6 HCF AND LCM

- Factorisation of Numbers
- Finding the HCF
- Finding the LCM
- LCM and HCF of Two Numbers
- LCM and HCF of Coprime Numbers

Introduction

A number that divides another number exactly, without leaving any remainder, is known as a **factor** of the second number.

$$n_1 \times n_2 = n_3$$

As factor n_1 multiplied by factor n_2 results in n_3 as the product, the number n_3 is known as a **multiple** of its factors.

The first counting number or 1 has only one factor in itself.

All other natural numbers have two or more factors.

Prime numbers are natural numbers which have only 1 and the number itself as their factors.

Set P = {2, 3, 5, 7, 11, ...}

Even numbers are natural numbers that are multiples of 2.

Set E = {2, 4, 6, 8, 10, ...}

Odd numbers are natural numbers that do not have 2 as a factor.

Set O = {1, 3, 5, 7, 9, 11, ...}

Composite numbers are natural numbers which have at least one factor, other than 1 and the number itself.

Set C = {4, 6, 8, 9, 10, 12, ...}

Two natural numbers are known as **coprime numbers** when they do not have any factors in common, other than 1.

Set C_P = {{5, 7}, {9, 10}, {14, 15}, ...}

Factorisation of Numbers

Set A = $\{x \mid x = \frac{72}{a}, a \in N, x \in N\}$

Representing set A in Roster form, we have:

a	1	2	3	4	6	8	9	12	18	24	36	72
x	72	36	24	18	12	9	8	6	4	3	2	1

72 is not exactly divisible by any other value of a.

Thus, set A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}.

To find all the factors of a number, divide it by the smallest natural number, till the divisor is less than the quotient. All the divisors and quotients will be the factors of the number.

Prime Factorisation

The expression of a number as a product of its prime factors is called its prime factorisation.

Example 1: $210 = 2 \times 3 \times 5 \times 7$

Example 2: $10296 = 2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 13$
$= 2^3 \times 3^2 \times 11 \times 13$

Common Factors

Set A $= \{x \mid x = \frac{168}{a}, a \in N, x \in N\}$
$= \{1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168\}$

Set B $= \{x \mid x = \frac{144}{b}, b \in N, x \in N\}$
$= \{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144\}$

$A \cap B = \{1, 2, 3, 4, 6, 8, 12, 24\}$,
which are the common factors.

The ***highest common factor*** (**HCF**) of 168 and 144 is 24.

Common Multiples

Set A $= \{x \mid x = 6a, a \in N, x \in N\}$
$= \{6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, \ldots\}$

Set B $= \{x \mid x = 8b, b \in N, x \in N\}$
$= \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, \ldots\}$

$A \cap B = \{24, 48, 72, \ldots\}$,
which are the common multiples.

The ***lowest common multiple*** (**LCM**) of 6 and 8 is 24.

Finding the HCF

Prime Factorisation Method

Write down the common prime factors of the given numbers. The HCF will be the product of these common prime factors.

Example 3: Find the HCF of 216, 360, and 432.

Prime factorisation of 216 $= \underline{2\times2\times2\times3\times3}\times3$
Prime factorisation of 360 $= \underline{2\times2\times2\times3\times3}\times5$
Prime factorisation of 432 $= 2\times\underline{2\times2\times2\times3\times3}\times3$

The product of the underlined common factors
$= 2 \times 2 \times 2 \times 3 \times 3 = 72$

Thus, the HCF of 216, 360, and 432 is 72.

Cross Division Method

The Cross Division Method is used to find the HCF for large numbers, when finding all their prime factors is a lengthy process.

HCF of Two Numbers

Divide the greater number by the smaller number and find the remainder. Take the remainder as the new divisor and the old divisor as the new dividend and divide. Continue this process till the remainder is 0. The last divisor is the HCF of the two numbers.

Example 4: Find the HCF of 1144 and 1287.

Step 1:

$$1144 \overline{)1287}\ (1,\quad -1144,\quad \text{remainder } 143$$

Step 2:

$$143 \overline{)1144}\ (8,\quad -1144,\quad \text{remainder } 0$$

143, being the last divisor, is the HCF of 1144 and 1287.

HCF of Three Numbers

Find the HCF of any two numbers. Then find the HCF of the third number and the HCF already obtained.

Example 5: Find the HCF of 6300, 7560, and 8820.

Step 1:

$$6300 \overline{)7560}\ (1,\quad -6300,\quad \text{remainder } 1260$$

Step 2:

$$1260 \overline{)6300}\ (5,\quad -6300,\quad \text{remainder } 0$$

Thus, the HCF of 6300 and 7560 is 1260.

Step 3:

$$1260 \overline{)8820}\ (7,\quad -8820,\quad \text{remainder } 0$$

Thus, the HCF of 6300, 7560, and 8820 is 1260.

HCF of Four Numbers

Pair the numbers in groups of two and find the HCFs of both pairs. Then find the HCF of the two HCFs already obtained.

Example 6: Find the HCF of 6720, 7840, 8400, and 8960.

Step 1:

$$\begin{array}{r|l} & 1 \\ \hline 6720 & 7840 \\ & -6720 \\ \hline & 1120 \\ \hline \end{array}$$

Step 2:

$$\begin{array}{r|l} & 6 \\ \hline 1120 & 6720 \\ & -6720 \\ \hline & 0 \\ \hline \end{array}$$

Thus, the HCF of 6720 and 7840 is 1120.

Step 3:

$$\begin{array}{r|l} & 1 \\ \hline 8400 & 8960 \\ & -8400 \\ \hline & 560 \\ \hline \end{array}$$

Step 4:

$$\begin{array}{r|l} & 15 \\ \hline 560 & 8400 \\ & -560 \\ \hline & 2800 \\ & -2800 \\ \hline & 0 \\ \hline \end{array}$$

Thus, the HCF of 8400 and 8960 is 560.

Now find the HCF of 560 and 1120.

Step 5:

$$\begin{array}{r|l} & 2 \\ \hline 560 & 1120 \\ & -1120 \\ \hline & 0 \\ \hline \end{array}$$

Thus, the HCF of 6720, 7840, 8400, and 8960 is 560.

HCF of Five Numbers

First, find the HCF of any four numbers. Then find the HCF of the fifth number and the HCF of the first four numbers.

Example 7: Find the HCF of 10080, 11760, 12600, 13440, and 15120.

Step 1:

$$\begin{array}{r|l} & 1 \\ \hline 10080 & 11760 \\ & -10080 \\ \hline & 1680 \\ \hline \end{array}$$

Step 2:

$$\begin{array}{r|l} & 6 \\ \hline 1680 & 10080 \\ & -10080 \\ \hline & 0 \\ \hline \end{array}$$

Thus, the HCF of 10080 and 11760 is 1680.

Now find the HCF of 13440 and 15120.

Step 3:

$$\begin{array}{r|l} & 1 \\ \hline 13440 & 15120 \\ & -13440 \\ \hline & 1680 \\ \hline \end{array}$$

Step 4:

$$\begin{array}{r|l} & 8 \\ \hline 1680 & 13440 \\ & -13440 \\ \hline & 0 \\ \hline \end{array}$$

We find that the HCF of 13440 and 15120 is also 1680.

Now find the HCF of 1680 and 12600.

Step 5:

$$\begin{array}{r|l} & 7 \\ \hline 1680 & 12600 \\ & -11760 \\ \hline & 840 \\ \hline \end{array}$$

Step 6:

$$\begin{array}{r|l} & 2 \\ \hline 840 & 1680 \\ & -1680 \\ \hline & 0 \end{array}$$

Thus, the HCF of 10080, 11760, 12600, 13440, and 15120 is 840.

Example 8: Find the greatest number that divides 15604, 16386, and 17168 leaving exactly 4, 6, and 8 as remainders, respectively.

When15604 is divided by the required number, a remainder of 4 is left. So, 15604 – 4 = 15600, when divided by the required number, will leave no remainder. Similarly,

16386 – 6 = 16380 and 17168 – 8 = 17160, when divided by the required number, will leave no remainder.

The greatest number that divides 15600, 16380, and 17160 exactly is their HCF.

Step 1:

$$\begin{array}{r|l} & 1 \\ \hline 15600 & 16380 \\ & -15600 \\ \hline & 780 \end{array}$$

Step 2:

$$\begin{array}{r|l} & 20 \\ \hline 780 & 15600 \\ & -1560 \\ \hline & 0 \\ & 0 \\ \hline & 0 \end{array}$$

Now find the HCF of 17160 and 780.

Step 3:

$$\begin{array}{r|l} & 22 \\ \hline 780 & 17160 \\ & -1560 \\ \hline & 1560 \\ & -1560 \\ \hline & 0 \end{array}$$

Thus, when 15604, 16386, and 17168 are divided by 780, 4, 6, and 8 are left as remainders, respectively.

CHECK : $15604 = 780 \times 20 + 4$,
$16386 = 780 \times 21 + 6$,
$17168 = 780 \times 22 + 8$

Try this!

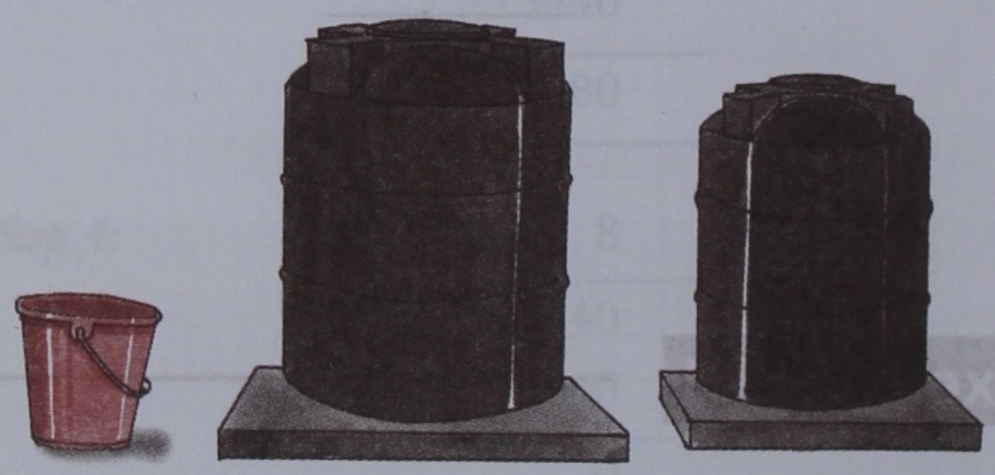
1. Find the HCF of 108, 288, and 360.
2. Find the HCF of 1056, 1584, and 2178.

Exercise 6.1

1. Find all the factors of the following numbers.
   (i) 42 (ii) 66 (iii) 180
   (iv) 84 (v) 150
2. Obtain the prime factorisation of the following numbers.
   (i) 162 (ii) 240 (iii) 252
   (iv) 546 (v) 1530
3. Find the HCF of the following numbers by prime factorisation method.
   (i) 88 and 99 (ii) 84 and 108
   (iii) 80 and 96 (iv) 208 and 234
   (v) 432, 540, and 648
   (vi) 408, 476, and 510
4. Find the HCF of the following numbers by cross division method.
   (i) 1350 and 1800 (ii) 3600 and 5040
   (iii) 7560 and 8820 (iv) 7920 and 8910
   (v) 14112 and 12936
   (vi) 25740 and 24024
5. Find the HCF of the following numbers by cross division method.
   (i) 1701, 1575, and 2016

(ii) 4680, 4160, and 5200
(iii) 3168, 3432, and 3696
(iv) 4752, 5184, and 5616
(v) 8640, 10368, and 12096
(vi) 9072, 8400, and 9744

6. Find the HCF of the following numbers by cross division method.
   (i) 3024, 2772, 2352, and 2100
   (ii) 1890, 2025, 2160, and 2430
   (iii) 7920, 8640, 7560, and 9000
   (iv) 6552, 7560, 7056, and 6048
   (v) 10584, 9072, 12096, and 13608
   (vi) 13440, 22176, 22848, and 23520
7. Find the HCF of the following numbers by cross division method.
   (i) 882, 1008, 756, 924, and 630
   (ii) 1092, 1404, 936, 1248, and 1716
   (iii) 2184, 1848, 2688, 2520, and 2352
   (iv) 4375, 4900, 4725, 5250, and 4200
   (v) 7392, 12474, 11550, 9702, and 8316
   (vi) 15708, 7854, 19635, 18326, and 9163
8. Find the greatest number that divides 10368, 9504, and 11232 exactly, leaving no remainders.
9. Find the greatest number that divides 7355, 8580, and 9805, leaving exactly 5 as a remainder in each case.
10. Find the greatest number that divides 9243 and 12325 leaving exactly 3 and 5 as remainders, respectively.
11. What would be the length of the longest tape that can be used to measure the length and breadth of an auditorium 204 feet wide and 486 feet long in an exact number of times.
12. A big cardboard picture 126 cm wide and 135 cm long is to be cut up into square pieces to create a jigsaw puzzle. How many small pieces would go on to make the jigsaw puzzle if each piece is to be equal and of the maximum possible size?
13. Square placards need to be cut out from a rectangular piece of cardboard 60 inches wide and 72 inches long. What is the maximum number of equal sized placards of the biggest possible size that can be cut out? What would be the length of each placard?
14. Three ribbons, 171 cm, 185 cm, and 199 cm, long are to be cut into equal pieces of maximum possible length, leaving bits of ribbons 3 cm long from each. What would be the length of each piece of ribbon and how many such pieces can one get?
15. The capacities of two empty water tanks are 504 ℓ and 490 ℓ. What would be the maximum capacity of a bucket that can be used an exact number of times to fill the tanks? How many buckets full of water will be needed?



Finding the LCM

Prime Factorisation Method

Express the numbers as a product of prime factors in exponential form. The product of the highest power of all the factors is the LCM of the given numbers.

Example 9: Find the LCM of 250 and 200.

$250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$

$200 = 2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$

LCM of 250 and 200 $= 2^3 \times 5^3$

$= 8 \times 125 = 1000$

Example 10: Find the LCM of 252, 756, and 840.

$252 = 2 \times 2 \times 3 \times 3 \times 7 = 2^2 \times 3^2 \times 7^1$

$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 2^2 \times 3^3 \times 7^1$

$840 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 2^3 \times 3^1 \times 5^1 \times 7^1$

LCM of 252, 756, and 840 $= 2^3 \times 3^3 \times 5^1 \times 7^1$

$= 8 \times 27 \times 35 = 7560$

Common Division Method

Begin with the smallest prime divisor and go on dividing all the numbers till all quotients are 1. The product of all the prime divisors of the given numbers is their LCM.

Example 11: Find the LCM of 4200, 3360, and 5040.

2	4200,	3360,	5040
2	2100,	1680,	2520
2	1050,	840,	1260
2	525,	420,	630
2	525,	210,	315
3	525,	105,	315
3	175,	35,	105
5	175,	35,	35
5	35,	7,	7
7	7,	7,	7
	1,	1,	1

The LCM of 4200, 3360, and 5040 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 50400$

Example 12: Find the greatest 6-digit number that can be divided by 350 as well as 525.

The LCM of 350 and 525 will be divisible by both the numbers.

2	350,	525
3	175,	525
5	175,	175
5	35,	35
7	7,	7,
	1,	1,

LCM of 350 and 525 = $2 \times 3 \times 5 \times 5 \times 7 = 1050$.

All multiples of 1050 will be divisible by both 350 as well as 525. To find the greatest 6-digit number that is a multiple of 1050, divide 999999 by 1050. We get 952 as quotient and 399 as remainder.

Thus, $1050 \times 952 = 999600$ is the greatest 6-digit number that is divisible by 350 as well as 525.

Note:
The next multiple of 1050 or $1050 \times 953 = 1000650$ is also divisible by 350 and 525 but it is a 7-digit number.

Example 13: Find the smallest 5-digit number which when divided by 42, 56, and 70 leaves exactly 3 as a remainder in each case.

2	42,	56,	70
2	21,	28,	35
2	21,	14,	35
3	21,	7,	35
5	7,	7,	35
7	7,	7,	7
	1,	1,	1

LCM of 42, 56, and $70 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$
Smallest 5-digit multiple of 840 = 10080.

Now 10080 when divided by 42, 56, and 70 will leave no remainder. 10080 + 3 = 10083, when divided by 42, 56, and 70 will leave exactly 3 as remainder in each case.

CHECK: $10083 = 240 \times 42 + 3$
$10083 = 180 \times 56 + 3$
$10083 = 144 \times 70 + 3$

LCM and HCF of Two Numbers

Given two numbers 150 and 180

$150 = \underline{2 \times 3 \times 5} \times 5 = 2^1 \times 3^1 \times 5^2$

$180 = 2 \times \underline{2 \times 3} \times 3 \times \underline{5} = 2^2 \times 3^2 \times 5^1$

HCF = $2 \times 3 \times 5 = 30$ LCM = $2^2 \times 3^2 \times 5^2$
$= 900$

Observe that

1. $900 = 30 \times (5 \times 2 \times 3)$
   or LCM = HCF × Product of prime factors that are not common
2. $30 \times 900 = 27000 = 150 \times 180$
   or HCF × LCM = Product of the two numbers

LCM and HCF of Coprime Numbers

Coprime numbers have no factor other than 1 in common. Therefore, 1 is the HCF of two coprime numbers.

- As LCM = HCF × product of factors that are not common, the LCM of two coprime numbers is their product.
- As LCM × HCF = product of the two numbers, when HCF = 1, LCM = product of the two numbers.

Example 14: Find the LCM and HCF of 24 and 35.

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$35 = 5 \times 7 = 5^1 \times 7^1$

HCF = 1 LCM $= 2^3 \times 3^1 \times 5^1 \times 7^1$

$= 840 = 24 \times 35$

Example 15: Find the LCM and HCF of 54 and 65.

$54 = 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3$

$65 = 5 \times 13 = 5^1 \times 13^1$

HCF = 1 LCM $= 2^1 \times 3^3 \times 5^1 \times 13^1$

$= 3510$

$= 54 \times 65$

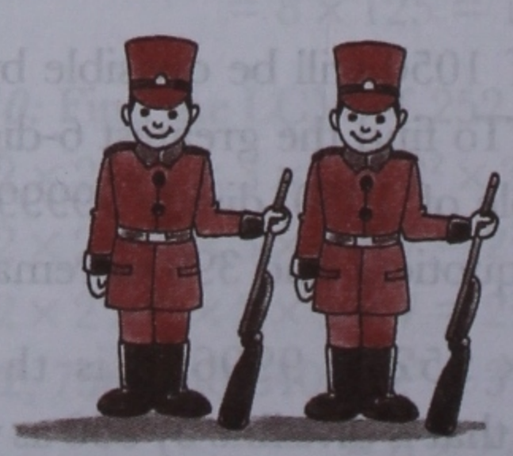
Try this!

1. Find the LCM of 72, 108, and 2100.
2. Find the LCM of 852 and 1491.

Exercise 6.2

1. Find the LCM of the following numbers by the prime factorisation method.
   (i) 24, 36, and 72 (ii) 84 and 112
   (iii) 144 and 192 (iv) 624 and 520
   (v) 225 and 270 (vi) 1008 and 1512
   (vii) 2310, 1540, and 770
   (viii) 840, 504, and 672
   (ix) 528, 396, and 352
   (x) 6552, 4368, and 9828
2. Find the LCM of the following numbers by the common division method.
   (i) 336 and 224
   (ii) 840 and 1260
   (iii) 630, 840, and 504
   (iv) 864, 1296, and 576
   (v) 144, 216, and 384
   (vi) 1764, 1176, and 2352
   (vii) 260, 390, 156, and 104
   (viii) 1170, 780, 1755, and 2340
   (ix) 2520, 1680, 3780, and 3024
   (x) 2730, 1950, 3822, and 1820
3. Find the smallest number that is exactly divisible by 2016 as well as 3024.
4. Find the greatest 5-digit number that is exactly divisible by 420, 490, and 280.
5. Find the smallest 6-digit number which, when divided by 96, 144, 72, and 192, leaves exactly 8 as a remainder.
6. The LCM of two coprime numbers is 70560. If one of the numbers is 245, find the other number.
7. The LCM of 42 and another number is 168. If the HCF of the two numbers is 14, find the other number.
8. Four bells begin to toll together. The bells toll after 8, 10, 12, and 15 seconds, respectively. After how long will all four bells toll together?

9. A toy soldier salutes after taking 14 steps while another salutes after every 21 steps. If both toy soldiers take a step every second, how long will it take for the toy soldiers to salute together five times after starting off together?

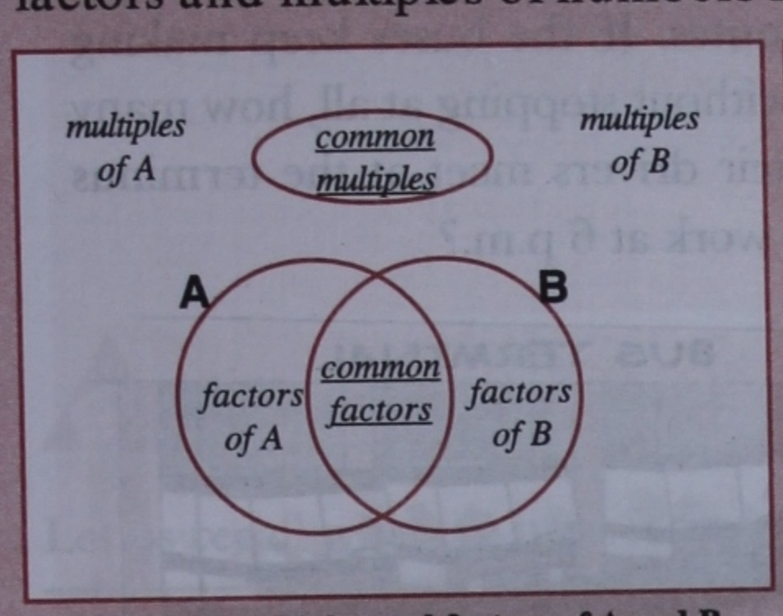


HOT TIPS - LCM and HCF

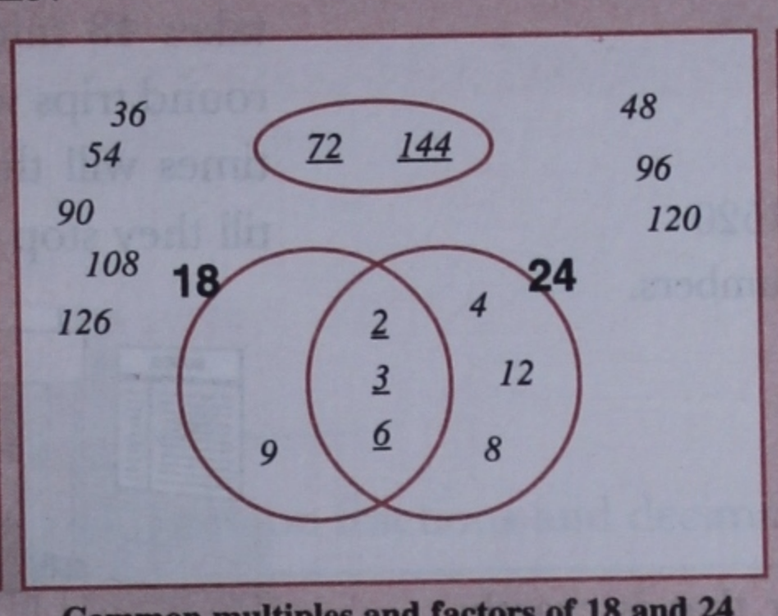
The more you practice solving problems, the better you will understand the concepts of LCM and HCF. Meanwhile, here are a few tips that should help you identify the problems correctly :

- LCM is greater than or equal to the given numbers.
- HCF is less than or equal to the given numbers.

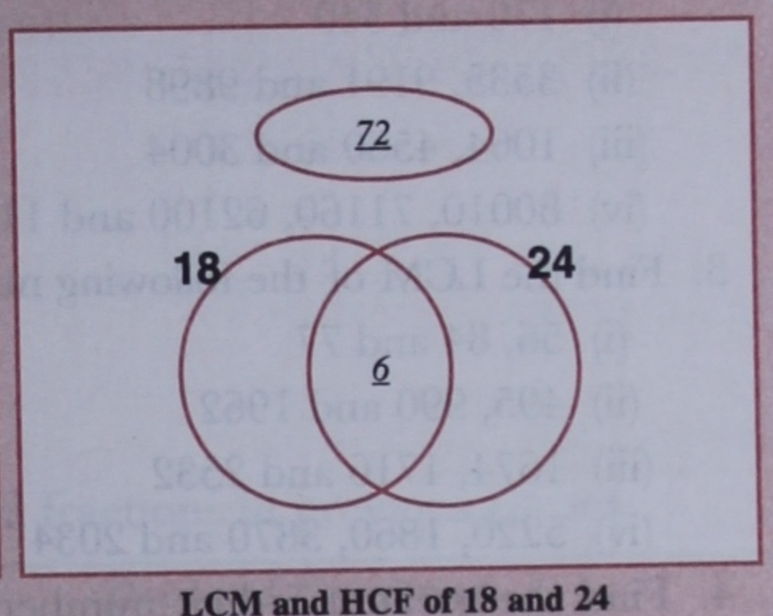
Although what is shown below is not an accurate Venn Diagram, it does show us where the common factors and multiples of numbers lie.



Common multiples and factors of A and B



Common multiples and factors of 18 and 24



LCM and HCF of 18 and 24

Recognise the indicative words in the word problems :

	LANGUAGE	TO FIND
1.	.... greatest number that divides a, b and c ...	HCF
2.	.... smallest number that can be divided by a, b and c ...	LCM
3.	.... greatest 5-digit number that can be divided by a, b and c ...	LCM

NOTE : Do not get misled by the greatest and smallest words. Instead remember that a factor divides while a multiple can be divided.

4.	... What should be the maximum height of the brick that can be used to build the two walls, the height of which are given as ...	HCF
5.	.... What is the minimum number of samosas to be prepared such that each guest can be served an equal number of samosas ...	LCM

NOTE : Do not get misled by the maximum and minimum words. Instead think :

4. The height of the brick has to be equal to or less than the height of both the walls, and HCF is less than or equal to the given numbers.
5. The number of samosas has to be equal to or more than the number of guests that may come, and LCM is greater than or equal to the given numbers.

Revision Exercise

1. Find the prime factorisation of the following numbers:
   (i) 420 (ii) 995
   (iii) 1224 (iv) 8712
2. Find the HCF of the following numbers:
   (i) 170 and 340
   (ii) 3535, 9191 and 9898
   (iii) 1064, 4560 and 3004
   (iv) 80010, 71160, 62100 and 11520
3. Find the LCM of the following numbers.
   (i) 56, 84 and 77
   (ii) 495, 990 and 1962
   (iii) 1674, 1716 and 2532
   (iv) 5220, 1860, 3870 and 2034
4. Find the greatest 5-digit number that is exactly divisible by 135, 225 and 405.
5. The length, breadth and height of a room are 1750cm, 7050cm and 4025cm respectively. Find the length of the longest tape which can measure the three dimensions of the room exactly.
6. Two buses start off together from the terminus at 6 a.m. on different routes. A round trip by one bus takes 36 minutes while the other bus takes 48 minutes. If the buses keep making round trips without stopping at all, how many times will their drivers meet at the terminus till they stop work at 6 p.m.?