UNIT
FIVE

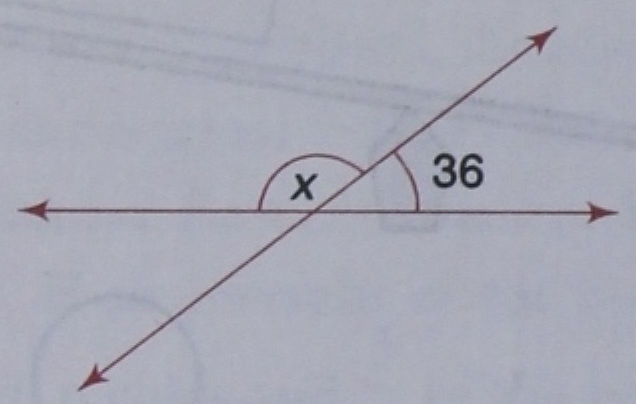
- Points and Planes, Lines and Angles
- Triangles
- Construction of Triangles
- Quadrilaterals
- Construction of Quadrilaterals
- Polygons
- Symmetry, Reflection and Rotation

Geometry

Let's Recap

1. Find the complement of $\angle A = 57°$.
2. Find the supplement of $\angle B = 87°$.
3. Find the measure of reflex $\angle ABC$, given $\angle CBA = 93°$.
4. Is ΔABC with sides AB = 9.33 cm, BC = 2.66 cm and CA = 6.66 cm possible?
5. Is ΔABC with $\angle ABC = x + 30°$, $\angle BCA = 2x + 60°$ and $\angle CAB = 3x + 90°$ possible?
6. Find the value of x in the following figures:

(i)



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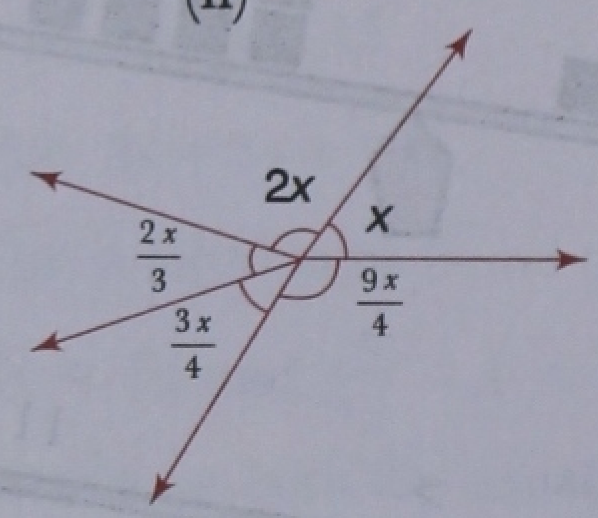
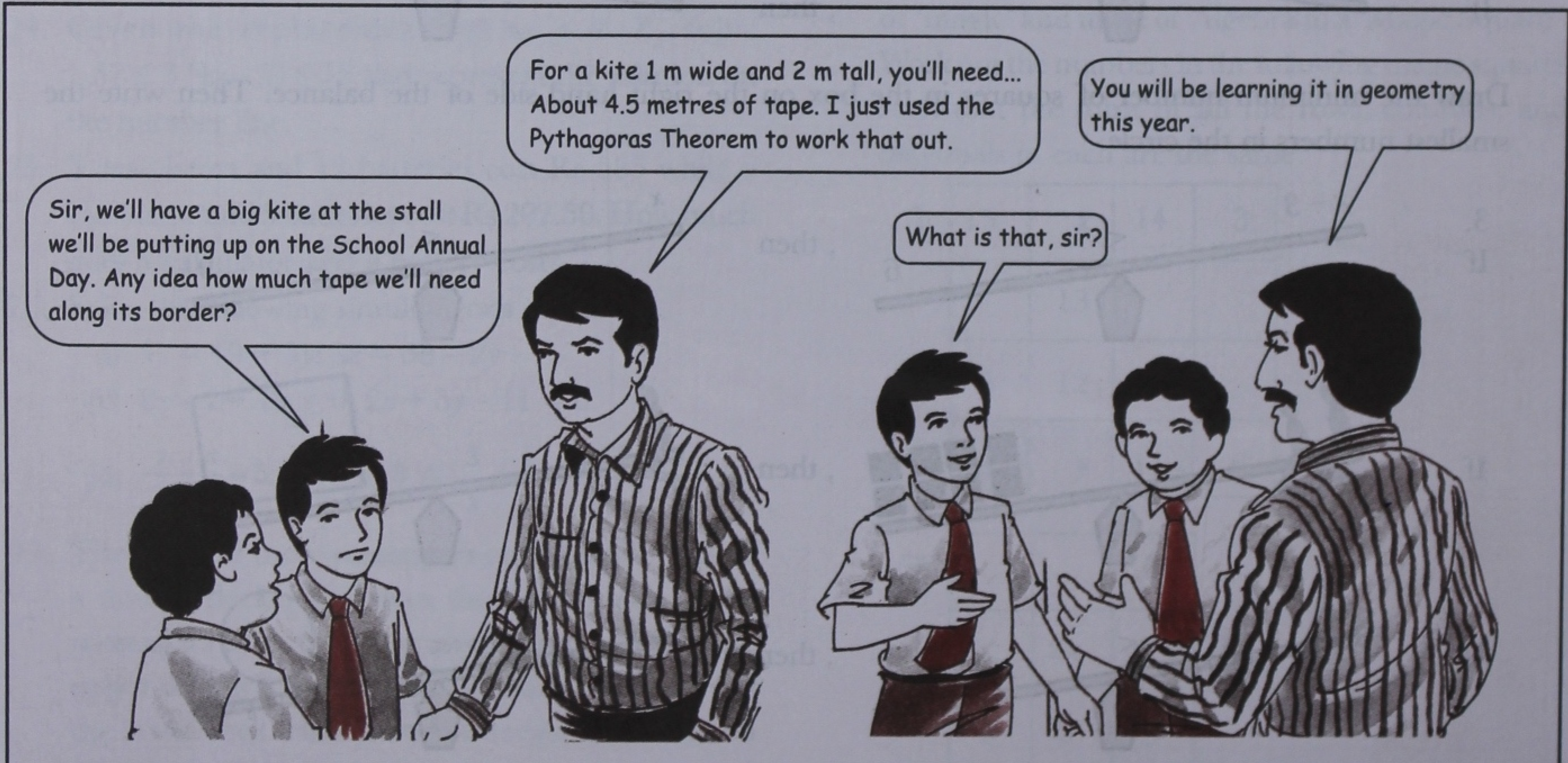




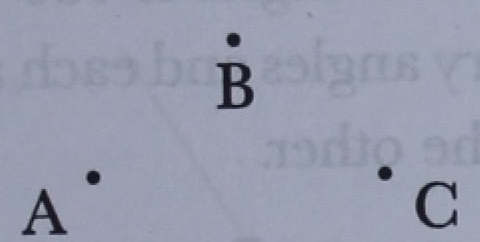
25 POINTS AND PLANES, LINES AND ANGLES

- Points and Planes
- Complementary Angles
- Supplementary Angles
- Adjacent Angles
- Linear Pair
- Vertically Opposite Angles
- Perpendicular Lines
- Parallel Lines

Points and Planes

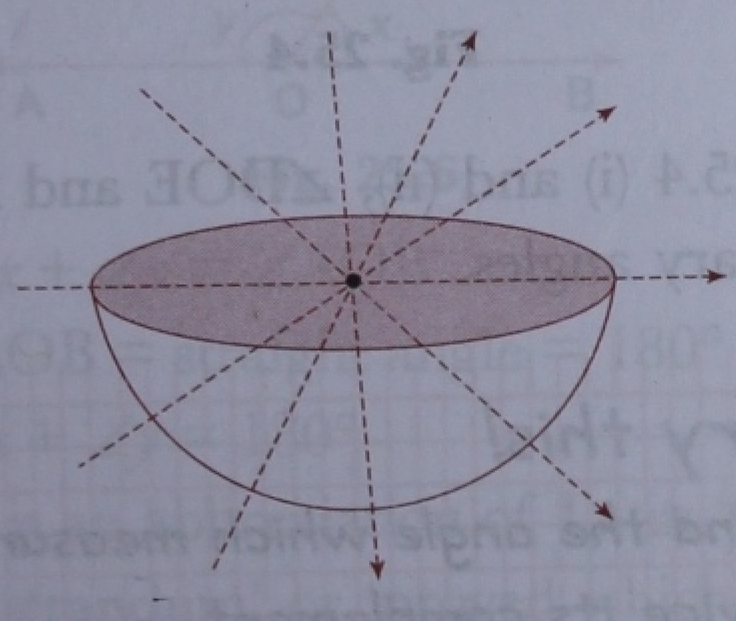
Geometry is the study of position, size, and shape of points, planes, lines, and angles.

A point is a small mark that has position, but no magnitude. It has neither shape nor size. On paper, a point is represented by a small dot and denoted by a capital letter from the English alphabet, like point 'A', point 'B', and point 'C' shown here:



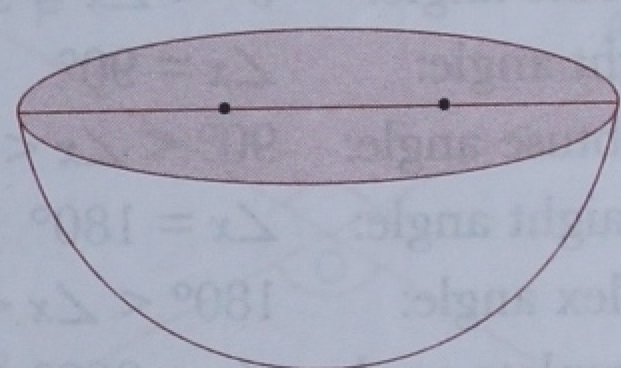
A flat surface that extends indefinitely in all directions is known as a plane.

Now, to relate a point to a plane, think of a single seed inside a big watermelon as a point. In how many ways can a knife cut through the watermelon so as to cut through that seed? Yes, there can be an infinite number of ways.



This is because a single point can lie on an infinite number of planes. Similarly, two points can lie on an infinite number of planes. But three points can lie on one plane only.

The line segment that joins two points on a plane is the shortest distance between them. If the line segment is extended in both directions endlessly, we realize that two points on a plane can be connected by one and only one line passing through them.



In geometry the word **space** is used to describe all points, lines, and planes that lie in the entire universe.

Remember
- A single point can lie on an infinite number of planes.
- Two points can lie on an infinite number of planes.
- There or more points can lie on only one plane.

Angles

Two lines on the same plane may intersect or be parallel to each other. *An angle is formed when two lines intersect.* In Figure 25.1, lines ℓ and m intersect at point O to form ∠AOC. AO and OC are the **arms** of the angle while point O is known as its **vertex**. The distance arm OC will have to cover, on being rotated, to be in line with arm OA is known as the magnitude of ∠AOC. The magnitude of an angle is measured in degrees. A full sweep of one arm of an angle measures 360° or a complete angle.

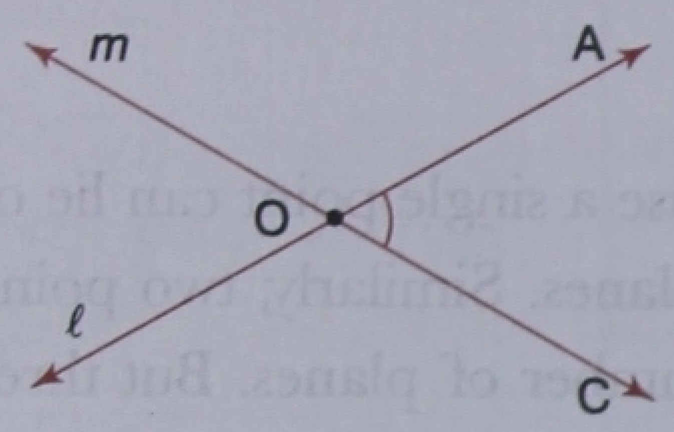


Fig. 25.1

Magnitude and Types of Angles

Remember

For any angle x, magnitude of:

- a zero angle: $\angle x = 0°$
- an acute angle: $0° < \angle x < 90°$
- a right angle: $\angle x = 90°$
- an obtuse angle: $90° < \angle x < 180°$
- a straight angle: $\angle x = 180°$
- a reflex angle: $180° < \angle x < 360°$
- a complete angle: $\angle x = 360°$

The sum of angles about a point is thus 360°.

In Figure 25.2, angles 1, 2, 3, 4, and 5 are all angles about the same vertex O.

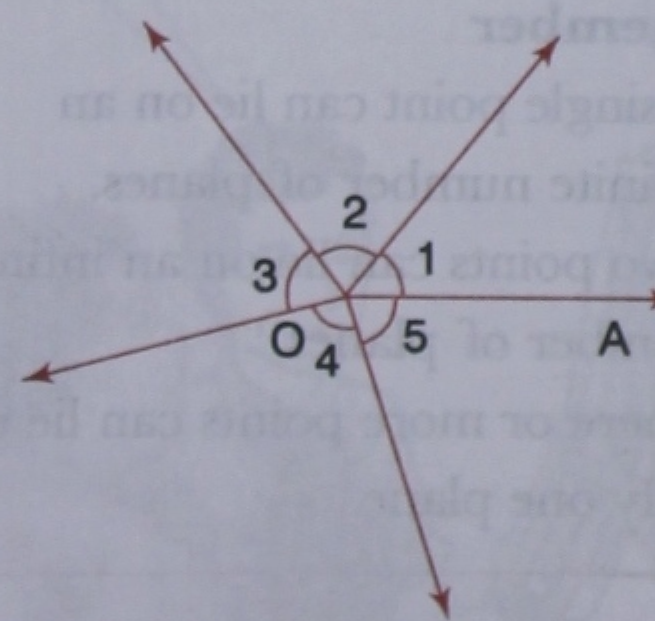


Fig. 25.2

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = \angle AOA$

= complete angle

As the measure of a complete angle = 360°,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$

Special Pairs of Angles

Complementary Angles

When the sum of two angles is 90°, they are said to be complementary angles and each angle is known as a *complement* of the other.

(i) (ii)

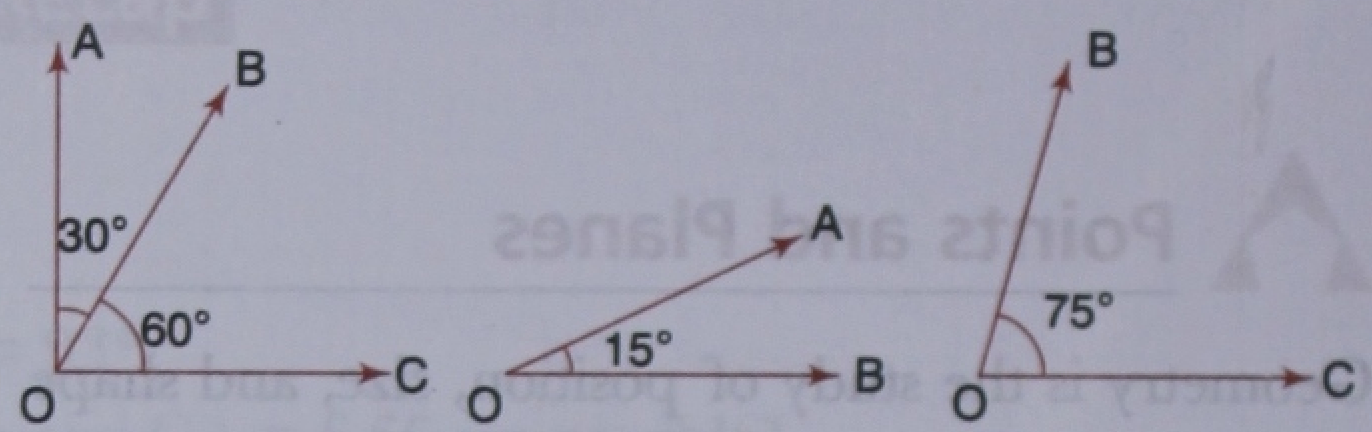


Fig. 25.3

In Figure 25.3 (i) and (ii), ∠AOB and ∠BOC are complementary angles.

Supplementary Angles

When the sum of two angles is 180°, they are said to be supplementary angles and each angle is known as a *supplement* of the other.

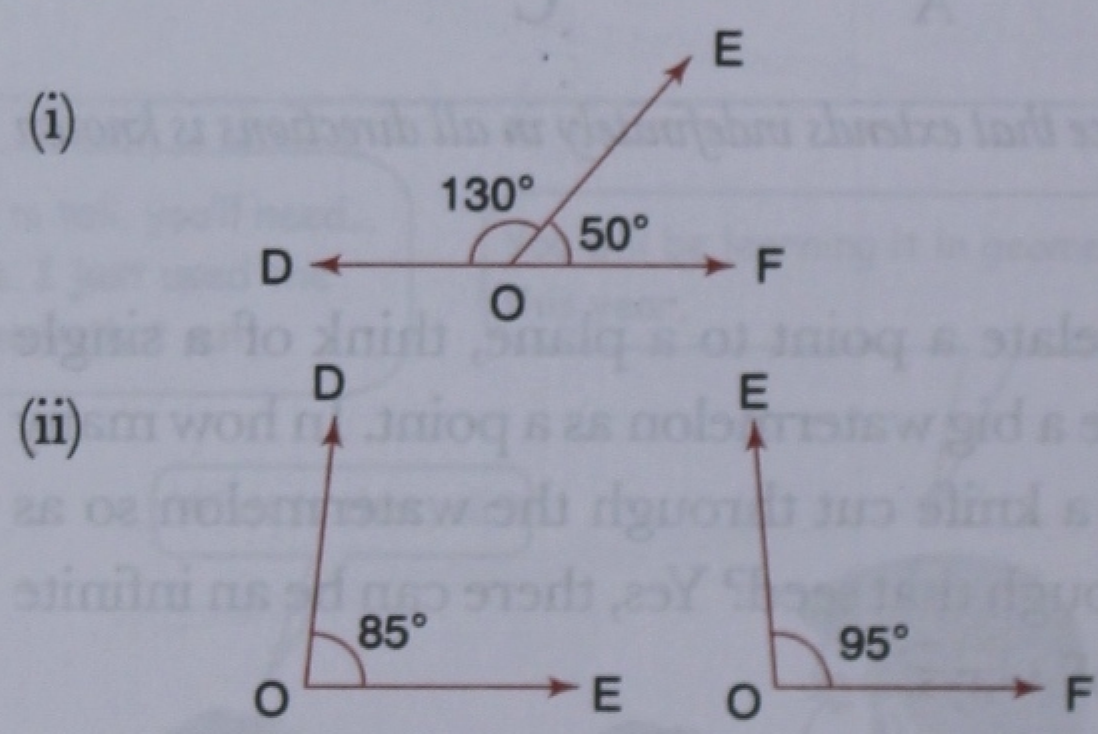


Fig. 25.4

In Figure 25.4 (i) and (ii), ∠DOE and ∠EOF are supplementary angles.

Try this!

Find the angle which measures twice its complement.

Adjacent Angles

When two angles lie on opposite sides of a common arm, they are known to be *adjacent* to each other. In Figure 25.5 (i) and (ii), ∠1 and ∠2 are adjacent angles about the common arm OA.

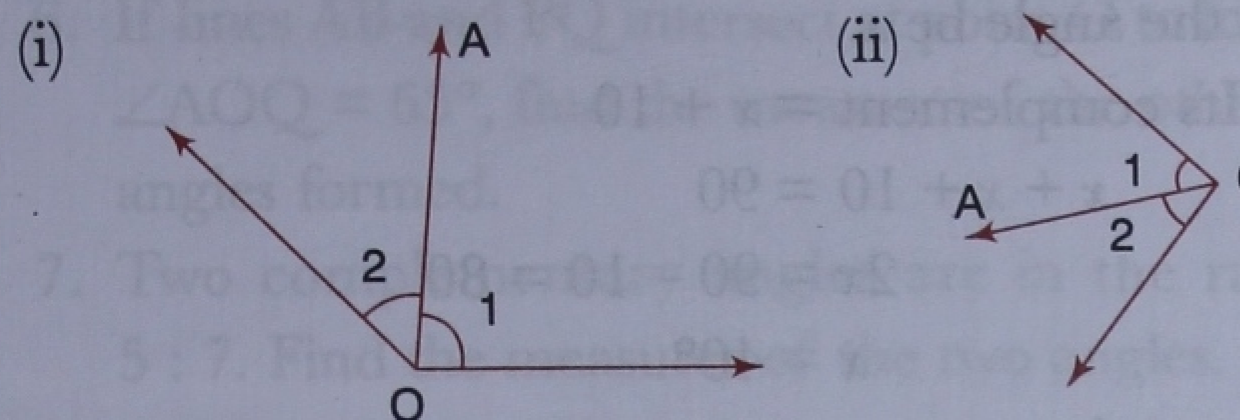
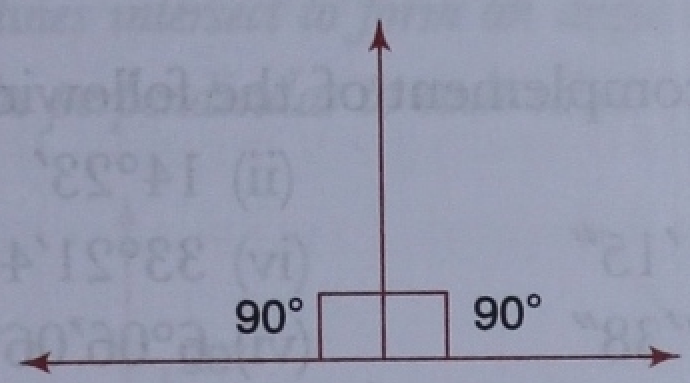


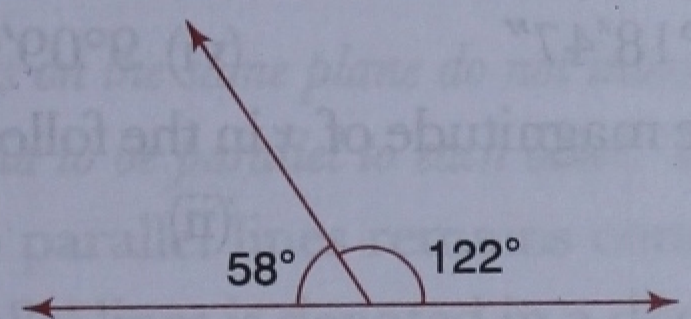
Fig. 25.5

Linear Pair

Two adjacent supplementary angles form a *linear pair*.



1. *To prove:* If two lines intersect, the adjacent angles are supplementary.



Proof: In Figure 25.6, lines ℓ and *m* intersect at point O to form ∠*x* and ∠*y* adjacent to each other.

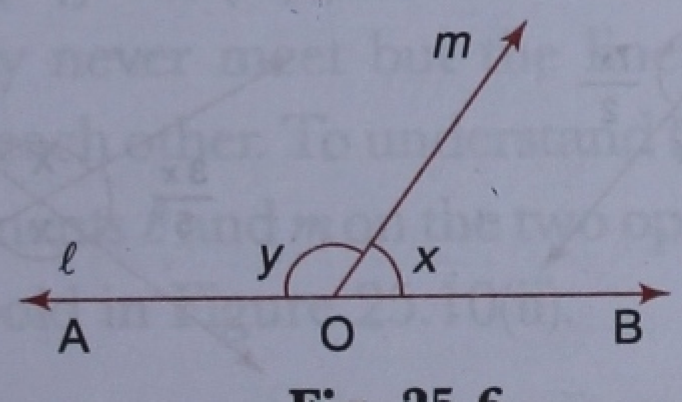


Fig. 25.6

Now ∠*x* + ∠*y* = ∠AOB

But ∠AOB = straight angle = 180°

⇒ ∠*x* + ∠*y* = 180° Q.E.D

(Q.E.D is an abbreviation of Latin 'Quod Erat Demonstrandum' or 'proved which was to be proved'.)

2. *To prove:* If two angles having a common arm are supplementary, the other two arms lie in a straight line.

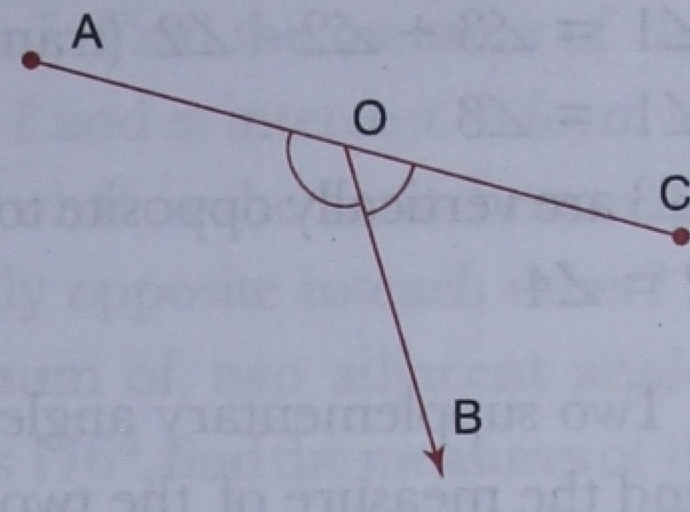


Fig. 25.7

Proof: In Figure 25.7, ∠AOB + ∠BOC = 180°

⇒ ∠AOC = 180°

⇒ ∠AOC = Straight angle

⇒ AC is a line segment Q.E.D

Vertically Opposite Angles

When two lines intersect, the two pairs of angles with no common arm are known to be vertically opposite to each other.

In Figure 25.8, lines AB and PQ intersect at point O to form four angles. Here ∠AOQ and ∠POB are vertically opposite to each other. ∠AOP and ∠BOQ are also vertically opposite angles.

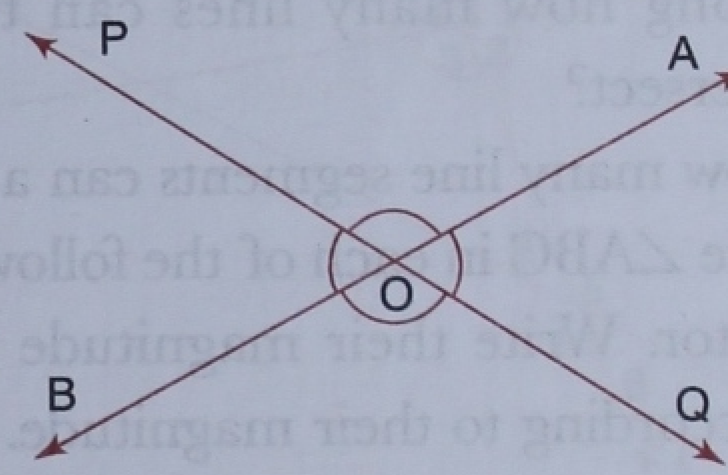


Fig. 25.8

To prove: If two lines intersect, the vertically opposite angles are equal.

Proof: In Figure 25.9, lines *l* and *m* intersect to form ∠1, ∠2, ∠3, and ∠4.

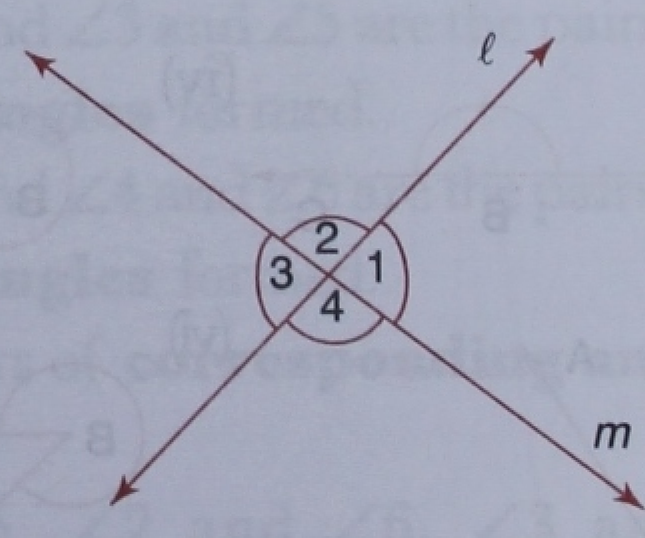


Fig. 25.9

$\angle 1 + \angle 2 = 180°$ (being a linear pair)

$\angle 2 + \angle 3 = 180°$ (also a linear pair)

$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3 + \angle 2 - \angle 2$ (transposing $\angle 2$)

$\Rightarrow \angle 1 = \angle 3$

But $\angle 1$ and $\angle 3$ are vertically opposite to each other.

Similarly, $\angle 2 = \angle 4$ Q.E.D

Example 1: Two supplementary angles are in the ratio 2 : 7. Find the measure of the two angles.

Let the angles be $2x$ and $7x$.

$2x + 7x = 180°$

$9x = 180°$

$\Rightarrow x = 20$ $\therefore 2x = 40$

and $7x = 140$

$\therefore$ The angles are 40° and 140°.

Example 2: The complement of an angle is 10° more than the angle. Find the angles.

Let the angle be x.

$\therefore$ Its complement $= x + 10$

$x + x + 10 = 90$

$2x = 90 - 10 = 80$

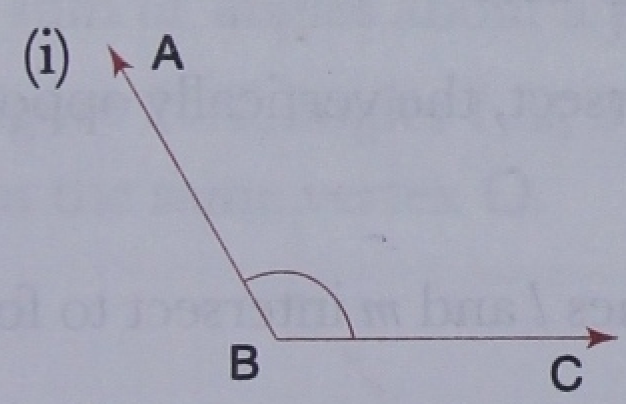
$x = 40°$

Its complementary angle = 50°.

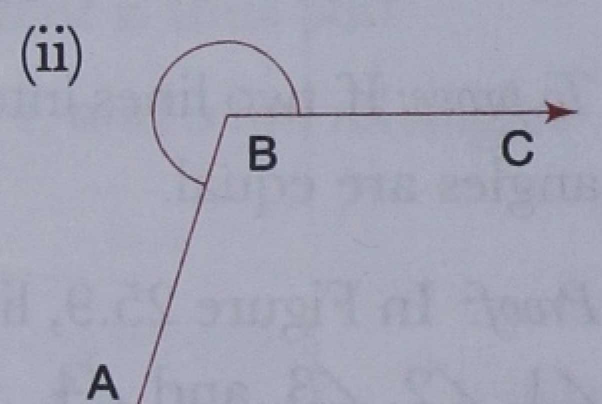
Exercise 25.1

1. (i) If an infinite number of points lie in the space inside a watermelon, how many points lie in the space inside a lemon?
   (ii) How many straight line segments can join two given points inside a lemon?
   (iii) If 3 points are given, on how many surfaces can all three lie together?
   (iv) Along how many lines can two planes intersect?
   (v) How many line segments can a line have?
2. Measure $\angle ABC$ in each of the following with a protractor. Write their magnitude and name them according to their magnitude.

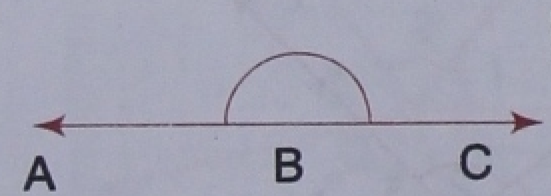
(i)



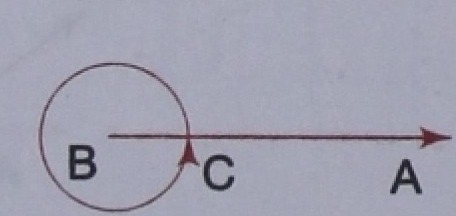
(ii)



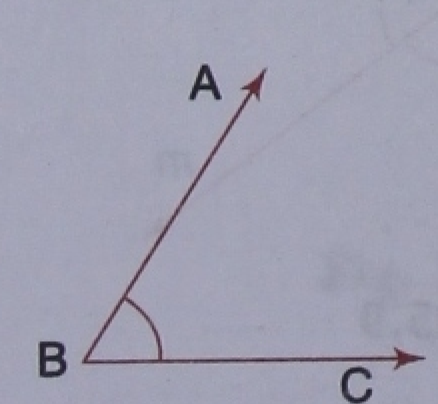
(iii)



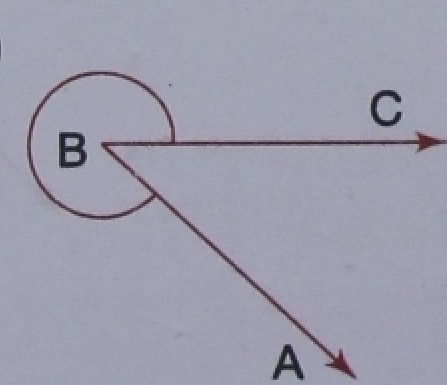
(iv)



(v)

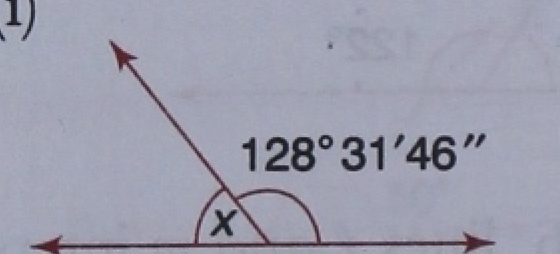


(vi)

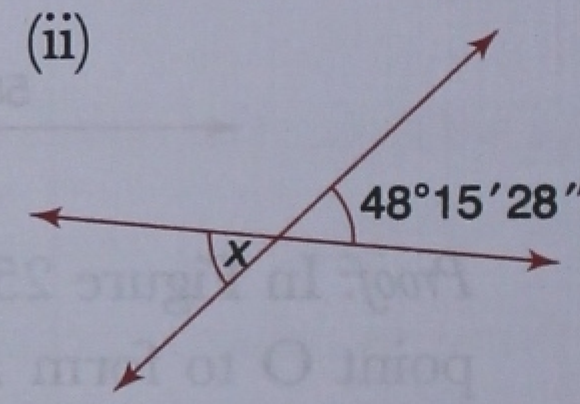


3. Find the complement of the following angles.
   (i) 79° (ii) 14°23′
   (iii) 28°11′15″ (iv) 33°21′46″
   (v) 52°27′38″ (vi) 6°06′06″
4. Find the supplement of the following angles.
   (i) 27° (ii) 35°10′
   (iii) 63°36′40″ (iv) 18°26′53″
   (v) 51°18′47″ (vi) 9°09′09″
5. Find the magnitude of x in the following figures.

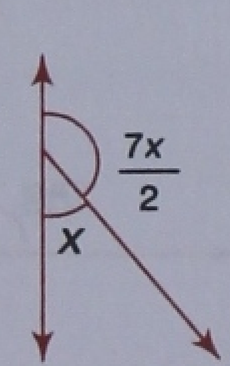
(i)



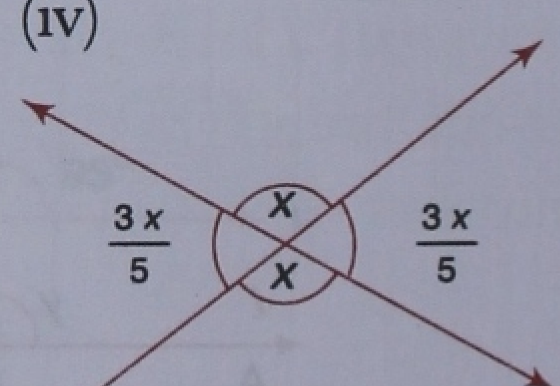
(ii)



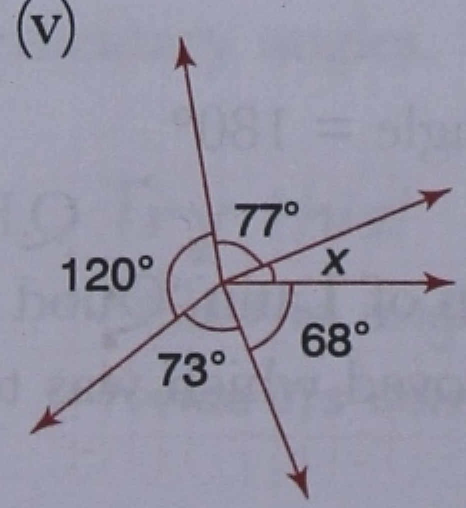
(iii)



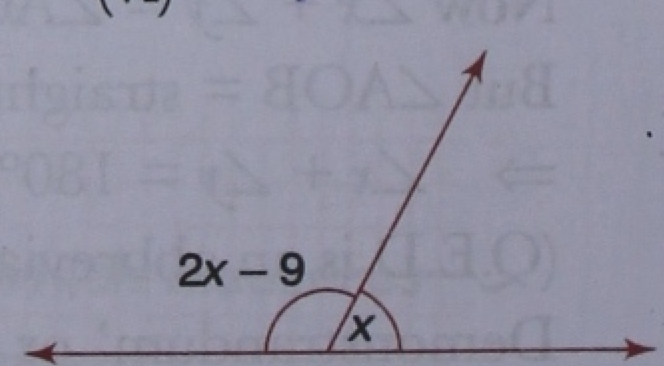
(iv)



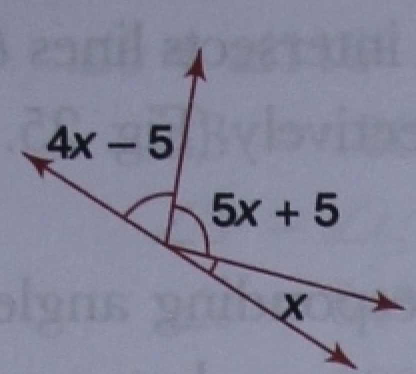
(v)



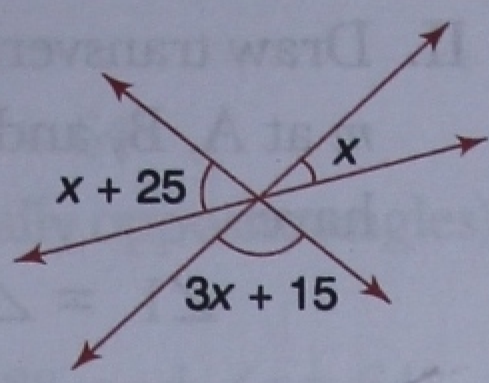
(vi)



(vii)



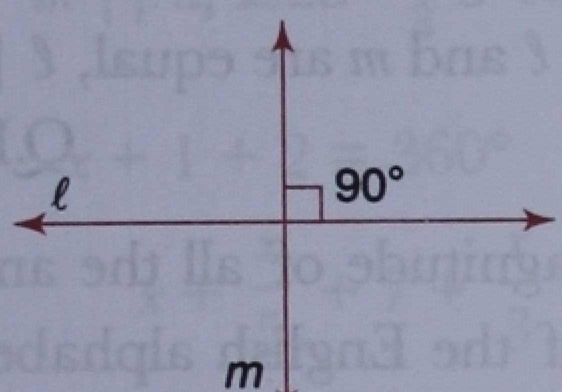
(viii)

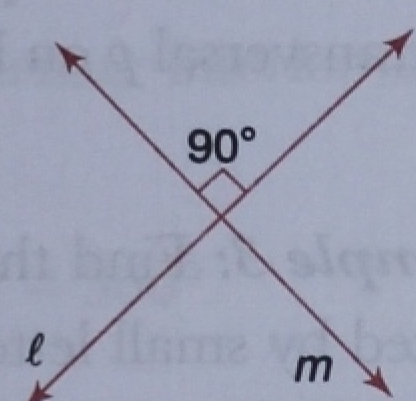


6. If lines AB and PQ intersect at point O to form ∠AOQ = 65°, find the measure of the other 3 angles formed.
7. Two complementary angles are in the ratio 5 : 7. Find the measures of the two angles.
8. Two supplementary angles are in the ratio 1 : 7. Find the measures of the two angles.
9. Acute ∠AOC and its reflex ∠COA are in the ratio 2 : 7. Find the measure of the two angles.
10. If lines ℓ and m intersect to form angles 1, 2, 3, and 4, what is the ratio between ∠2 and ∠4 vertically opposite to each other?
11. If the sum of two adjacent angles $x - 3$ and $2x + 2$ is 176°, find the measures of the two angles.
12. The complement of ∠A is one-third of its supplement. Find ∠A.

Perpendicular and Parallel Lines

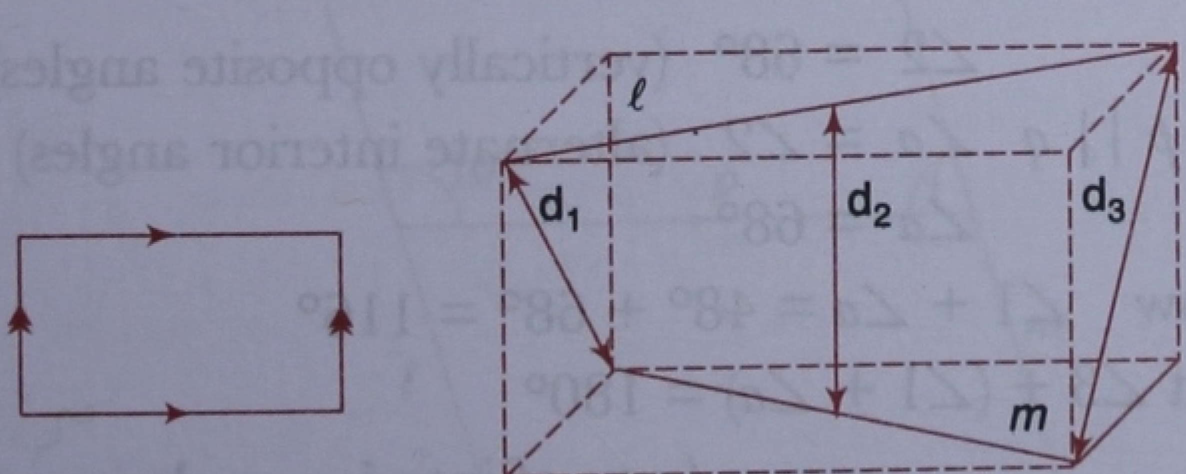
When two lines intersect to form an angle of 90°, the lines are said to be perpendicular to each other.





When two lines on the same plane do not intersect each other, the lines are said to be parallel to each other. The distance between two parallel lines remains constant.

A set of parallel lines is denoted in a diagram with a set of single arrow heads pointing in the same direction. Another set of parallel lines in the same diagram is denoted with a set of double arrow heads and so on (Fig. 25.10 (i)). Two lines on two different planes may never meet but the lines need not be parallel to each other. To understand this better, look at line segments ℓ and m on the two opposite surfaces of the cuboid in Figure 25.10(ii).



(i) (ii)

Fig. 25.10

As the planes are parallel to each other, ℓ and m will never intersect. However, notice that the distance 'd' between the lines changes as $d_1 \neq d_2 \neq d_3$.

Angles Formed by a Transversal

A transversal is a line that intersects a set of two or more lines, all the lines and the transversal being on the same plane.

In Figure 25.11, 8 angles are formed when transversal ℓ intersects lines m and n.

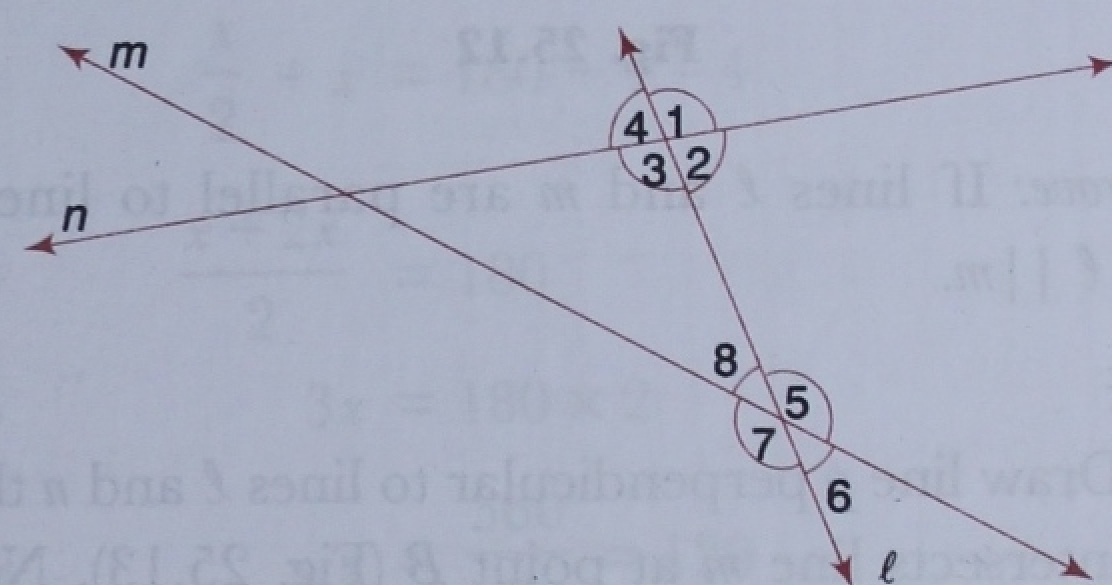


Fig. 25.11

∠2, ∠3, ∠5, and ∠8 are the **interior angles** formed.

∠1, ∠4, ∠7, and ∠6 are the **exterior angles** formed.

∠2 and ∠8 and ∠3 and ∠5 are the pairs of **interior alternate angles** formed.

∠1 and ∠7 and ∠4 and ∠6 are the pairs of **exterior alternate angles** formed.

The four pairs of **corresponding angles** formed are:

∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, and ∠4 and ∠8.

When a pair of parallel lines are intersected by a transversal,

1. the alternate angles are equal to each other,
2. the corresponding angles are equal to each other, and
3. the interior angles on the same side of the transversal are supplementary to each other.

Two lines are said to be parallel to each other if any one of the three conditions are fulfilled.

In Figure 25.12, transversal ℓ intersects parallel lines m and n to form angles 1 to 8.

Alternate angles formed are:

$\angle 2 = \angle 8$; $\angle 3 = \angle 5$; $\angle 1 = \angle 7$ and $\angle 4 = \angle 6$

Corresponding angles formed are:

$\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

Sum of interior angles on the same side of ℓ:

$\angle 2 + \angle 5 = 180°$ and $\angle 3 + \angle 8 = 180°$

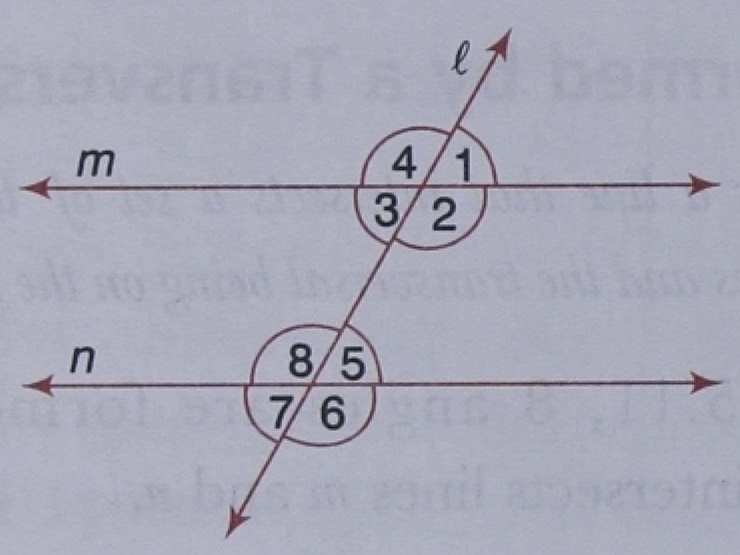


Fig. 25.12

To prove: If lines ℓ and m are parallel to line n, then $\ell \,||\, m$.

Proof:

I. Draw line p perpendicular to lines ℓ and n that intersects line m at point B (Fig. 25.13). Now distance AC between lines ℓ and n is constant (as lines are given to be parallel).

Thus AC + BC = constant + constant

$\Rightarrow$ AB = constant

As the distance between lines ℓ and m is constant, $\ell \,||\, m$. Q.E.D.

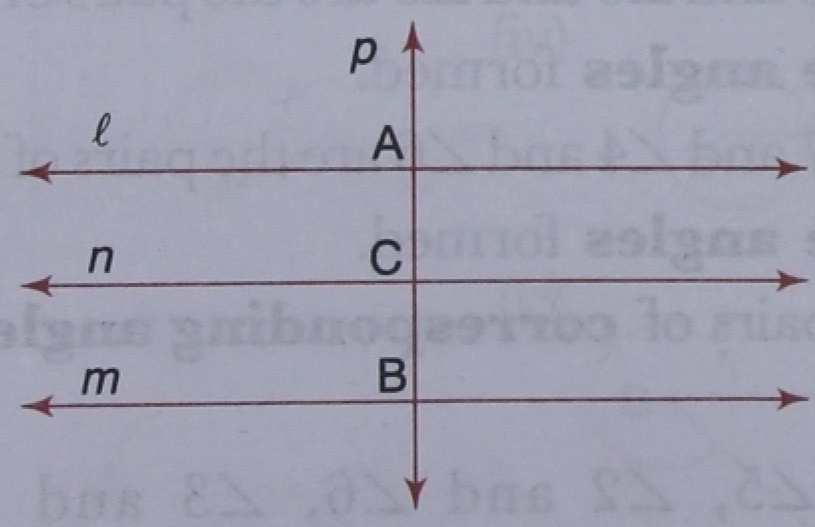


Fig. 25.13

II. Draw transversal p that intersects lines ℓ, n, and m at A, B, and C, respectively (Fig. 25.14). We have

$\angle 1 = \angle 2$ (corresponding angles between $\ell \,||\, n$)

$\angle 2 = \angle 3$ (corresponding angles between $m \,||\, n$)

$\Rightarrow \angle 1 = \angle 2 = \angle 3$

$\Rightarrow \angle 1 = \angle 3$

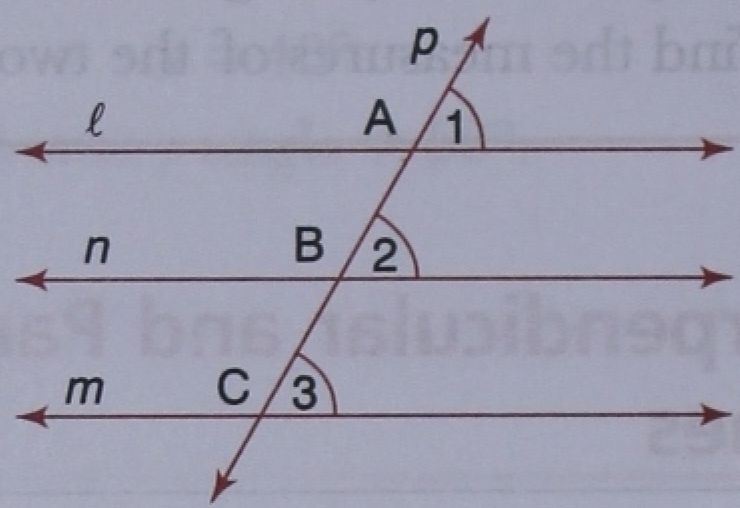


Fig. 25.14

As the corresponding angles formed by transversal p on lines ℓ and m are equal, $\ell \,||\, m$. Q.E.D.

Example 3: Find the magnitude of all the angles marked by small letters of the English alphabet in Figure 25.15.

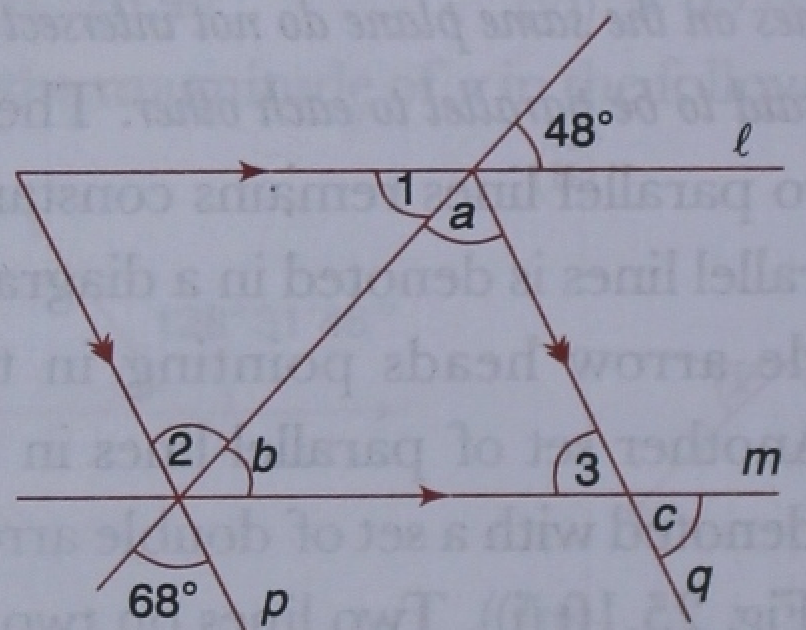


Fig. 25.15

$\angle 1 = 48°$ (vertically opposite angles)

As $\ell \,||\, m$ $\angle b = \angle 1$ (alternate interior angles)

$\Rightarrow \angle b = 48°$

$\angle 2 = 68°$ (vertically opposite angles)

As $p \,||\, q$ $\angle a = \angle 2$ (alternate interior angles)

$\Rightarrow \angle a = 68°$

Now $\angle 1 + \angle a = 48° + 68° = 116°$

But $\angle 3 + (\angle 1 + \angle a) = 180°$

(sum of interior angles on the same side of the transversal q)

$\Rightarrow \quad \angle 3 + \angle 116° = 180°$

$\Rightarrow \quad \angle 3 = 180° - 116° = 64°$

As $\quad \angle 3 = \angle c$, (vertically opposite angles)

$\Rightarrow \quad \angle c = 64°$

Thus $\quad \angle a = 68°, \angle b = 48°$, and $\angle c = 64°$

Example 4: Find the value of x in Figure 25.16.

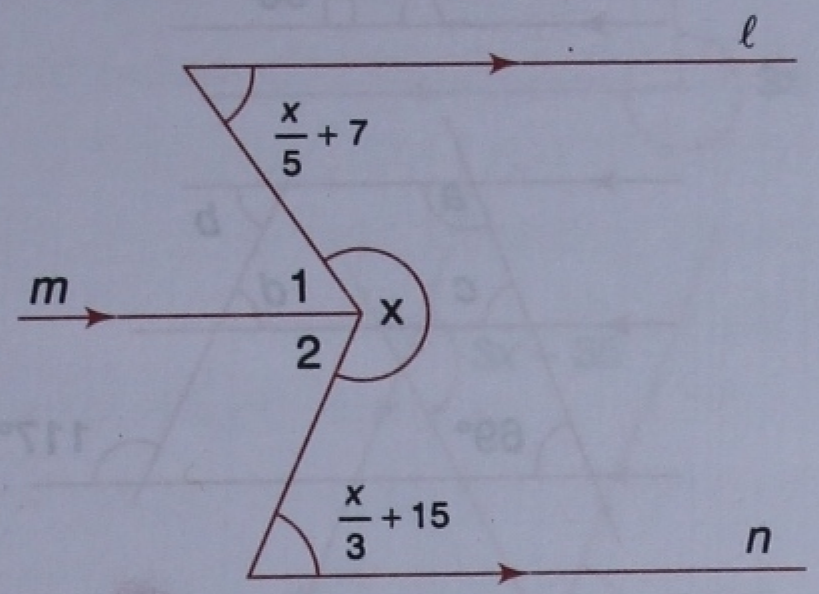


Fig. 25.16

As $\quad \ell \,||\, m, \angle 1 = \frac{x}{5} + 7$ (alternate interior angles)

As $\quad m \,||\, n, \angle 2 = \frac{x}{3} - 15$ (alternate interior angles)

But $\quad x + 1 + 2 = 360°$

$$\Rightarrow \quad x + \frac{x}{5} + 7 + \frac{x}{3} - 15 = 360°$$

$$\Rightarrow \quad \frac{15x + 3x + 5x}{15} - 8 = 360°$$

$$\Rightarrow \quad \frac{23x}{15} = 368° \Rightarrow 23x = 5520°$$

$$\Rightarrow \quad x = \frac{5520}{23} = 240°$$

Example 5: In Figure 25.17, $p \,||\, q$. Is $\ell \,||\, m$? Why?

As $\quad p \,||\, q, \angle 1 = 73°$ (corresponding angles)

As $\quad \angle 1 + \angle 107° = 73° + 107° = 180°$ (interior angles on the same side of transversal p)

$\Rightarrow \quad \ell \,||\, m$

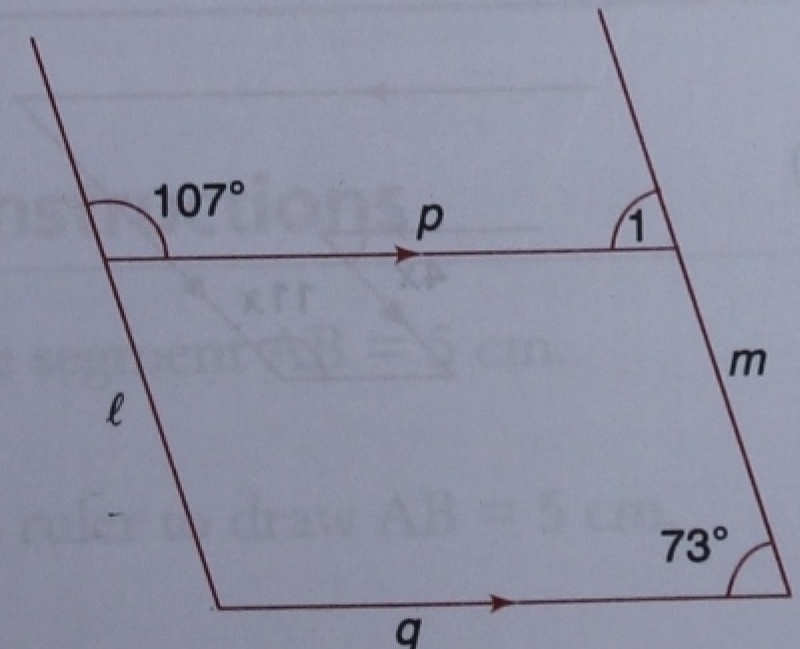


Fig. 25.17

Example 6: For what value of x will the lines l and m be parallel in Figure 25.18?

As $\quad p \,||\, q, x - 4 + a = 180°$

$\Rightarrow \quad a = 180 - (x - 4)$

$\quad = 180 - x + 4$

For lines ℓ and m to be parallel, the interior alternate angles formed by transversal q need to be equal.

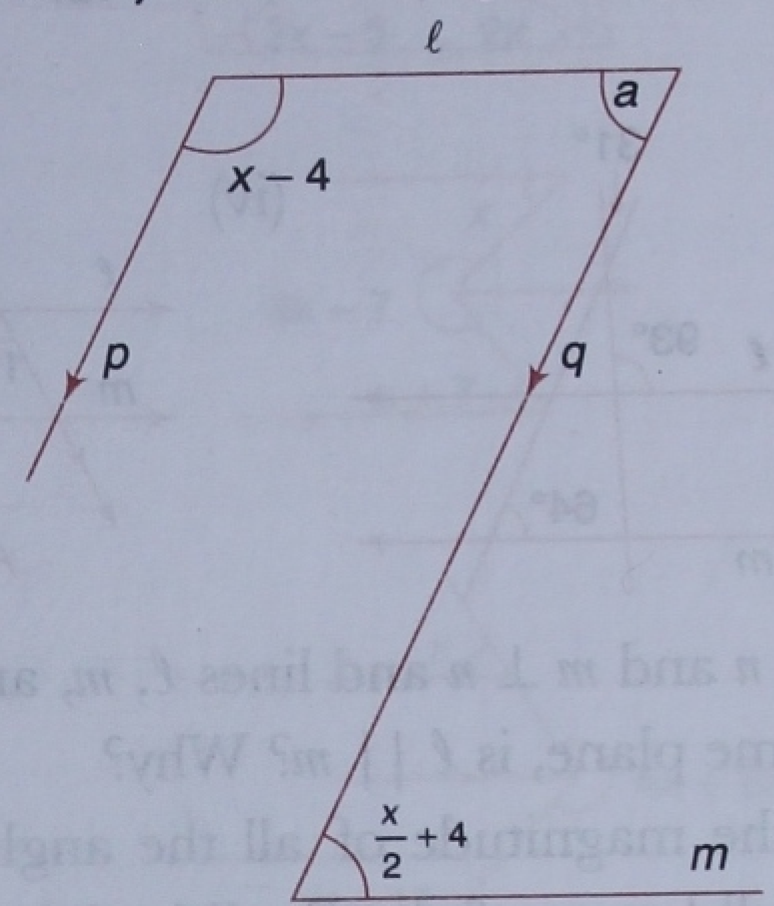


Fig. 25.18

Or $\quad \frac{x}{2} + 4 = 180 - x + 4$

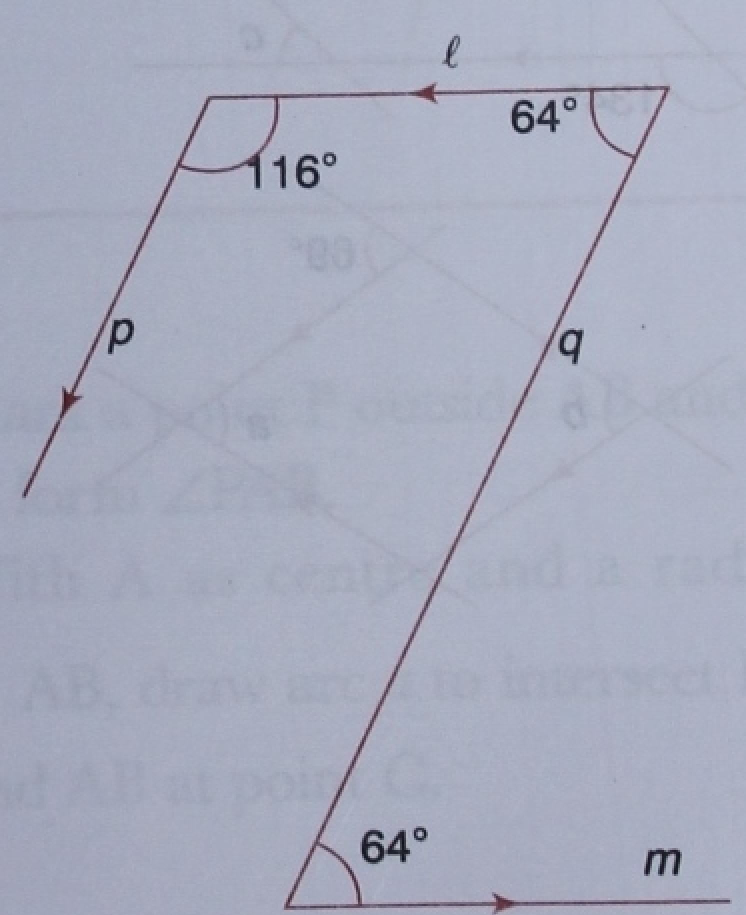
$$\Rightarrow \quad \frac{x}{2} + x = 180 + 4 - 4$$

$$\Rightarrow \quad \frac{x + 2x}{2} = 180$$

$$\Rightarrow \quad 3x = 180 \times 2$$

$$\Rightarrow \quad x = \frac{360}{3} = 120$$

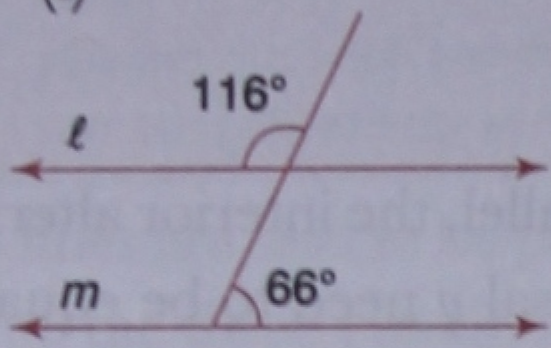
Thus when $x = 120°$, $\ell \,||\, m$ as the given angles in the figure would be as shown below:



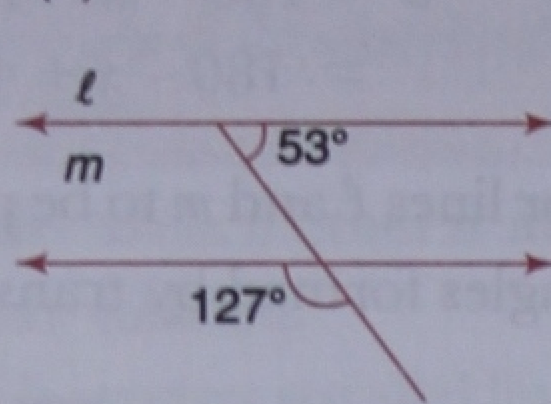
Exercise 25.2

1. In which of the following figures is $\ell \parallel m$?

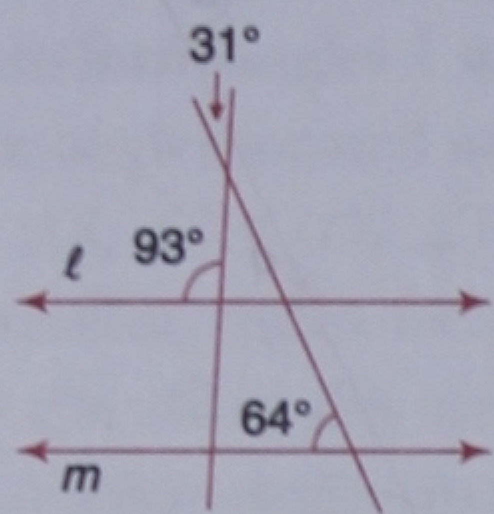
(i)



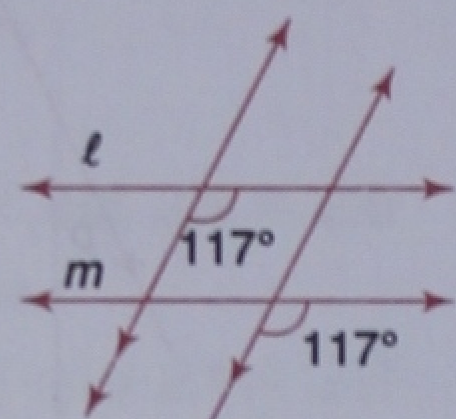
(ii)



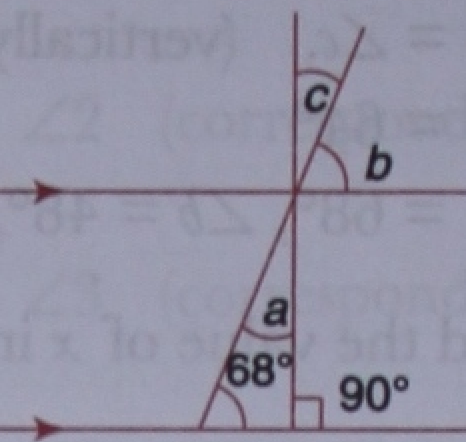
(iii)



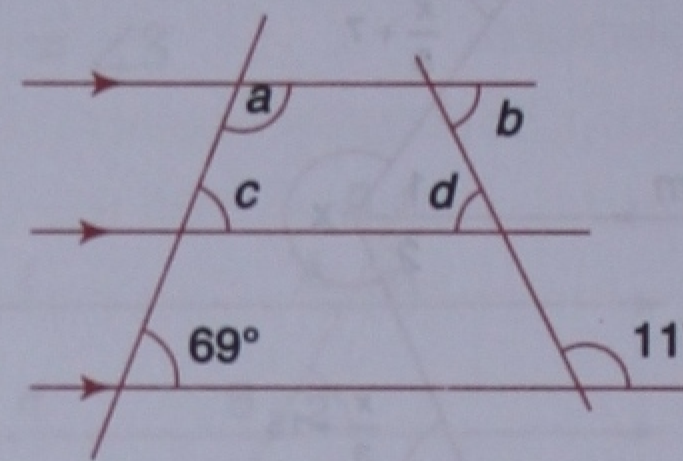
(iv)



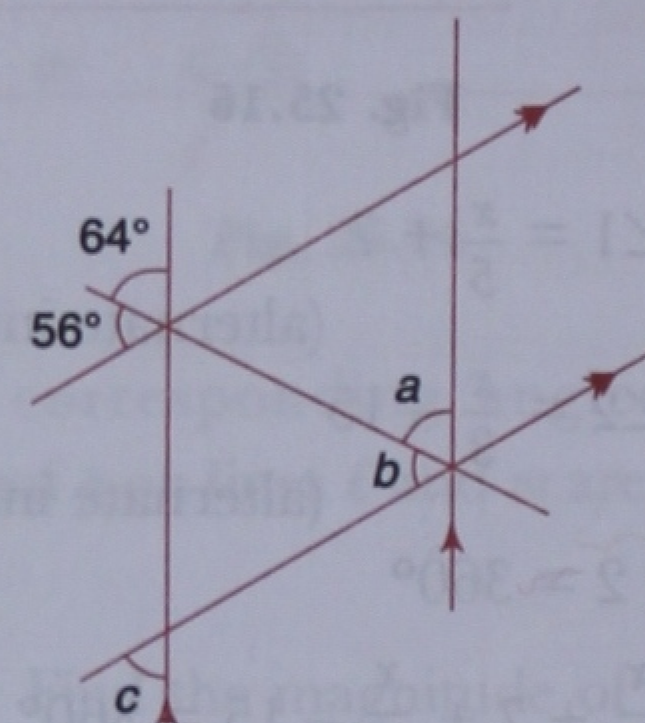
(v)



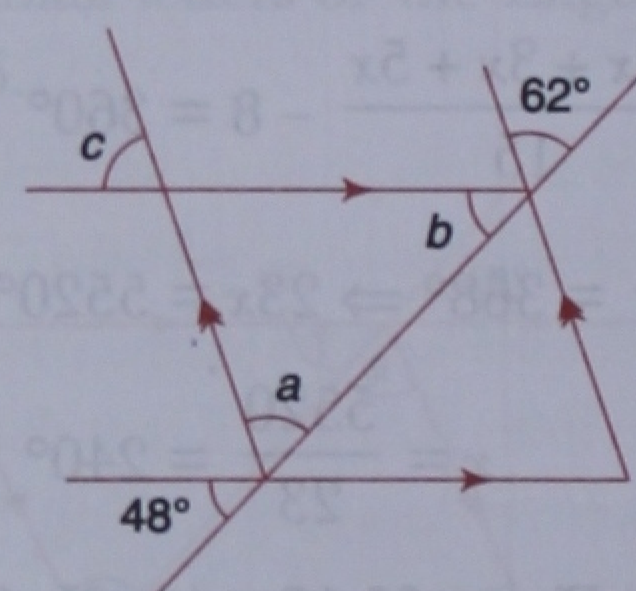
(vi)



(vii)



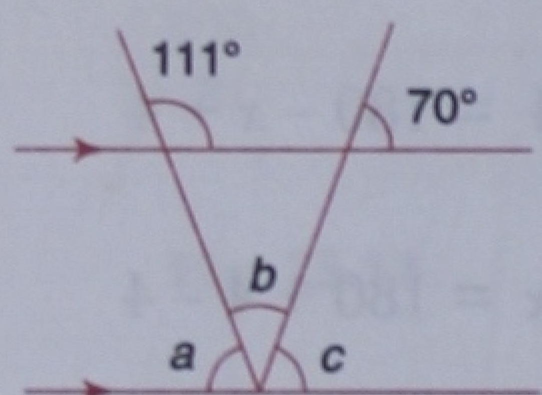
(viii)



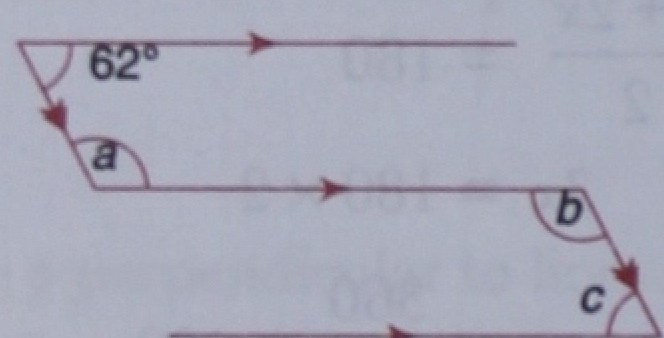
2. If $\ell \perp n$ and $m \perp n$ and lines ℓ, m, and n lie on the same plane, is $\ell \parallel m$? Why?

3. Find the magnitude of all the angles marked by small letters of the English alphabet in the following figures.

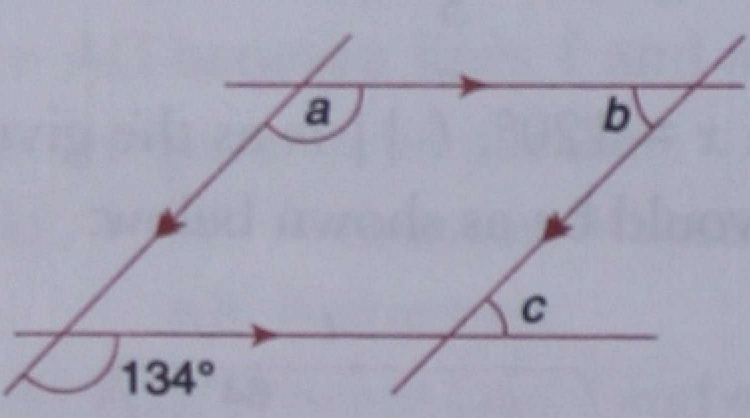
(i)



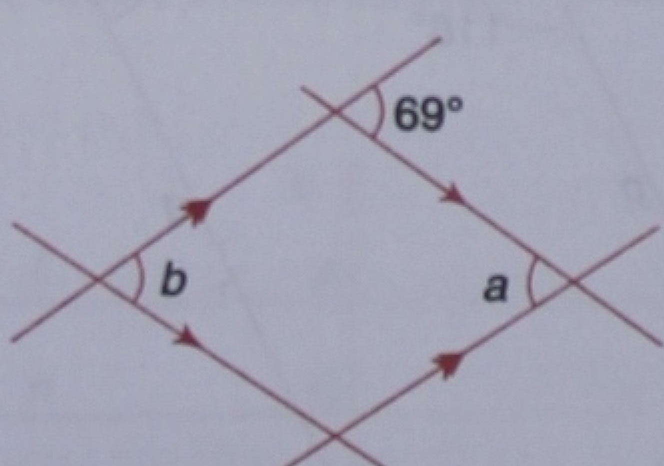
(ii)



(iii)

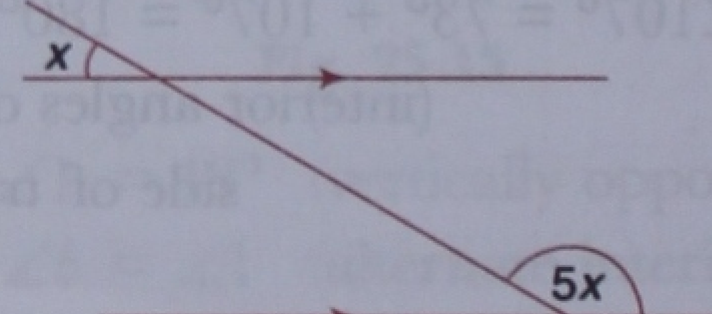


(iv)

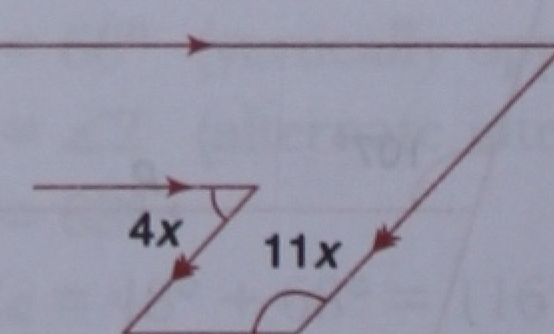


4. Find the value of x in the following figures.

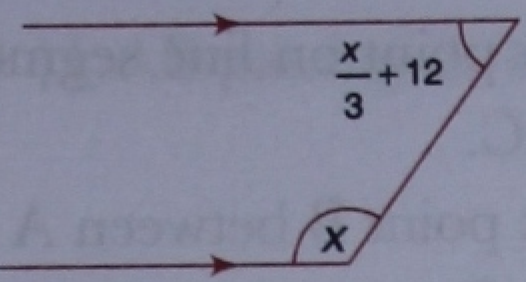
(i)



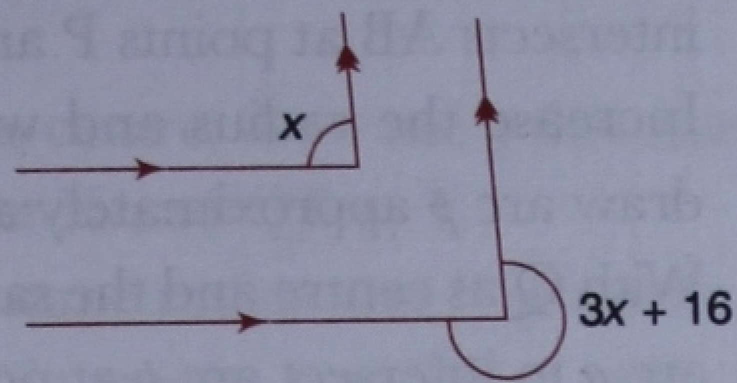
(ii)



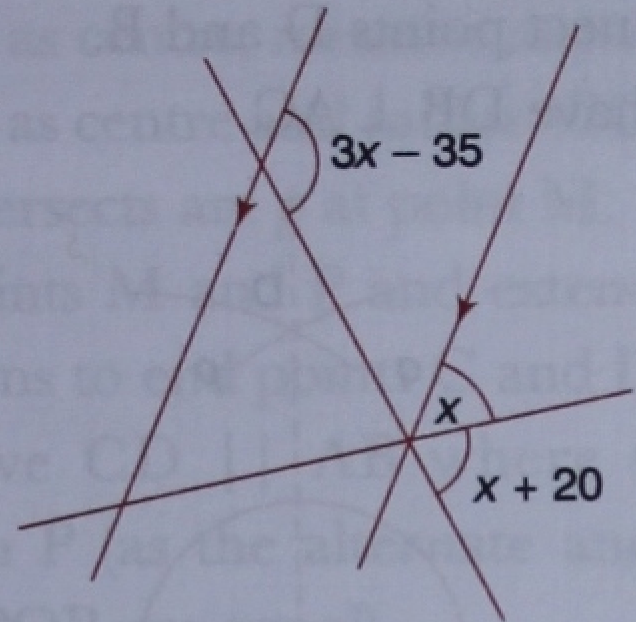
(iii)



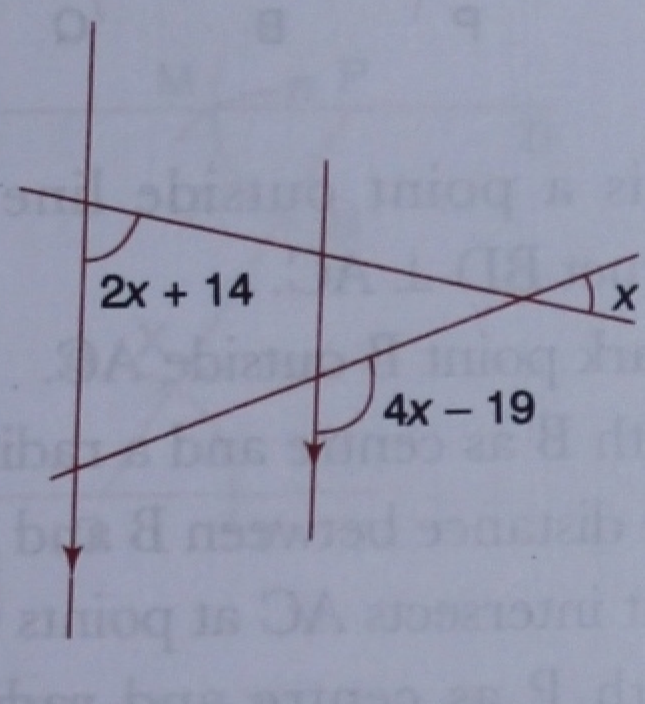
(iv)



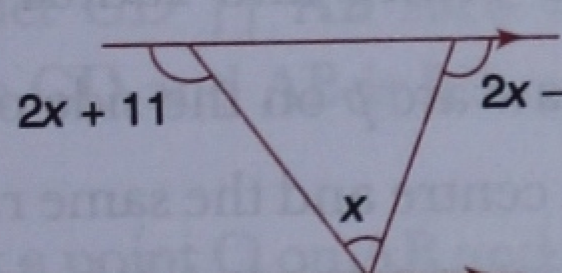
(v)



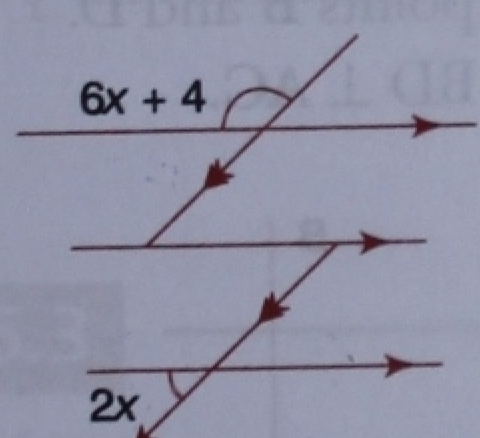
(vi)



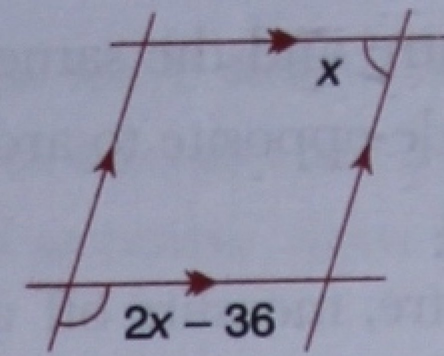
(vii)



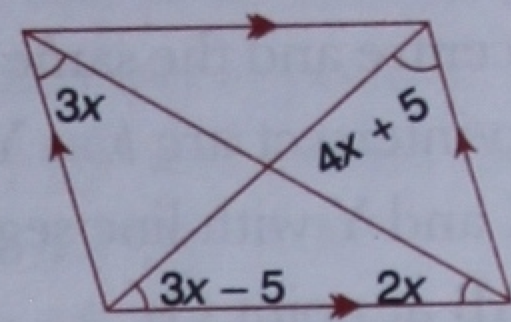
(viii)



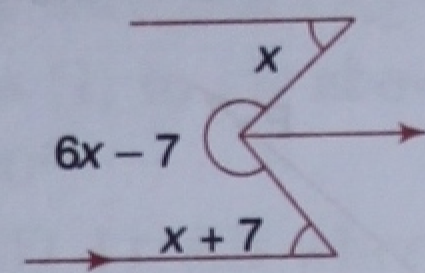
(ix)



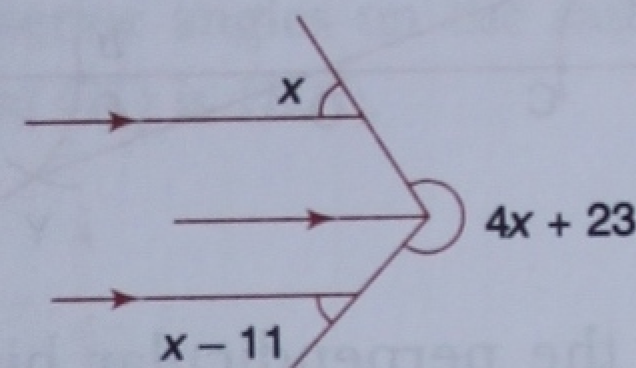
(x)



(xi)

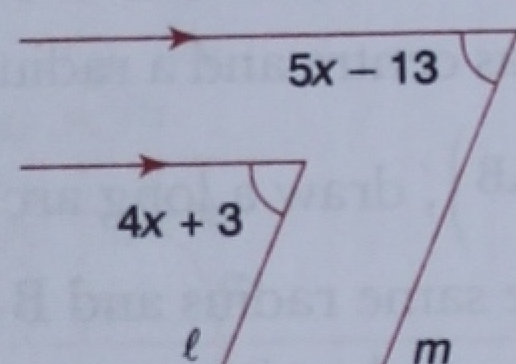


(xii)

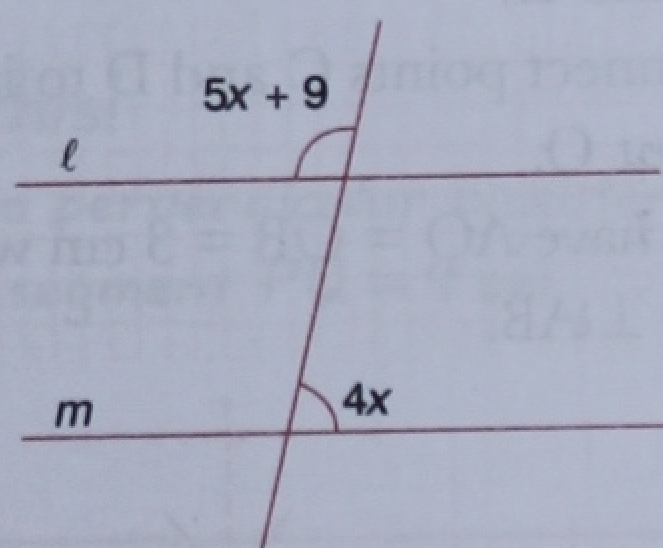


5. Find the value of x for which lines ℓ and m will be parallel to each other in the following figures.

(i)



(ii)



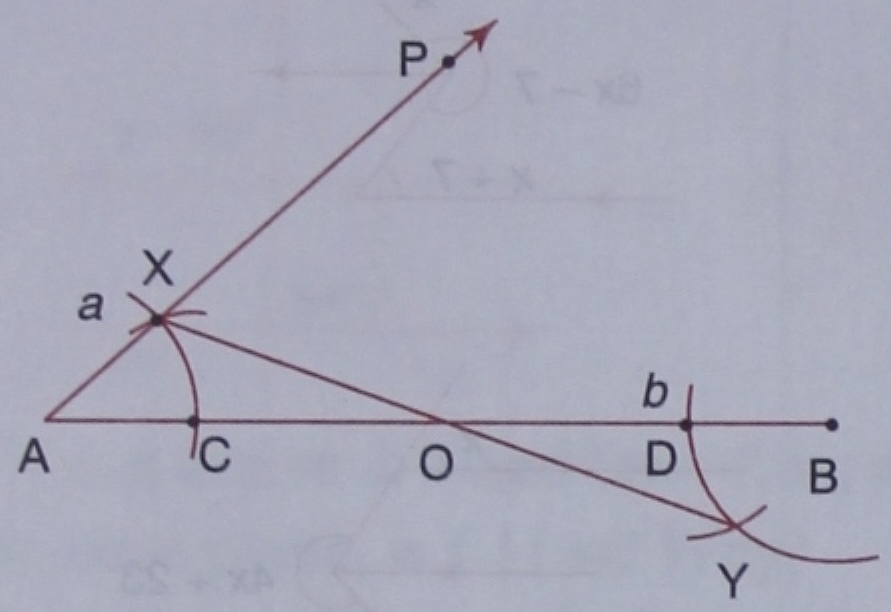
Constructions

I. Bisect line segment AB = 5 cm.

Steps:

1. Use a ruler to draw AB = 5 cm.
2. Mark a point P outside AB and join P and A to form ∠PAB.
3. With A as centre and a radius less than $\frac{1}{2}$ AB, draw arc a to intersect PA at point X and AB at point C.

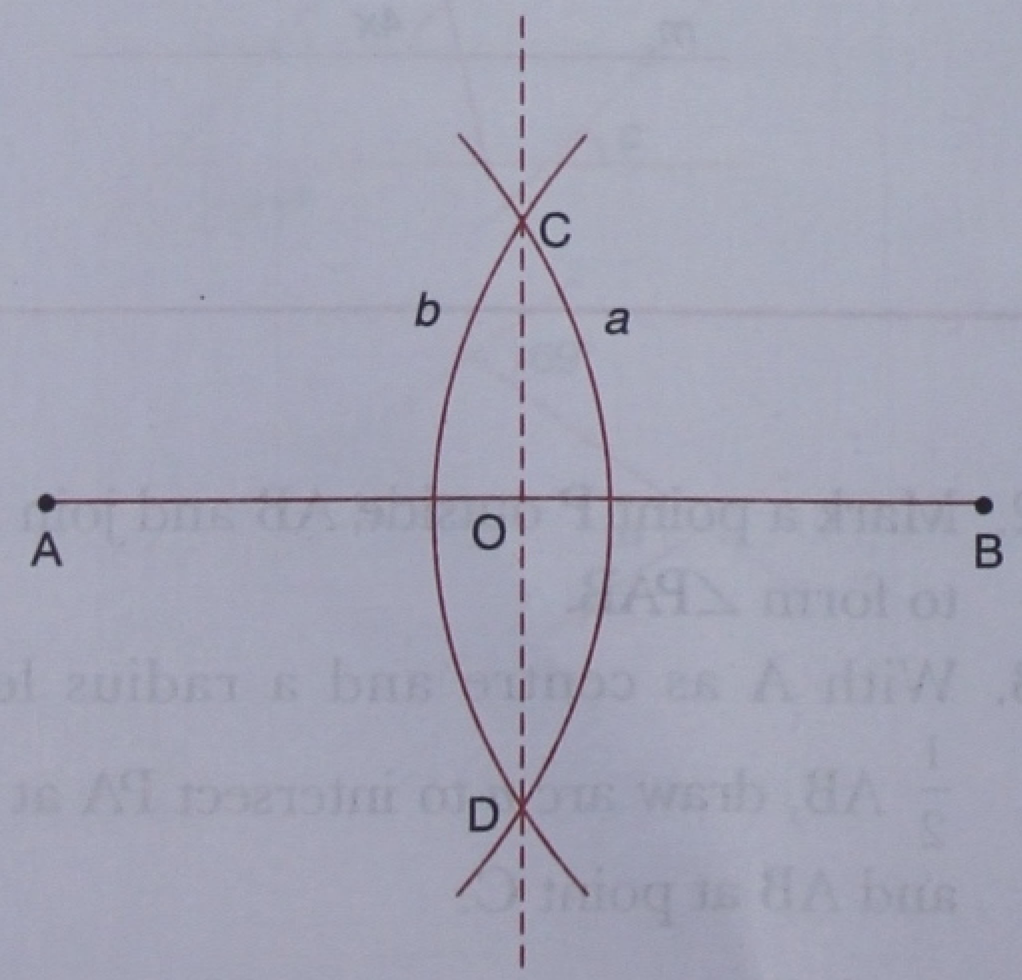
4. With B as centre and the same radius, draw arc *b* on the side opposite to arc *a* to intersect AB at point D.
5. With C as centre, measure off a radius equal to CX.
6. With D as centre and the same radius, draw a small arc to intersect arc *b* at Y.
7. Connect X and Y with line segment XY that intersects AB at point O.
8. We have AO = OB = 2.5 cm.



II. Draw the perpendicular bisector of line segment AB = 6 cm.

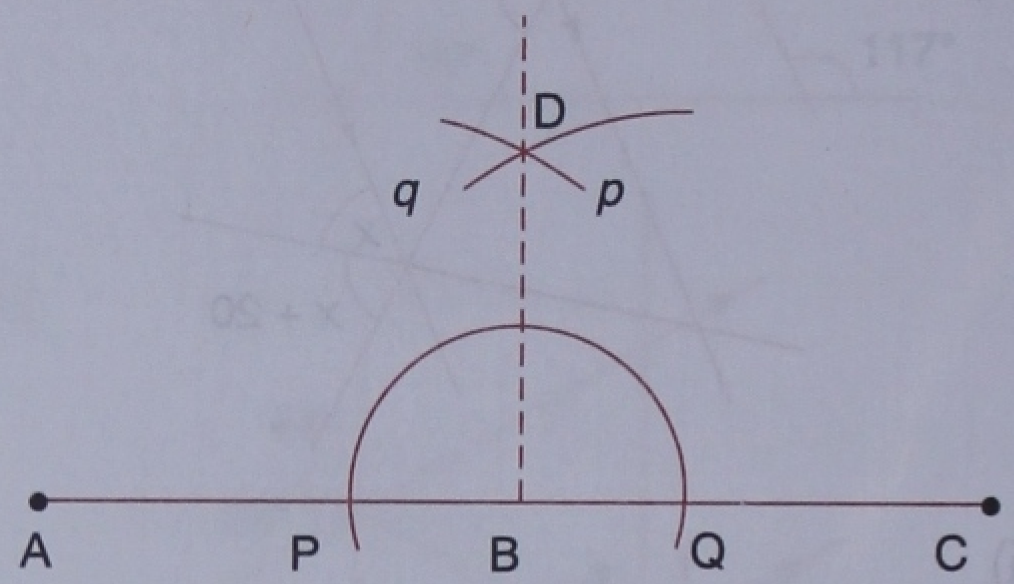
Steps:

1. Use a ruler to draw AB = 6 cm.
2. With A as centre and a radius greater than 3 cm $\left(\frac{1}{2}\text{AB}\right)$, draw a long arc *a*.
3. With the same radius and B as centre, draw a long arc *b* that intersects arc *a* at points C and D.
4. Connect points C and D to intersect AB at point O.
5. We have AO = OB = 3 cm where CD ⊥ AB.



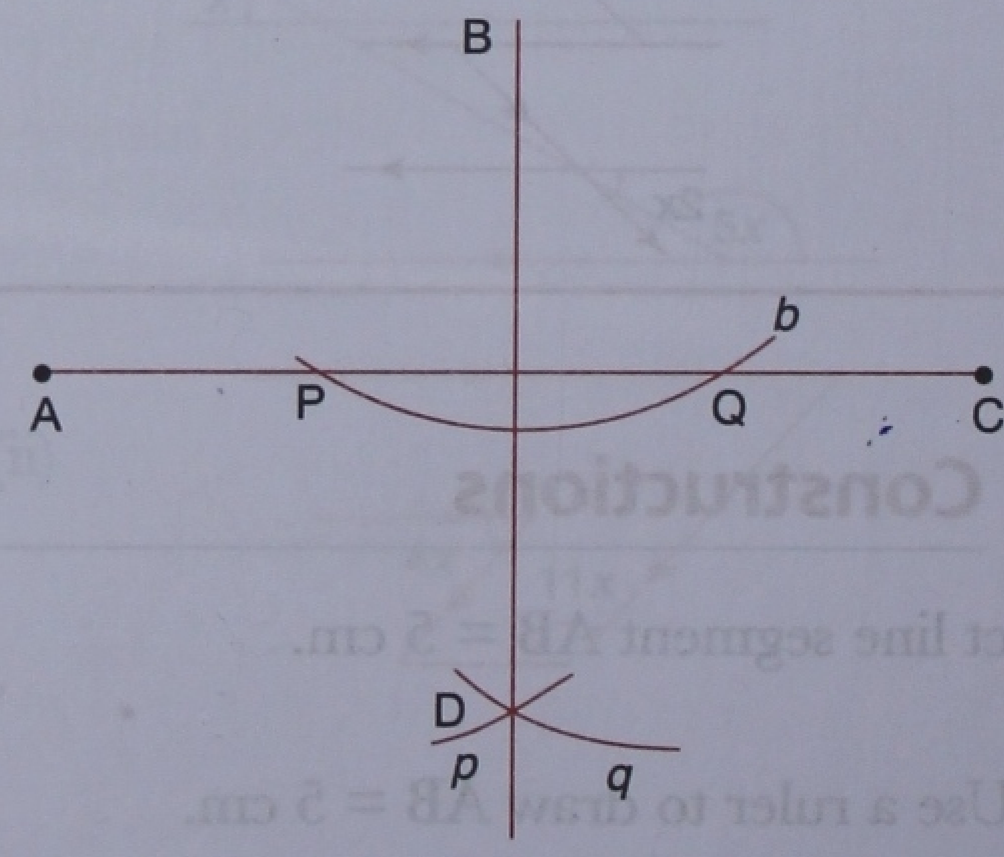
III. If B is a point on line segment AC, construct DB ⊥ AC.

1. Mark point B between A and C.
2. With B as centre, draw a long arc that intersects AB at points P and Q.
3. Increase the radius and with P as centre, draw arc *p* approximately above B.
4. With Q as centre and the same radius, draw arc *q* to intersect arc *p* at point D.
5. Connect points D and B.
6. We have DB ⊥ AC.



IV. If B is a point outside line segment AC, construct BD ⊥ AC.

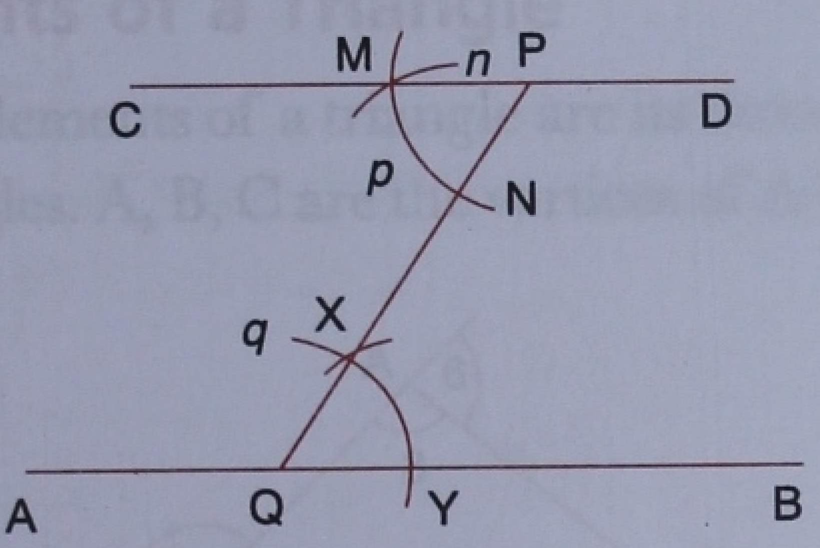
1. Mark point B outside AC.
2. With B as centre and a radius greater than the distance between B and AC, draw arc *b* that intersects AC at points P and Q.
3. With P as centre and radius more than $\frac{1}{2}$ PQ, draw arc *p* on the side opposite B.
4. With Q as centre and the same radius, draw arc *q* that intersects arc *p* at point D.
5. Connect points B and D.
6. We have BD ⊥ AC.



V. Construct CD || AB passing through point P outside AB.

Steps:

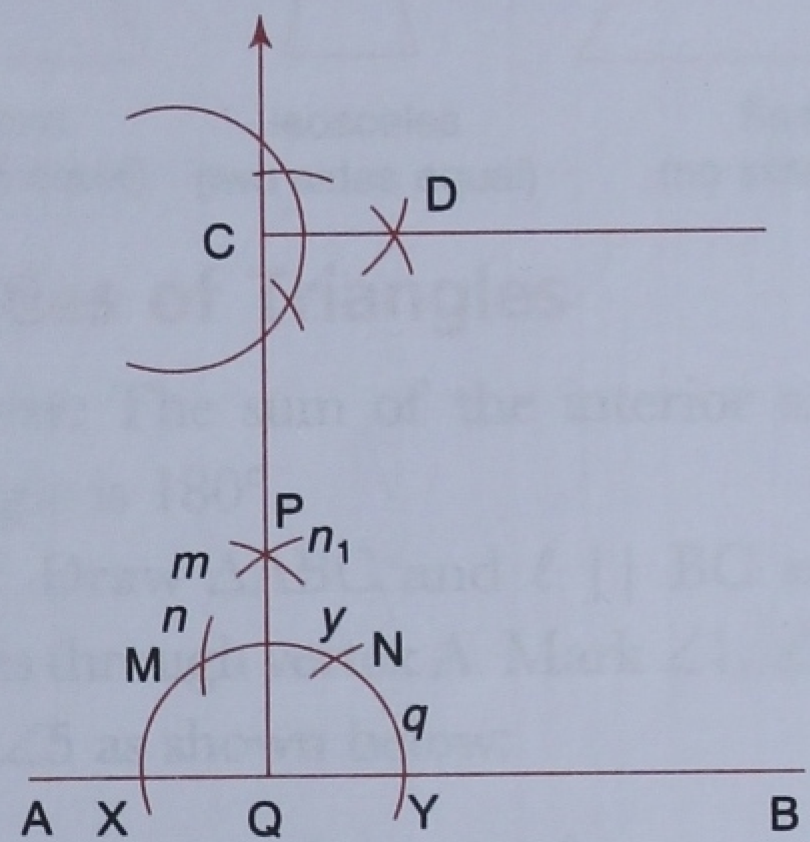
1. Mark a point Q on AB and connect it with point P.
2. With Q as centre, draw arc q that intersects arm PQ at X and QB at Y.
3. With the same radius and P as centre, draw arc p that intersects arm QP at point N.
4. With Y as centre, measure radius YX.
5. With N as centre and radius YX, draw arc n that intersects arc p at point M.
6. Join points M and P and extend it in both directions to end points C and D.
7. We have CD || AB where CD passes through P (as the alternate angles ∠CPQ and ∠PQB are equal).



VI. Construct CD || AB such that the distance between CD and AB is 3.5 cm.

Steps:

1. Mark a point Q on AB and with Q as centre, draw arc q that intersects AB at points X and Y.
2. With the same radius, draw arc y with Y as centre to intersect arc q at point N.
3. With N as centre, draw arc n to intersect arc q at M.
4. With M and N as centres, draw arcs m and n_1 that intersect at point P.
5. Connect QP and extend the line upwards.
6. We have ∠PQB = 90°
7. Measure off 3.5 cm from Q and mark point C on ray QP such that CQ = 3.5 cm.
8. Follow steps (1) to (5) above to construct ∠DCQ = 90°
9. We have CD || AB where the distance between CD and AB is 3.5 cm (as the sum of the interior angles on the same side of transversal CQ is 180°).



Try this!
Draw a perpendicular bisector of line segment PQ = 4 cm.

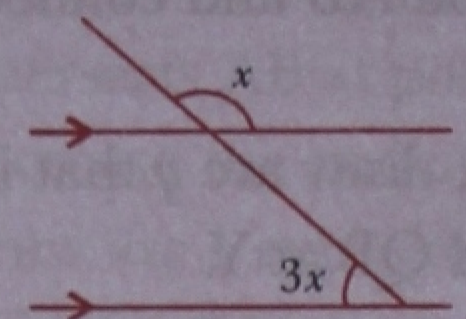
Exercise 25.3

1. Bisect line segment AB = 14 cm such that AO = OB = 7 cm and NO ⊥ AB.
2. Divide line segment AB = 20 cm into four equal parts such that AX = XY = YZ = ZB = 5 cm. (***Hint:*** *you will have to perform 2 bisections.*)
3. Construct CD ⊥ AB such that AD = 5 cm and DB = 6 cm.
4. Construct CD || AB such that the distance between AB and CD is 9 cm.
5. Construct CD ⊥ AB. Draw PQ || CD and QR || AB such that the distance between Q and CD = 6 cm and between Q and AB = 7 cm.

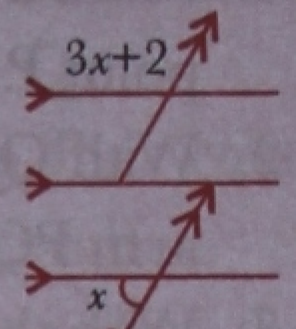
Revision Exercise

1. Two complementary angles are in the ratio of 1 : 5. Find the measure of the two angles.
2. If lines CD and XY intersect at point R to form $\angle CRY = 50°$, find the measure of the other 3 angles formed.
3. Two supplementary angles are in the ratio 3 : 5. Find the measures of the two angles.
4. Find the value of x in the following figures :

(i)



(ii)



5. Construct PQ || XY such that the distance between PQ and XY = 7 cm.