UNIT TWO

Numbers

- Real Numbers
- Directed Numbers
- HCF and LCM
- Fractions and Decimals
- Approximation
- Squares and Square Roots
- Powers and Roots

Let's Recap

1. Write the following numbers in ascending and descending order.

   (i) $1\frac{7}{8}, \frac{3}{10}, \frac{2}{7}, 1\frac{4}{5}, \frac{11}{24}, 1\frac{1}{3}$

   (ii) 1.7117, 7.1771, 1.7711, 7.7171, 1.7771

   (iii) $6.21\dot{3}, 6.23\dot{1}, 6.13\dot{2}, 6.32\dot{1}, 6.\dot{3}$

   (iv) $9, 9^3, \sqrt[3]{8}, 8^3, \sqrt{9}, 8$

2. The additive inverse of –24 is multiplied with the multiplicative inverse of $2\frac{2}{3}$. What is the product obtained?

3. By how much does $-4\frac{1}{3}$ have to be increased in order to get $-2\frac{5}{6}$?

4. Mango trees cover 0.35 portion of an orchard, while guava trees grow on 0.26 portion. If the rest of the orchard has 3042 litchi trees, how many mango trees are there in the orchard?

4 REAL NUMBERS

- Natural Numbers
- Whole Numbers
- Integers
- Fractions
- Rational Numbers
- Irrational Numbers
- Operations Involving Irrational Numbers
- Properties of Irrational Numbers
- Rationalising Factor
- Real Numbers
- Properties of Real Numbers

Introduction

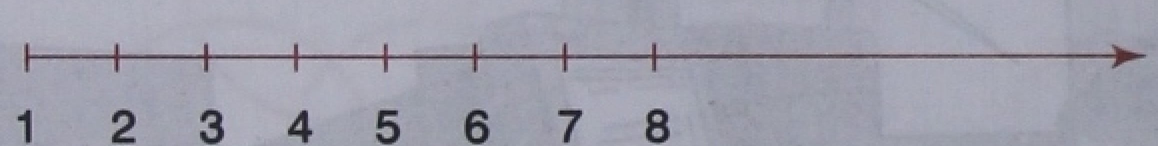
We have learnt about natural numbers, whole numbers, integers, and rational numbers in previous classes. In this chapter, we will briefly recall what was learnt earlier and extend the system of these numbers to the set of real numbers.

Natural Numbers

The set of natural numbers is the infinite set of counting numbers beginning with 1.

$$N = \{1, 2, 3, 4, 5, 6, \ldots\}$$

Natural numbers may be represented on a number ray with 1 as its end point and all subsequent numbers on its right at an equal distance from each other.

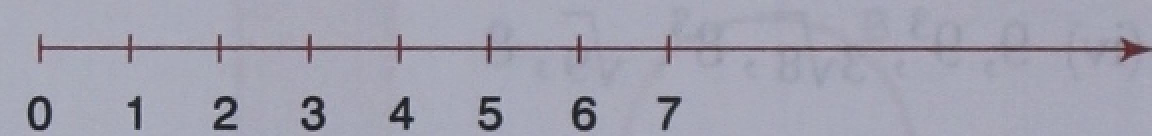


Whole Numbers

The set of natural numbers along with the digit 0 form the infinite set of whole numbers.

$$W = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

Whole numbers may be represented on a number ray with 0 as its end point and all subsequent numbers on its right at an equal distance from each other.



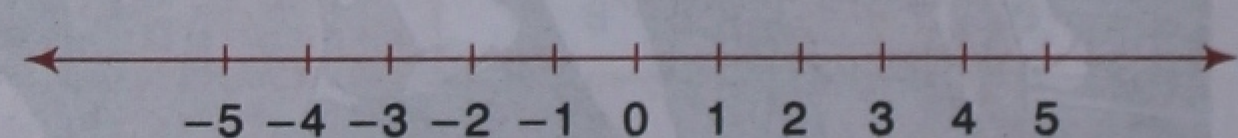
Integers

The additive inverse of a natural number is a negative integer. The sum of a number and its additive inverse is 0. The set of whole numbers, along with the set of the additive inverse of all natural numbers, forms the infinite set of integers.

Integers are represented by I or Z.

$$Z = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$$

Integers may be represented on a number line with 0 as its mid-point, all natural numbers to its right, and all negative integers to its left.

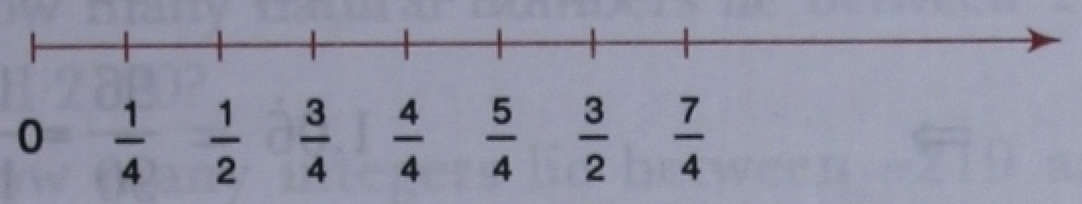


Natural numbers are positive integers. 0 is an integer which is neither positive nor negative.

Fractions

Fractions are numbers that are written in the form $\frac{a}{b}$, where a and b are natural numbers.

Fractions are represented on a number ray to the right of zero.



Rational Numbers

All numbers that can be written in the form $\frac{p}{q}$, where p and q are integers, but $q \neq 0$, form the set of rational numbers. It is represented by Q.

$$Q = \{\ldots, -\frac{3}{4}, \ldots, -\frac{1}{7}, \ldots, +2.4, \ldots, +5\frac{1}{5}, \ldots\}$$

Thus, rational numbers include:

1. $\frac{p}{q} = \frac{8}{2} = 4$ (natural numbers)
2. $\frac{p}{q} = \frac{0}{7} = 0$ (whole numbers)
3. $\frac{p}{q} = \frac{-14}{7} = -2$ (integers)
4. $\frac{p}{q} = \frac{5}{7}$ (fractions)
5. $\frac{p}{q} = \frac{-8}{9}$ (negative fractions)

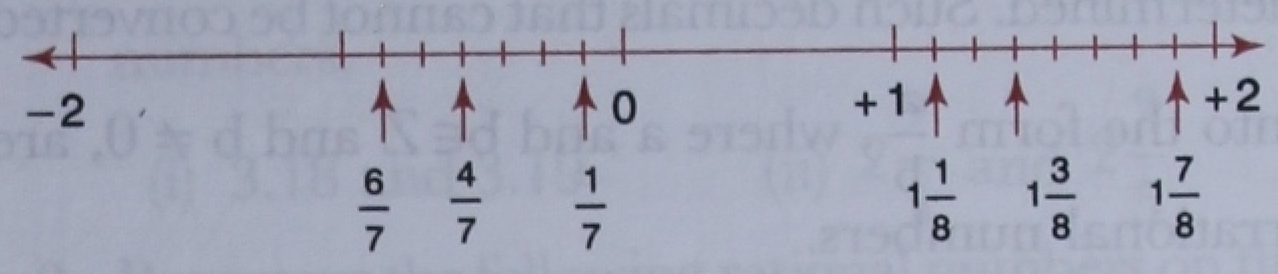
Representation of a Rational Number on the Number Line

Rational numbers may be represented on the number line with 0 as its mid-point.

Example 1: Represent $-\frac{6}{7}$ and $1\frac{3}{8}$ on the number line.

If –6 is divided by 7, it is apparent that the quotient will be less than 0 but more than –1. Thus $-\frac{6}{7}$ will lie between –1 and 0 on the number line. Similarly, $1\frac{3}{8}$ will lie between +1 and +2 on the number line.

Divide the distance between 0 and –1 into seven equal parts. The distance to the left of 0, till the sixth of these parts, represents $-\frac{6}{7}$. Now divide the distance between 1 and 2 into eight equal parts. The distance to the right of 1, till three of these parts, represents $+1\frac{3}{8}$.



Insertion of Rational Numbers

Rational numbers are very **densely packed** on the number line.

There can be infinite rational numbers between two given rational numbers. This is why there can be no predecessor or successor of a rational number.

The common fraction, also known as vulgar fraction, obtained by adding the numerators and denominators of any two given common fractions, will always lie between the two on the number line.

Example 2: Find two rational numbers between –8.17 and –8.18.

The average of the given numbers is given by

$$\frac{-8.17 + (-8.18)}{2} = \frac{-16.35}{2} = -8.175$$

The average of –8.17 and –8.175 = $\frac{-8.17 + (-8.175)}{2}$

$= \frac{-16.345}{2} = -8.1725$. Thus, 2 rational numbers between –8.17 and –8.18 are –8.175 and –8.1725.

Example 3: Find two rational numbers between $\frac{4}{7}$ and $\frac{5}{7}$.

The given fractions are $\frac{4}{7}$ and $\frac{5}{7}$.

Now, $\frac{4+5}{7+7} = \frac{9}{14}$ will lie between $\frac{4}{7}$ and $\frac{5}{7}$.

$\frac{4+9}{7+14} = \frac{13}{21}$ will lie between $\frac{4}{7}$ and $\frac{9}{14}$.

Thus, 2 rational numbers between $\frac{4}{7}$ and $\frac{5}{7}$ are $\frac{9}{14}$ and $\frac{13}{21}$.

Decimal Fractions as Rational Numbers

All decimal fractions that can be converted to the form $\frac{p}{q}$, q ≠ 0 where p and q are integers, are rational numbers. The exact value of non-terminating and non-repeating decimals cannot be determined. Such decimals that cannot be converted into the form $\frac{a}{b}$, where a and b∈Z and b ≠ 0, are irrational numbers.

Example 4: Convert the following terminating decimal fractions to the form $\frac{p}{q}$, q ≠ 0.

(i) 0.65

Multiplying and dividing 0.65 by 100

$$= \frac{0.65 \times 100}{100}$$

$$= \frac{65}{100}$$

$$= \frac{13}{20}$$

Try this!

Express 0.88 as a rational number.

....

(ii) 0.275

Multiplying and dividing 0.275 by 1000, we get

$$0.275 = \frac{0.275 \times 1000}{1000}$$

$$= \frac{275}{1000} = \frac{11}{40}$$

Thus, rational numbers $\frac{p}{q}$ where q is either 2 or 5, or their products, are all terminating decimals.

Example 5: Convert the following non-terminating recurring decimal fractions to the form $\frac{p}{q}$, q ≠ 0.

(i) $1.0\dot{6}$

$$100 \times 1.0\dot{6} = 106.6666...$$

$$10 \times 1.0\dot{6} = 10.66666...$$

$$(100 \times 1.0\dot{6}) - (10 \times 1.0\dot{6}) = 96.0000...$$

$$\Rightarrow \quad 1.0\dot{6}(100 - 10) = 96$$

$$\Rightarrow \quad 1.0\dot{6} = \frac{96}{90} = \frac{16}{15}$$

(ii) $1.\dot{2}\dot{3}$

$$100 \times 1.\dot{2}\dot{3} = 123.2323...$$

$$1 \times 1.\dot{2}\dot{3} = 1.2323...$$

$$(100 \times 1.\dot{2}\dot{3}) - (1 \times 1.\dot{2}\dot{3}) = 122.0000...$$

$$\Rightarrow \quad 1.\dot{2}\dot{3}(100 - 1) = 122$$

$$\Rightarrow \quad 1.\dot{2}\dot{3} = \frac{122}{99}$$

Note: $1.0\dot{6}$ can also be written as $1.0\overline{6}$, to represent a recurring decimal.

Remember

Natural numbers N = {1, 2, 3, 4, 5, 6, ...}

Whole numbers W = {0, 1, 2, 3, 4, 5, 6, . . .}

Integers (I or Z) = {. . . .. –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}

Rational Numbers Q is the set of numbers which can be expressed in the form $\frac{p}{q}$, where q ≠ 0

Every fraction is a rational number.

Every integer is a rational number.

∴ 0 is a rational number.

Every decimal fraction is a rational number.

Exercise 4.1

1. State whether the following statements are True or False.

   (i) 0 is to the left of all negative integers on the number line.

   (ii) 3 is greater than –3333.

   (iii) $1\frac{2}{5}$ will lie to the right of the mid-point between 1 and 2 on the number line.

   (iv) $1\frac{2}{5}$ will lie to the left of the mid-point between 1 and 2 on the number line.

(v) If a decimal fraction is non-terminating and non-recurring, it is known as a rational number.

(vi) The rational number $3\frac{1}{5}$ lies between $3\frac{2}{11}$ and $3\frac{3}{11}$.

(vii) Set W – Set N = {0}.

2. How many natural numbers lie between 212 and 2120?

3. How many integers lie between –219 and +2190?

4. Write the following numbers in descending order.

(i) –213, +126, –212, +127, –127

(ii) $-1\frac{7}{11}, -3\frac{7}{11}, -2\frac{7}{11}, -5\frac{7}{11}, -4\frac{7}{11}$

(iii) $\frac{3}{5}, -\frac{2}{9}, \frac{5}{7}, -\frac{3}{10}, \frac{11}{21}$

(iv) –2.3838, –2.3388, –2.8838, –2.8833, –2.3883

(v) $3.\dot{8}, 3.\dot{6}, 3.88, 3.6\dot{8}, 3.8\dot{6}$

5. Write the following numbers in ascending order.

(i) +418, –481, –418, +481, –841

(ii) $-1\frac{3}{11}, -1\frac{4}{11}, -1\frac{5}{11}, -1\frac{6}{11}, -1\frac{7}{11}$

(iii) $\frac{2}{5}, \frac{11}{23}, \frac{7}{15}, \frac{9}{20}, \frac{3}{7}$

(iv) 6.7134, 6.7431, 6.7341, 6.7413, 6.7143

(v) $7.9\dot{8}, 7.\dot{9}, 7.\dot{9}\dot{8}, 7.8\dot{9}, 7.\dot{8}$

6. Insert a rational number between the following pairs of numbers.

(i) – 0.001 and + 0.001

(ii) – 8 and – 3 (iii) 85 and 86

(iv) $5\frac{1}{2}$ and 6 (v) $\frac{1}{4}$ and $\frac{1}{5}$

(vi) $2\frac{2}{5}$ and $2\frac{3}{5}$ (vii) 3.0688 and 3.0699

(viii) 5.2168 and 5.2169

(ix) $1\frac{9}{13}$ and $1\frac{11}{15}$ (x) $-8\frac{6}{7}$ and $-8\frac{5}{7}$

7. Insert 3 rational numbers between the following numbers.

(i) 3.18 and 3.19 (ii) $2\frac{2}{5}$ and $2\frac{3}{5}$

8. Represent the following rational numbers on the number line.

(i) $2\frac{1}{3}$ (ii) $-\frac{5}{7}$

(iii) 3.7 (iv) 4.85

(v) $6\frac{7}{11}$

9. Which of the following rational numbers will have a terminating decimal value?

(i) $\frac{3}{5}$ (ii) $\frac{5}{6}$

(iii) $\frac{1}{2}$ (iv) $-\frac{7}{10}$

(v) $\frac{7}{15}$

10. Convert each of the following decimal fractions in the form $\frac{p}{q}$, where p and q $\in$ Z and q $\neq$ 0.

(i) 0.32 (ii) 0.42

(iii) 0.85 (iv) 1.875

(v) 0.4375 (vi) $3.\dot{7}$

(vii) $1.6\dot{4}$ (viii) $5.\dot{2}\dot{3}$

(ix) $7.11\dot{3}$ (x) $8.\dot{9}0\dot{5}$

Irrational Numbers

The set of irrational numbers includes all those numbers which cannot be written in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0. The exact values of irrational numbers cannot be determined.

$\overline{Q} = \{\ldots, -\sqrt{5} \ldots, 3\sqrt{2} \ldots, 4.020020002\ldots, \sqrt{37}, \ldots\}$

Example 6: Consider this: $\sqrt{64} = 8$

8 is a natural number or a rational number $\left(\frac{8}{1}\right)$ but $\sqrt{8} = 2.8284271\ldots$ is an irrational number as its exact value cannot be determined.

Example 7: Consider this: $7.7777\ldots = \frac{70}{9}$

$\frac{70}{9}$ is a rational number in the form $\frac{p}{q}$, $q \neq 0$ where p and $q \in Z$ but 7.77777... is an irrational number as it cannot be converted to the form $\frac{p}{q}$, $q \neq 0$, where p and $q \in Z$.

Operations Involving Irrational Numbers

The closure property of multiplication may or may not apply to irrational numbers.

For example,

$$\sqrt{3} \times \sqrt{2} = \sqrt{6} \text{ or } \bar{q}_1 \times \bar{q}_2 = \bar{q}_3$$

(an irrational number)

$$\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6 \text{ or } \bar{q}_1 \times \bar{q}_2 = q_3$$

(a rational number)

A rational number may be multiplied with an irrational number to form a new irrational number as product.

$$5 \times \sqrt{2} = 5\sqrt{2} \text{ or } 7 \times \sqrt{3} = 7\sqrt{3}$$

In an irrational product like $5\sqrt{2}$, $\sqrt{2}$ is known as the **radical**.

Irrational numbers with the same radicals are known as **like** irrational numbers, whereas irrational numbers with different radicals are known as **unlike** irrational numbers.

Like irrational numbers: $7\sqrt{6}$, $3\sqrt{6}$, $5\sqrt{6}$, $2\sqrt{6}$

Unlike irrational numbers: $7\sqrt{6}$, $6\sqrt{3}$, $6\sqrt{5}$, $3\sqrt{2}$

Addition and Subtraction of Irrational Numbers

Addition and subtraction can be carried out only between like irrational numbers.

Example 8: Evaluate $2\sqrt{5} + 3\sqrt{5}$

$$= (2 + 3)\sqrt{5} = 5\sqrt{5}$$

Example 9: Evaluate $4\sqrt{7} - 3\sqrt{7}$

$$= (4 - 3)\sqrt{7} = \sqrt{7}$$

Note:

$\sqrt{5} + \sqrt{5} \neq \sqrt{10}$ but $\sqrt{5} + \sqrt{5} = 1\sqrt{5} + 1\sqrt{5}$

$= (1 + 1)\sqrt{5}$

$= 2\sqrt{5}$

Similarly, always keep in mind that $\sqrt{1} + \sqrt{2} \neq \sqrt{3}$, $\sqrt{3} - \sqrt{2} \neq \sqrt{1}$, $2\sqrt{3} + 3\sqrt{2} \neq 5\sqrt{5}$ and so on.

Multiplication and Division of Irrational Numbers

Multiplication and division can be carried out between like as well as unlike irrational numbers.

Example 10: Find the product.

(i) $2 \times \sqrt{3} = 2\sqrt{3}$

(ii) $\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3$

(iii) $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

(iv) $2\sqrt{5} \times 3\sqrt{5} = (2 \times 3)(\sqrt{5} \times \sqrt{5}) = 6 \times 5 = 30$

(v) $2\sqrt{3} \times 3\sqrt{2} = (2 \times 3)(\sqrt{3} \times \sqrt{2})$

$= 6\sqrt{3 \times 2} = 6\sqrt{6}$

Example 11: Divide the following:

(i) $3 \div \sqrt{3} = 3 \div \sqrt{3} = \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3}$

(ii) $3\sqrt{3} \div \sqrt{6} = \frac{3\sqrt{3}}{\sqrt{6}} = 3 \times \sqrt{\frac{3}{6}}$

$= 3 \times \sqrt{\frac{1}{2}} = \frac{3\sqrt{1}}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

(iii) $6\sqrt{24} \div 2\sqrt{6} = \frac{6\sqrt{24}}{2\sqrt{6}} = 3 \times \sqrt{\frac{24}{6}}$

$= 3 \times \sqrt{4} = 3 \times 2 = 6$

Try this!

$7\sqrt{2} \times 2 =$ ______

$7\sqrt{2} \times \sqrt{2} =$ ______

$7\sqrt{2} \times \sqrt{1} =$ ______

$7\sqrt{2} \div 7 =$ ______

$7\sqrt{2} \div 2 =$ ______

$7\sqrt{2} \div \sqrt{7} =$ ______

Properties of Irrational Numbers

1. The negative of an irrational number is an irrational number too. $-1 \times \overline{q_1} = \overline{q_2}$ where $\overline{q_2} = -\overline{q_1}$.
2. The sum of a rational number and an irrational number is an irrational number.
$$q + \overline{q_1} = \overline{q_2}$$
3. The product of a rational number, other than 0, and an irrational number is an irrational number. $q \times \overline{q_1} = \overline{q_2}$, where $q \neq 0$
4. The sum and difference of two irrational numbers is a new irrational number.
$$\overline{q_1} + \overline{q_2} = \overline{q_3}$$
$$\overline{q_1} - \overline{q_2} = \overline{q_3} \text{ where } \overline{q_1} \neq \overline{q_2}$$
5. The product and quotient of two irrational numbers is a new rational or irrational number.
$$\overline{q_1} \times \overline{q_2} = q_3 \text{ or } \overline{q_4}$$
where $\overline{q_1} \neq \dfrac{1}{\overline{q_2}}$
$$\overline{q_1} \div \overline{q_2} = q_3 \text{ or } \overline{q_4} \text{ where } \overline{q_1} \neq \overline{q_2}$$

Rationalising Factor

To 'rationalise' is to make an irrational number a rational number. If the product of two irrational numbers $\overline{q_1}$ and $\overline{q_2}$ is a rational number, then q_1 and q_2 are rationalising factors of each other.

Example 12: $\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$

Thus $\sqrt{5}$ is the rationalising factor of $\sqrt{5}$.

Example 13: $3\sqrt{2} \times \sqrt{2} = 3\sqrt{4} = 3 \times 2 = 6$

Thus $\sqrt{2}$ is the rationalising factor of $3\sqrt{2}$.

Example 14: $(3 + \sqrt{2})(3 - \sqrt{2}) = 9 - (\sqrt{2})^2$
$$= 9 - 2 = 7$$
{using the identity $(a + b)(a - b) = a^2 - b^2$}

Thus $3 - \sqrt{2}$ is the rationalising factor of $3 + \sqrt{2}$.

Or, $3 + \sqrt{2}$ is the rationalising factor of $3 - \sqrt{2}$.

Example15: $(2\sqrt{6} - 3\sqrt{5})(2\sqrt{6} + 3\sqrt{5})$
$$= (2\sqrt{6})^2 - (3\sqrt{5})^2$$
$$= (4 \times 6) - (9 \times 5) = 24 - 45 = -21$$

Thus $2\sqrt{6} + 3\sqrt{5}$ is the rationalising factor of $2\sqrt{6} - 3\sqrt{5}$ or vice versa.

Rationalising the Denominator of an Irrational Number

The denominators of irrational numbers like $\dfrac{1}{\sqrt{5}}, \dfrac{5}{2\sqrt{6}}, \dfrac{3}{2+\sqrt{3}}, \dfrac{7}{\sqrt{5}-\sqrt{3}}$ can be rationalised by multiplying the numerator and denominator of the irrational number by the rationalising factor of the denominator.

Example 16: $\dfrac{1}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

Example 17: $\dfrac{5}{2\sqrt{6}} = \dfrac{5}{2\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{5\sqrt{6}}{2\sqrt{36}} = \dfrac{5\sqrt{6}}{12}$

Example 18: $\dfrac{3}{2+\sqrt{3}} = \dfrac{3}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$
$$= \frac{3(2-\sqrt{3})}{4-3} = \frac{6-3\sqrt{3}}{1} = 6 - 3\sqrt{3}$$

Example 19: $\dfrac{7}{\sqrt{5}-\sqrt{3}} = \dfrac{7}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
$$= \frac{7(\sqrt{5}+\sqrt{3})}{5-3} = \frac{7\sqrt{5}+7\sqrt{3}}{2}$$

Try this!

Rationalise the denominators of:

(i) $\dfrac{1}{3\sqrt{5}} =$

(ii) $\dfrac{1}{3\sqrt{6}+2\sqrt{3}} =$

Real Numbers

The set of real numbers is the union of the set of rational numbers and the set of irrational numbers. Every point on a number line represents a real number.

$R = Q \cup \bar{Q} = \ldots,\ 3\sqrt{2},\ 2\frac{1}{5},\ 1.678,\ \frac{7}{11},\ 0,\ \frac{3}{4},\ 1.5\dot{8},\ 3.212112111,\ 2\ \sqrt{24},$

Properties of Real Numbers

Let us consider operations on rational as well as irrational numbers to highlight the properties of operations on real numbers.

Closure Property

The sum, difference, product, or quotient of two real numbers is a real number.

1. $r_1 + r_2 = r_3$
2. $r_1 - r_2 = r_3$
3. $r_1 \times r_2 = r_3$
4. $r_1 \div r_2 = r_3$

Example 20: $-\frac{2}{3} + \frac{1}{5} = \frac{-10+3}{15} = -\frac{7}{15}$

Example 21: $3\sqrt{2} + 5\sqrt{2} = (3+5)\sqrt{2} = 8\sqrt{2}$

Example 22: $1\frac{2}{5} - 2\frac{1}{3} = \frac{7}{5} - \frac{7}{3} = \frac{21-35}{15} = -\frac{14}{15}$

Example 23: $3\sqrt{7} - \sqrt{7} = (3-1)\sqrt{7} = 2\sqrt{7}$

Example 24: $-1.88 \times -3.55 = +6.674$

Example 25: $4\sqrt{20} \times 2\sqrt{5} = (4\times 2)\sqrt{100}$
$= 8 \times 10 = 80$

Example 26: $3.762 \div 1.52 = 2.475$

Example 27: $15\sqrt{30} \div 3\sqrt{6} = \frac{15}{3}\sqrt{\frac{30}{6}} = 5\sqrt{5}$

Commutative Property of Addition and Multiplication

A change in the order of addition or multiplication of two real numbers does not change their respective sum or product.

1. $r_1 + r_2 = r_2 + r_1$
2. $r_1 \times r_2 = r_2 \times r_1$

Example 28: $\frac{1}{7} + \left(-\frac{1}{2}\right) = \left(-\frac{1}{2} + \frac{1}{7}\right) = -\frac{5}{14}$

Example 29: $-3\sqrt{5} + \sqrt{5} = \sqrt{5} + (-3\sqrt{5})$
$= -2\sqrt{5}$

Example 30: $6.94 \times -1.5 = -1.5 \times 6.94 = -10.41$

Example 31: $\sqrt{2} \times -\sqrt{8} = -\sqrt{8} \times \sqrt{2}$
$= -\sqrt{16} = -4$

Associative Property of Addition and Multiplication

A change in the grouping of three real numbers while adding or multiplying does not change their respective sum or product.

1. $(r_1 + r_2) + r_3 = r_1 + (r_2 + r_3)$
2. $(r_1 \times r_2) \times r_3 = r_1 \times (r_2 \times r_3)$

Example 32: $\left(-\frac{4}{5} + \frac{2}{3}\right) + \left(-\frac{1}{3}\right)$

$= \left(-\frac{2}{15}\right) + \left(-\frac{1}{3}\right) = -\frac{7}{15}$

or $-\frac{4}{5} + \left(\frac{2}{3} + \left(-\frac{1}{3}\right)\right) = \left(-\frac{4}{5}\right) + \left(\frac{1}{3}\right) = -\frac{7}{15}$

Example 33: $(2\sqrt{6} + \sqrt{6}) + 3\sqrt{6} = 3\sqrt{6} + 3\sqrt{6}$
$= 6\sqrt{6}$

or $2\sqrt{6} + (\sqrt{6} + 3\sqrt{6}) = 2\sqrt{6} + 4\sqrt{6} = 6\sqrt{6}$

Example 34: $(7.8 \times 3.5) \times 2.2 = 27.3 \times 2.2 = 60.06$

or $7.8 \times (3.5 \times 2.2) = 7.8 \times 7.7 = 60.06$

Example 35: $(\sqrt{2} \times 2\sqrt{3}) \times \sqrt{5} = 2\sqrt{6} \times \sqrt{5}$
$= 2\sqrt{30}$

or $\sqrt{2} \times (2\sqrt{3} \times \sqrt{5}) = \sqrt{2} \times 2\sqrt{15} = 2\sqrt{30}$

Distributive Property of Multiplication over Addition

When a real number is multiplied by the sum of two or more real numbers, the product is the same as the sum of the individual products of the real number and each addend.

$$r_1 \times (r_2 + r_3) = (r_1 \times r_2) + (r_1 \times r_3)$$

Example 36: $2.85 \times (3.61 + 2.51) = 2.85 \times 6.12$
$= 17.442$

or $(2.85 \times 3.61) + (2.85 \times 2.51)$
$= 10.2885 + 7.1535 = 17.442$

Thus $2.85 \times (3.61 + 2.51)$
$= (2.85 \times 3.61) + (2.85 \times 2.51)$

Example 37: $3\sqrt{2} \times (\sqrt{2} + 2\sqrt{2}) = 3\sqrt{2} \times 3\sqrt{2}$
$= 9 \times 2 = 18$

or $(3\sqrt{2} \times \sqrt{2}) + (3\sqrt{2} \times 2\sqrt{2}) = 6 + 12 = 18$

Thus $3\sqrt{2} \times (\sqrt{2} + 2\sqrt{2}) = (3\sqrt{2} \times \sqrt{2})$
$+ (3\sqrt{2} \times 2\sqrt{2})$

Identity Property of Real Numbers

The addition of 0 or the multiplication with 1 does not change a real number.

$$r + 0 = 0 + r = r$$
$$r \times 1 = 1 \times r = r$$

Inverse Property of Real Numbers

1. Corresponding to every real number, there exists another real number of opposite sign such that the sum of the two real numbers is 0.

$$r + (-r) = 0$$

2. Corresponding to every (non-zero) real number, there exists a real number, known as its reciprocal, such that the product of the two real numbers is 1.

$$r \times \frac{1}{r} = 1, r \neq 0$$

Cancellation Property of Real Numbers

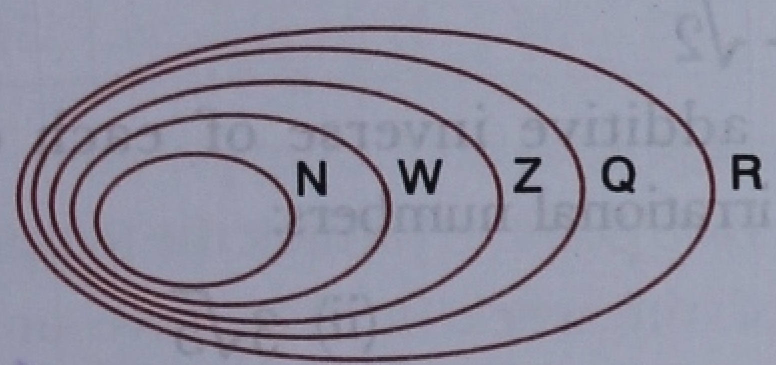
For any three real numbers,

if $r_1 + \cancel{r_2} = \cancel{r_2} + r_3 \Rightarrow r_1 = r_3$

if $r_1 \times \cancel{r_2} = \cancel{r_2} \times r_3 \Rightarrow r_1 = r_3$

Exercise 4.2

1. Write the following numbers in the smallest set or subset in the Venn diagram below.



(i) 8 (ii) -8
(iii) $+478$ (iv) -2191
(v) -21.91 (vi) $+3.6$
(vii) 0 (viii) $+4.\dot{6}$
(ix) $-6.\dot{7}$ (x) 8.292992999...
(xi) $\frac{3}{8}$ (xii) $\frac{8}{2}$
(xiii) $0\frac{0}{7}$ (xiv) $-3\frac{1}{5}$
(xv) $\frac{22}{33}$ (xvi) $\sqrt{64}$
(xvii) $\sqrt{6.4}$ (xviii) $2 + \sqrt{3}$
(xix) $6\sqrt{4}$ (xx) $4\sqrt{6}$

2. If $\frac{22}{7} = 3.1428571...$, is $\frac{22}{7}$ an irrational number?

3. Fill in the boxes with the correct real numbers in the following statements:

(i) $2\sqrt{7} + \sqrt{7} = \square + 2\sqrt{7}$

(ii) $3.\dot{8} + 4.65 = 4.65 + \square$

(iii) $\square + 29 = 29 + 5\sqrt{10}$

(iv) $3.\dot{9} + (4.69 + 2.12) = (\square + 4.69) + 2.12$

(v) $\left(\frac{7}{8} + \frac{3}{7}\right) + \frac{6}{5} = \left(\frac{6}{5} + \frac{3}{7}\right) + \square$

(vi) $3\sqrt{2}(\sqrt{3} + 2\sqrt{5}) = (3\sqrt{2} \times 2\sqrt{5}) + (\square \times \square)$

(vii) $1\frac{3}{7}\left(2\frac{7}{11}+\square\right)$

$=\left(1\frac{3}{7}\times1\frac{8}{9}\right)+\left(1\frac{3}{7}\times2\frac{7}{11}\right)$

(viii) $2\frac{1}{3}+\square=0$

(ix) $\frac{7}{-8}\times\square=1$

(x) $-7.3\dot{6}+\square=0$

4. Find the answers to the following expressions by using the properties of addition and multiplication of real numbers.
   (i) $283 + (717 + 386)$
   (ii) $(2154 - 1689) + 1689$
   (iii) $3.18 + (6.82 + 1.35)$
   (iv) $(6.784 - 3.297) + 3.297$
   (v) $\frac{7}{13}+\left(\frac{6}{13}-1\right)$
   (vi) $0.25 \times (4.17 - 0.17)$
   (vii) $(6.6 \times 6.6) + (6.6 \times 3.4)$
   (viii) $\left(\frac{2}{3}\times5\right)-\left(\frac{2}{3}\times2\right)$
   (ix) $(6.\dot{8}\times5)-(6.\dot{8}\times4)$
   (x) $\frac{6}{7}\times\frac{7}{6}\times\frac{6}{7}$

5. Which of the following operations on irrational numbers are correct?
   (i) $6\sqrt{5}-4\sqrt{3}=2\sqrt{2}$ (ii) $\sqrt{7}\times\sqrt{7}=7$
   (iii) $3\sqrt{3}+3\sqrt{3}=6\sqrt{3}$ (iv) $\sqrt{7}\times\sqrt{7}=49$
   (v) $\sqrt{7}+\sqrt{2}=\sqrt{9}$ (vi) $2\sqrt{8}\times3\sqrt{2}=24$
   (vii) $8\sqrt{2}+8\sqrt{2}=32$ (viii) $2\sqrt{3}+3\sqrt{6}=\frac{2}{3\sqrt{2}}$
   (ix) $5+\sqrt{3}=5\sqrt{3}$ (x) $3\sqrt{20}+3\sqrt{5}=2$

6. Find the rationalising factors of the following irrational numbers:
   (i) $\sqrt{10}$ (ii) $\sqrt{7}$
   (iii) $2\sqrt{5}$ (iv) $3\sqrt{7}$
   (v) $-2\sqrt{8}$ (vi) $-6\sqrt{7}$
   (vii) $\frac{1}{\sqrt{2}}$ (viii) $\frac{2}{\sqrt{3}}$
   (ix) $2\sqrt{3}+4\sqrt{3}$ (x) $7\sqrt{5}-2\sqrt{5}$
   (xi) $1+\sqrt{2}$ (xii) $3-\sqrt{5}$
   (xiii) $3\sqrt{2}+6$ (xiv) $4\sqrt{7}+6\sqrt{2}$
   (xv) $3\sqrt{6}-2\sqrt{3}$

7. Rationalise the denominators of the following numbers:
   (i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{3}{\sqrt{3}}$
   (iii) $\frac{3}{\sqrt{5}}$ (iv) $\frac{8}{\sqrt{6}}$
   (v) $\frac{3}{2\sqrt{5}}$ (vi) $\frac{\sqrt{5}}{\sqrt{7}}$
   (vii) $\frac{3\sqrt{3}}{3\sqrt{5}}$ (viii) $\frac{3}{\sqrt{5}-\sqrt{3}}$
   (ix) $\frac{5}{\sqrt{3}+\sqrt{2}}$ (x) $\frac{17}{4\sqrt{6}+3\sqrt{5}}$
   (xi) $\frac{3}{3+\sqrt{3}}$ (xii) $\frac{11}{3\sqrt{5}-2\sqrt{3}}$
   (xiii) $\frac{\sqrt{5}}{3\sqrt{5}-3\sqrt{2}}$ (xiv) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
   (xv) $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

8. Find the additive inverse of each of the following irrational numbers:
   (i) $\sqrt{7}$ (ii) $3\sqrt{5}$
   (iii) $-6\sqrt{7}$ (iv) $5+\sqrt{7}$
   (v) $3\sqrt{7}-2\sqrt{8}$

9. Find the multiplicative inverse of each of the following irrational numbers:
   (i) $\sqrt{6}$ (ii) $\frac{1}{2\sqrt{7}}$
   (iii) $\frac{3\sqrt{8}}{2\sqrt{7}}$ (iv) $\frac{4}{3+\sqrt{2}}$
   (v) $\frac{2\sqrt{5}+3\sqrt{6}}{5\sqrt{8}-4\sqrt{7}}$

Challenge

1. Illustrate the closure property of addition of real numbers using the irrational numbers $\sqrt{5}$ and $2\sqrt{5}$.
2. Illustrate that the closure property does not apply on subtraction of real numbers using two rational numbers $2\frac{1}{7}$ and $-3\frac{2}{5}$.
3. Illustrate the distributive property of multiplication over addition of real numbers using three irrational numbers $3\sqrt{7}$, $-2\sqrt{7}$ and $\sqrt{7}$.

Revision Exercise

1. Find two rational numbers between $\frac{7}{9}$ and $\frac{8}{9}$.
2. How many integers lie between –416 and +4160?
3. Write the following numbers in descending order:
   (i) –312, + 621, –412, + 721, –721
   (ii) $-7\frac{1}{9}, -7\frac{3}{9}, -7\frac{2}{9}, -7\frac{5}{9}, -7\frac{4}{9}$
4. Find the rationalising factors of the following irrational numbers.
   (i) $5\sqrt{7}-2\sqrt{7}$ (ii) $7-\sqrt{13}$
   (iii) $2 \quad \sqrt{5}$ (iv) $6\sqrt{11} \quad 4\sqrt{7}$
4. Rationalize the denominators of the following.
   (i) $\frac{7}{7 \quad \sqrt{7}}$ (ii) $\frac{\sqrt{3}}{2\sqrt{7}-2\sqrt{3}}$
   (iii) $\frac{\sqrt{5} \quad 1}{\sqrt{5}-1}$ (iv) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2} \quad \sqrt{3}}$