UNIT SIX

- Area and Perimeter of Plane Figures
- Volume and Surface Area of Cuboids

Mensuration

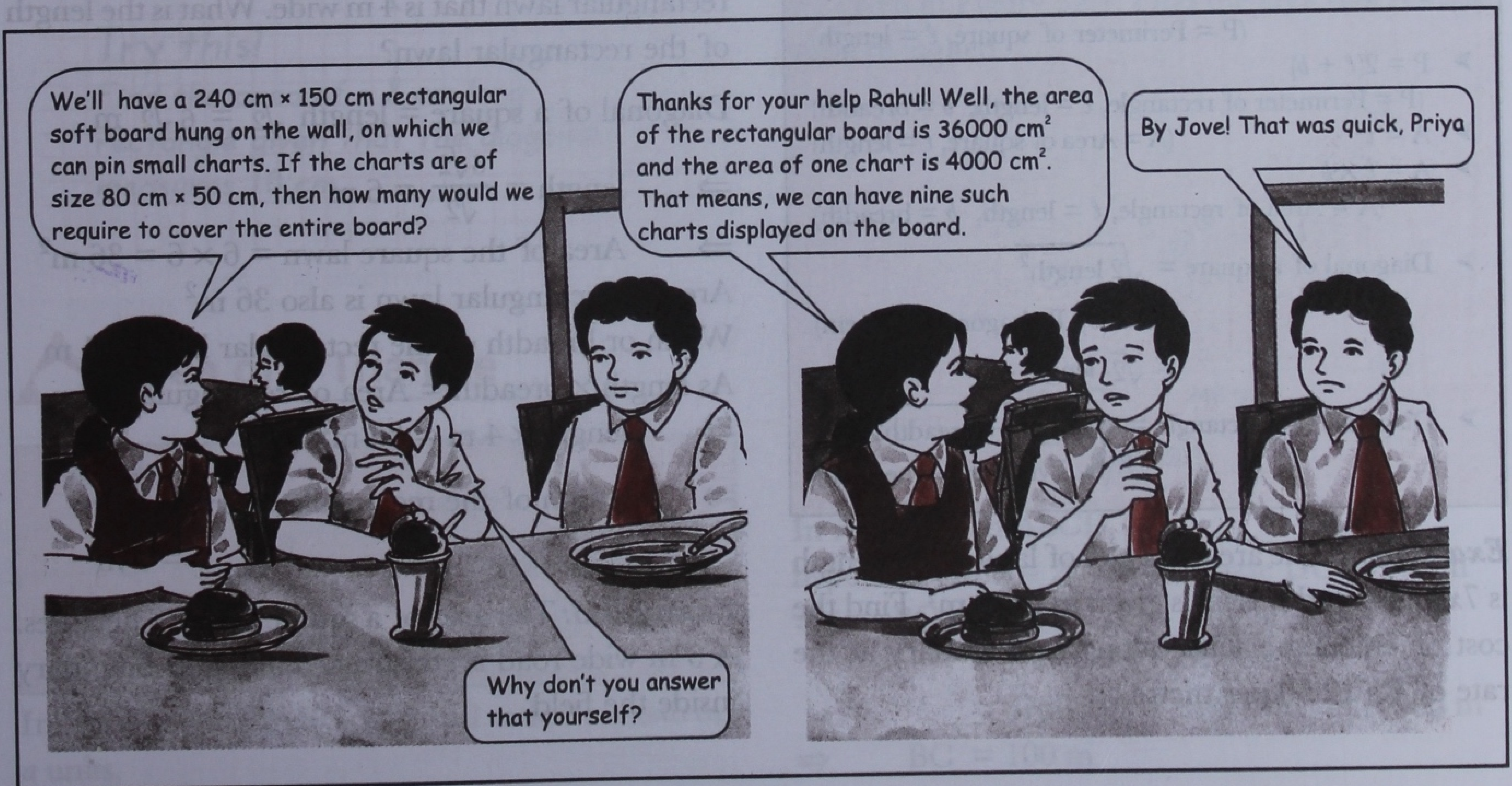
Let's Recap

1. Find the area of a square, given its perimeter is 15.2 cm.
2. Find the perimeter of a 9.8 cm long rectangle, given its area is 53.9 cm^2.
3. Find the total surface area of a cube, given its volume is 216 cm^3.

Are *An are is a unit of measure for area equal to 100 square metres. The word, and the unit of measure, seems to have been created by the French and derived from the Latin word 'area' with its current meaning. The are is seldom used today, but its derivative form, the hectare is still a common unit of land measure in some countries.*

Perimeter *The word perimeter comes from the Greek word 'peri' (around) + 'metron' (measure).*

Volume *Volume is derived from the Latin word 'volvere', which means to turn or roll. The idea comes from the practice of writing on scrolls that were then rolled into a cylinder.*



32 AREA AND PERIMETER OF PLANE FIGURES

- Area
- Perimeter
- Circumference

Let us first recall the units of area and their conversions.

$1\ km^2 = 1\ km \times 1\ km = 1000\ m \times 1000\ m = 1000000\ m^2$
1 hectare $= 1\ hm \times 1\ hm = 100\ m \times 100\ m = 10000\ m^2$
1 are $= 1\ dcm \times 1\ dcm = 10\ m \times 10\ m = 100\ m^2$
$1\ m^2 = 1\ m \times 1\ m = 100\ cm \times 100\ cm = 10000\ cm^2$
$1\ m^2 = 1\ m \times 1\ m = 1000\ mm \times 1000\ mm$
$= 1000000\ mm^2$
$1\ cm^2 = 1\ cm \times 1\ cm = 10\ mm \times 10\ mm = 100\ mm^2$

Area and Perimeter of Squares and Rectangles

Formulae

- $P = 4\ell$ (P = Perimeter of square, ℓ = length)
- $P = 2(\ell + b)$ (P = Perimeter of rectangle, ℓ = length, b = breadth)
- $A = \ell^2$ (A = Area of square, ℓ = length)
- $A = \ell \times b$ (A = Area of rectangle, ℓ = length, b = breadth)
- Diagonal of a square $= \sqrt{2\ \text{length}^2}$ (by Pythagoras' theorem) $= \sqrt{2}$ length
- Diagonal of a rectangle $= \sqrt{\text{length}^2 + \text{breadth}^2}$ (by Pythagoras' theorem)

Example 1: The area of a plot of land whose length is $7x$ and breadth is $5x$ is given as $2835\ m^2$. Find the cost of erecting a fence along its boundary at the rate of Rs 12.50 per metre.

Area = length × breadth = $2835\ m^2$

$\Rightarrow\ 7x \times 5x = 2835\ m^2$

$\Rightarrow\ x^2 = \frac{2835}{35}\ m^2 = 81\ m^2$

$\Rightarrow\ x = \sqrt{81}\ m = 9\ m$

Thus, the length of the plot = $7 \times 9 = 63$ m and breadth of the plot = $5 \times 9 = 45$ m

Perimeter = 2(length + breadth)
= $2(63 + 45)\ m = 2 \times 108 = 216$ m

Cost of erecting fence along the boundary of the plot = 216 m × Rs 12.50 = Rs 2700.00

Example 2: The grass from a square lawn whose diagonal measures $6\sqrt{2}$ m is transplanted to a rectangular lawn that is 4 m wide. What is the length of the rectangular lawn?

Diagonal of a square = length $\sqrt{2} = 6\sqrt{2}$ m

$\Rightarrow$ length $= \frac{6\sqrt{2}}{\sqrt{2}} = 6$ m

$\Rightarrow$ Area of the square lawn $= 6 \times 6 = 36\ m^2$

Area of rectangular lawn is also $36\ m^2$

Width or breadth of the rectangular lawn = 4 m

As length × breadth = Area of rectangular lawn,

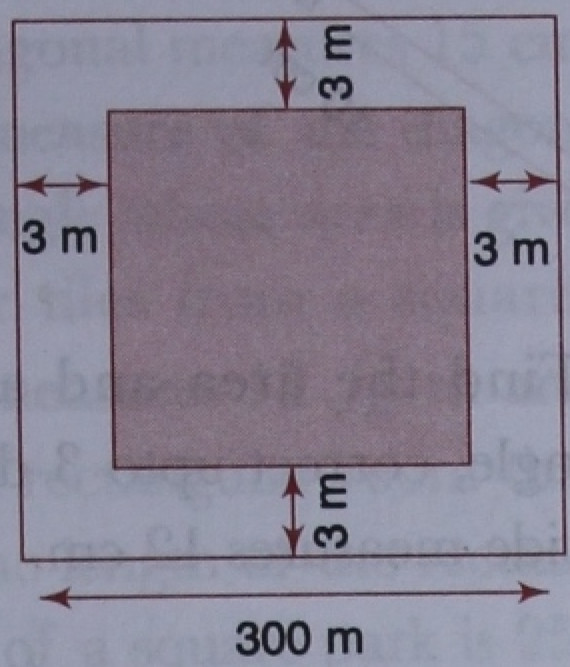
$\Rightarrow$ length × 4 m = $36\ m^2$

$\Rightarrow$ length of the rectangular lawn $= \frac{36}{4}$ m
$= 9$ m

Example 3: The area of a square field is 9 hectares. A 3 m wide road is constructed along its boundary inside the field.

(i) How much area remains free for cultivation?

(ii) At Rs 18.50 per sq. m, how much did it cost to construct the road?



(i) Area of the field

$= 9$ hectares $= 90000\ m^2$

$\Rightarrow$ length of the field $= \sqrt{90000}$ m $= 300$ m

Length of cultivable land

$= 300\ m - 3\ m - 3\ m = 294\ m$

Area free for cultivation $= 294\ m \times 294\ m$

$= 86436\ m^2$ or 8.6436 hectares

(ii) Area of road constructed

= Area of the field – Area free for cultivation

$= 90000\ m^2 - 86436\ m^2 = 3564\ m^2$

At Rs 18.50 per sq. m, cost of constructing road $= 3564\ m^2 \times$ Rs 18.50 = Rs 65934.00

Try this!

Find the area of a 8 cm long rectangle given that the diagonal measures 10 cm.

Area of a Triangle

Formulae

- Area of a right-isosceles triangle $= \frac{1}{2}a^2$ (being $\frac{1}{2}$ the area of a square with length a)
- Area of a triangle $= \frac{1}{2}$ base × altitude

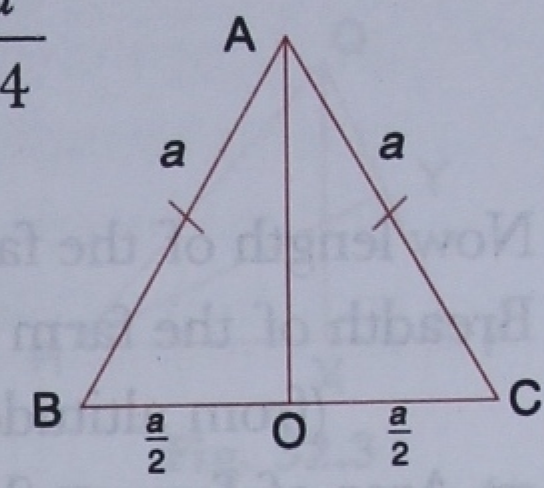
In equilateral ΔABC with all sides measuring a units,

$AB^2 = AO^2 + OB^2 \Rightarrow AO^2 = AB^2 - OB^2$ (by Pythagoras' theorem)

$$\Rightarrow AO^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4}$$

$$= \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$$

$$\Rightarrow AO = \sqrt{\frac{3a^2}{4}} = \frac{a}{2}\sqrt{3}$$



- Altitude of an equilateral triangle $= \frac{a}{2}\sqrt{3}$ (a = length of a side)

As area of equilateral ΔABC $= \frac{1}{2}$ base × altitude

$= \frac{1}{2} \times a \times \frac{a}{2}\sqrt{3}$

- Area of equilateral triangle $= \frac{a^2}{4}\sqrt{3}$ (a = length of a side)
- The Hero's formula, discovered by a Greek mathematician, gives the area of a triangle with sides a, b, c, and semi-perimeter s as

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ unit²

where $s = \frac{a+b+c}{2}$

Example 4: In rectangular farm ABCE, a tractor has ploughed plot BCD, the dimensions of which are given in Figure 32.1. Find the area that remains to be ploughed.

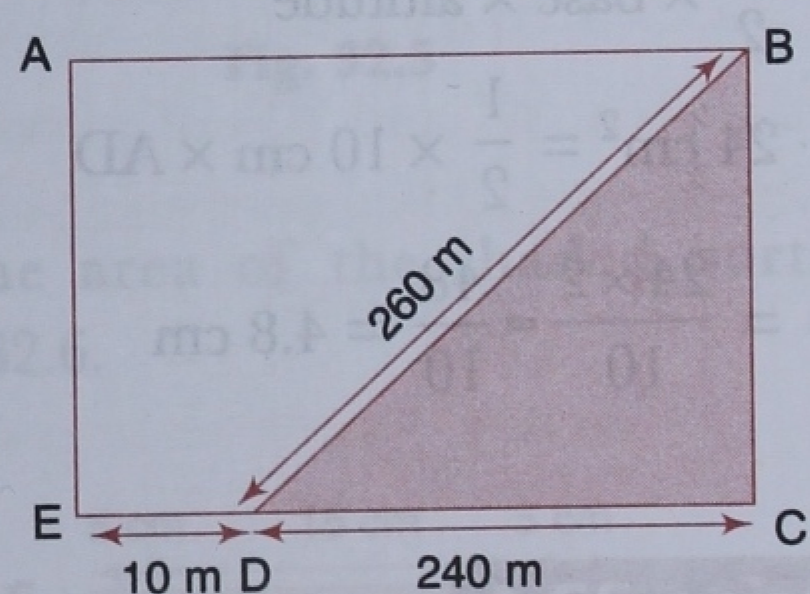


Fig. 32.1

In right-angled ΔBCD,

hypotenuse BD = 260 m and base DC = 240 m

$\Rightarrow\ BC^2 = BD^2 - DC^2$

$= 260^2 - 240^2$

$= 67600\ m^2 - 57600\ m^2 = 10000\ m^2$

$\Rightarrow\ BC = 100$ m

Thus, area ploughed = $\frac{1}{2}$ base × altitude

$$= \frac{1}{2} \times 240 \times 100 = 12000\ m^2$$

= 1.2 hectares

Now length of the farm = 240 + 10 = 250 m

Breadth of the farm = 100 m

(from altitude of triangle calculated above)

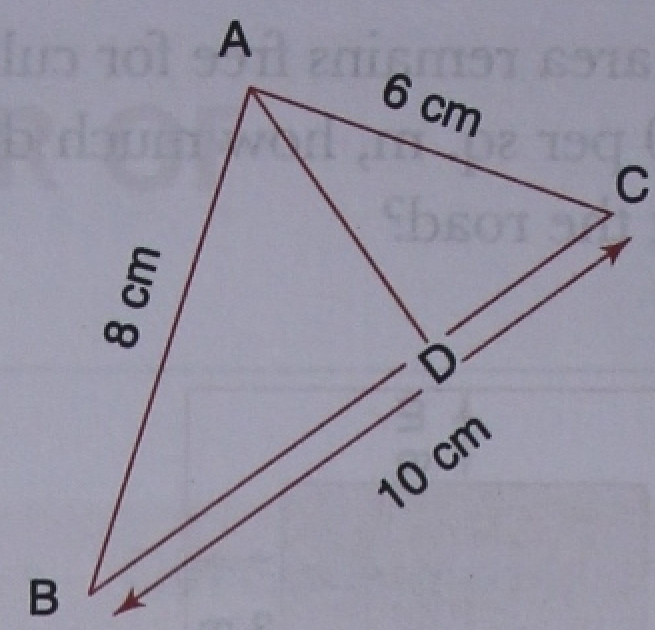
⇒ Area of farm = 250 m × 100 m = 25000 m^2

= 2.5 hectares

Thus, area to be ploughed = 2.5 – 1.2

= 1.3 hectares

Example 5: AD is the altitude of ΔABC, drawn from A to side BC. Find the measure of AD, given AB = 8 cm, BC = 10 cm, and AC = 6 cm.



Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ unit^2

(Hero's formula)

⇒ Area of ΔABC

$$= \sqrt{12(12-10)(12-8)(12-6)}\ cm^2$$

$$\left(\text{as } s = \frac{10+8+6}{2} = \frac{24}{2} = 12\ cm\right)$$

$$= \sqrt{12 \times 2 \times 4 \times 6} = \sqrt{576}\ cm^2$$

⇒ Area of ΔABC = 24 cm^2

But area of ΔABC

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\Rightarrow \quad 24\ cm^2 = \frac{1}{2} \times 10\ cm \times AD$$

$$\Rightarrow AD = \frac{24 \times 2}{10} = \frac{48}{10} = 4.8\ cm$$

Example 6: Find the area and altitude of an equilateral triangle, correct upto 3 decimal places, in which each side measures 12 cm.

Area of an equilateral triangle = $\frac{a^2}{4}\sqrt{3}$ unit^2

When a = 12 cm, area of the equilateral triangle

$$= \frac{12^2}{4}\sqrt{3} = 36\sqrt{3}\ cm^2$$

$\sqrt{3}$ = 1.732 (from the chapter on powers asnd roots)

⇒ Area of the equilateral triangle = 36 × 1.732

= 62.352 cm^2

Altitude of an equilateral triangle = $\frac{a}{2}\sqrt{3}$ units

When a = 12 cm, altitude of the equilateral triangle

$$= \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}\ cm$$

⇒ Altitude of the equilateral triangle = 6 × 1.732

= 10.392 cm

Try this!

Find the area of a right-angled triangle given its base measures 5 cm and its altitude is 6 cm.

Exercise 32.1

1. Find the area of a square whose perimeter is 27.6 cm.
2. Find the area of a 7.45 cm long rectangle whose perimeter is 24.5 cm.
3. The area of a rectangle whose sides are in the ratio 5 : 2 is given as 3.6 m^2. Find the perimeter of the rectangle.
4. The area of a park whose length is $5x$ and breadth is $4x$ is given as 1280 m^2. Find the cost of erecting a fence along its boundary at the rate of Rs 11.35 per metre.
5. Find the area of a square, given that its diagonal measures $5\sqrt{2}$ cm.

6. Find the measure of the diagonal of a square, whose area is given as 100 cm^2, correct up to 2 decimal places.
7. Find the area of a 14.4 cm long rectangle, given that its diagonal measures 15 cm.
8. Find the measure of the diagonal of a 30 cm long rectangle whose area is given as 480 cm^2.
9. The floor tiles from a square room whose diagonal measures $12\sqrt{2}$ m are removed and fitted in a rectangular room that is 8 m wide. What is the length of the rectangular room?
10. The area of a square park is 25 ares. A gravel path, 1.5 m wide, is laid along its boundary inside the park. At Rs 6.50 per sq. m, how much did it cost to lay the gravel path?
11. Three 2 m wide paths criss-cross each other, as shown in Figure 32.2, in a rectangular field whose area is 6 hectares. If the field is 500 m long, find the area that remains free for cultivation.

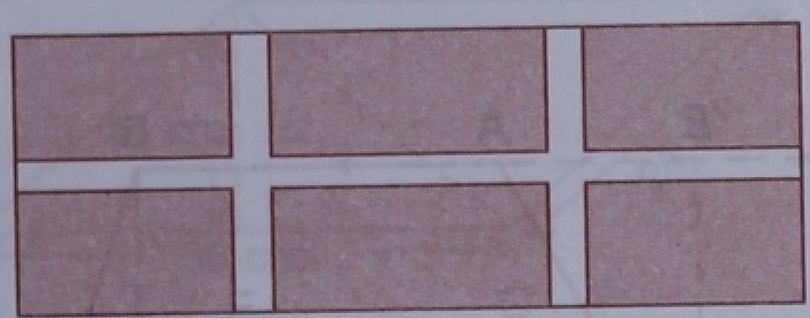


Fig. 32.2

12. At Rs 8.50 per sq. m, it costs Rs 2754 to lay grass in a square lawn. At Rs 11.50 per metre, find how much it would cost to erect a fence along its boundary.
13. Find the area of a right-angled triangle, given its base measures 14 cm and its altitude is 0.6 times its base.
14. Find the area of a right-isosceles triangle, given that one of its equal sides measures 11 cm.
15. Find the area of an equilateral triangle, given that one of its equal sides measures 6 cm. (Take $\sqrt{3} = 1.73$)
16. Find the altitude of an equilateral triangle, given that one of its equal sides measures $5\sqrt{3}$ cm.
17. Find the area of a scalene triangle, given the measures of its three sides as 9 cm, 12 cm, 15 cm.
18. The three sides of a scalene triangle measure 3 cm, 4 cm, and 5 cm. Find the altitude of the triangle when the shortest side is its base.
19. In ΔPQR (Figure 32.3), PQ = 8 cm and PR = 10 cm. If altitude RY on PQ measures 9 cm, find the measure of altitude QX on PR.

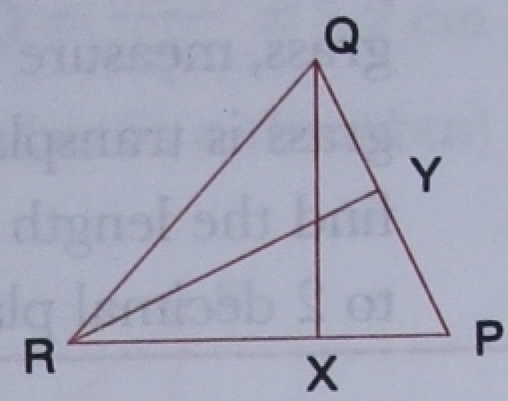


Fig. 32.3

20. In ΔABC (Figure 32.4), AB = 6 cm and AC = 9 cm. If altitude BX on AC measures 5.5 cm, find the measure of altitude CY on side AB.

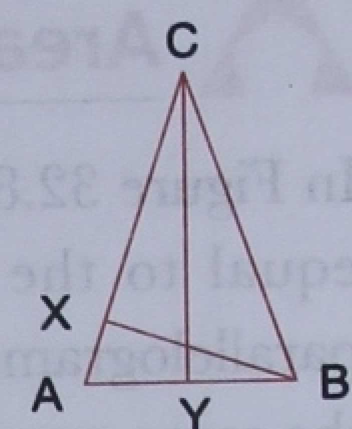


Fig. 32.4

21. The base of a triangle is given as $7x$ while its altitude is given as $5x$. If the area of the triangle is given as 2117.5 cm^2, find its base and altitude.
22. Find the area of trapezium ABCE (Figure 32.5), given ABCD is a square and ADE is a right-angled triangle.

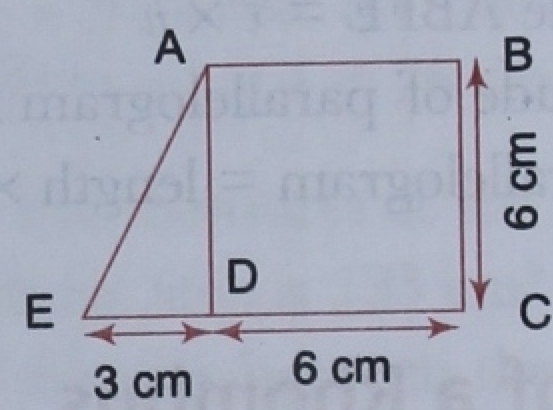


Fig. 32.5

23. Find the area of the shaded portion in Figure 32.6.

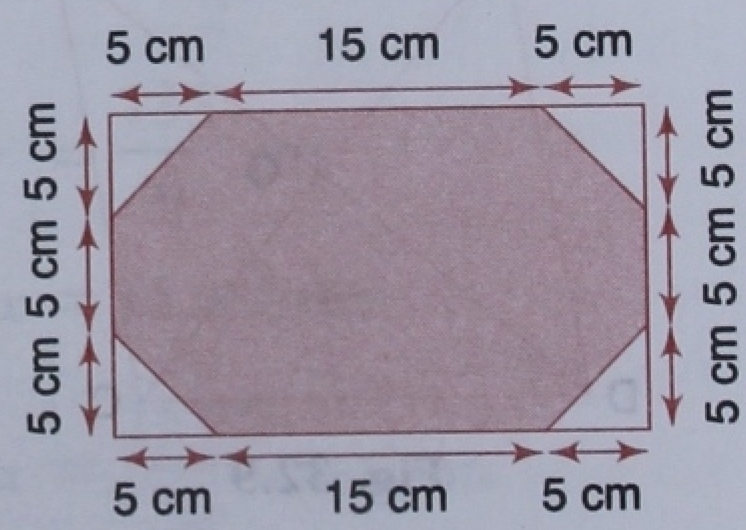


Fig. 32.6

24. Find the area of the shaded portion in Figure 32.7.
25. The sides of a triangular lawn, covered with grass, measure 28 m, 21 m, and 35 m. If all the grass is transplanted to a square-shaped lawn, find the length of the square lawn, correct up to 2 decimal places.

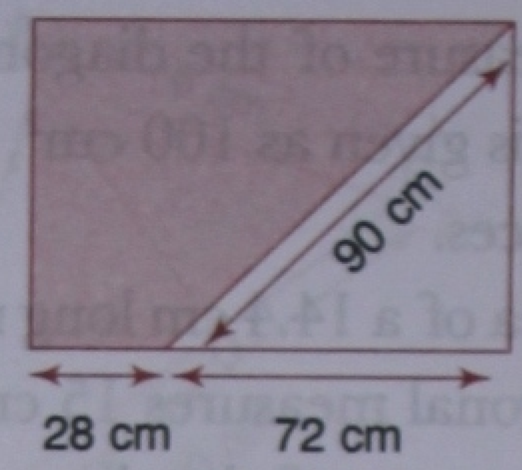


Fig. 32.7

Area of a Parallelogram

In Figure 32.8, the area of parallelogram ABCD is equal to the area of rectangle ABFE as both parallelograms are on the same base and between the same parallel lines.

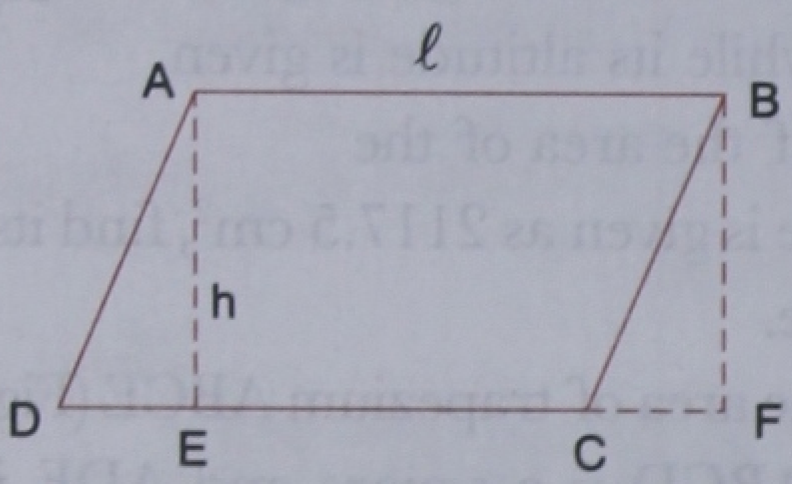


Fig. 32.8

Area of rectangle ABFE = $\ell \times h$

But h is the altitude of parallelogram ABCD.

$\therefore$ Area of a parallelogram = length × altitude

Area of a Rhombus

The diagonals AC and BD of rhombus ABCD, shown in Figure 32.9, bisect each other and are perpendicular to each other.

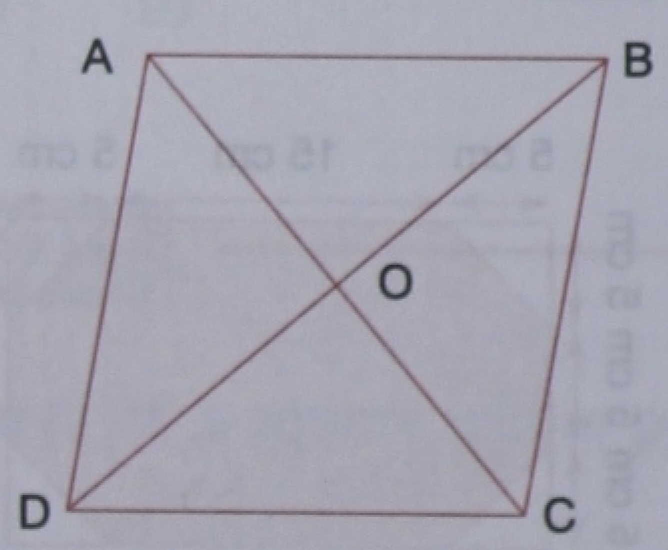


Fig. 32.9

Let the measure of AC = a and BD = b

$\Rightarrow$ AO = OC = $\frac{a}{2}$ and DO = OB = $\frac{b}{2}$

Area of right $\Delta AOD = \frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)$

As $\Delta AOD \cong \Delta AOB \cong \Delta COB \cong \Delta COD$

$\Rightarrow$ Area of rhombus ABCD $= 4 \times \frac{ab}{8} = \frac{ab}{2}$

$\therefore$ Area of a rhombus $= \frac{\text{product of its diagonals}}{2}$

Area of a Trapezium

To find the area of trapezium ABCD in Figure 32.10, we draw BF ⊥ DC.

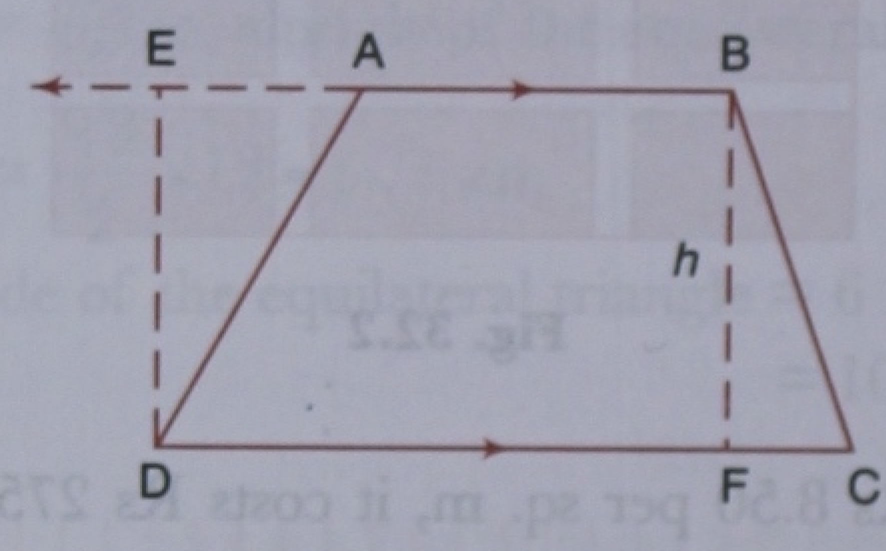


Fig. 32.10

Extend AB to E and draw DE ⊥ EB

BF = h = altitude of trapezium

= DE (distance between parallel lines)

Area of trapezium ABCD

= Area of ΔABD + Area of ΔBCD

$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$

$= \frac{1}{2} h\,(AB + DC)$

Thus, area of a trapezium

$= \frac{1}{2}$ (sum of parallel sides) × (distance between them)

Example 7: The base of a parallelogram is $8x$ while its altitude is $4x$. If the area of the parallelogram is given to be 1152 cm^2, find the measures of its base and altitude.

Area of parallelogram = base × altitude

$\Rightarrow \quad 1152 \text{ cm}^2 = 8x \times 4x$

$\Rightarrow \quad 1152 = 32x^2$

$\Rightarrow \quad x^2 = \dfrac{1152}{32} = 36 \text{ cm}^2$

$\Rightarrow \quad x = 6 \text{ cm}$

Thus, the base of the parallelogram = 8 × 6 = 48 cm and its altitude = 4 × 6 = 24 cm

Example 8: Two adjacent sides in a parallelogram measure 36 cm and 24 cm. If the distance between the longer sides of the parallelogram measures 15 cm, find the distance between its shorter sides.

In parallelogram ABCD (Figure 32.11), when base is 36 cm, altitude is 15 cm.

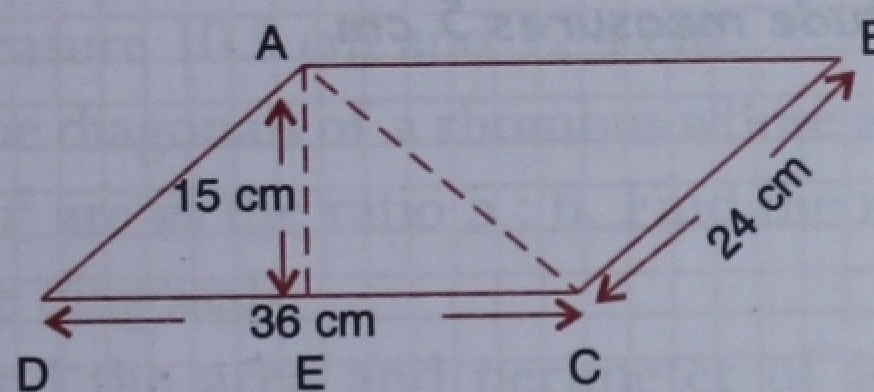


Fig. 32.11

$\Rightarrow$ Area of parallelogram = 36 × 15 = 540 cm^2

When base = 24 cm,

$\Rightarrow \quad 24 \times \text{altitude} = 540 \text{ cm}^2$

$\Rightarrow \quad \text{altitude} = \dfrac{540}{24} = 22.5 \text{ cm}$

Thus, the distance between the shorter sides of the parallelogram is 22.5 cm.

Example 9: Find the area and perimeter of a rhombus whose diagonals measure 10.4 cm and 7.8 cm.

$$\text{Area of a rhombus} = \frac{\text{Product of its diagonals}}{2}$$

$$= \frac{10.4 \times 7.8}{2} \text{ cm}^2$$

$$= \frac{81.12}{2} = 40.56 \text{ cm}^2$$

In rhombus ABCD (Figure 32.12), given AC = 7.8 cm and BD = 10.4 cm

$\Rightarrow \quad AO = \dfrac{7.8}{2} = 3.9 \text{ cm and } DO = \dfrac{10.4}{2} = 5.2 \text{ cm}$

(as diagonals bisect each other in a rhombus)

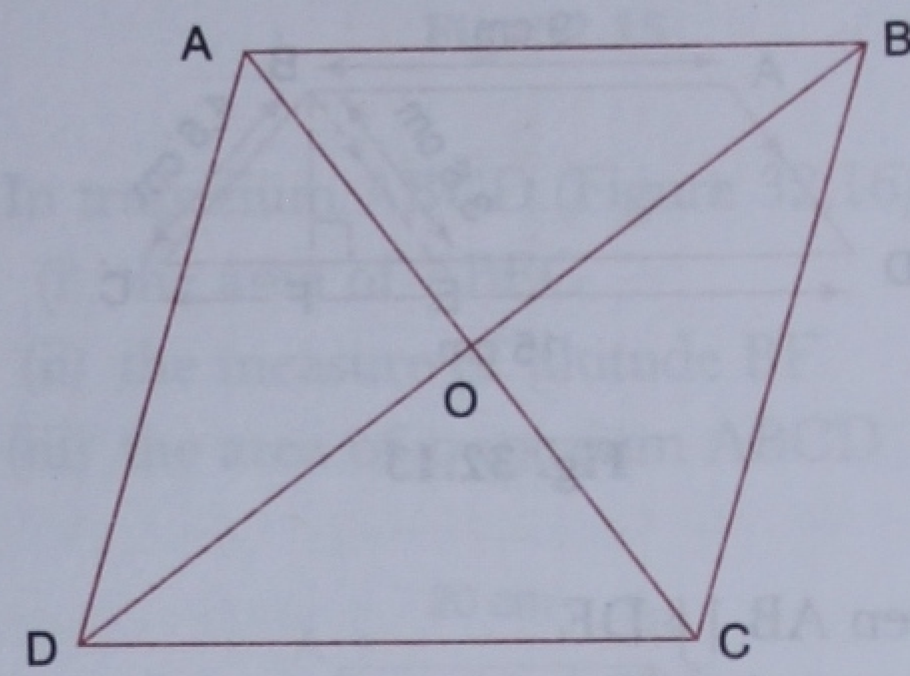


Fig. 32.12

As the diagonals of a rhombus are perpendicular to each other, in right-angled ΔAOD,

$AD^2 = AO^2 + DO^2 = 3.9^2 + 5.2^2$

$= 15.21 + 27.04 \text{ cm}^2$

$= 42.25 \text{ cm}^2$

$\Rightarrow \quad AD = \sqrt{42.25} \text{ cm} = 6.5 \text{ cm}$

As all sides of a rhombus are equal, perimeter of rhombus ABCD = 6.5 × 4 = 26 cm

Example 10: The measures of the parallel sides of a trapezium whose area is 135 cm^2 are given as $3x$ and $2x$. If the distance between them is 9 cm, find the measure of the parallel sides.

$$\text{Area of a trapezium} = \frac{1}{2}(\text{sum of parallel sides}) \times (\text{distance between them})$$

$\Rightarrow \quad 135 \text{ cm}^2 = \dfrac{2x + 3x}{2} \times 9 \text{ cm}$

$\Rightarrow \quad \dfrac{5x}{2} = \dfrac{135}{9} \text{ cm}$

$\Rightarrow \quad 5x = 15 \times 2 \text{ cm}$

$\Rightarrow \quad x = \dfrac{15 \times 2}{5} = 6 \text{ cm}$

Thus, the measures of the parallel sides are 3 × 6 = 18 cm and 2 × 6 = 12 cm.

Example 11: In trapezium ABCD shown in Figure 32.13, find:

(i) the area of ΔBEC
(ii) altitude BF
(iii) area of trapezium ABCD

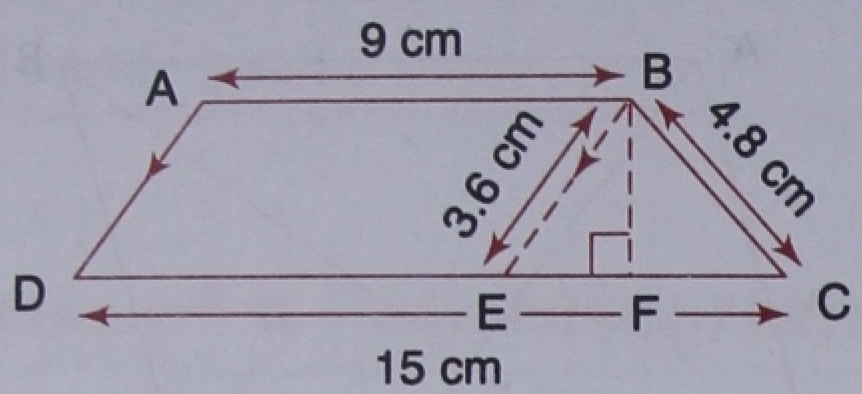


Fig. 32.13

(i) Given AB || DE (parallel sides of trapezium ABCD)

and AD || BE (given in figure)

⇒ ABED is a parallelogram

⇒ AB = DE = 9 cm (opposite sides of a parallelogram)

⇒ EC = DC – DE = 15 – 9 = 6 cm

Area of ΔBEC = $\sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = \frac{3.6+4.8+6}{2} = \frac{14.4}{2} = 7.2 \text{ cm}$

⇒ Area of ΔBEC

$= \sqrt{7.2(7.2-3.6)(7.2-4.8)(7.2-6)}$

$= \sqrt{7.2 \times 3.6 \times 2.4 \times 1.2}$

$= \sqrt{74.6496}$

$= 8.64 \text{ cm}^2$

(ii) As area of ΔBEC = $\frac{1}{2}$ base × altitude

$\Rightarrow \frac{1}{2} \times 6 \times \text{altitude BF} = 8.64 \text{ cm}^2$

$\Rightarrow \text{altitude BF} = \frac{8.64 \times 2}{6} \text{ cm} = 2.88 \text{ cm}$

(iii) Area of trapezium ABCD = $\frac{1}{2}$(sum of parallel sides) × (distance between parallel sides)

$= \frac{1}{2}(9 + 15) \times 2.88 \text{ cm}^2$

$= \frac{24 \times 2.88}{2} = 34.56 \text{ cm}^2$

Try this!

Find the area of a parallelogram, given its base measures 10 cm and its altitude measures 5 cm.

Exercise 32.2

1. Find the area of a parallelogram, given its base measures 21 cm and its altitude measures 9 cm.
2. Find the area of a parallelogram in which one of the longer sides measures 14.5 cm and the distance between them is 6.8 cm.
3. The base and altitude of a parallelogram are in the ratio 13 : 5. If the area of the parallelogram is given as 3185 cm², find the measures of its base and altitude.
4. The longer side and the distance between the longer sides in a parallelogram are in the ratio 11 : 7. If the area of the parallelogram is given as 693 cm², find the measure of one of the longer sides and the distance between them.
5. In parallelogram ABCD (Figure 32.14), the altitude on side AD measures 10 cm. Find the measure of altitude AE on side DC.

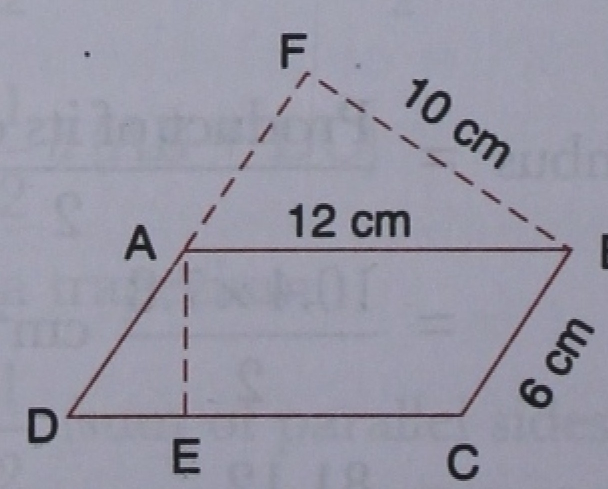


Fig. 32.14

6. Two adjacent sides in a parallelogram measure 14 cm and 21 cm. If the distance between the longer sides of the parallelogram measures 9 cm, find the distance between its shorter sides.
7. Two adjacent sides of a parallelogram measure 15 cm and 20 cm while the diagonal opposite their common vertex measures 25 cm. Find the area of the parallelogram.
8. Two adjacent sides of a parallelogram measure 24 cm and 45 cm while the diagonal opposite their common vertex measures 51 cm. Find the area of the parallelogram.
9. Two adjacent sides of a parallelogram measure 39 cm and 52 cm while the diagonal opposite their common vertex measures 65 cm. Find the area of the parallelogram.
10. Find the area of rhombus ABCD, given AB = 10 cm, BC = 10 cm, and diagonal AC = 12 cm.
11. Find the area of a rhombus whose diagonals measure 10.5 cm and 12.4 cm.
12. The diagonals of a rhombus whose area is 375 cm² are in the ratio 5 : 6. Find the measure of the diagonals.
13. Find the area and perimeter of a rhombus whose diagonals measure 18 cm and 24 cm.
14. Find the area and perimeter of a rhombus whose diagonals measure 28 cm and 96 cm.
15. The parallel sides of a trapezium measure 8 cm and 12 cm. If the distance between the parallel sides is 6 cm, find the area of the trapezium.
16. The mid-points D and E of the equal sides AB and AC of isosceles ΔABC are joined. If side BC measures 9 cm and the distance between line segments DE and BC is 4 cm, find the area of trapezium DECB.
17. The parallel sides of a trapezium whose area is 99 cm² are in the ratio 4 : 7. If the distance between them is 9 cm, find the measures of the parallel sides.
18. Find the area of trapezium ABCD, shown in Figure 32.15, given ∠DAB and ∠ABC are right angles.

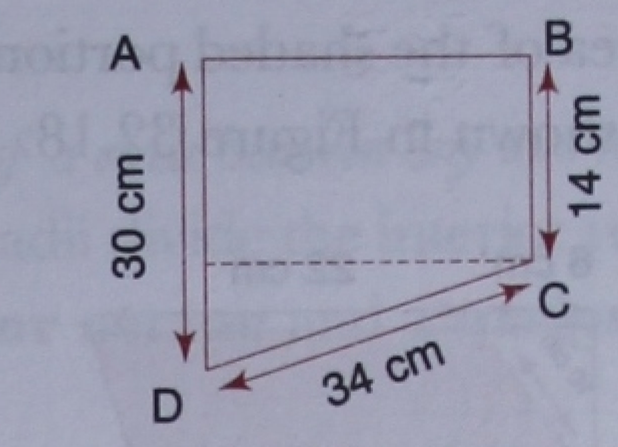


Fig. 32.15

19. In trapezium ABCD (Figure 32.16), find
    (i) the area of ΔBEC
    (ii) the measure of altitude BF
    (iii) the area of trapezium ABCD

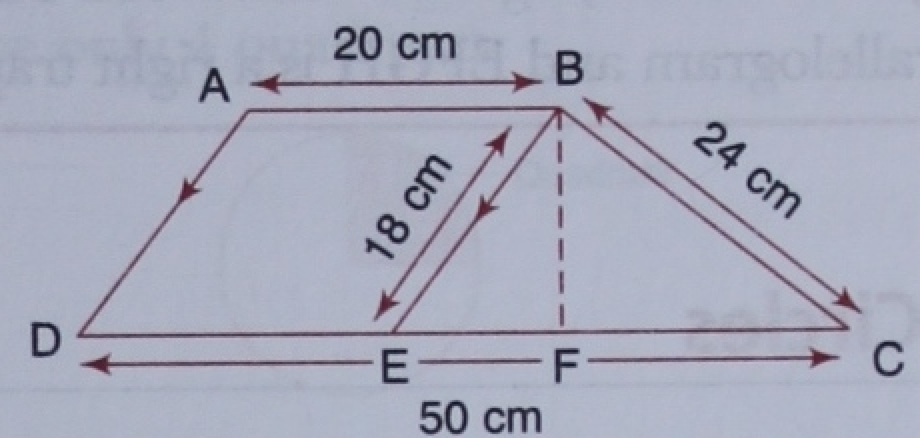
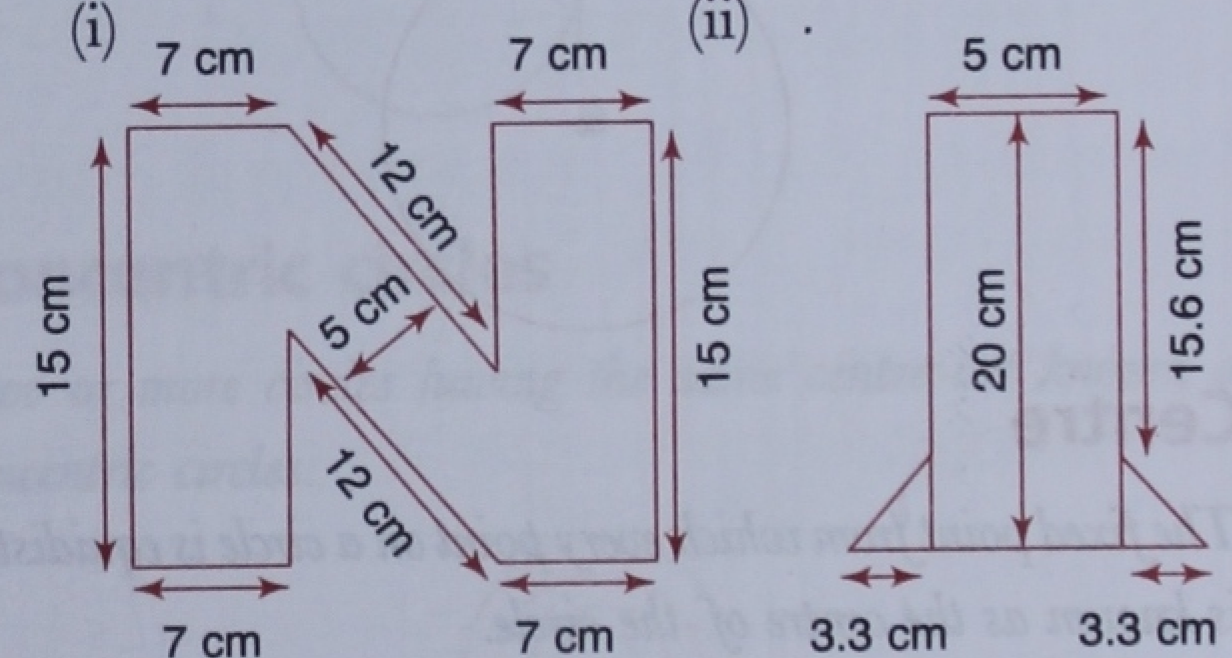


Fig. 32.16

20. The parallel sides of a trapezium, whose area is 162 cm², are given as $x + 2$ and $x - 2$. If the distance between them is 9 cm, find the measures of the parallel sides.
21. Find the area of the following closed figures.



(i) (ii)

22. Find the area of right trapezium ABCD shown in Figure 32.17, given AEFD is a rhombus.

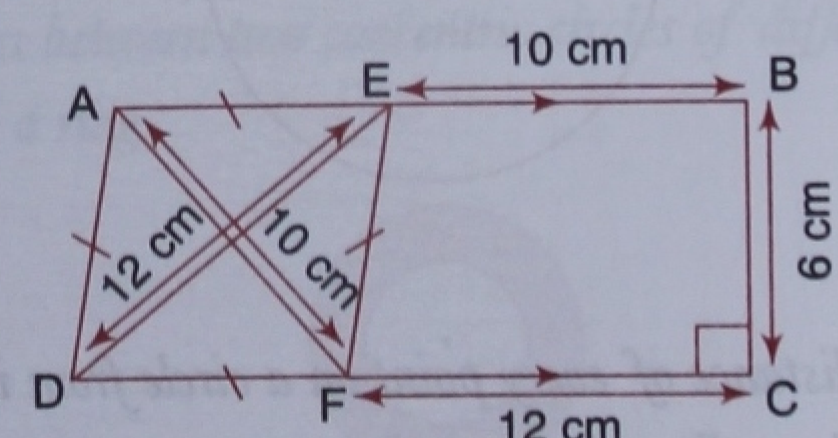


Fig 32.17

23. Find the area of the shaded portion in the right trapezium shown in Figure 32.18.

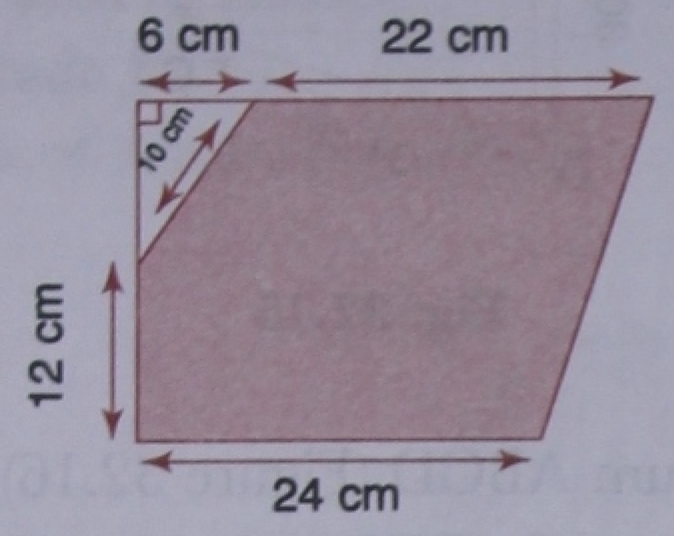


Fig. 32.18

24. Find the area of the shaded portion in Figure 32.19, given that ABCD is a parallelogram and EFGH is a right trapezium.

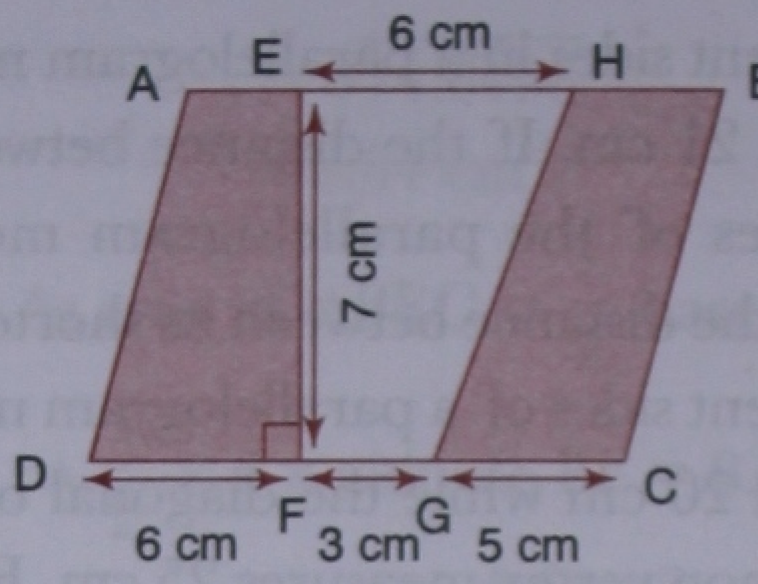


Fig. 32.19

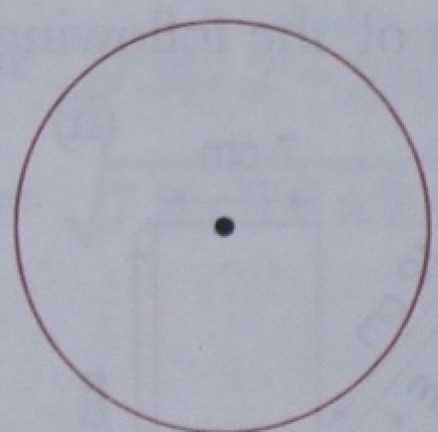
25. An equilateral ΔABC with each side measuring $2\sqrt{3}$ cm is reflected about its side AC to form its image AB_1C. Find the measure of the diagonal BB_1 in rhombus AB_1CD.

Circles

Let us recapitulate some terms associated with circles.

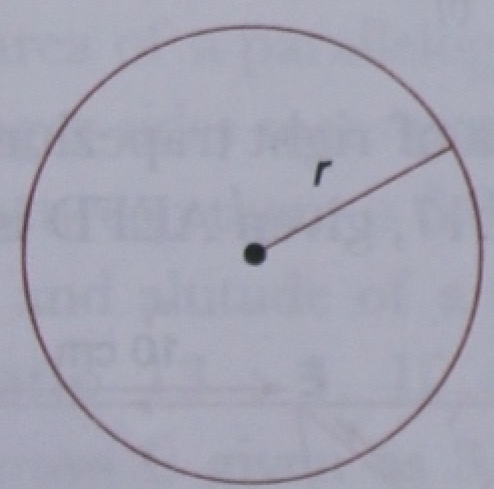
Circle

A circle is a plane figure bounded by a curved line, every point of which is equidistant from a fixed point.



Centre

The fixed point from which every point on a circle is equidistant is known as the centre of the circle.



Radius

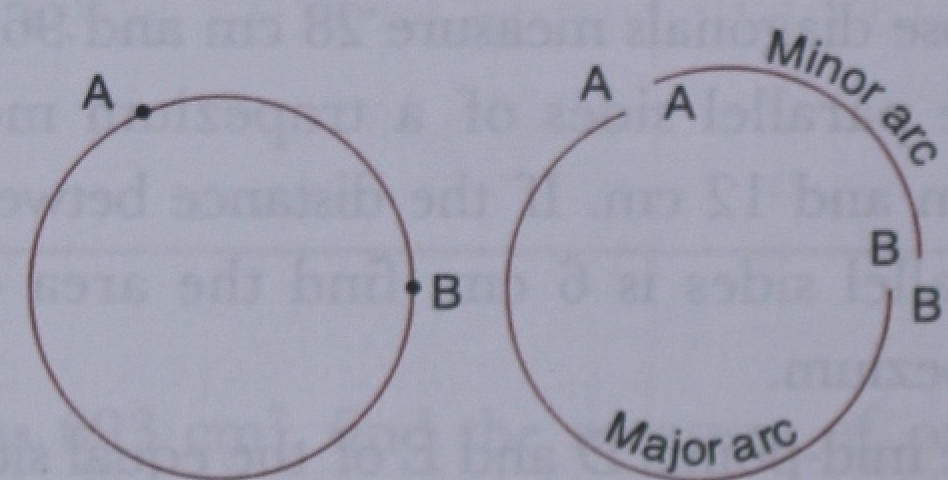
The fixed distance of every point on a circle from its centre is known as the radius of the circle.

Circumference

The length of the curved line traced by a point moving at a fixed distance from its centre is called the circumference of the circle. The circumference is the length of the boundary of a circle.

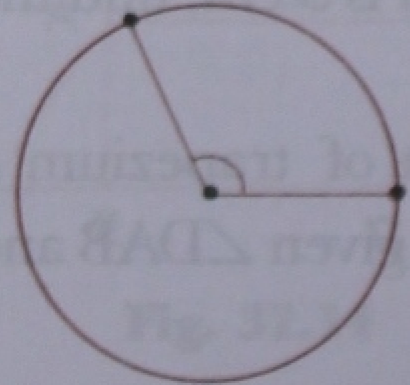
Arc

An arc is a part of the circumference of a circle that lies between any two points on the circle. Two points A and B that lie on a circle divide its circumference into a **major arc** and a **minor arc.**



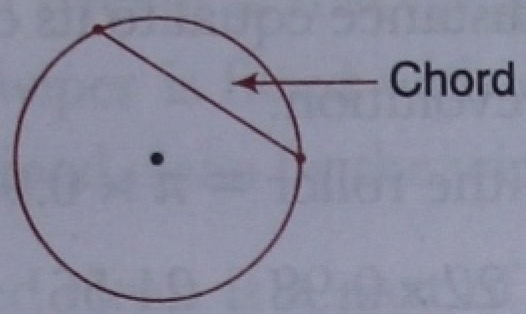
Angle Subtended by an Arc

When the end points of an arc are joined with the centre, the two radii intersect to form a central angle that is subtended by the arc.



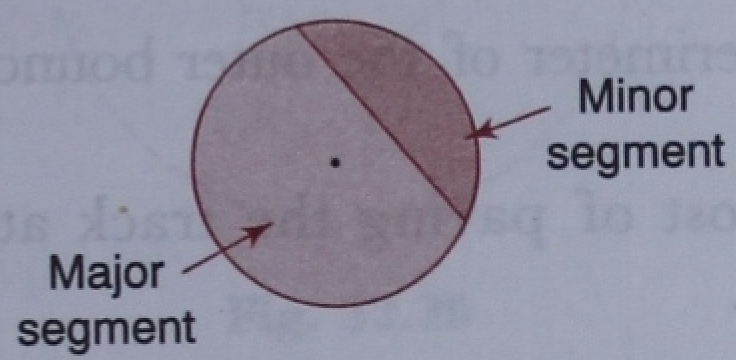
Chord

A chord is a straight line segment with its end points on the boundary of a circle.



Segment

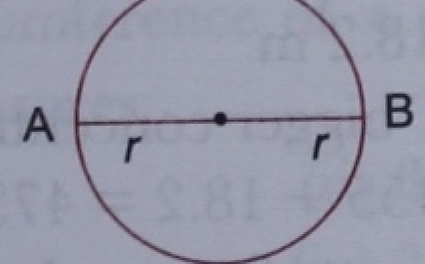
The interior region of a circle that lies between a chord and an arc is known as a segment. A chord divides the interior region of a circle into a **minor segment** and a **major segment.**



Diameter

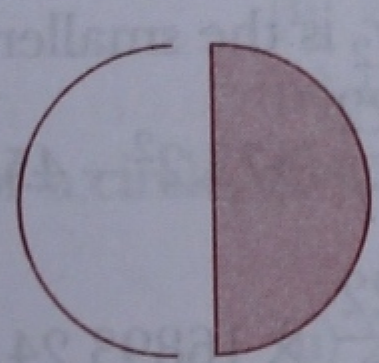
The diameter of a circle is a chord that passes through its centre. It is the longest chord in a circle and an axis about which a circle is symmetrical. The length of the diameter of a circle is twice the length of its radius.

$$d = 2r$$



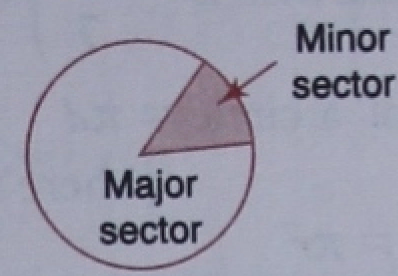
Semicircle

The diameter divides a circle into two equal arcs, each of which is known as a semicircle. The diameter also divides the interior region of a circle into two equal **semicircular regions.**



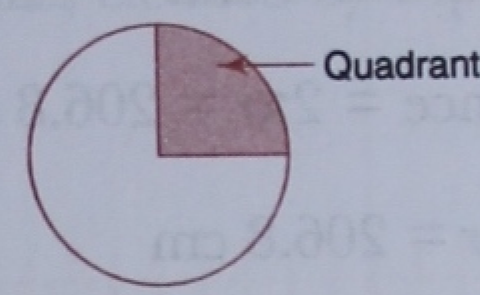
Sector

The interior of a circle between any two radii is known as a sector. Two radii divide the interior region of a circle into a **minor sector** and a **major sector.**



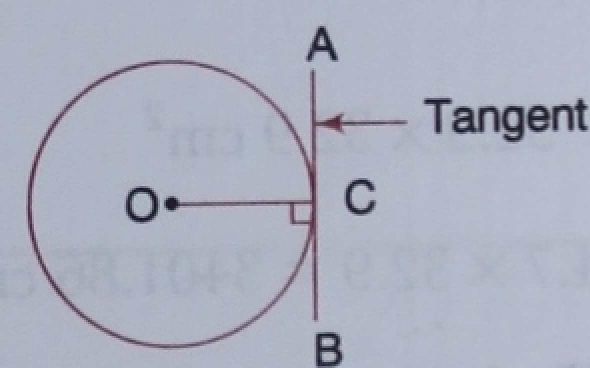
Quadrant

The minor sector between two radii that are perpendicular to each other is known as a quadrant. Given a radius, a circle can have only 4 quadrants.



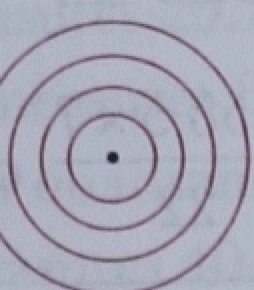
Tangent

A straight line with only one point on a circle is known as its tangent. If line AB is a tangent at point C, then radius OC is perpendicular to AB.



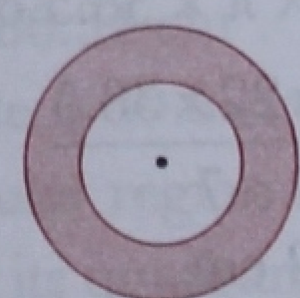
Concentric circles

Two or more circles having the same centre are known as concentric circles.



Ring

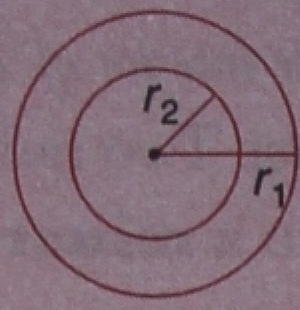
The region between two concentric circles of different radii is known as a ring.



Formulae

- Circumference of a circle = $2\pi r$

  (where r is the radius, and unless mentioned otherwise $\pi = \frac{22}{7}$)



- Circumference of a circle = πd (where d is the diameter)
- Area of a circle = πr^2
- Area of a ring = $\pi r_1^2 - \pi r_2^2$ (where r_1 is the greater radius)

  $= \pi(r_1^2 - r_2^2)$

Example 12: Find the area of a circle, given that its circumference is 206.8 cm.

Given circumference = $2\pi r$ = 206.8 cm

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 206.8$ cm

$\Rightarrow \quad r = \frac{206.8 \times 7}{2 \times 22} = \frac{1447.6}{44} = 32.9$ cm

Area of a circle = πr^2

Area of a circle with 32.9 cm radius = $\pi \times 32.9 \times 32.9$ cm^2

$= \frac{22}{7} \times 32.9 \times 32.9$ cm^2

$= 22 \times 4.7 \times 32.9 = 3401.86$ cm^2

Example 13: Find the circumference of a circle, given that its area is 4658.5 cm^2.

Given area = πr^2 = 4658.5 cm^2

$\Rightarrow \quad \frac{22}{7} r^2 = 4658.5$ cm^2

$\Rightarrow \quad r^2 = \frac{4658.5 \times 7}{22}$ cm^2

$\Rightarrow \quad r = \sqrt{1482.25}$ cm

$\Rightarrow \quad r = 38.5$ cm

Circumference of a circle = $2\pi r$

Circumference of a circle with 38.5 cm radius

$= 2 \times \pi \times 38.5$ cm

$= \frac{2 \times 22 \times 38.5}{7}$

$= 242$ cm

Example 14: The diameter of a heavy roller is 0.98 m. How far will it move in 50 complete revolutions?

A wheel covers a distance equal to its circumference in one complete revolution.

Circumference of the roller = $\pi \times 0.98$ m

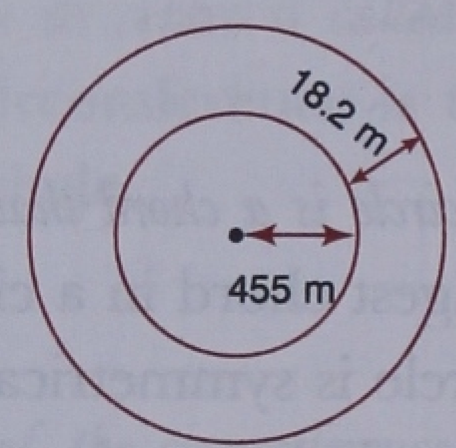
$= \frac{22 \times 0.98}{7} = \frac{21.56}{7} = 3.08$ m

In 50 complete revolutions, the roller will cover 50 times its circumference = $3.08 \times 50 = 154$ m

Example 15: A circular race track is 18.2 m wide. If the radius of the inner boundary of the track is 455 m, find:

(i) the perimeter of the outer boundary of the track.

(ii) the cost of paving the track at Rs 5 per sq. m.



(i) Given radius of smaller concentric circle = 455 m

Given distance between concentric circles = 18.2 m

$\Rightarrow$ radius of bigger concentric circle = 455 + 18.2 = 473.2 m

Circumference of outer boundary of race track = $2\pi r$

$= 2 \times \frac{22}{7} \times 473.2$ m

$= \frac{20820.8}{2} = 2974.4$ m

(ii) Area of ring = $\pi(r_1^2 - r_2^2)$, where r_1 is the greater radius, and r_2 is the smaller radius

$= \frac{22}{7}(473.2^2 - 455^2)$ m^2

$= \frac{22}{7} \times 16893.24$ m^2

$= 53093.04$ m^2

At Rs 5 per sq. m, paving 53093.04 m^2 would cost 53093.04 × Rs 5 = Rs 265465.20.

Example 16: A 42 cm long windshield wiper covers a quadrant as it sweeps across the glass. If the outer 35 cm of the wiper is fitted with srubber, find the area of glass wiped clean by the wiper in one sweep.

The area wiped clean is a part of a ring made by two circles as shown in Figure 32.20.

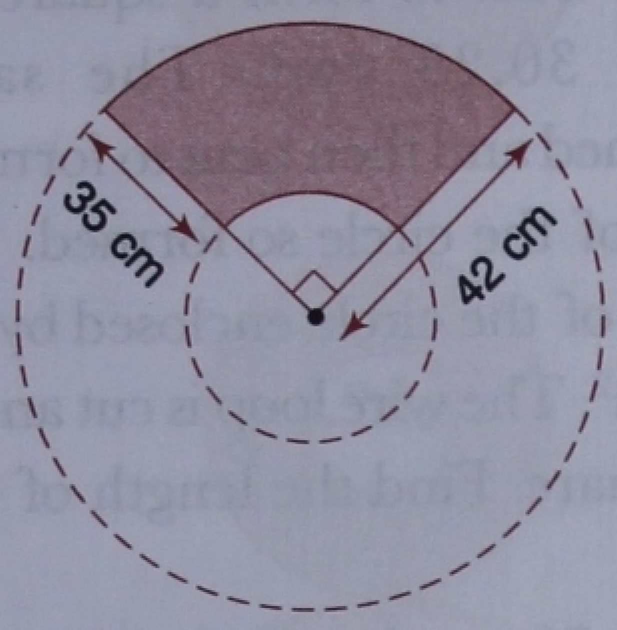


Fig. 32.20

Greater radius

$r_1 = 42$ cm

Smaller radius

$r_2 = 42 - 35 = 7$ cm

$$\text{Area of ring} = (r_1^2 - r_2^2)\ \text{cm}^2$$

$$= \frac{22}{7}(42^2 - 7^2)\ \text{cm}^2$$

$$= \frac{22}{7} \times 1715\ \text{cm}^2$$

$$= 5390\ \text{cm}^2$$

As the wiper covers a quadrant, the area of which is one-fourth the area of the circle $\left(\text{as } \frac{360°}{4} = 90°\right)$, the area cleaned by the wiper is also one-fourth the area of the ring.

Thus, area of glass cleaned by wiper $= \frac{5390}{4}$ cm^2

$= 1347.5\ \text{cm}^2$

Try this!

Find the area of a circle, given that its radius measures 21 cm.

Exercise 32.3

1. Find the circumference of a circle, given that its radius measures:
   (i) 7 cm (ii) 3.5 cm
   (iii) 8.4 cm (iv) 7.35 cm
2. Find the circumference of a circle, given that its diameter measures:
   (i) 21 cm (ii) 4.2 cm
   (iii) 9.1 cm (iv) 10.85 cm
3. Find the area of a circle, given that its radius measures:
   (i) 3.5 cm (ii) 28 cm
   (iii) 4.9 cm (iv) 0.63 m
4. Find the area of a circle, given that its diameter measures:
   (i) 14 cm (ii) 21 cm
   (iii) 3.5 cm (iv) 2.8 m
5. Find the length of a semicircle, given:
   (i) its radius measures 5.6 cm.
   (ii) its diameter measures 31.5 cm.
6. Find the area of the semicircular region, given:
   (i) its radius measures 25.2 cm.
   (ii) its diameter measures 60.2 cm.
7. Find the area of a quadrant in a circle, given:
   (i) its radius measures 40.6 cm.
   (ii) its diameter measures 44.8 cm.
8. Find the radius of a circle, given:
   (i) its diameter is 10.5 cm.
   (ii) its circumference is 35.2 cm.
   (iii) its area is 186.34 cm^2.
   (iv) its semicircle is 39.6 cm.
   (v) its semicircular region is 481.25 cm^2.
   (vi) the area of its quadrant is 9.625 cm^2.

9. Find the area of a circle, given that its circumference is 66 cm.
10. Find the area of a circle, given that its circumference is 299.2 cm.
11. Find the circumference of a circle, given that its area is 55.44 cm^2.
12. Find the circumference of a circle, given that its area is 2847.46 cm^2.
13. The radius of a wheel is 39.9 cm. How much distance will the wheel cover in 25 complete revolutions?
14. How many times will a 89.6 cm wide cycle wheel revolve in covering a distance of 42.24 m?
15. If a bicycle wheel covers 51.92 m in 20 revolutions, find its radius.
16. If a child covers 5.06 m on pedalling 5 times on his tricycle, find the diameter of the tricycle's front wheel.
17. A circular garden has a 2.1 m wide bed of flowers along its boundary on the inside encircling a grass lawn (Figure 32.21). If the total area of the garden is 260.26 m^2, find the area of the grass lawn.

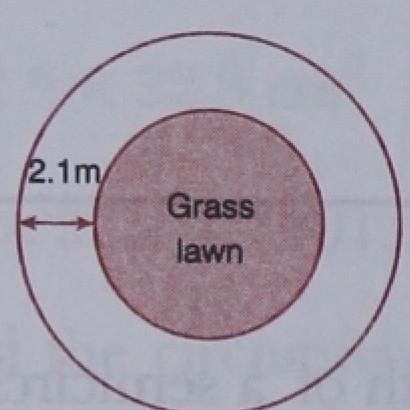


Fig. 32.21

18. A 1.75 m path runs along the inside of the boundary of a circular park. If the maximum length between any two points in the park is 84 m, find how much it would cost to pave the path at Rs 20 per m^2.
19. A circular railway track runs through an amusement park. If the distance between the tracks is 1.4 m and the radius of the inner track is 175 m, find:
    (i) the perimeter of the inner track.
    (ii) the perimeter of the outer track.
    (iii) the area between the two tracks.
    (iv) the cost of laying stone chips in the area between the tracks at Rs 6.75 per m^2.
20. A wire is bent to form a 90 cm long rectangle whose breadth measures 64 cm. The same wire is straightened and then bent to form a circle. Find the area of the circle so formed.
21. A wire is bent to form a square enclosing an area of 30.25 cm^2. The same wire is straightened and then bent to form a circle. Find the area of the circle so formed.
22. The area of the circle enclosed by a wire loop is 18634 cm^2. The wire loop is cut and then shaped into a square. Find the length of the square so formed.
23. A truck's 70 cm long windshield wiper's tip covers an arc that equals one-third the circumference of the circle. If the outer 56 cm of the wiper is fitted with rubber, find the area of glass cleaned by the wiper in one sweep.
24. A baseball field is shaped like a quadrant with a radius of 56 m (Figure 32.22). A 7 m wide viewing area is to be cleared outside the field along the quadrant's arc. At Rs 12.50 per sq.m, what would be the total cost of clearing the viewing area?

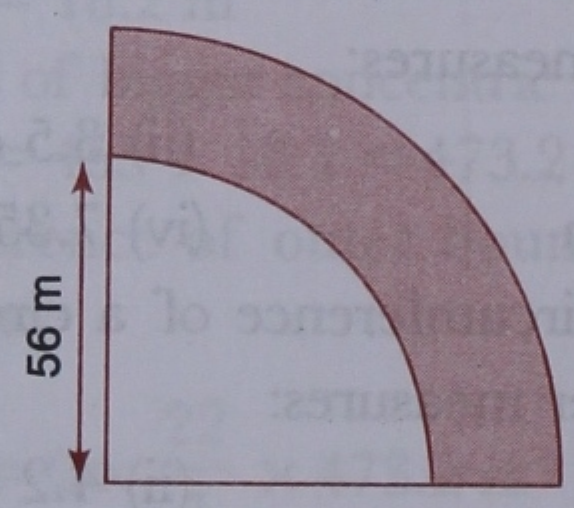
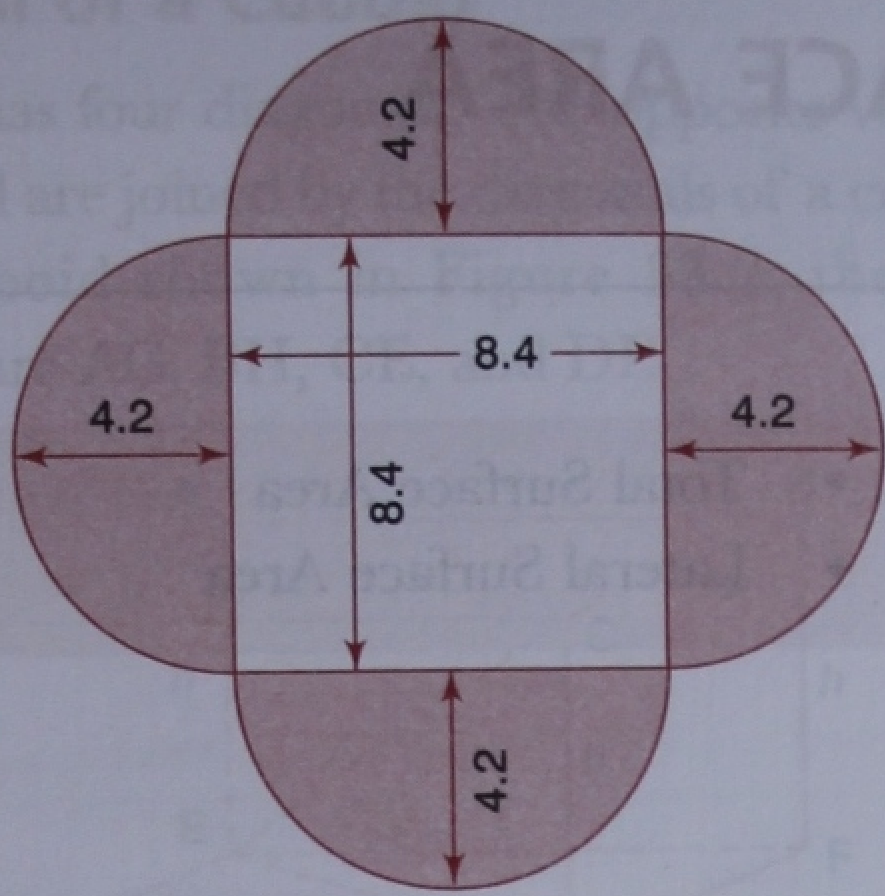


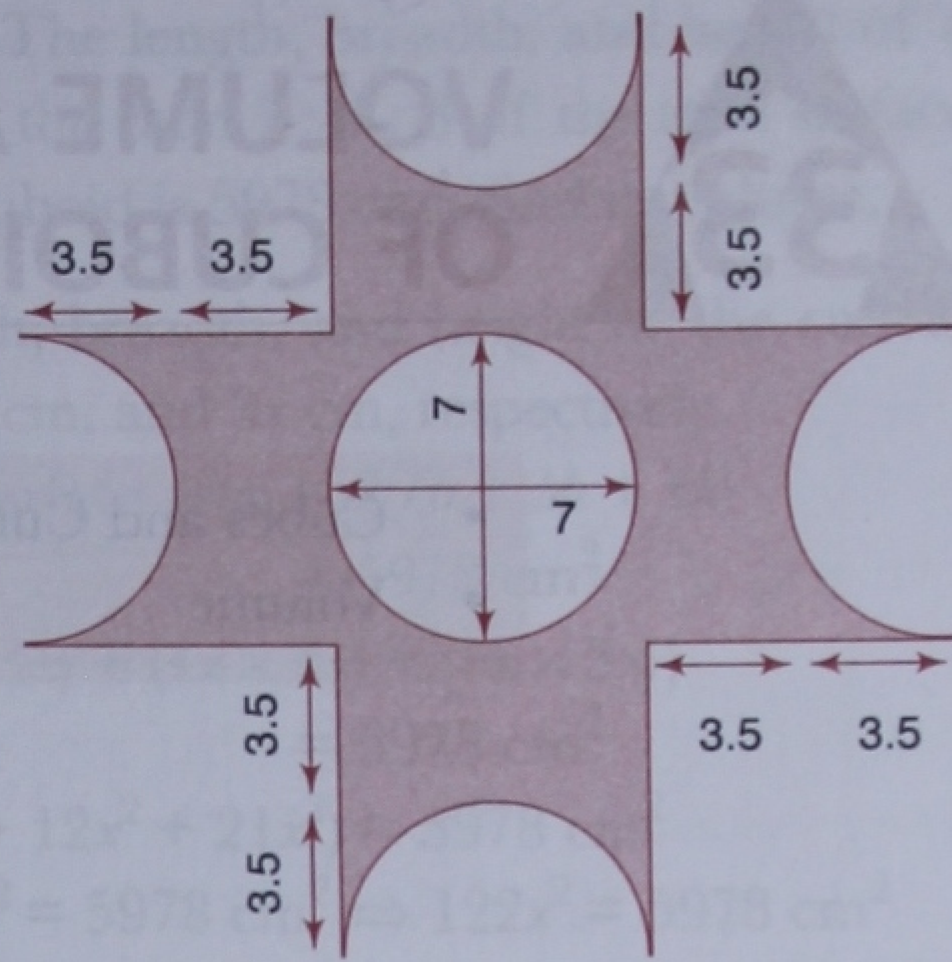
Fig. 32.22

25. Find the area of the shaded portions in the following figures.
    All given measurements are in centimetres.

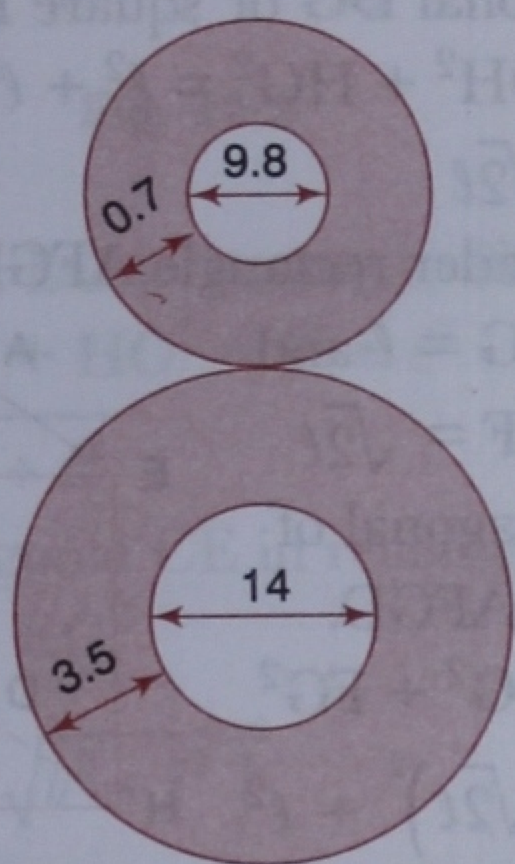
(i)



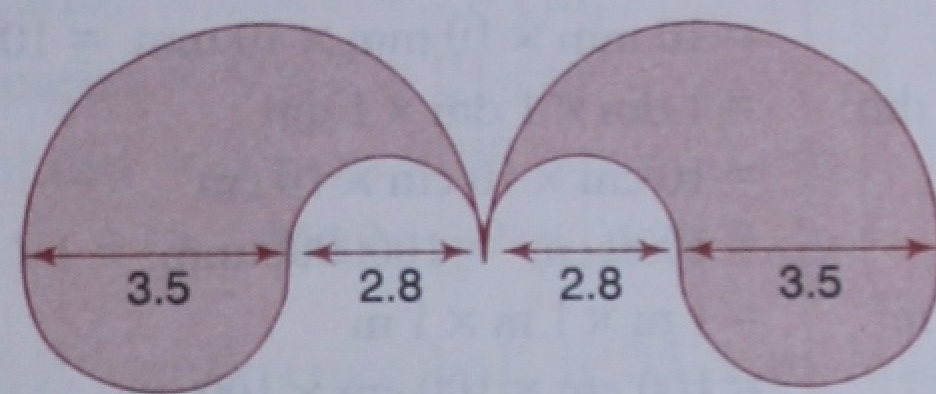
(ii)



(iii)



(iv)



Revision Exercise

1. Find the area of a square whose perimeter is 48.4 cm.
2. Find the area of a right-isosceles triangle, given that one of its equal sides measures 15 cm.
3. Find the area of a parallelogram in which one of the longer sides measures 18.5 cm and the distance between them is 9.4 cm.
4. Find the area of a rhombus whose diagonals measure 13.2 and 15.6.
5. The radius of a wheel is 26.4 cm. How much distance will the wheel cover in 35 complete revolutions?