1 Number Systems

You are already familiar with numbers. You have read about natural numbers, whole numbers and integers, and the four fundamental operations on them. Let us briefly revise what you have already learnt.

The numbers used to count things are called natural numbers. The set of natural numbers $N = \{1, 2, 3, 4, \ldots\}$.

The number 0 (zero) together with the natural numbers gives us the set of whole numbers $W = \{0, 1, 2, 3, 4, \ldots\}$.

The members of the collection of numbers ..., –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ... are called integers. 1, 2, 3, 4, ... are called positive integers while –1, –2, –3, –4, ... are called negative integers. The integer 0 (zero) is neither positive nor negative. The set of integers is denoted by I or Z. So, $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. Integers are also called signed numbers or directed numbers.

Four fundamental operations on directed numbers

The directed numbers and the integers are the same and you are already familiar with the four fundamental operations on integers.

Simplification

An expression involving integers (i.e., directed numbers) is simplified by the rule of BODMAS.

Steps 1. First, carry out the operations inside the brackets (B).

Brackets are removed in the order: line bracket (or vinculum), first (or common) brackets, second (or curly) brackets, third (or rectangular or square) brackets.

2. Then change 'of' into '×' and multiply (O).
3. Carry out the operations of division and multiplication, in that order (DM).
4. Finally, carry out the operations of addition and subtraction (AS).

EXAMPLE **Simplify $14 + 16$ of $[625 \div \{55 - 5 \times (9 - \overline{6 - 3})\}]$.**

Solution

$$\text{The given expression} = 14 + 16 \text{ of } [625 \div \{55 - 5 \times (9 - 3)\}]$$
$$= 14 + 16 \text{ of } [625 \div \{55 - 5 \times 6\}] = 14 + 16 \text{ of } [625 \div \{55 - 30\}]$$
$$= 14 + 16 \text{ of } [625 \div 25] = 14 + 16 \text{ of } 25 = 14 + 400 = 414.$$

Solved Examples

EXAMPLE 1 **Write all possible three-digit numbers using the digits 0, 3 and 5 when the digits are (i) not repeated, (ii) repeated.**

Solution The digit 0 cannot be placed in the leftmost place as it would yield a two-digit number.

(i) When the digits are not repeated

Digit at units place	Numbers formed
0	350, 530
3	503
5	305

(ii) When the digits are repeated

Digit at units place	Numbers formed
0	300, 330, 350, 500, 530, 550
3	303, 333, 353, 503, 533, 553
5	305, 335, 355, 505, 535, 555

EXAMPLE 2 **Find the smallest five-digit number which is exactly divisible by 241.**

Solution The smallest five-digit number = 10000.

$$\begin{array}{r} 241)\,10000\,(41 \\ -\,964 \\ \hline 360 \\ -\,241 \\ \hline 119 \end{array}$$

10000 divided by 241 gives the remainder 119.

So, the required number must be greater than 10000 by (241 – 119).

Hence, the required number = 10000 + (241 – 119) = 10000 + 122 = 10122.

EXAMPLE 3 **Find the largest six-digit number which is exactly divisible by 459.**

Solution The largest six-digit number = 999999.

$$\begin{array}{r} 459)\,999999\,(2178 \\ -\,918 \\ \hline 819 \\ -\,459 \\ \hline 3609 \\ -\,3213 \\ \hline 3969 \\ -\,3672 \\ \hline 297 \end{array}$$

999999 divided by 459 gives the remainder 297.

Hence, the required number = 999999 – remainder = 999999 – 297 = 999702.

EXAMPLE 4 **Simplify 4 of [16 – {72 ÷ (–3 + 15 × 3)} + 2].**

Solution The given expression = 4 of [16 – {72 ÷ (12 × 3)} + 2]

= 4 of [16 – {72 ÷ 36} + 2] = 4 of [16 – 2 + 2]

= 4 of 16 = 4 × 16 = 64.

EXAMPLE 5 **Simplify $24-[36\div 9-4 \text{ of } \{11-\overline{5-13}\}]$.**

Solution The given expression $= 24-[36\div 9-4 \text{ of } \{11-(-8)\}] = 24-[36\div 9-4 \text{ of } \{11+8\}]$

$= 24-[36\div 9-4 \text{ of } 19] = 24-[36\div 9-76] = 24-[4-76]$

$= 24-[-72] = 24+72 = 96.$

EXAMPLE 6 **Fill in the blanks by selecting from the brackets.**

(i) If a profit of ₹ 100 is described as + 100 then a loss of ₹ 80 will be described as ..... . [–20, +80, –80] **(We use ₹ to represent Rs or Re.)**

(ii) $-15-(-18)$ ..... $(-2)\times 5$. [=, <, >]

(iii) $-7\times(-4)+56\div(-2)=$ ..... . [–56, 0, 56]

Solution (i) –80

(ii) > {∵ $-15-(-18)=-15+18=3$; $(-2)\times 5=-10$; and $3>-10$}

(iii) 0 {∵ $-7\times(-4)+56\div(-2)=28+(-28)=0$}

EXERCISE 1

1. Find (i) the sum and (ii) the difference of the smallest whole number and the smallest natural number.
2. Write all the possible 4-digit numbers using each of the digits 4, 0, 5, 9 only once.
3. Write all the possible 3-digit numbers using each of the digits 2, 5, 9 only once.
4. Write all the possible 2-digit numbers using digits from 2, 0, 5 when the digits (i) may be repeated, (ii) are not repeated.
5. Find the sum of the smallest and the greatest numbers of 6 digits using each of the digits 0, 1, 2, 7, 5, 9 only once.
6. Find the smallest six-digit number which is exactly divisible by 471.
7. Find the largest seven-digit number which is exactly divisible by 313.
8. Find the largest five-digit number which is exactly divisible by 37.
9. Fill in the blanks by using = or ≠ .

   (i) $100-8\times 24\div 3$ ..... $100-(8\times 24)\div 3$ (ii) $4\times 5-3+12\div 6$ ..... $4\times\overline{5-3}+12\div 6$
10. Use > or < to fill in the blanks.

    (i) $2+5\times 8+(-24)\div 4$ ..... $2+5\times\overline{8+(-24)}\div 4$ (ii) $2\times 3+9-6\div 3$ ..... $2\times 3+\overline{9-6}\div 3$
11. Simplify the following.

    (i) $260\div 5$ of $[50-\{16+(24\div\overline{4-1})\}]$ (ii) $350-[64\div 8-7 \text{ of } \{4\times 6\div 2-(9-\overline{1-3})\}]$
12. Simplify $-7-[(-8)-6\div(-2)-\{10 \text{ of } (-2)-(7-17)\}]$.

ANSWERS

1. (i) 1 (ii) –1

2. 4590, 4509, 4950, 4905, 4059, 4095, 5490, 5409, 5940, 5904, 5049, 5094, 9450, 9405, 9540, 9504, 9045, 9054

3. 259, 295, 529, 592, 925, 952 **4.** (i) 20, 22, 25, 50, 52, 55 (ii) 20, 25, 50, 52

5. 1077789 **6.** 100323 **7.** 9999724 **8.** 99974

9. (i) = (ii) ≠ **10.** (i) > (ii) > **11.** (i) 2 (ii) 349 **12.** –12

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