29 CONSTRUCTION OF QUADRILATERALS

- Construction of Quadrilaterals

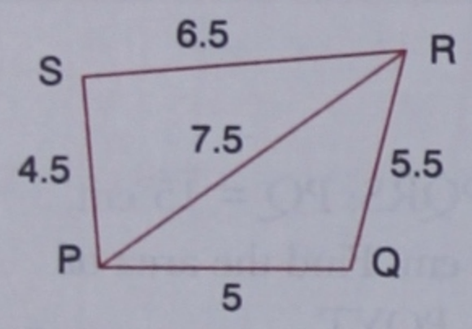
This chapter deals with the steps of construction of various types of quadrilaterals. It is advisable to draw a rough sketch of the required quadrilaterals to help plan the constructions.

Construction of Quadrilaterals

I. Given the measure of 4 sides and 1 diagonal.

Construct quadrilateral PQRS, given PQ = 5 cm, QR = 5.5 cm, RS = 6.5 cm, SP = 4.5 cm, and PR = 7.5 cm.

SKETCH



CONSTRUCTION

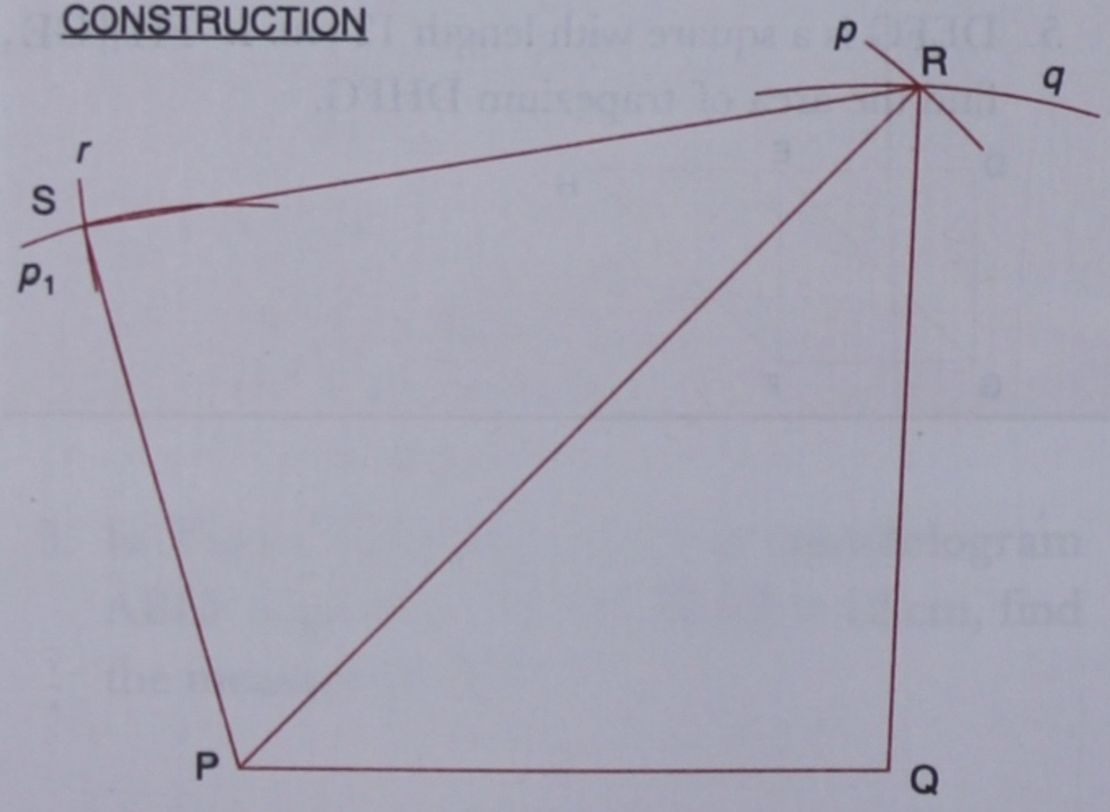


Fig. 29.1

Steps:

1. Draw PQ = 5 cm.
2. With Q as centre and radius 5.5 cm, draw arc *q*.
3. With P as centre and radius 7.5 cm, draw arc *p* that intersects arc *q* at point R.
4. With P as centre and radius 4.5 cm, draw arc p_1.
5. With R as centre and radius 6.5 cm, draw arc *r* that intersects arc p_1 at point S.
6. Connect point S with P and R and point R with P and Q.
7. We have quadrilateral PQRS (Figure 29.1), where PQ = 5 cm, QR = 5.5 cm, RS = 6.5 cm, SP = 4.5 cm, and diagonal PR = 7.5 cm.

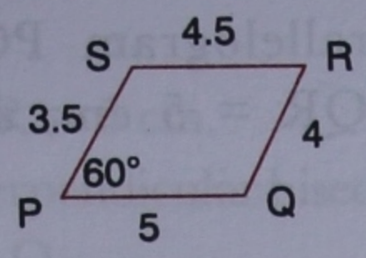
II. Given the measure of 4 sides and 1 angle

Construct quadrilateral PQRS, given PQ = 5 cm, QR = 4 cm, RS = 4.5 cm, SP = 3.5 cm, and ∠SPQ = 60°.

Steps:

1. Draw PQ = 5 cm.
2. At point P construct ∠APQ = 60°.
3. With P as centre and radius 3.5 cm, draw arc *p* that intersects arc p at point S.
4. With Q as centre and radius 4 cm, draw arc *q*.
5. With S as centre and radius 4.5 cm, draw arc *s* that intersects arc *q* at point R.
6. Connect point S with R and connect point R with Q.
7. We have quadrilateral PQRS (Figure 29.2), where PQ = 5 cm, QR = 4 cm, RS = 4.5 cm, SP = 3.5 cm, and ∠SPQ = 60°.

SKETCH



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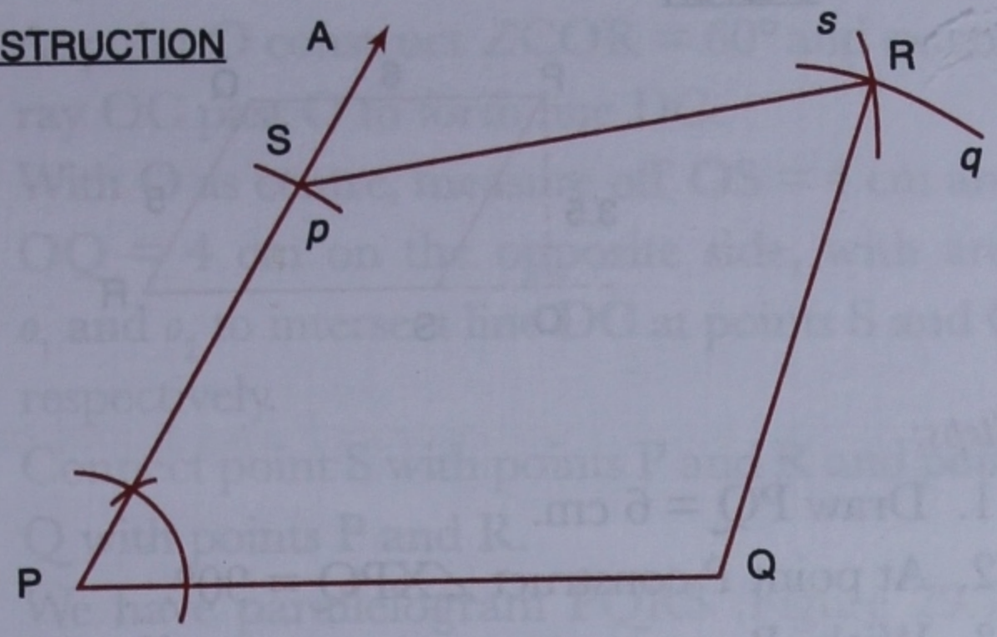


Fig. 29.2

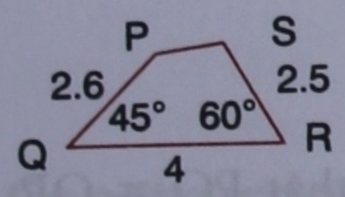
III. Given 3 consecutive sides and the included angles.

Construct quadrilateral PQRS, given PQ = 2.6 cm, QR = 4 cm, RS = 2.5 cm, ∠PQR = 45°, and ∠QRS = 60°.

Steps:

1. Draw QR = 4 cm.
2. At point Q construct ∠AQR = 45° and at point R construct ∠QRB = 60°.
3. With R as centre and radius 2.5 cm, draw arc *r* that intersects ray RB at point S.
4. With Q as centre and radius PQ = 2.6 cm, draw arc *q* that intersects ray QA at point P.
5. Connect points P and S.
6. We have quadrilateral PQRS (Figure 29.3), where PQ = 2.6 cm, QR = 4 cm, RS = 2.5 cm, ∠PQR = 45°, and ∠QRS = 60°.

SKETCH



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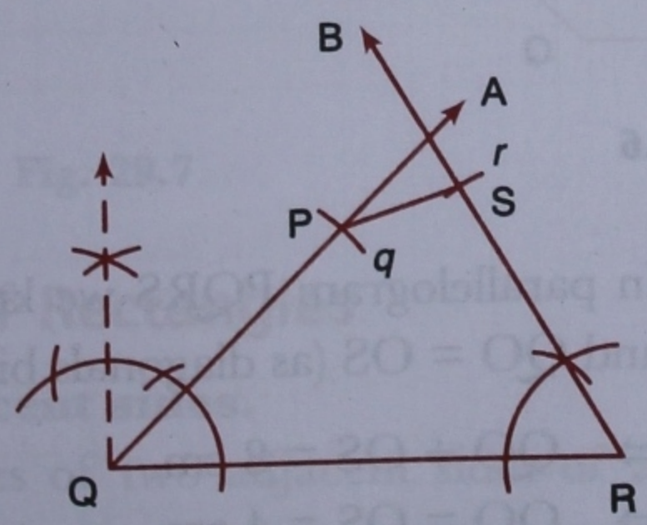


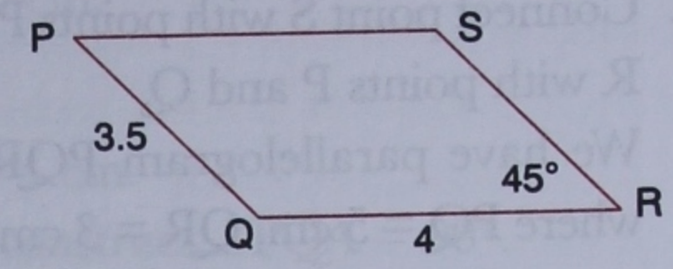
Fig. 29.3

Construction of Parallelograms

I. Given 2 consecutive sides and 1 angle.

Construct parallelogram PQRS, given PQ = 3.5 cm, QR = 4 cm, and ∠QRS = 45°.

SKETCH



From the above sketch we can see that the construction would be easier if the included angle is found.

$$\angle PQR = 180° - 45° = 135°$$

Steps:

1. Draw QR = 4 cm.
2. At point Q construct ∠AQR = 135°.
3. With Q as centre and radius 3.5 cm, draw arc *q* that intersects ray QA at point P.
4. With R as centre and radius 3.5 cm, draw arc *r*.
5. With P as centre and radius 4 cm, draw arc *p* that intersects arc *r* at point S.
6. Connect point S with points P and R.
7. We have parallelogram PQRS (Figure 29.4), where PQ = 3.5 cm, QR = 4 cm, and ∠QRS = 45°.

CONSTRUCTION

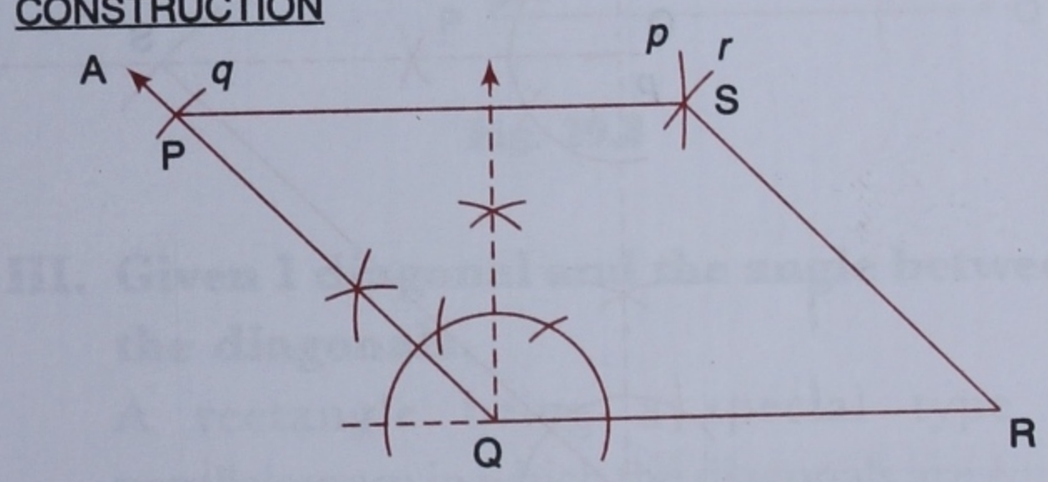


Fig. 29.4

II. Given 2 adjacent sides and a diagonal.

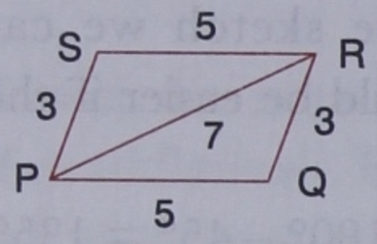
Construct parallelogram PQRS, given PQ = 5 cm, QR = 3 cm, and PR = 7 cm.

Steps:

1. Draw PQ = 5 cm.
2. With P as centre, measure off diagonal PR = 7 cm with arc *p*.

3. With Q as centre and radius 3 cm, draw arc q that intersects arc p at point R.
4. With P as centre and radius 3 cm, draw arc p_1.
5. With R as centre and radius 5 cm, draw arc r that intersects arc p_1 at point S.
6. Connect point S with points P and R and point R with points P and Q.
7. We have parallelogram PQRS (Figure 29.5), where PQ = 5 cm, QR = 3 cm, and PR = 7 cm.

SKETCH



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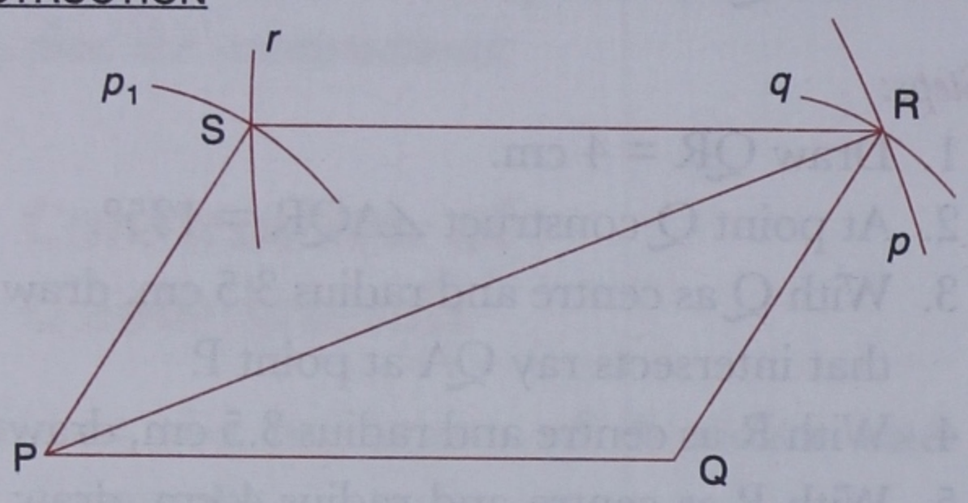
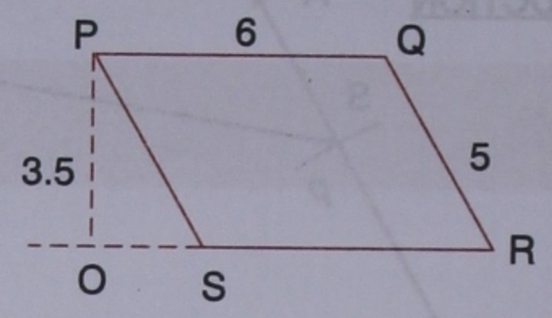


Fig. 30.5

III. Given 2 adjacent sides and the distances between the longer sides.

Construct parallelogram PQRS, given PQ = 6 cm, QR = 5 cm, and altitude PO = 3.5 cm.

SKETCH



Steps:

1. Draw PQ = 6 cm.
2. At point P construct ∠XPQ = 90°.
3. With P as centre, measure off altitude PO = 3.5 cm with arc p.
4. At point O construct ∠YOP = 90°.
5. With P as centre and radius 5 cm, draw arc p_1 that intersects ray OY at point S.
6. With S as centre and radius 6 cm, draw arc s that intersects ray OY at point R.
7. Connect point S with point P and point R with point Q.
8. We have parallelogram PQRS (Figure 29.6), where PQ = 6 cm, QR = 5 cm, and altitude PO = 3.5 cm.

CONSTRUCTION

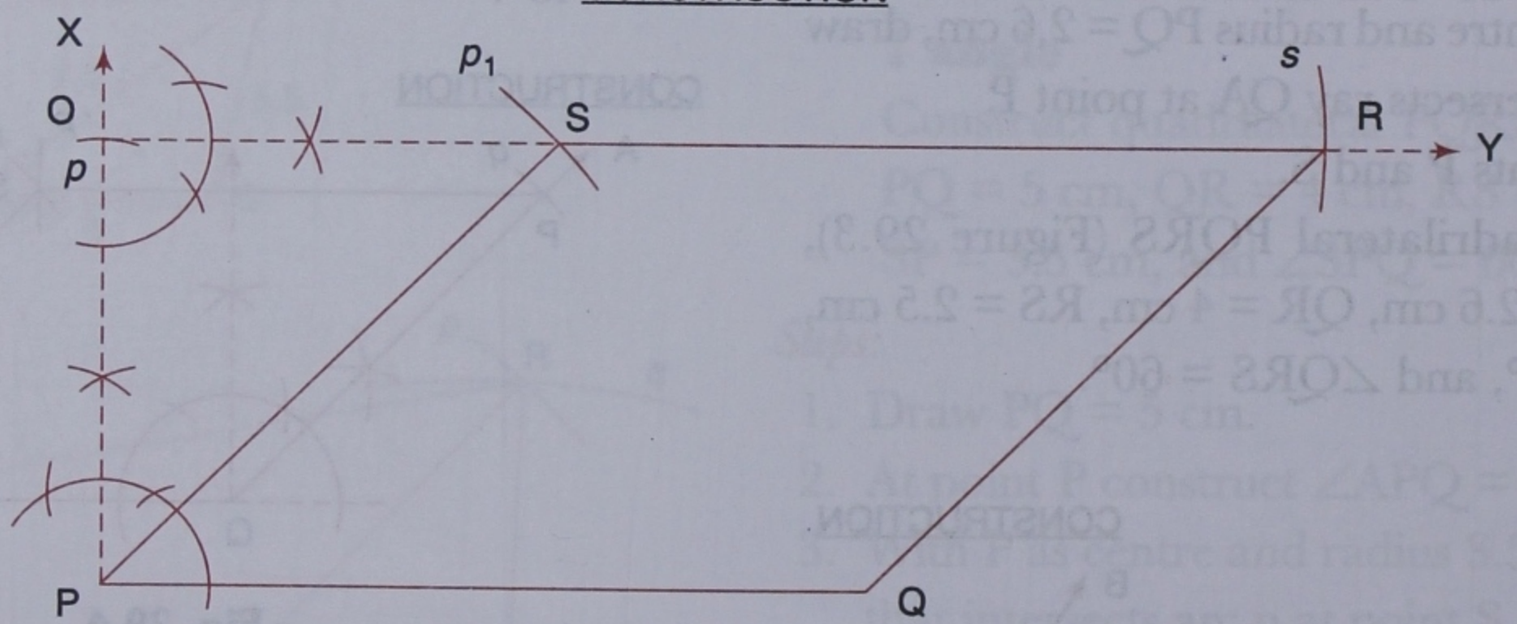


Fig. 29.6

IV. Given 2 diagonals and angle between the two diagonals.

Construct parallelogram PQRS, given PR = 5 cm, SQ = 8 cm, and ∠POQ = 60°, where O is the point of intersection of the two diagonals.

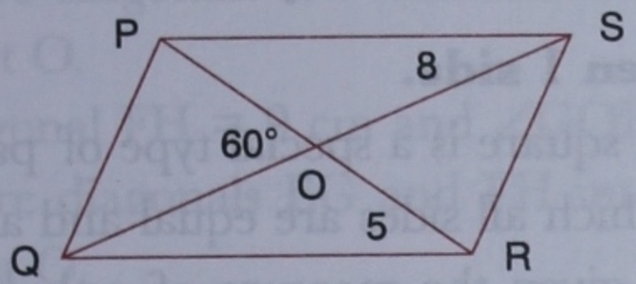
In parallelogram PQRS we know that PO = OR and QO = OS (as diagonals bisect each other).

⇒ QO + OS = 8 cm
⇒ QO = OS = 4 cm

Steps:

1. Draw PR = 5 cm.
2. Draw perpendicular bisector AB to intersect PR at point O.
3. At point O construct ∠COR = 60° and extend ray OC past O to form line DC.
4. With O as centre, measure off OS = 4 cm and OQ = 4 cm on the opposite side, with arcs o_1 and o_2 to intersect line DC at points S and Q respectively.
5. Connect point S with points P and R and point Q with points P and R.
6. We have parallelogram PQRS (Figure 29.7), where PR = 5 cm, SQ = 8 cm, and ∠POQ = 60°.

SKETCH



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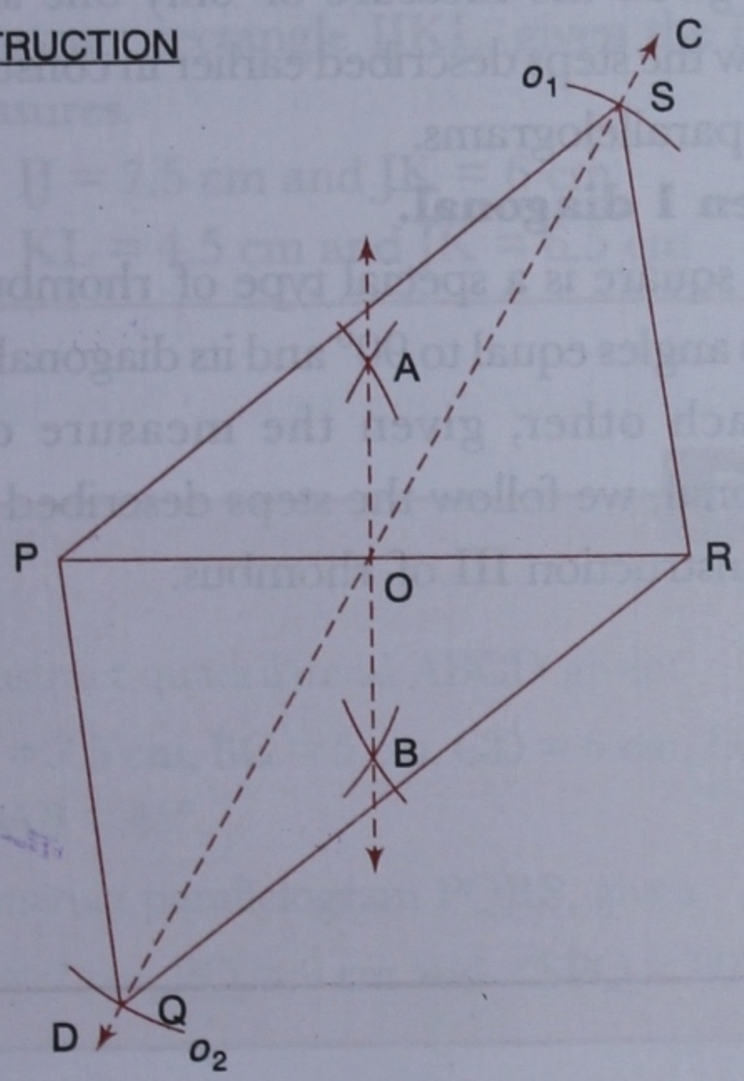


Fig. 29.7

Construction of Rectangles

I. Given 2 adjacent sides.

If the measures of two adjacent sides of a rectangles are given, we know that the angle included between them is 90°. Thus, we follow the steps described earlier in construction I of parallelograms.

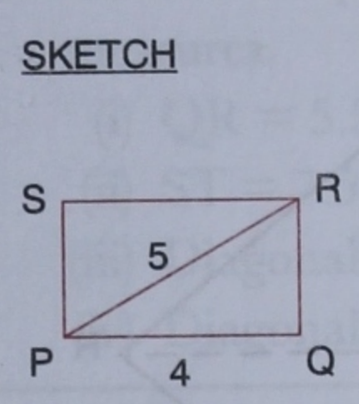
II. Given 1 side and 1 diagonal.

Construct rectangle PQRS, given PQ = 4 cm and PR = 5 cm.

Steps:

1. Draw PQ = 4 cm.
2. At point Q construct ∠PQA = 90°.
3. With P as centre, measure off diagonal PR = 5 cm with arc p that intersects ray QA at point R.
4. With R as centre and radius 4 cm, draw arc r.
5. With P as centre, and a radius equal to line segment QR, measure off SP with arc p_1 that intersects arc r at point S.
6. Connect point S with points P and R.
7. We have rectangle PQRS (Figure 29.8), where PQ = 4 cm and diagonal PR = 5 cm.

SKETCH



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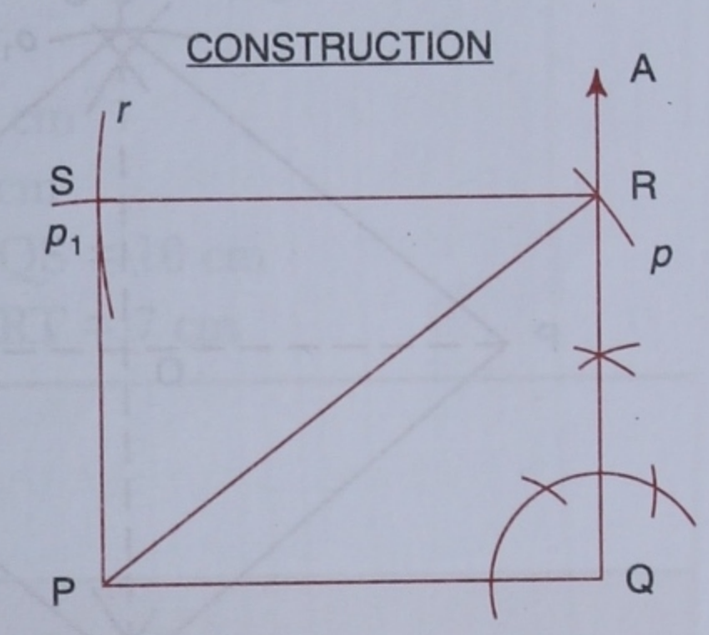


Fig. 29.8

III. Given 1 diagonal and the angle between the diagonals.

A rectangle being a special type of parallelogram in which the diagonals are equal in measure, given the measure of 1 diagonal and the angle between the diagonals, we follow the steps described earlier in construction IV of parallelograms.

Construction of Rhombuses

I. Given 1 side and 1 angle.

As a rhombus is a parallelogram in which all sides are equal to each other, given the measure

of one side, its adjacent side will also measure the same. Constructing the given angle included between them we follow the steps described earlier in construction I of parallelograms.

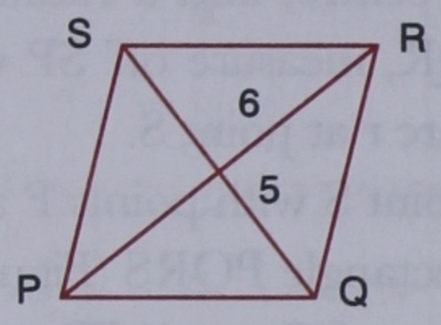
II. Given 1 side and 1 diagonal.

As the adjacent sides of a rhombus are equal in measure, we follow the steps as described earlier in construction II of parallelograms.

III. Given 2 diagonals.

Construct rhombus PQRS, given PR = 6 cm and QS = 5 cm.

SKETCH



CONSTRUCTION

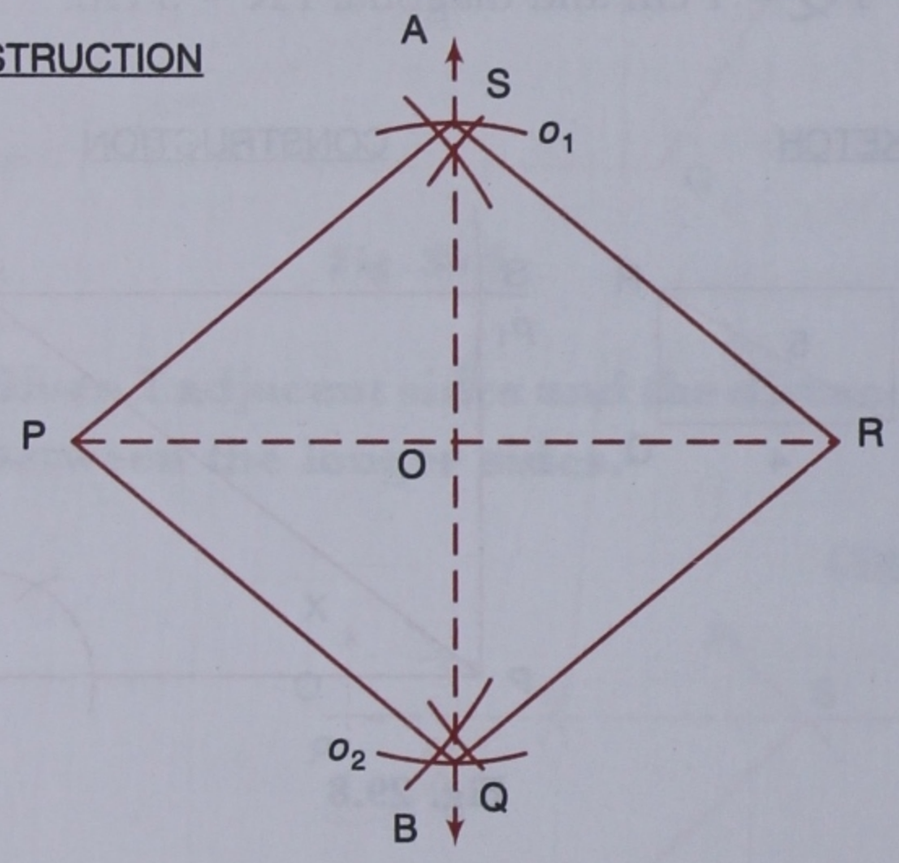


Fig. 29.9

Steps:

1. Draw PR = 6 cm.
2. Draw perpendicular bisector AB that intersects PR at point O.
3. With O as centre, measure off OS = $\frac{5}{2}$ = 2.5 cm with arc o_1 that intersects AB at point S and OQ = $\frac{5}{2}$ = 2.5 cm with arc o_2 that intersects AB at point Q.
4. Connect point S with points P and R and point Q with points P and R.
5. We have rhombus PQRS (Figure 29.9), where PR = 6 cm and QS = 5 cm.

Construction of Squares

I. Given 1 side.

As a square is a special type of parallelogram in which all sides are equal and all angles are 90°, given the measure of only one side, we follow the steps described earlier in construction I of parallelograms.

II. Given 1 diagonal.

As a square is a special type of rhombus with all its angles equal to 90° and its diagonals equal to each other, given the measure of one diagonal, we follow the steps described earlier in construction III of rhombus.

Exercise 29.1

1. Construct quadrilateral ABCD, given the following measures.
   (i) AB = 8 cm, BC = 10 cm, CD = 7 cm, DA = 7 cm, and diagonal AC = 12 cm
   (ii) AB = 6 cm, BC = 5 cm, CD = 7 cm, DA = 6 cm, and diagonal BD = 9 cm
   (iii) AB = 7 cm, BC = 3 cm, CD = 6 cm, DA = 5 cm, and ∠ABC = 120°
   (iv) AB = 6.5 cm, BC = 4 cm, CD = 5 cm, DA = 4 cm, and ∠DAB = 60°
   (v) AB = 9 cm, BC = 6 cm, CD = 8 cm, ∠ABC = 90°, and ∠BCD = 60°

(vi) BC = 5 cm, CD = 8 cm, DA = 8 cm, ∠BCD = 75°, and ∠CDA = 60°

2. Construct parallelogram EFGH, given the following measures.
   (i) HE = 5 cm, EF = 7 cm, and ∠HEF = 45°
   (ii) FG = 8 cm, GH = 5 cm, and ∠FGH = 120°
   (iii) GH = 7 cm, FG = 4 cm, and diagonal EG = 9 cm
   (iv) HE = 5 cm, EF = 6 cm, and diagonal EG = 7.5 cm
   (v) EF = 6.5 cm, FG = 4.5 cm, and altitude GO = 3.5 cm
   (vi) EF = 5.5 cm, FG = 4.5 cm, and altitude HO = 3 cm
   (vii) Diagonal EG = 10 cm and ∠GOF = 75° where diagonals EG and FH intersect at point O.
   (viii) Diagonal FH = 9 cm and ∠GOH = 120° where diagonals EG and FH intersect at point O.

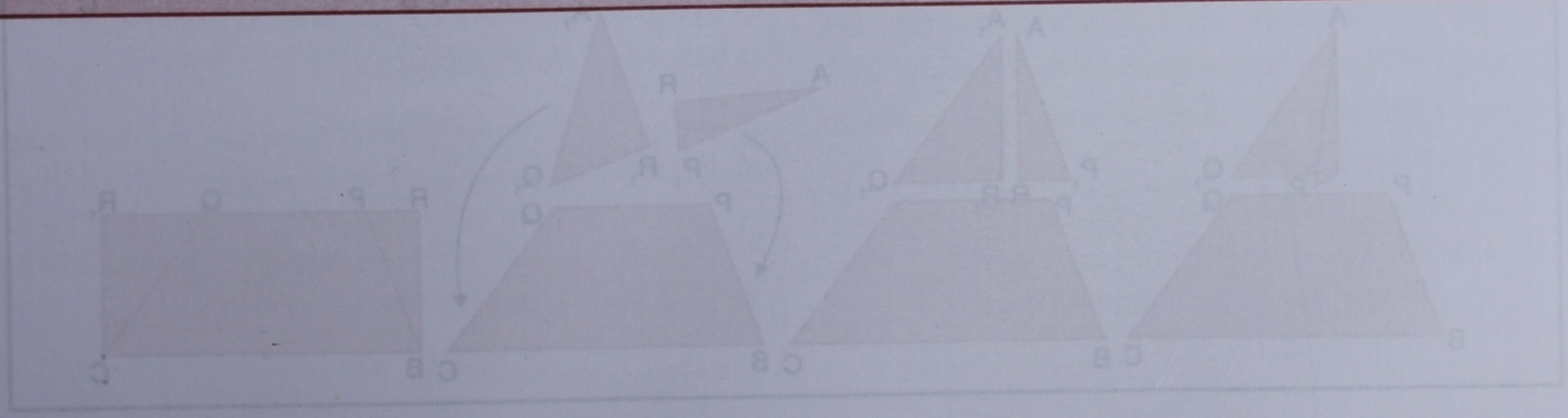
3. Construct rectangle IJKL, given the following measures.
   (i) IJ = 7.5 cm and JK = 6 cm
   (ii) KL = 4.5 cm and JK = 6.5 cm
   (iii) IJ = 9.5 cm and diagonal IK = 11.5 cm
   (iv) JK = 3.5 cm and diagonal JL = 6 cm
   (v) Diagonal IK = 13 cm and ∠KOL = 120° where diagonals IK and JL intersect at point O.
   (vi) Diagonal JL = 7 cm and ∠JOK = 30° where diagonals IK and JL intersect at point O.

4. Construct rhombus MNOP, given the following measures.
   (i) MN = 8 cm and ∠PMN = 60°
   (ii) PM = 5.5 cm and ∠MNO = 45°
   (iii) MN = 7 cm and diagonal MO = 9 cm
   (iv) NO = 6.5 cm and diagonal NP = 9.5 cm
   (v) Diagonal MO = 8 cm and diagonal NP = 6 cm
   (vi) Diagonal MO = 7.5 cm and diagonal NP = 9.5 cm

5. Construct square QRST, given the following measures.
   (i) QR = 5.5 cm
   (ii) ST = 7.5 cm
   (iii) Diagonal QS = 10 cm
   (iv) Diagonal RT = 7 cm

Revision Exercise

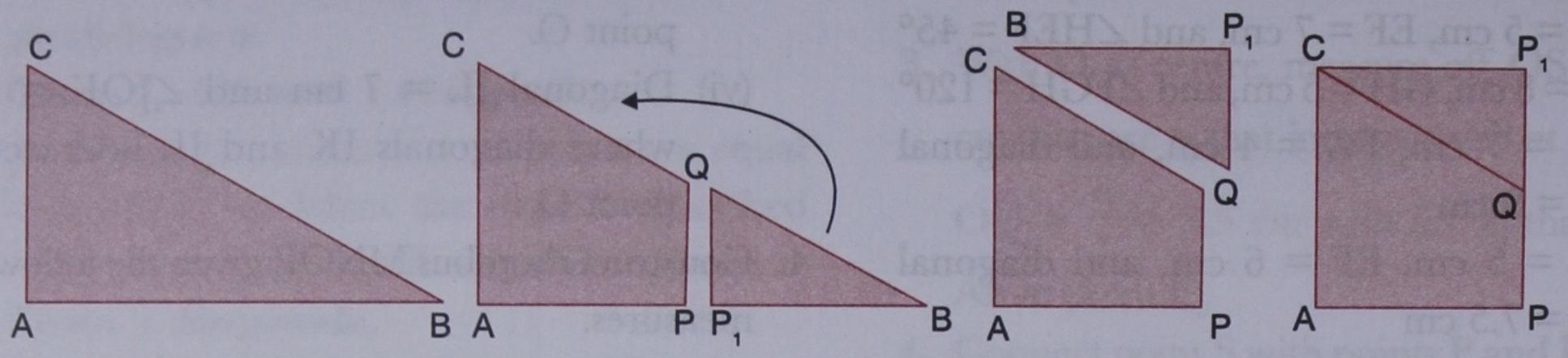
1. Construct quadrilateral ABCD given:
   AB = 7.5 cm, BC = 5 cm, CD = 6 cm, DA = 5 cm, ∠DAB = 45°
2. Construct parallelogram PQRS, given:
   PS = 7 cm, PQ = 9 cm and ∠SPQ = 60°.
3. Construct rectangle DEFG, given:
   DE = 8.5 cm and EF = 7 cm.
4. Construct rhombus ABCD, given:
   AB = 6 cm and ∠DAB = 30°.
5. Construct square KLMN, given:
   Diagonal 12 cm.



Show and prove that any triangle can be converted into a rectangle.

Let us demonstrate this using just a piece of paper

I Reforming a right angled triangle



(i) Fold a paper along its opposite vertices and tear into two to get a right angled triangle.

(ii) Fold and crease the triangle along its base such that vertex B touches vertex A. Now tear the triangle along the crease that bisects base AB.

(iii) We have a new triangle P_1BQ and trapezium APQC. Move triangle P_1BQ such that point B matches point C

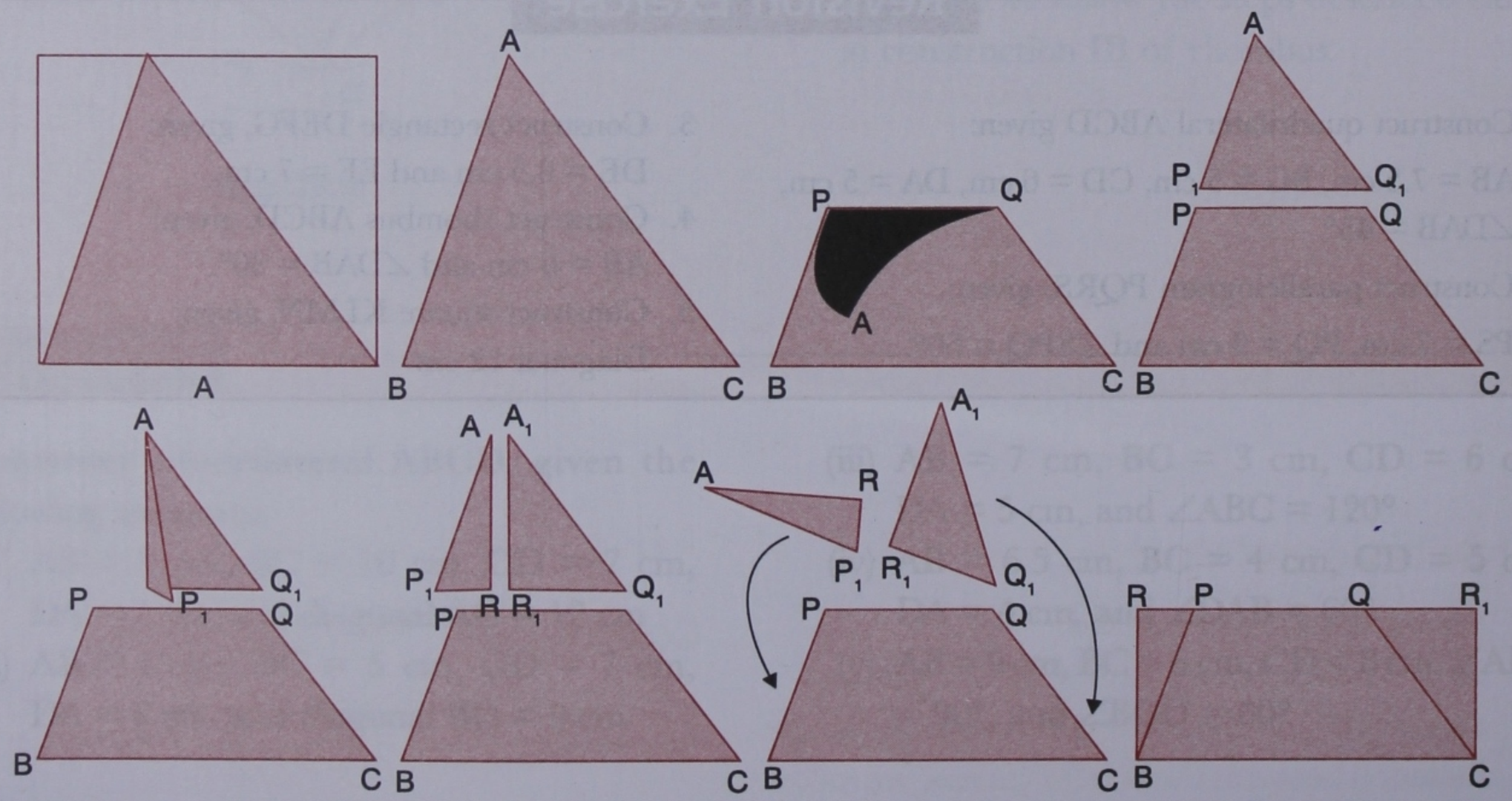
(iv) We have rectangle APP_1C.

Proof

- In quadrilateral APP_1C, $\angle CAP = 90°$ (one corner of rectangular paper), $\angle APQ = \angle QP_1C = 90°$ (paper creased and torn along perpendicular to base).
- Thus, $\angle P_1CA = 360° - (90° \times 3) = 90°$ or all angles of the quadrilateral APP_1C are right angles.
- As the base was torn in half, $AP = P_1C$ or the opposite sides are equal to each other.
- Hence quadrilateral APP_1C is a rectangle.

II Reforming a scalene triangle

- Cut a paper in the shape of a scalene triangle.
- Fold and crease the triangle along sides AB and AC to mark their mid-points. Now tear the triangle along the crease that joins mid-points P and Q.



- We have a new triangle AP_1Q_1 and a trapezium BCQP.
- Fold and crease triangle AP_1Q_1 and tear along perpendicular AR on side P_1Q_1.
- We have two new triangles AP_1R and $A_1R_1Q_1$.
- Rotate both triangles by 180°. Move triangle AP_1R such that point A matches point B and move triangle $A_1R_1Q_1$ such that point A1 matches point C.
- We have rectangle BCR_1R.

Proof

- PQ connects the mid-points of sides AB and AC. Thus PQ || BC.
- AR is perpendicular to PQ, thus || PRB = $\angle QR_1C = 90°$.
- AS PQ || BC and RB and RC are transversals $\angle RBC = 180° - \angle PRB = 90°$. Similarly $\angle BCR = 180° - \angle QR1C = 90°$. Thus, all angles of the quadrilateral BCR_1R are right angles.
- $BR = CR_1$ = (form the same side AR)
- Hence quadrilateral BCR_1R is a rectangle.