subseteq R$
 (v) $P \leftrightarrow Q$ (vi) $Q \leftrightarrow R$ (vii) $Q = R$
7. Find the subsets of the following.
 (i) $A = \{4, 8\}$ (ii) $B = \{3, 6, 9\}$ (iii) $C = \{0, 3, 4, 5\}$
8. If $P = \{\text{letters of the word ONE}\}$, $Q = \{\text{letters of the word NONE}\}$ and, $R = \{\text{letters of the word GONE}\}$ then which of the following statements are true and which are false?
 (i) $P \subset Q$ (ii) $Q \subset R$ (iii) $P \subseteq R$ (iv) $P \subset R$
 (v) $P = Q$ (vi) $Q = R$

ANSWERS

1. (i) Infinite set (ii) Infinite set (iii) Finite set (iv) Infinite set (v) Finite set (vi) Infinite set (vii) Infinite set
2. $\{6, 12, 18, 24, 30, 36\}$ 3. $P = \{1, 4, 9\}$, $Q = \{1, 2, 4, 8\}$, $n(P) = 3$, $n(Q) = 4$
4. (i) $n(R) = 4$, $n(S) = 4$; R and S are equivalent sets; $R = S$ (ii) Yes, Yes
5. (i) True (ii) True (iii) False (iv) True (v) True (vi) True (vii) False
6. (iii) and (vi) are true
7. (i) ϕ , $\{4\}$, $\{8\}$, $\{4, 8\}$ (ii) ϕ , $\{3\}$, $\{6\}$, $\{9\}$, $\{3, 6\}$, $\{3, 9\}$, $\{6, 9\}$, $\{3, 6, 9\}$
 (iii) ϕ , $\{0\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{0, 3\}$, $\{0, 4\}$, $\{0, 5\}$, $\{3, 4\}$, $\{3, 5\}$, $\{4, 5\}$, $\{0, 3, 4\}$, $\{0, 3, 5\}$, $\{0, 4, 5\}$,
 $\{3, 4, 5\}$, $\{0, 3, 4, 5\}$
8. (i) False (ii) True (iii) True (iv) True (v) True (vi) False

