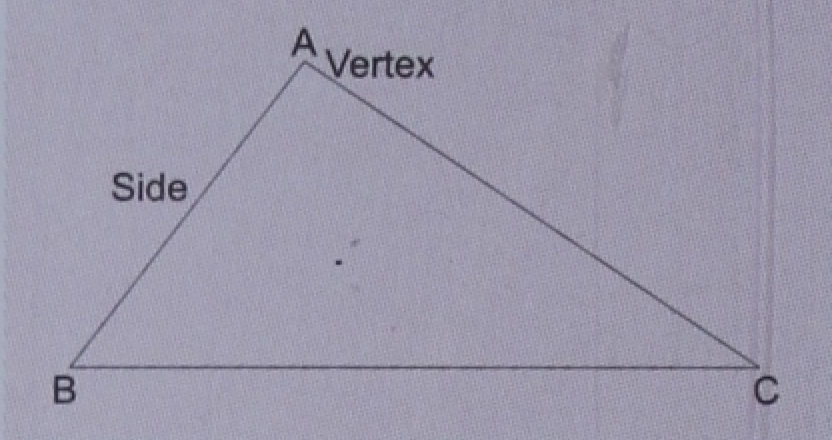
2

Triangles

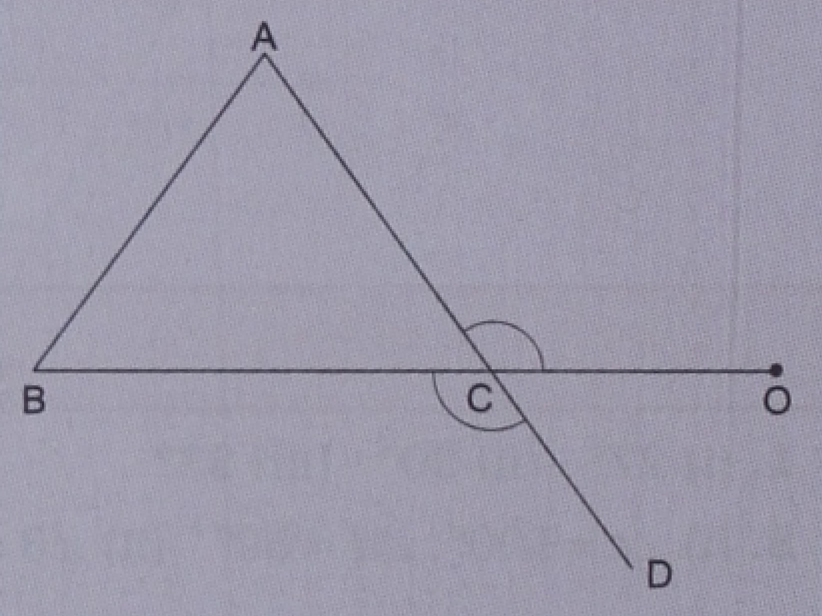
About Triangles

A closed plane figure bounded by three line segments is called a triangle. In the figure, ABC is a triangle bounded by three line segments AB, BC and CA. These are called the sides of the triangle. The points A, B and C are called the vertices of the triangle. The angles $\angle BAC$, $\angle ABC$ and $\angle BCA$ are called the interior angles or simply, the angles of the triangle. The three sides and the three angles of a triangle are together called the six parts (or elements) of the triangle.



Exterior angles

If the side BC of $\triangle ABC$ is produced to the point O then $\angle ACO$ is called an exterior angle of $\triangle ABC$ at C. $\angle ACB$ and $\angle ACO$ are adjacent angles, so, $\angle ACB + \angle ACO = 180°$.

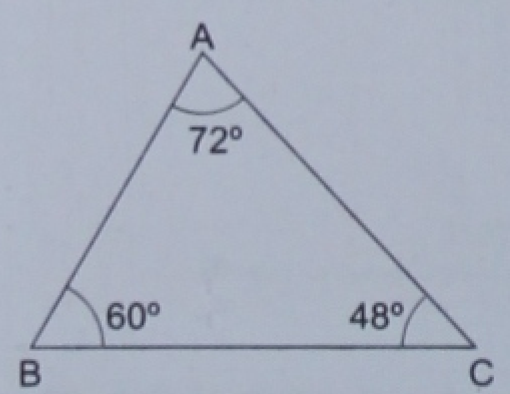


Exterior angle + adjacent interior angle = 180°

If AC is produced to D then $\angle BCD$ will be another exterior angle of the $\triangle ABC$ at C. Being vertically opposite angles, $\angle BCD = \angle ACO$. Similarly, there are two exterior angles of equal magnitude at each vertex.

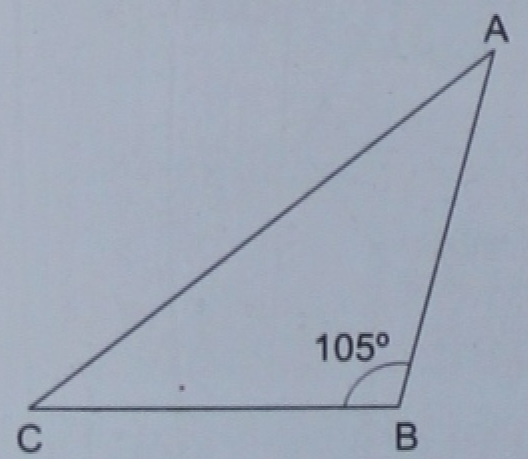
Classification of triangles on the basis of angles

Acute-angled



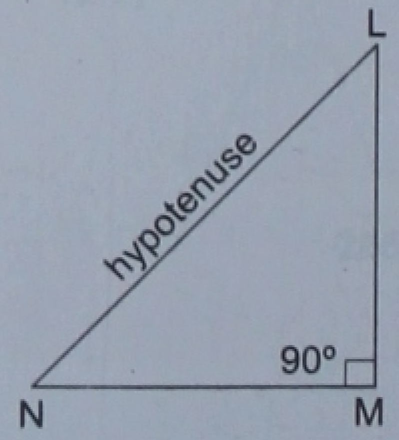
In an acute-angled or acute triangle, each of the angles is less than 90°.

Obtuse-angled



In an obtuse-angled or obtuse triangle, one of the angles is greater than 90°.

Right-angled



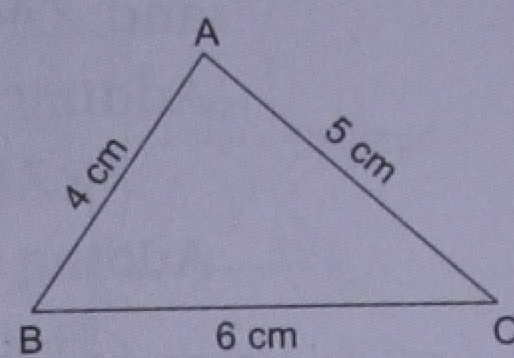
In a right-angled or right triangle, one of the angles is a right angle. The side opposite to it is called the hypotenuse.

Classification of triangles on the basis of sides

Scalene triangle

No two sides of a scalene triangle are equal.

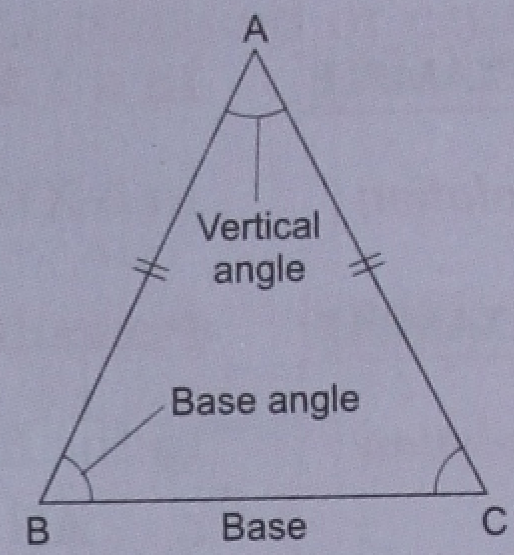
In the figure, ABC is a scalene triangle as $AB \neq BC \neq CA$.



Scalene triangle

Isosceles triangle

An isosceles triangle has two equal sides. In the figure, ABC is an isosceles triangle in which $AB = AC$ (equal sides are marked by an equal number of strokes). The third side BC is called the base of the triangle, while $\angle ABC$ and $\angle ACB$ are called the base angles. $\angle BAC$ is called the vertical angle.



Isosceles triangle

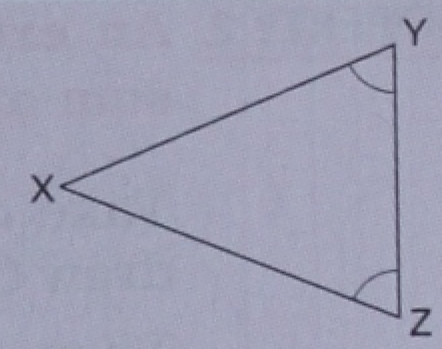
PROPERTY **The angles opposite to the equal sides of an isosceles triangle are equal.**

In the figure, $AB = AC$. So, $\angle ABC = \angle ACB$.

CONVERSE **The converse, or opposite, of this is also true. Thus, if two angles of a triangle are equal, the sides opposite to them are equal.**

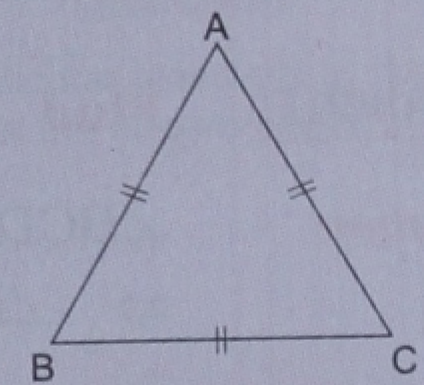
In the figure, $\angle XYZ = \angle XZY$. So, $XZ = XY$.

This also implies that the angles of a scalene triangle are all unequal.



Equilateral triangle

All the three sides of an equilateral triangle are equal. In the adjoining figure, ABC is an equilateral triangle as $AB = BC = CA$.



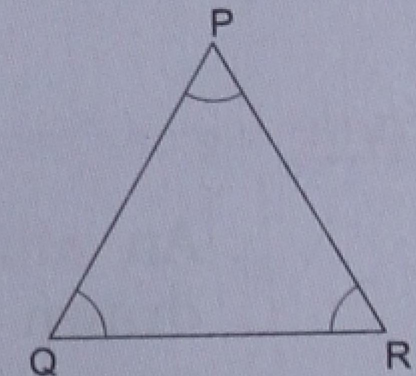
Equilateral triangle

PROPERTY **All the angles of an equilateral triangle are equal.**

In the figure, $\angle BAC = \angle ABC = \angle ACB$.

CONVERSE **If all the angles of a triangle are equal, it must be an equilateral triangle.**

In the adjoining figure, $\angle P = \angle Q = \angle R$. Hence, PQR is an equilateral triangle.

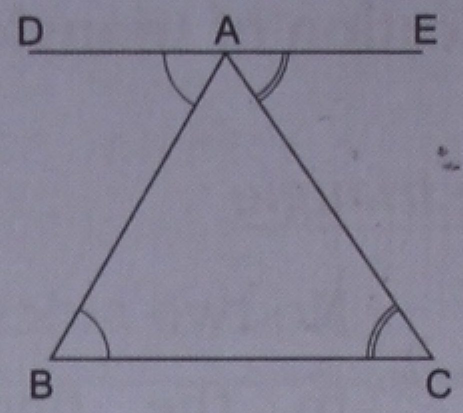


Angle properties of a triangle

PROPERTY 1 **The sum of the three angles of a triangle is 180°.**

Take a triangle ABC.

Draw the line DE parallel to BC through A.



Then $\angle ABC$ = alternate $\angle DAB$ ... (1)

and $\angle ACB$ = alternate $\angle EAC$ ... (2)

Adding (1) and (2),

$$\angle ABC + \angle ACB = \angle DAB + \angle EAC.$$

Adding $\angle BAC$ to both sides of the above,

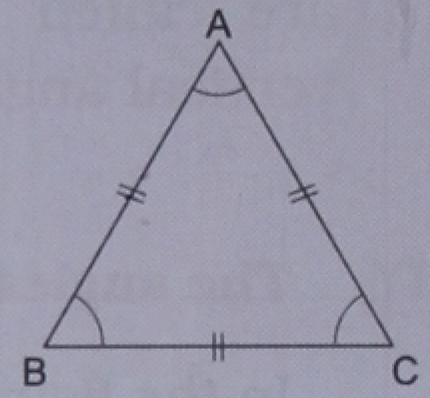
$$\angle BAC + \angle ABC + \angle ACB = \angle DAB + \angle BAC + \angle EAC$$
$$= \text{a straight angle} = 180°.$$

Hence, $\angle BAC + \angle ABC + \angle ACB = 180°$.

EXAMPLE **In a $\triangle XYZ$, $\angle X = 45°$ and $\angle Y = 75°$. Find $\angle Z$.**

Solution In $\triangle XYZ$, $\angle X + \angle Y + \angle Z = 180°$ or $45° + 75° + Z = 180°$ or $\angle Z = 180° - 120° = 60°$.

EXAMPLE **Prove that each angle of an equilateral triangle is 60°.**



Solution In the figure, ABC in an equilateral triangle.

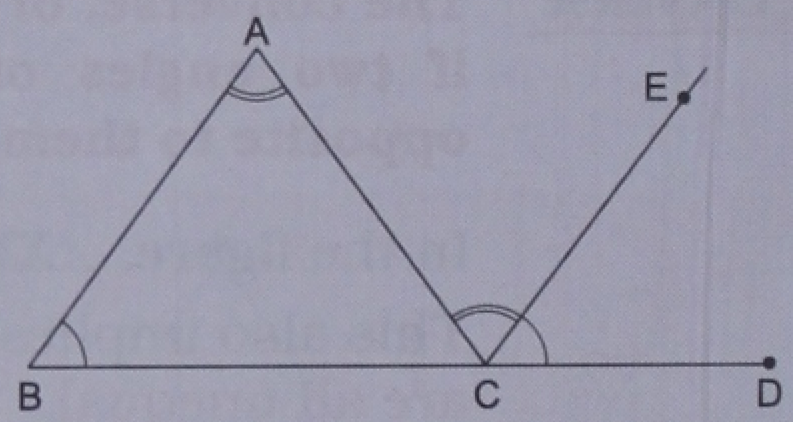
$\therefore \quad AB = BC = CA \Rightarrow \angle A = \angle B = \angle C.$

But $\angle A + \angle B + \angle C = 180 \Rightarrow \angle A + \angle A + \angle A = 180°$

$\Rightarrow \quad 3\angle A = 180° \Rightarrow \angle A = \dfrac{180°}{3} = 60°.$

Hence, $\angle A = \angle B = \angle C = 60°$.

PROPERTY 2 **An exterior angle of a triangle is equal to the sum of the two opposite interior angles.**

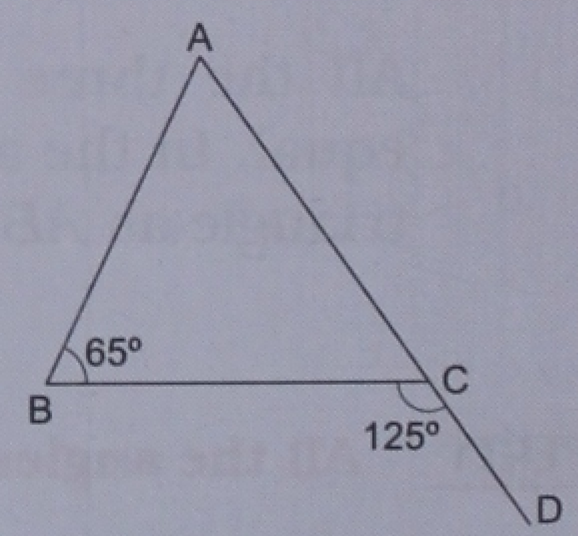


Take $\triangle ABC$. Produce BC to D and through C, draw $CE \parallel BA$.

Then $\angle A$ = alternate $\angle ACE$ and $\angle B$ = corresponding $\angle ECD$.

But, $\angle ACE + \angle ECD = \angle ACD$. So, $\angle ACD = \angle A + \angle B$.

EXAMPLE **Find $\angle BAC$ from the figure.**



Solution $\angle BCD = \angle BAC + \angle ABC \Rightarrow 125° = \angle BAC + 65°$

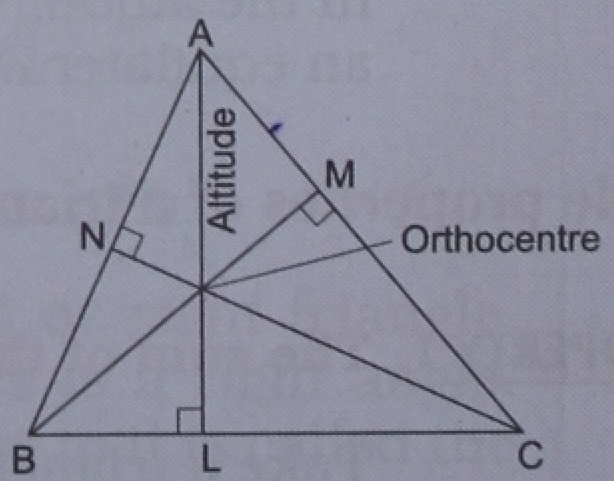
$\Rightarrow \quad \angle BAC = 125° - 65° = 60°.$

Some important terms

Altitude

An altitude of a triangle is the perpendicular drawn from any vertex of the triangle to the opposite side. A triangle has three altitudes. In the figure, AL, BM and CN are the altitudes of $\triangle ABC$.

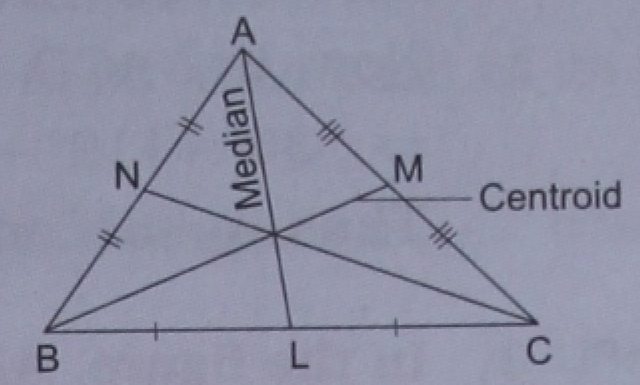
The three altitudes of a triangle pass through a common point called the 'orthocentre' of the triangle.



The altitudes pass through the orthocentre.

Median

The line segment joining a vertex of a triangle to the mid-point of the opposite side is called a median of the triangle. A triangle has three medians. In the figure, L, M and N are the mid-points of the sides BC, CA and AB respectively. So, AL, BM and CN are the medians of $\triangle ABC$.

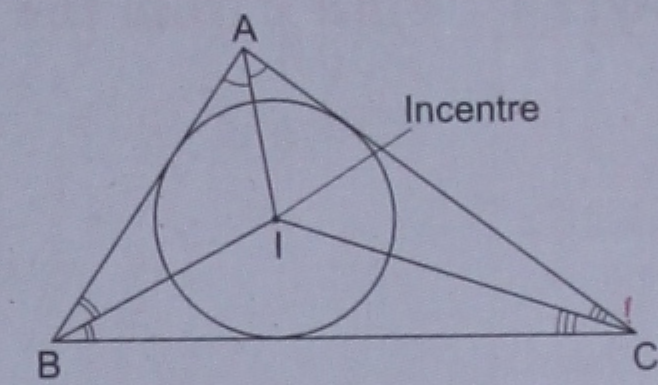


The medians pass through the centoid.

The three medians of a triangle intersect at a point called the centroid or centre of gravity of the triangle.

Incircle

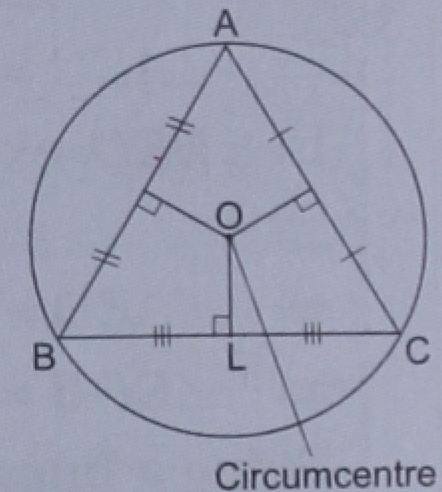
The circle that lies inside a triangle and touches its three sides is called the incircle. The centre of the incircle is called the incentre. The incentre is the point at which the three (internal) bisectors of the angles of the triangle meet. In the figure, AI, BI and CI bisect $\angle A$, $\angle B$ and $\angle C$ respectively and meet at I, which is the incentre.



The internal bisectors of the angles meet at the incentre.

Circumcircle

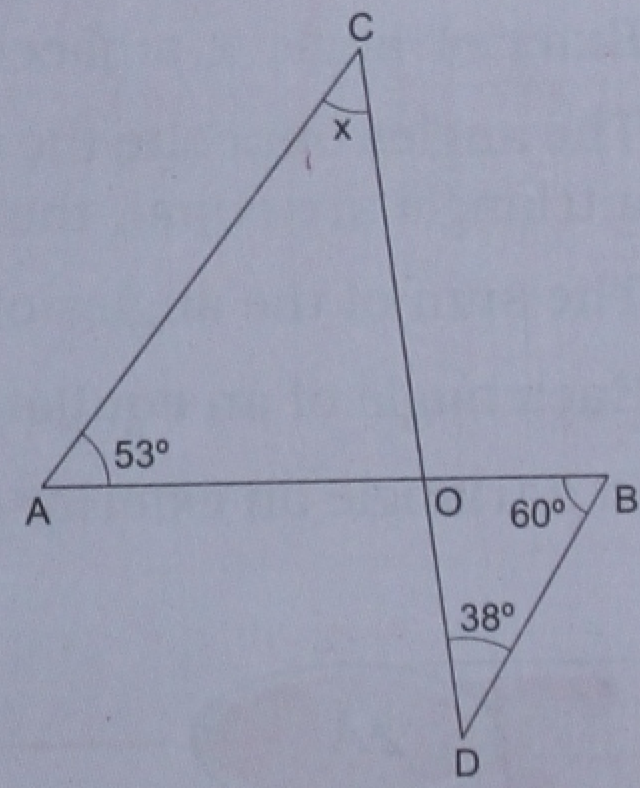
The circumcircle of a triangle is the circle that passes through its three vertices. Its centre is called the circumcentre, The point at which the perpendicular bisectors of the sides of the triangle meet is the circumcentre.



The perpendicular bisectors of the sides meet at the circumcentre.

Solved Examples

EXAMPLE 1 **Find x from the figure.**



Solution The sum of the angles of a triangle = 180°.

$\therefore$ in $\triangle OBD$, $\angle BOD + 60° + 38° = 180°$

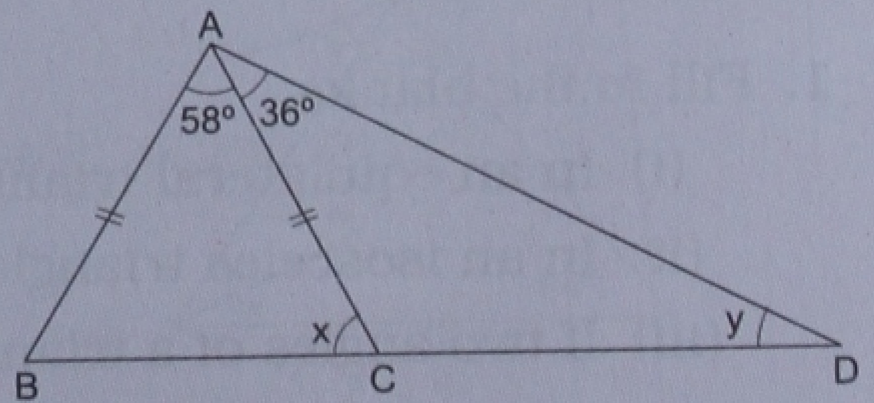
or $\angle BOD = 180° - 98° = 82°$.

$\therefore$ $\angle AOC$ = vertically opposite $\angle BOD = 82°$.

Now, in $\triangle AOC$, $x + 53° + 82° = 180°$

or $x = 180° - 135° = 45°$.

EXAMPLE 2 **Find x and y from the adjoining figure.**



Solution In $\triangle ABC$, $AB = AC \Rightarrow \angle ABC = \angle ACB = x$.

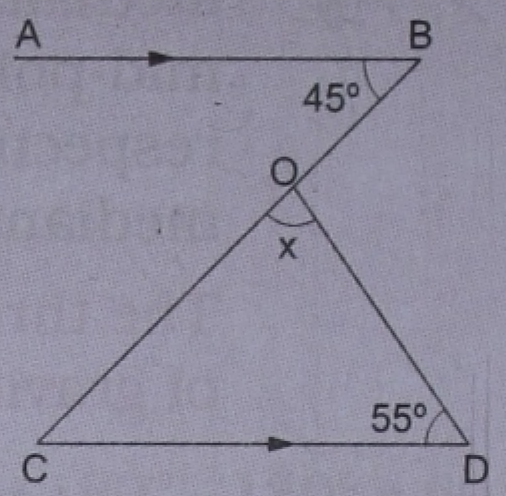
Also, $\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow 58° + x + x = 180° \Rightarrow 2x = 180° - 58° = 122°$

$\Rightarrow x = 61°$.

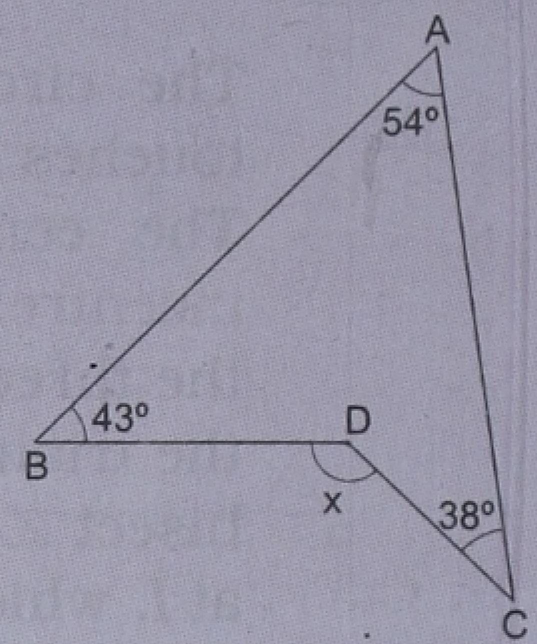
Also, $\angle ACD = 180° - x = 180° - 61° = 119°$
Now, in $\triangle ACD$, $\angle CAD + \angle ACD + \angle ADC = 180°$.
$\Rightarrow \quad 36° + 119° + y = 180° \Rightarrow y = 180° - 155° = 25°$.
Thus, $x = 61°$ and $y = 25°$.

EXAMPLE 3 **In the figure, *AB* || *CD*. Find *x*.**

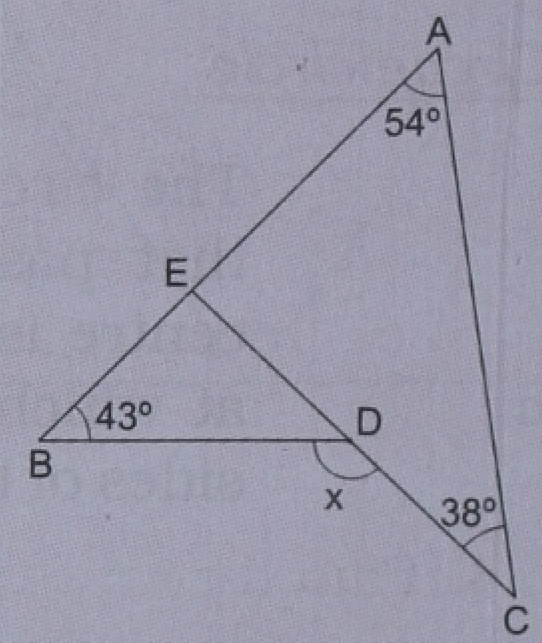


Solution Since $AB \parallel CD$, $\angle OCD$ = alternate $\angle ABO = 45°$.
Now, in $\triangle OCD$, $\angle COD + \angle OCD + \angle ODC = 180°$
$\Rightarrow \quad x + 45° + 55° = 180° \Rightarrow x = 180° - 100° = 80°$.

EXAMPLE 4 **Find *x* from the adjoining figure.**



Solution Extend the line *CD* so that it meets *AB* at the point *E*.
In $\triangle AEC$, $\angle EAC + \angle AEC + \angle ACE = 180°$
$\Rightarrow \quad 54° + \angle AEC + 38° = 180°$
$\Rightarrow \quad \angle AEC = 180° - 92° = 88° = \angle AED$.
In $\triangle BED$, exterior $\angle AED$
= sum of two opposite interior angles
$\Rightarrow \quad \angle AED = \angle EBD + \angle EDB \Rightarrow 88° = 43° + \angle EDB$
$\Rightarrow \quad \angle EDB = 88° - 43° = 45°$.
Being a linear pair, $\angle EDB + \angle BDC = 180°$
$\Rightarrow \quad 45° + x = 180° \Rightarrow x = 180° - 45° = 135°$.



Remember These

1. Exterior angle + adjacent interior angle = 180°.
2. The angles opposite the equal sides of an isosceles triangle are equal. Also, if two angles of a triangle are equal, the sides opposite the angles are equal.
3. The sum of the angles of a triangle = 180°.
4. Each angle of an equilateral triangle = 60°.
5. In a triangle an exterior angle = the sum of the opposite interior angles.

EXERCISE 2A

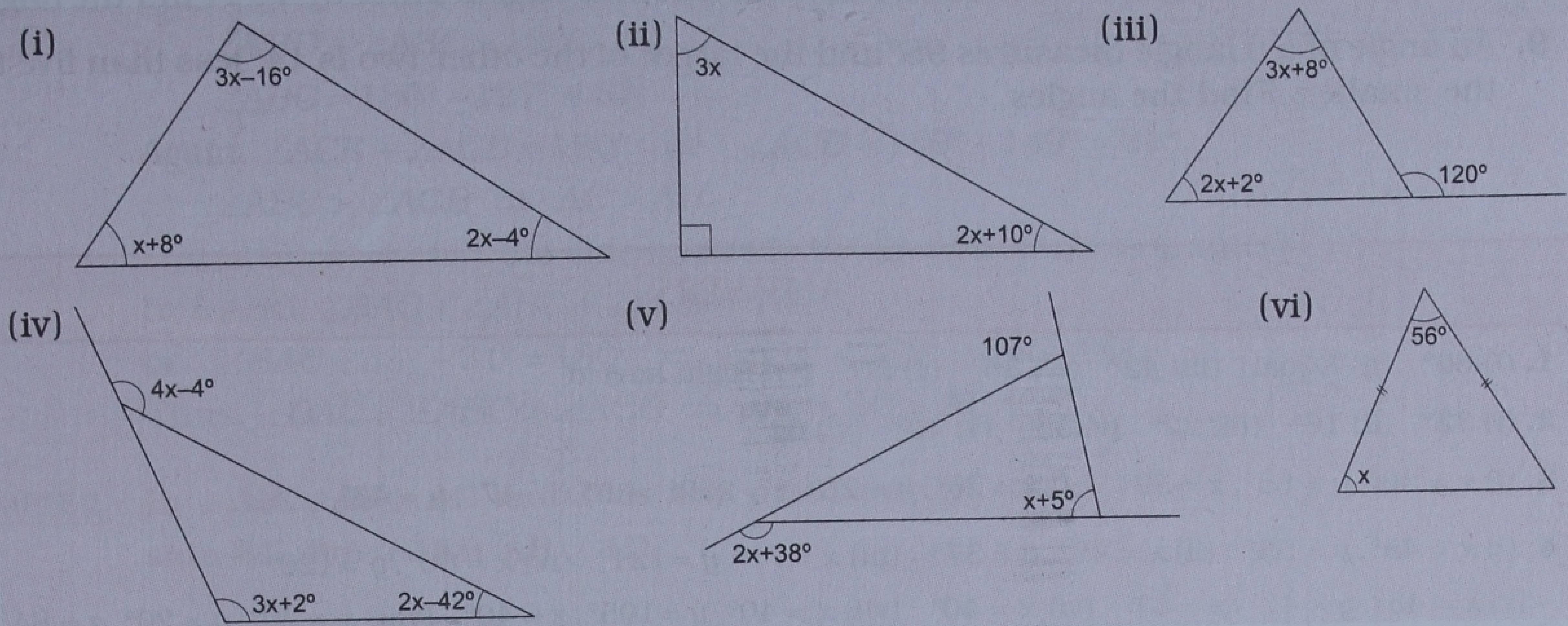
1. Fill in the blanks.
   (i) In an equilateral triangle, each angle = ...
   (ii) In an isosceles triangle, at least two sides are ....
   (iii) If two angles of a triangle are 63° and 75°, its third angle = ...

(iv) If two angles of a triangle are 72° each, its third angle = ...

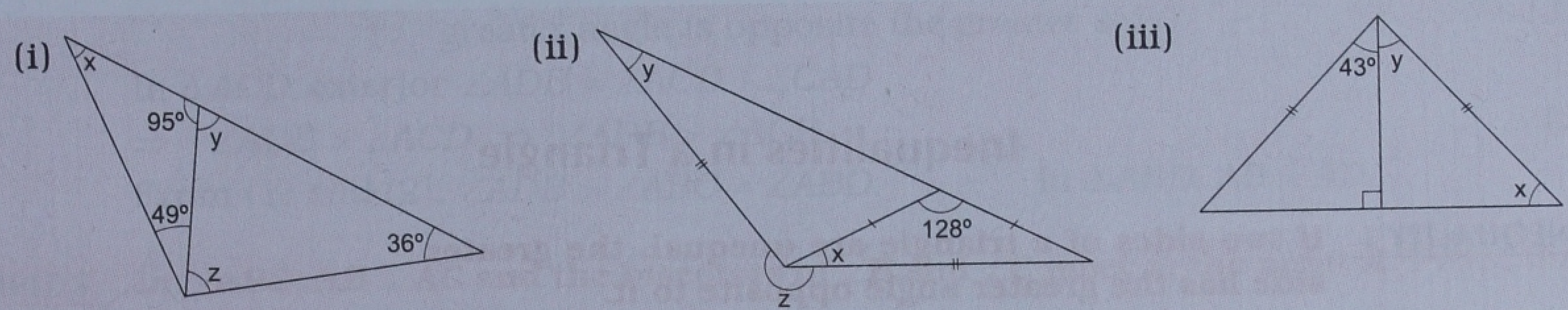
(v) If two angles of a triangle are equal and its third angle is 68° then each of its equal angles = ...

(vi) If the angles of a triangle are in the ratio 1 : 2 : 3 then the triangle is a ...... triangle.

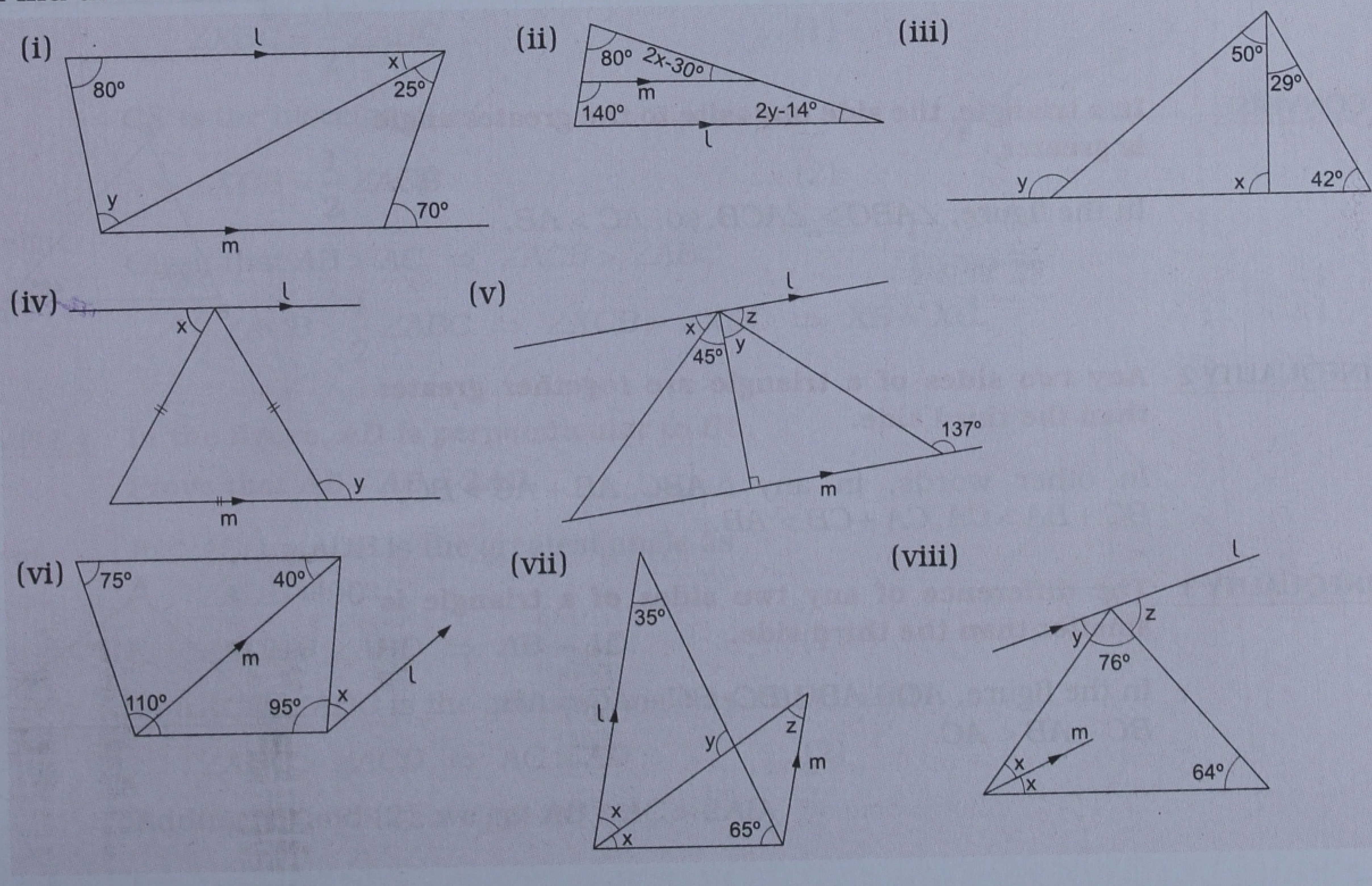
2. Find x in each of the following figures.



3. Calculate the measures of the lettered angles.



4. Find the measures of the lettered angles when $l \parallel m$.



5. If the angles of a triangle are in the ratio 7 : 11 : 18, find the angles.

6. If the acute angles of a right-angled triangle are in the ratio 5 : 13, find the acute angles.

7. Find the angles of an isosceles triangle if the ratio of the base angle to the vertical angle is 2 : 5.

8. An angle of a triangle measures 68° and the other two angles differ by 16°. Find the angles.

9. An angle of a triangle measures 98° and the larger of the other two is 14° less than five times the smaller. Find the angles.

ANSWERS

1. (i) 60° (ii) Equal (iii) 42° (iv) 36° (v) 56° (vi) Right angled

2. (i) 32° (ii) 16° (iii) 22° iv) 36° (v) 40° (vi) 62°

3. (i) $x = 36°, y = 85°, z = 59°$ (ii) $x = 26°, y = 26°, z = 232°$ (iii) $x = 47°, y = 43°$

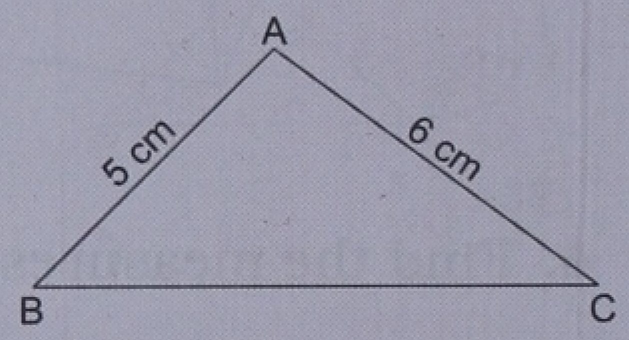
4. (i) $x = 45°, y = 55°$ (ii) $x = 45°, y = 37°$ (iii) $x = 71°, y = 121°$ (iv) $x = 60°, y = 120°$
(v) $x = 45°, y = 47°, z = 43°$ (vi) $x = 40°$ (vii) $x = 40°, y = 105°, z = 40°$ (viii) $x = 20°, y = 20°, z = 84°$

5. 35°, 55°, 90° **6.** 25°, 65° **7.** 40°, 40°, 100° **8.** 64°, 48° **9.** 66°, 16°

Inequalities in a Triangle

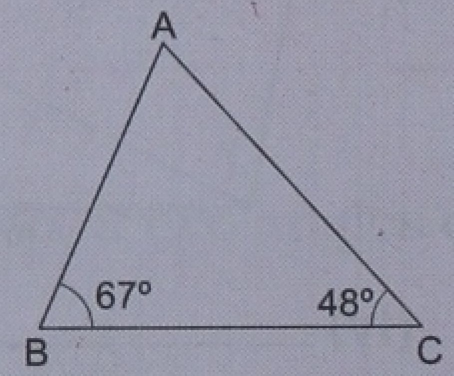
INEQUALITY 1 **If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.**

In the figure, $AC > AB$, so, $\angle ABC > \angle ACB$.



CONVERSE **In a triangle, the side opposite to the greater angle is greater.**

In the figure, $\angle ABC > \angle ACB$, so, $AC > AB$.

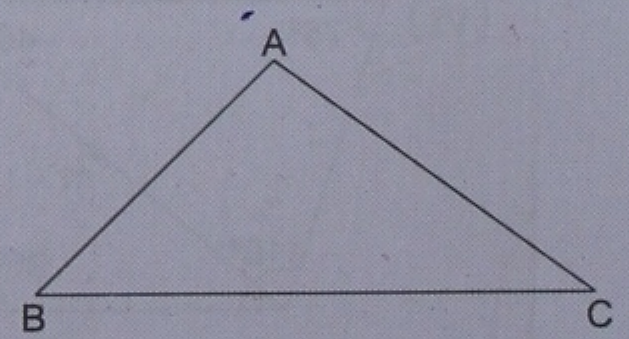


INEQUALITY 2 **Any two sides of a triangle are together greater than the third side.**

In other words, in any $\triangle ABC$, $AB + AC > BC$, $BC + BA > CA$, $CA + CB > AB$.

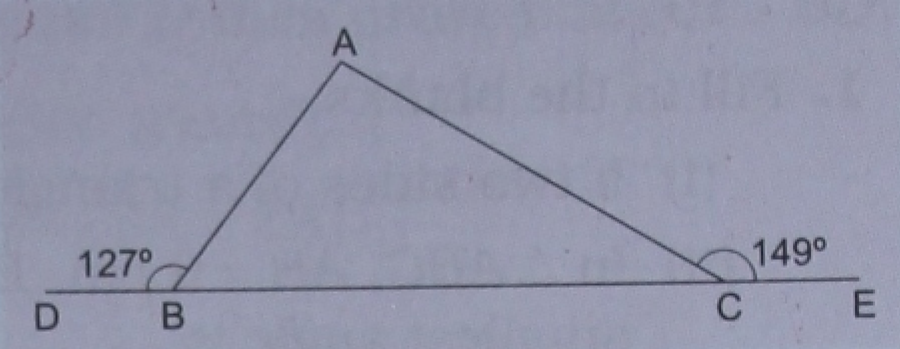
INEQUALITY 3 **The difference of any two sides of a triangle is smaller than the third side.**

In the figure, $AC - AB < BC$, $BC - AC < AB$, $BC - AB < AC$.



Solved Examples

EXAMPLE 1 **In the adjoining figure, $\angle ABD = 127°$ and $\angle ACE = 149°$. Prove that $AC > AB$. Also, arrange the sides of $\triangle ABC$ in descending order.**



Solution $\because$ $\angle ABD + \angle ABC = 180°$,

$\angle ABC = 180° - 127° = 53°$.

Again, $\angle ACE + \angle ACB = 180°$ or $\angle ACB = 180° - 149° = 31°$.

$\therefore$ $\angle ABC > \angle ACB \Rightarrow AC > AB$.

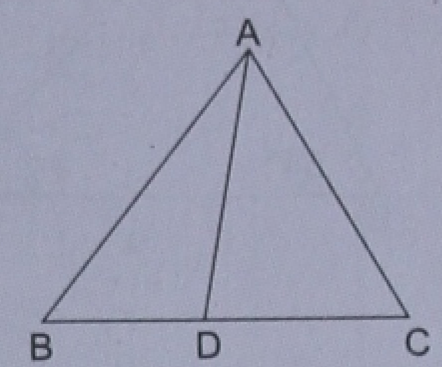
($\because$ the side opposite the greater angle is greater)

In $\triangle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180°$.

or $\angle BAC + 53° + 31° = 180° \Rightarrow \angle BAC = 180° - 84° = 96°$.

Thus, $\angle BAC > \angle ABC > \angle ACB \Rightarrow BC > AC > AB$.

EXAMPLE 2 **In $\triangle ABC$, $AB > AC$ and D is any point on the side BC. Prove that $AB > AD$.**



Solution $AB > AC \Rightarrow \angle ACB > \angle ABC$. ... (1)

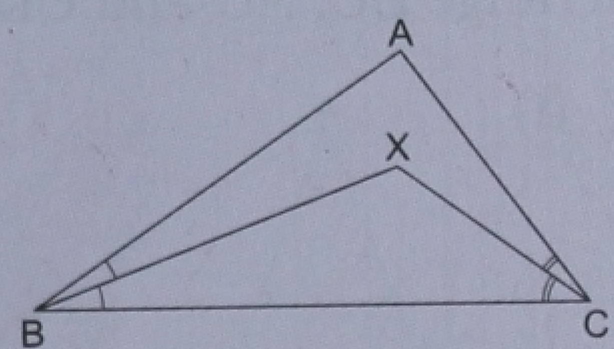
($\because$ greater angle is opposite the greater side)

In $\triangle ACD$, exterior $\angle ADB = \angle ACD + \angle CAD$

$\Rightarrow \angle ADB > \angle ACD \Rightarrow \angle ADB > \angle ACB$. ... (2)

From (1) and (2), $\angle ADB > \angle ABC = \angle ABD$. $\therefore$ In $\triangle ABD$, $AB > AD$.

EXAMPLE 3 **In $\triangle ABC$, $AB > AC$ and the bisectors of $\angle B$ and $\angle C$ meet at the point X. Prove that $XB > XC$.**



Solution BX is the bisector of $\angle B$.

$\therefore$ $\angle XBC = \frac{1}{2}\angle ABC$ ... (1)

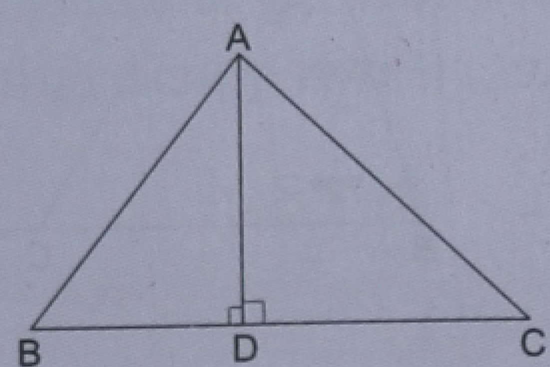
CX is the bisector of $\angle C$.

$\therefore$ $\angle XCB = \frac{1}{2}\angle ACB$ ... (2)

Given that $AB > AC \Rightarrow \angle ACB > \angle ABC$

$\therefore$ $\frac{1}{2}\angle ACB > \frac{1}{2}\angle ABC \Rightarrow \angle XCB > \angle XBC \Rightarrow XB > XC$.

EXAMPLE 4 **In the figure, AD is perpendicular to BC. Prove that $AB + AC > 2AD$.**



Solution In $\triangle ABD$, $\angle ADB$ is the greatest angle as $\angle ADB = 90°$.

$\therefore$ $\angle ADB > \angle ABD \Rightarrow AB > AD$. ... (1)

In $\triangle ACD$, $\angle ADC$ is the greatest angle as $\angle ADC = 90°$.

$\therefore$ $\angle ADC > \angle ACD \Rightarrow AC > AD$ ... (2)

Adding (1) and (2), we get $AB + AC > 2AD$.

EXERCISE 2B

1. Fill in the blanks.

(i) If two sides of a triangle are unequal, the angle opposite the greater side is ....

(ii) In $\triangle ABC$, $AB = 6$ cm, $BC = 8$ cm and $AC = 9$ cm. The greatest angle is ...... and the smallest angle is ....

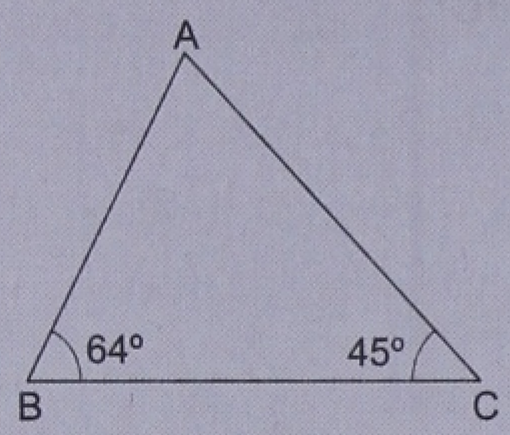
(iii) In $\triangle XYZ$, $\angle X = 60°$, $\angle Y = 75°$. The greatest side is ...... and the smallest side is ....

(iv) In a triangle, the sum of the lengths of any two sides is ...... than the third side.

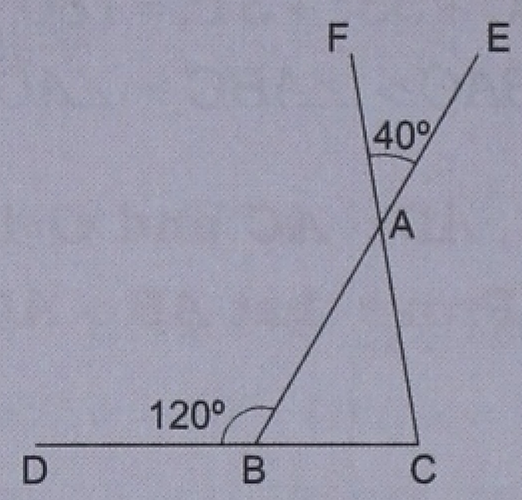
(v) In a triangle, the difference of any two sides is ...... than the third side.

2. Find the greatest side and the smallest side of $\triangle ABC$ in each of the following figures.

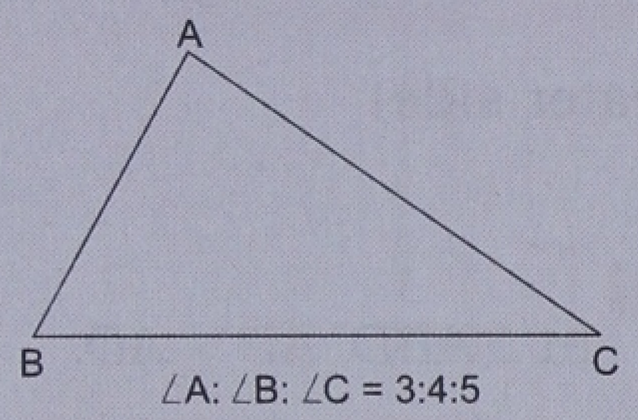
(i)



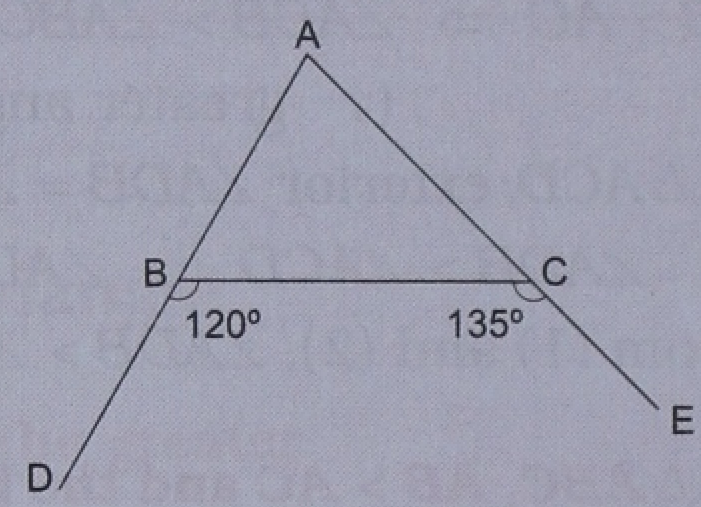
(ii)



(iii)

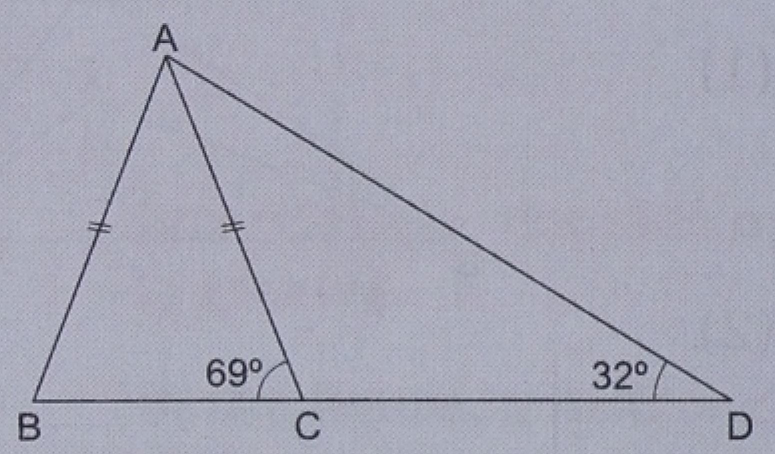


(iv)

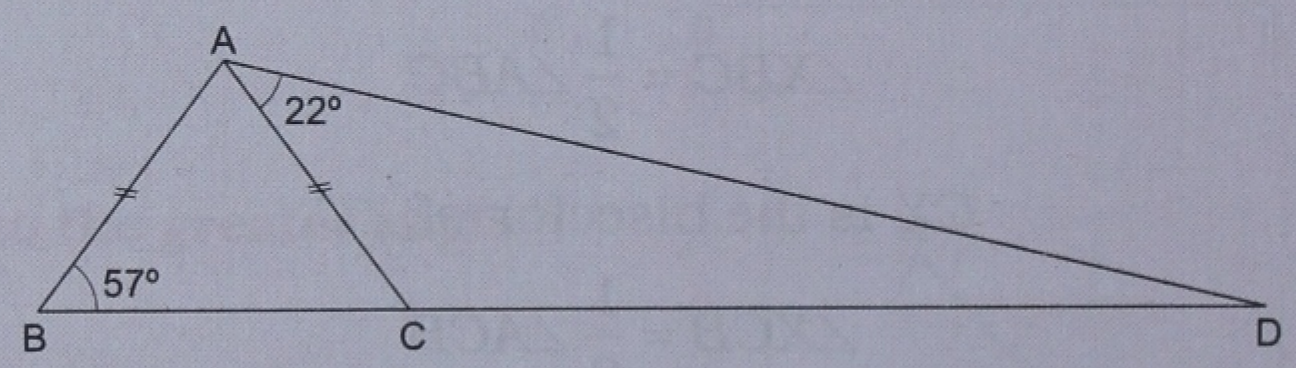


3. Arrange BC, AC and CD in descending order.

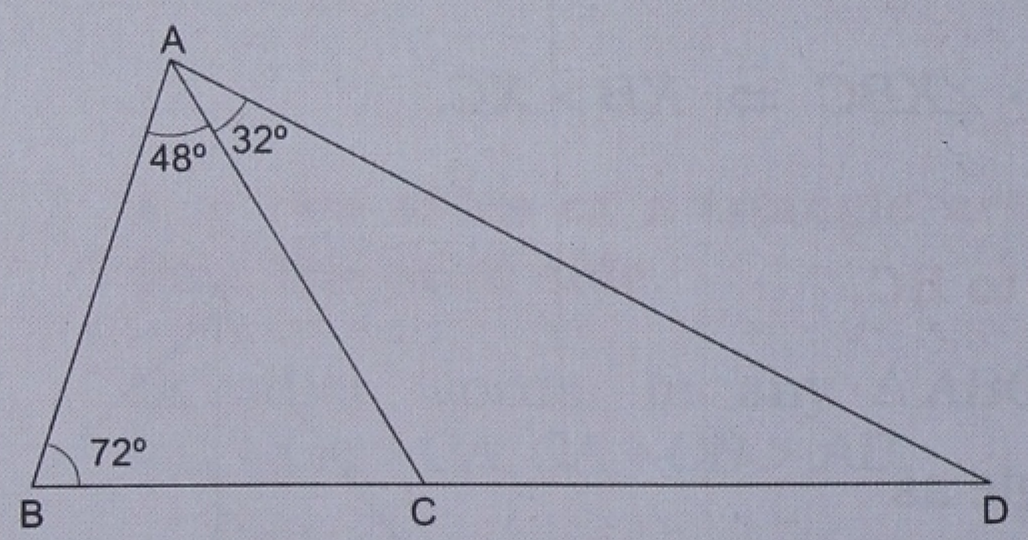
(i)



(ii)



(iii)



4. In $\triangle ABC$, $\angle B = 83°$ and $\angle C = 33°$. The bisector of $\angle A$ meets BC at the point D. Then which is larger, AD or DC?

5. O is any point on the side AB of an equilateral triangle ABC. Arrange AC, AO, CO in descending order.

6. O is any point on the side AB of an isosceles triangle ABC in which $CA = CB$. Prove that $CA > CO$.
7. AD is the bisector of $\angle A$ in $\triangle ABC$. If AD meets BC at the point D then prove that $AB > BD$.
8. Prove that the hypotenuse is the largest side of a right-angled triangle.
9. If D is a point on the side BC of $\triangle ABC$, prove that $AB + BC + CA > 2AD$.

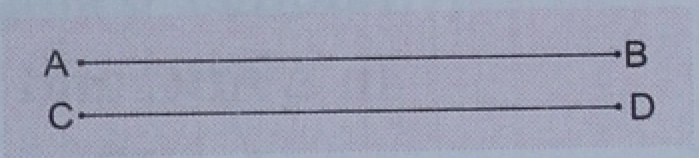
ANSWERS

1. (i) Greater (ii) $\angle B$, $\angle C$ respectively (iii) XZ, XY respectively (iv) Greater (v) Smaller
2. (i) BC, AB (ii) AB, BC (iii) AB, BC (iv) BC, AB
3. (i) $CD > AC > BC$ (ii) $BC > AC > CD$ (iii) $CD > AC > BC$
4. $AD > DC$
5. $AC > CO > AO$

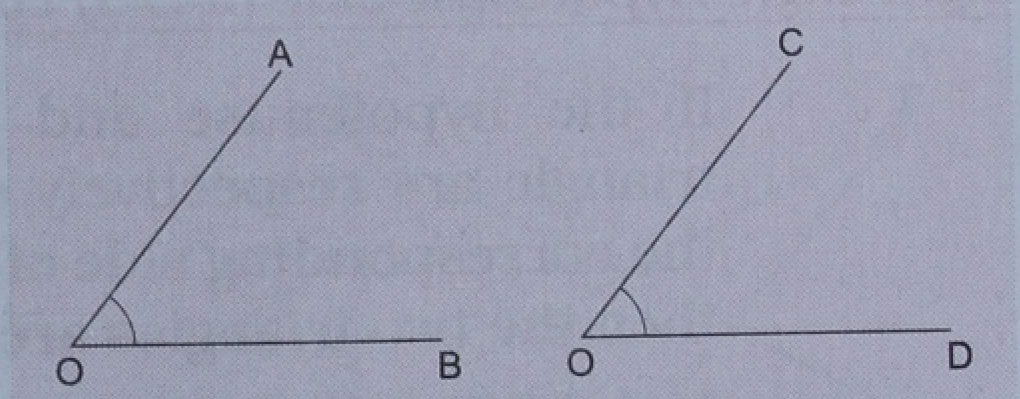
Congruence

If the shape and the size of two geometrical figures l and m be the same then we say that l and m are congruent and denote this as $l \cong m$. Two congruent figures can be superimposed on each other.

Examples (i) When two line segments are of equal length, they are congruent. Also, if two line segments are congruent, their lengths are equal.



(ii) If two angles are of equal measure, they are congruent. If $\angle AOB = \angle COD$ then $\angle AOB \cong \angle COD$. Also, if two angles are congruent then they are of equal measure.

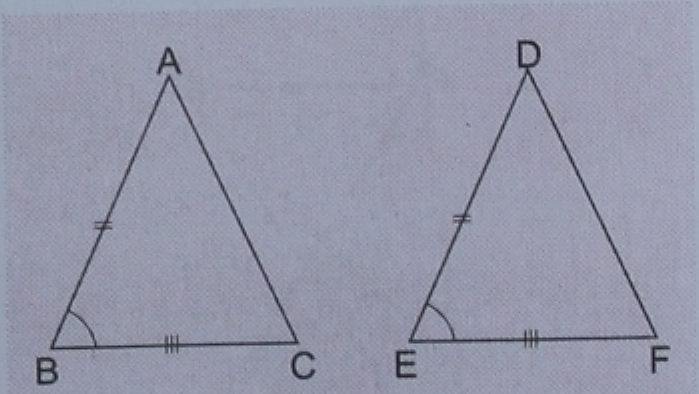


Congruency of two triangles

To prove that two triangles are congruent, we need not prove that the six elements of one are equal to the corresponding six elements of the other. Any of the following four conditions is enough to prove the congruency two triangles.

Side-Angle-Side (S-A-S) condition

If any two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle then the two triangles are congruent.

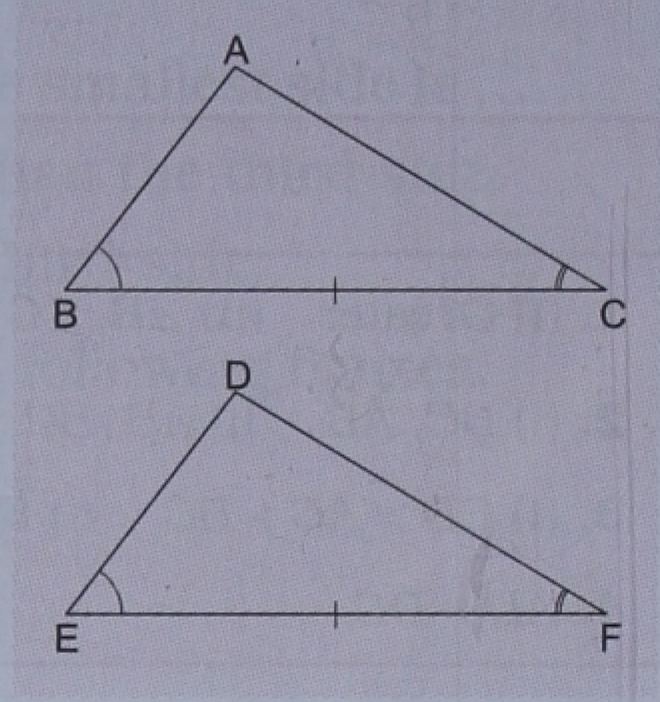


In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $BC = EF$ and $\angle ABC = \angle DEF$.

$\therefore \quad \triangle ABC \cong \triangle DEF$.

Angle-Side-Angle (A-S-A) condition

If two angles and the included side of a triangle are equal to the corresponding two angles and the included side of the other triangle then the two triangles are congruent.

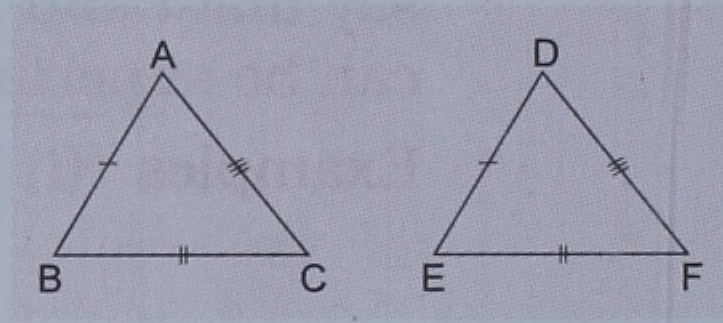


In $\triangle ABC$ and $\triangle DEF$, if $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BC = EF$ then $\triangle ABC \cong \triangle DEF$.

If two angles of $\triangle ABC$ are equal to two angles of $\triangle DEF$ then the third angle of $\triangle ABC$ will be equal to the third angle of $\triangle DEF$ because the sum of the three angles of any triangle is 180°. Thus, we can say that two triangles are congruent if any two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle. This condition for congruence is denoted by Angle-Angle-Side (A-A-S).

Side-Side-Side (S-S-S) condition

If three sides of one triangle are equal to three sides of another triangle then the two triangles are congruent.

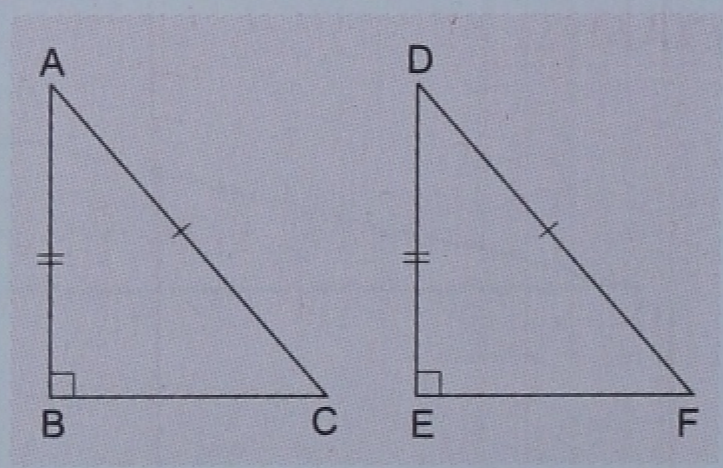


In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $BC = EF$ and $CA = FD$.

$\therefore \quad \triangle ABC \cong \triangle DEF$.

Right angle-Hypotenuse-Side (R-H-S) condition

If the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and the corresponding side of another right-angled triangle then the two triangles are congruent.

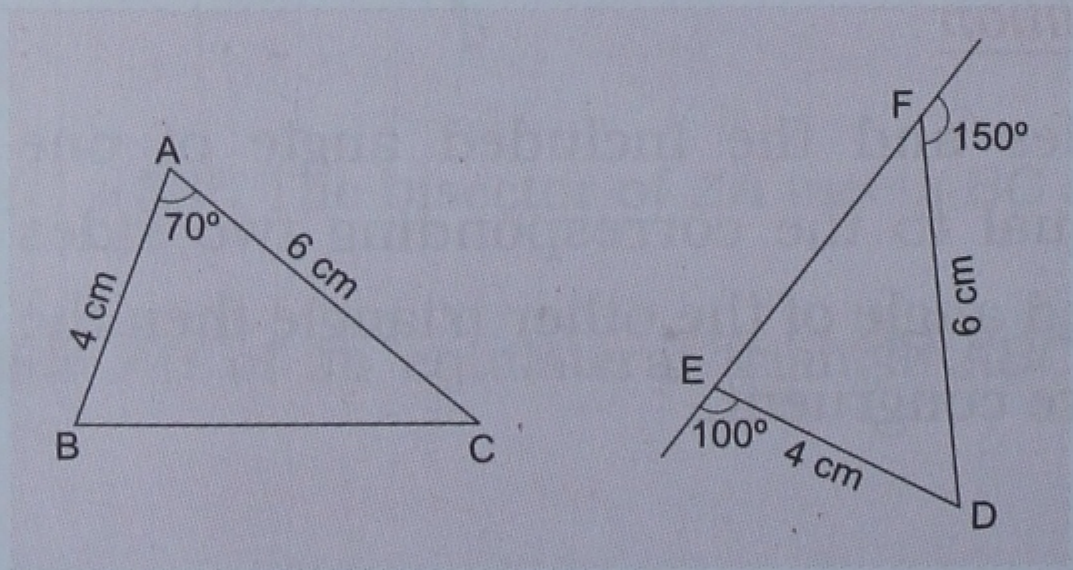


In $\triangle ABC$ and $\triangle DEF$, $\angle ABC = \angle DEF = 90°$,
hypotenuse AC = hypotenuse DF and $AB = DE$.

$\therefore \quad \triangle ABC \cong \triangle DEF$.

Solved Examples

EXAMPLE 1 **State whether the triangles in the figure are congruent.**



Solution In $\triangle DEF$, $\angle DEF = 180° - 100° = 80°$, $\angle EFD = 180° - 150° = 30°$.

$\therefore \quad \angle EDF = 180° - (80° + 30°) = 70°$.

In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $\angle BAC = \angle EDF (= 70°)$ and $AC = DF$.

So, the S-A-S condition of congruence is satisfied.

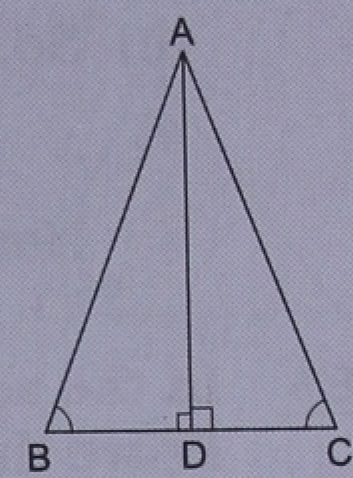
$\therefore \quad \triangle ABC \cong \triangle DEF$.

EXAMPLE 2 **In the adjoining figure, $\angle ABD = \angle ACD$ and $AD \perp BC$. Prove that**

(i) $\triangle ABD \cong \triangle ACD$

(ii) D is the mid-point of BC and

(iii) $\angle BAD = \angle CAD$, that is, AD bisects $\angle BAC$.



Solution (i) Given, $\angle ABD = \angle ACD \Rightarrow AC = AB$.

In $\triangle ABD$ and $\triangle ACD$, $\angle ABD = \angle ACD$, $\angle ADB = \angle ADC (= 90°)$ and $AB = AC$.

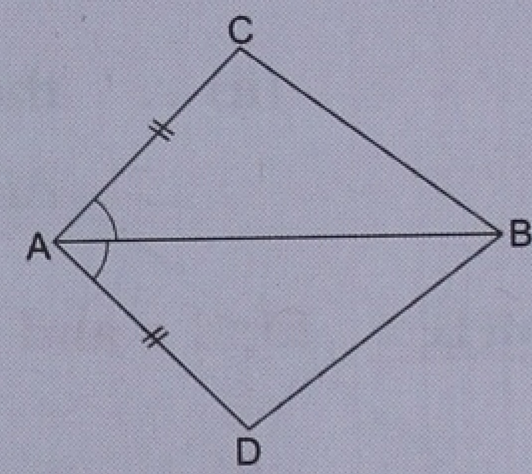
So, the A-A-S condition of congruence is satisfied. $\therefore \quad \triangle ABD \cong \triangle ACD$.

(ii) $\therefore$ the corresponding parts of $\triangle ABD$ and $\triangle ACD$ are equal.

$\therefore \quad BD = CD$, that is, D is the mid-point of BC.

(iii) Also, $\angle BAD = \angle CAD$, that is, AD bisects $\angle BAC$.

EXAMPLE 3 **In the adjoining figure, AB bisects $\angle CAD$ and $AC = AD$. Prove that (i) $\triangle ABC \cong \triangle ABD$ and (ii) $BC = BD$.**



Solution (i) In $\triangle ABC$ and $\triangle ABD$, $AC = AD$ (given),

$\angle BAC = \angle BAD$ ($\because$ AB bisects $\angle CAD$)

and $AB = AB$.

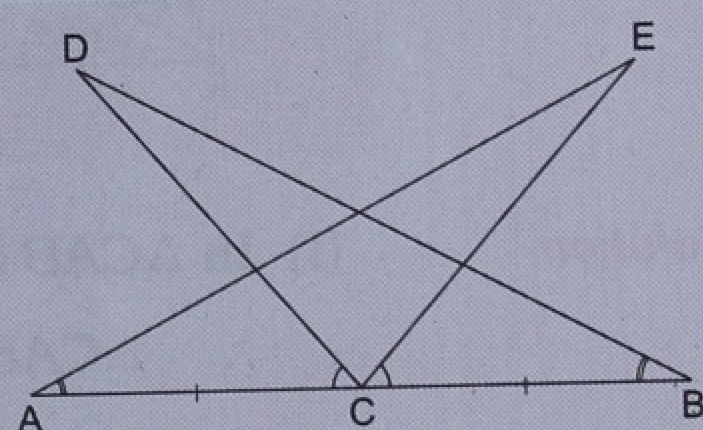
$\therefore$ the S-A-S condition is satisfied.

$\therefore \quad \triangle ABC \cong \triangle ABD$.

(ii) $\therefore$ the corresponding sides of $\triangle ABC$ and $\triangle ABD$ are equal. So, $BC = BD$.

EXAMPLE 4 **In the adjoining figure, prove that**

(i) $\triangle DBC \cong \triangle EAC$ and (ii) $DC = EC$.



Solution (i) Given that $\angle ECB = \angle DCA$.

Adding $\angle DCE$ to both sides,

$\angle ECB + \angle DCE = \angle DCA + \angle DCE$

$\Rightarrow \quad \angle DCB = \angle ECA$. ... (1)

In $\triangle DBC$ and $\triangle EAC$, $\angle DCB = \angle ECA$ [using (1)], $BC = AC$ (given)

and $\angle DBC = \angle EAC$ (given).

$\therefore$ the A-S-A condition is satisfied.

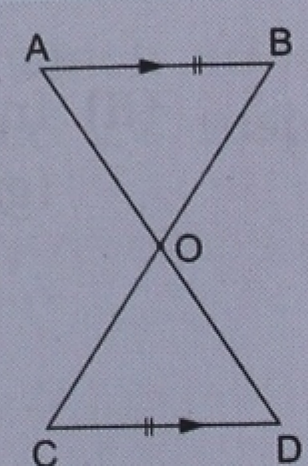
$\therefore \quad \triangle DBC \cong \triangle EAC$.

(ii) $\therefore$ the corresponding sides of $\triangle DBC$ and $\triangle EAC$ are equal. So, $DC = EC$.

EXAMPLE 5 **In the adjoining figure, $AB = CD$ and $AB \parallel CD$. Prove that**

(i) $\triangle AOB \cong \triangle DOC$ and

(ii) AD and BC bisect each other at the point O.



Solution $AB \parallel CD \Rightarrow \angle OAB$ = alternate $\angle ODC$ and $\angle OBA$ = alternate $\angle OCD$.

(i) In $\triangle AOB$ and $\triangle DOC$, $\angle OAB = \angle ODC$, $\angle OBA = \angle OCD$ and $AB = CD$.
So, the A-S-A condition of congruence is satisfied.

$\therefore \quad \triangle AOB \cong \triangle DOC$.

(ii) So, the corresponding sides of $\triangle AOB$ and $\triangle DOC$ are equal.

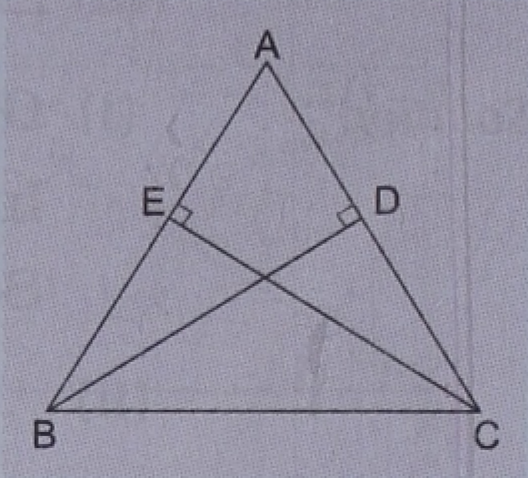
$\therefore \quad AO = DO$ and $BO = CO$.

Hence, AD and BC bisect each other at the point O.

EXAMPLE 6 **In the adjoining figure, $BD = CE$ and $\angle ADB = 90° = \angle AEC$. Prove that**

(i) $\triangle ABD \cong \triangle ACE$ and

(ii) ABC is an isosceles triangle in which $AB = AC$.



Solution (i) In $\triangle ABD$ and $\triangle ACE$, $\angle ADB = \angle AEC$ (= 90°) (given),
$\angle BAD = \angle CAE$ and $BD = CE$ (given).

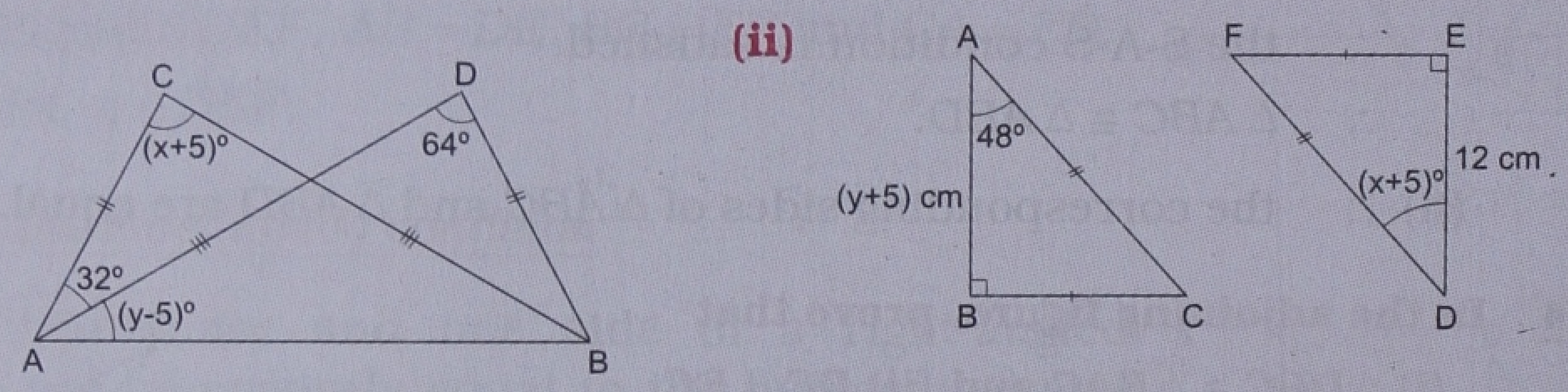
$\therefore$ the A-A-S condition of congruence is satisfied.

$\therefore \quad \triangle ABD \cong \triangle ACE$.

(ii) $\therefore$ the corresponding sides of $\triangle ABD$ and $\triangle ACE$ are equal.

$\therefore \quad AB = AC$, that is, ABC is an isosceles triangle.

EXAMPLE 7 **Find x and y in each of the following figures.**



Solution (i) In $\triangle CAB$ and $\triangle DBA$, $AC = BD$ (given), $BC = AD$ (given) and $AB = AB$.

$\therefore \quad \triangle CAB \cong \triangle DBA$ (S-S-S condition)

$\therefore \quad \angle ACB = \angle BDA$, $\angle CBA = \angle DAB = (y-5)°$ and $\angle CAB = \angle DBA$

$\therefore \quad x + 5 = 64$ or $x = 59$.

In $\triangle ABC$, $(x+5)° + 32° + (y-5)° + (y-5)° = 180°$

$\Rightarrow 59 + 5 + 32 + y - 5 + y - 5 = 180$

$\Rightarrow 2y + 86 = 180 \Rightarrow y = \frac{180-86}{2} = \frac{94}{2} = 47.$

$\therefore \quad x = 59, y = 47.$

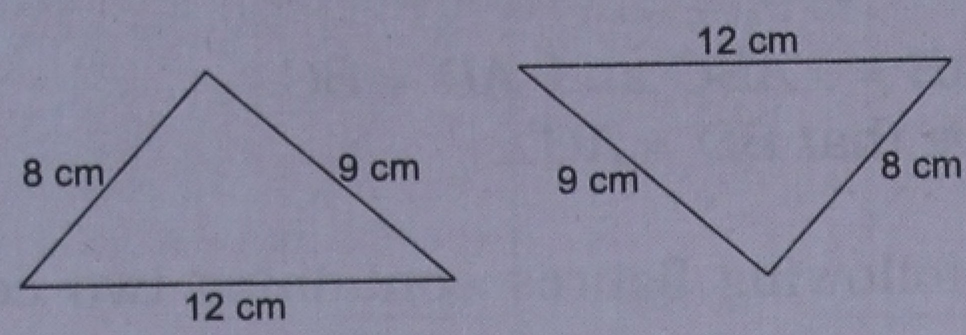
(ii) In $\triangle ABC$ and $\triangle DEF$, $\angle ABC = \angle DEF = 90°$, hypotenuse AC = hypotenuse DF (given) and $BC = EF$ (given).

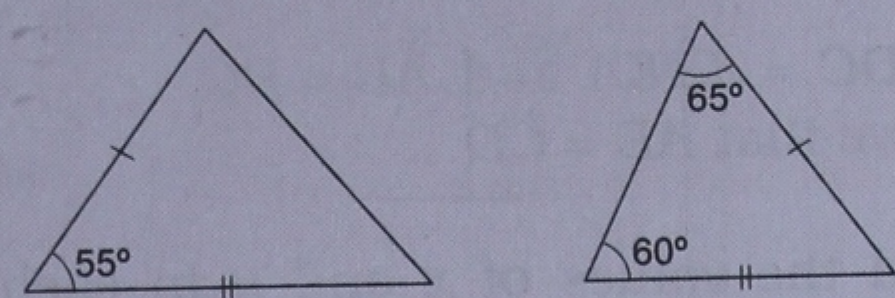
$\therefore \quad \triangle ABC \cong \triangle DEF$. (R-H-S condition)

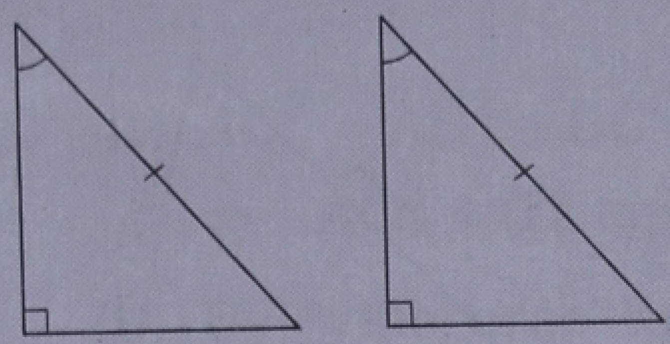
$\therefore \quad 48° = x + 5°$ and $y + 5 = 12 \Rightarrow x = 43°$ and $y = 7$.

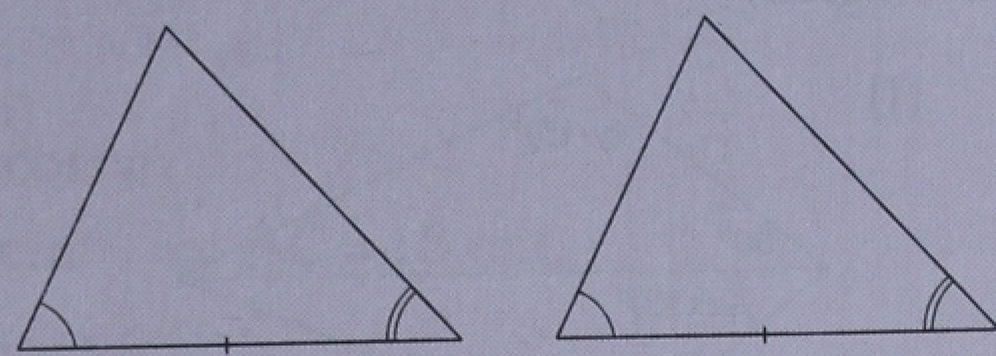
EXERCISE 2C

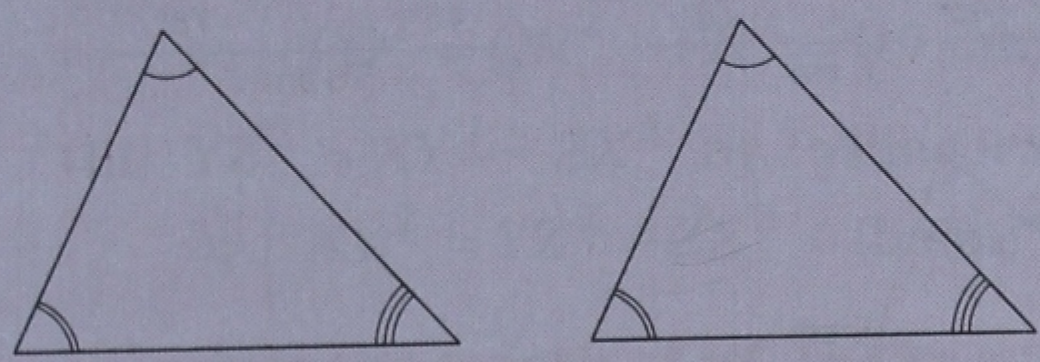
1. Which of the following pairs of triangles are congruent? Mention the condition for congruence.

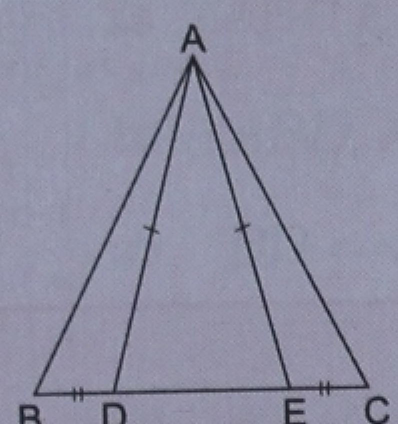
(i)


(ii)


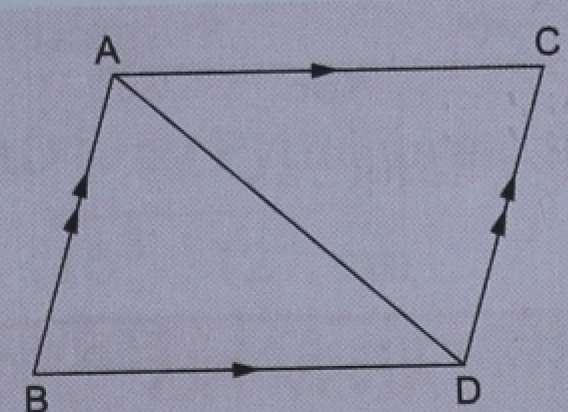
(iii)


(iv)


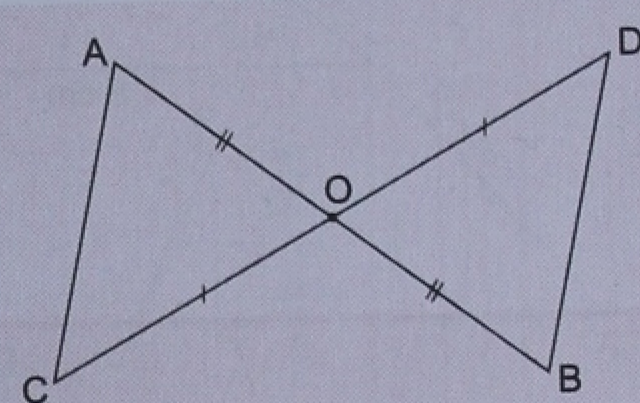
(v)


2.


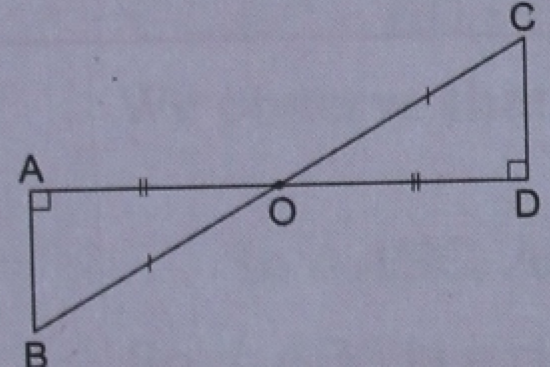
Prove that $AB = AC$.

3.


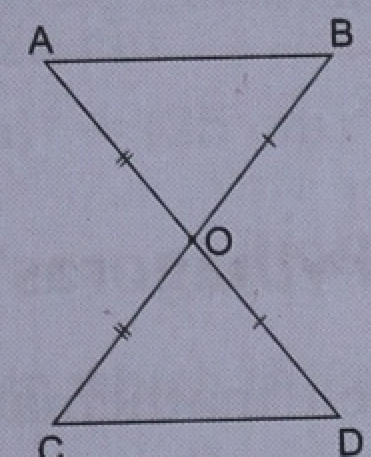
Prove that $AB = CD$ and $AC = BD$.

4.


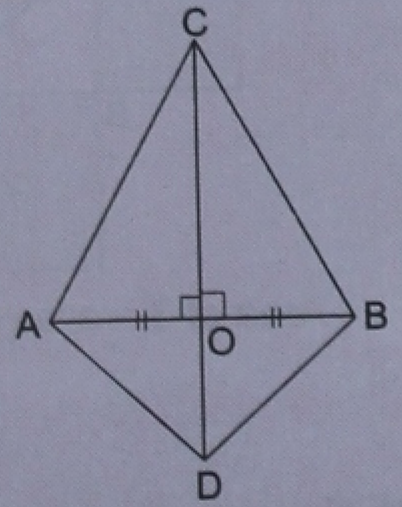
Prove that $AC = DB$.

5.


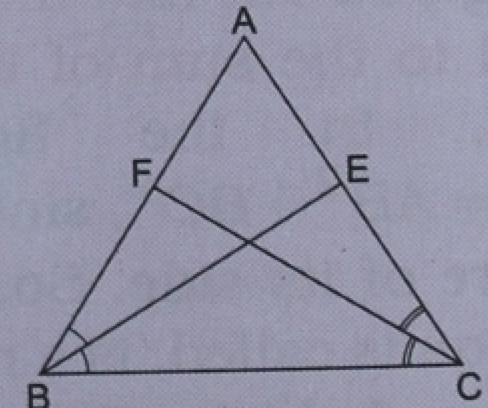
Prove that $\angle ABO = \angle DCO$.

6.


Prove that $AB = CD$.

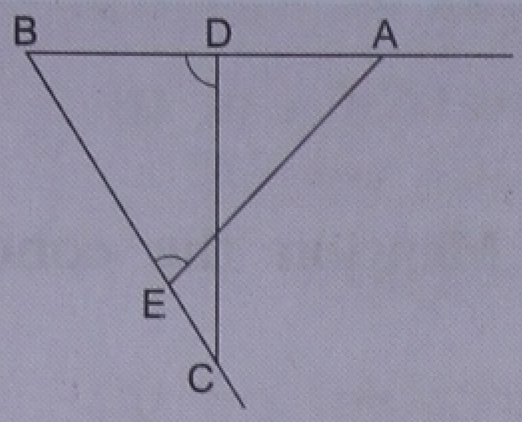
7.


Prove that (i) $\triangle AOC \cong \triangle BOC$
(ii) $\triangle AOD \cong \triangle BOD$

8.


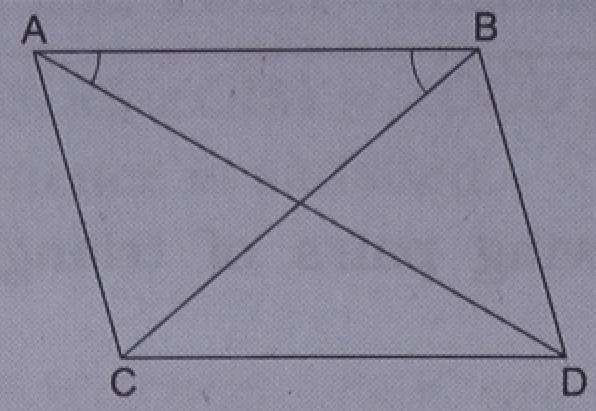
ABC is an isosceles triangle in which $AB = AC$; BE and CF are bisectors of $\angle B$ and $\angle C$ respectively. Prove that $BE = CF$.

9.



$\angle BDC = \angle BEA$ and $AB = BC$.
Prove that $AE = CD$.

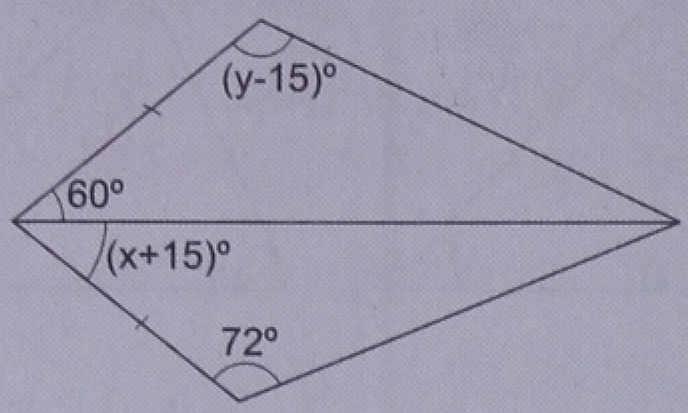
10.



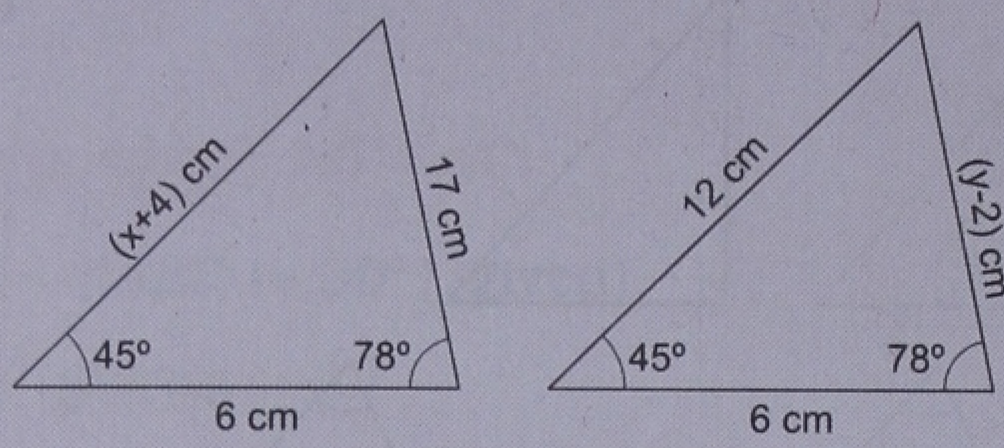
$\angle DAB = \angle ABC$ and $AD = BC$.
Prove that $BD = AC$.

11. Find the values of x and y in each of the following figures containing two congruence triangles.

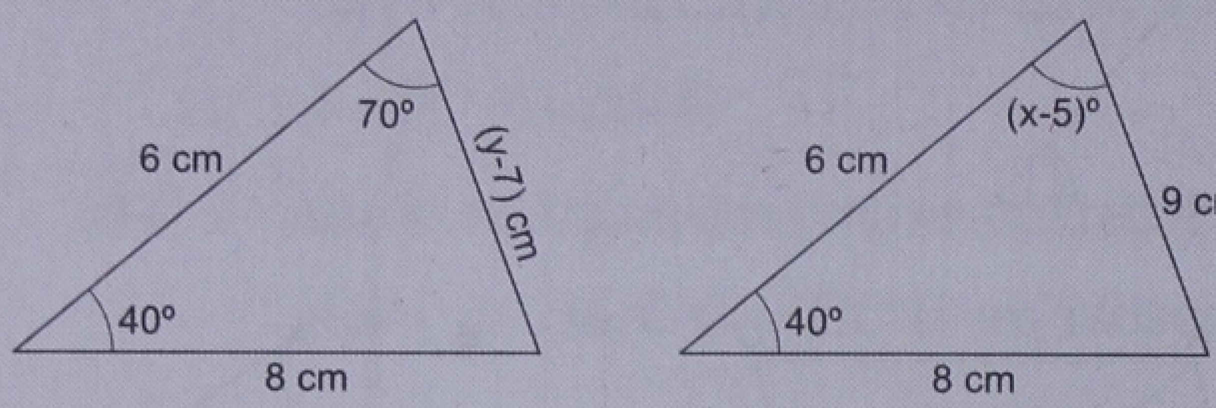
(i)



(ii)



(iii)



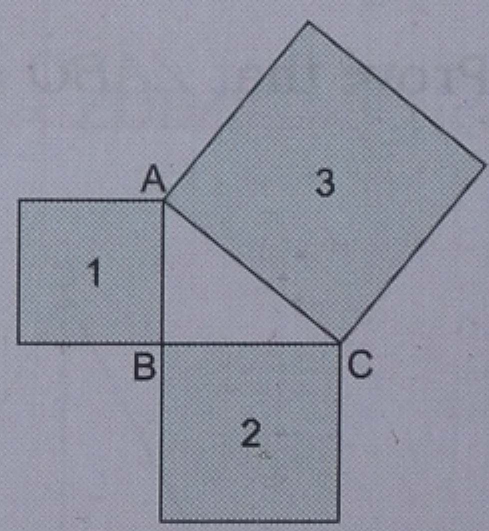
ANSWERS

1. (i) S-S-S (ii) S-A-S (iii) A-A-S (iv) A-S-A (v) Not necessarily congruent

4. (i) $x = 45, y = 87$ (ii) $x = 8, y = 19$ (iii) $x = 75, y = 16$

Pythagoras' Theorem

Pythagoras was a Greek mathematician who came up with a relation between the areas of the squares (constructed) on the sides of a right-angled triangle. He figured out that the square on the hypotenuse is equal to the sum of the squares on the other two sides. In the figure, this would mean $AC^2 = AB^2 + BC^2$, since the area of a square is the square of its side. So, Pythagoras' theorem, as the relation is called, is a relation between the sides of a right-angled triangle as well.

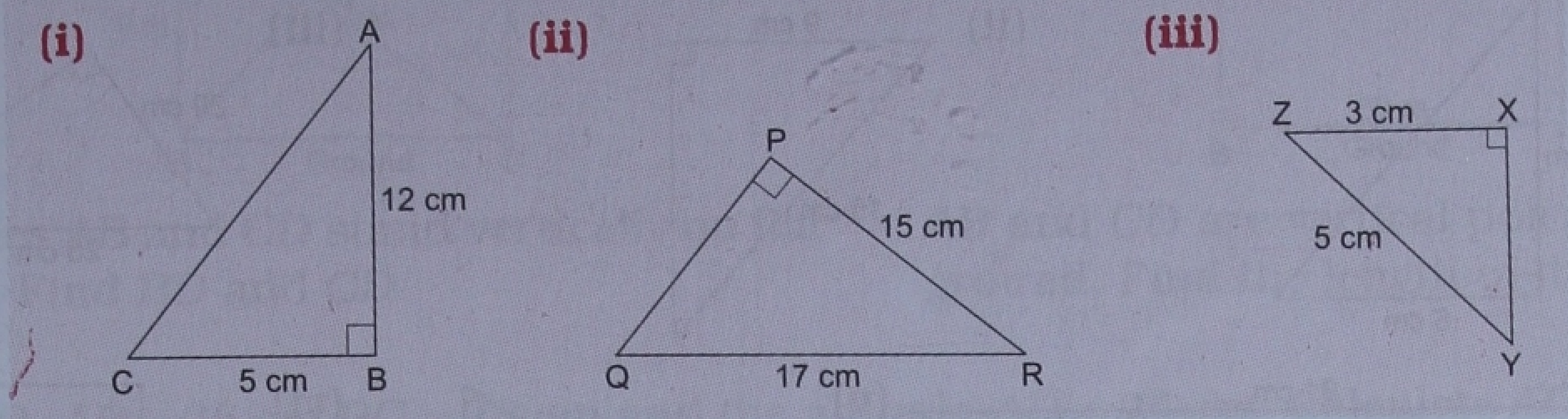


Area of 3 = area of 1 + area of 2

The converse of the theorem is also true. That means if the square on one side of a triangle is equal to the sum of the squares on the other two sides then those two sides contain a right angle and the triangle is a right-angled triangle. To put it simply, if a $\triangle ABC$ is such that $AC^2 = AB^2 + BC^2$ then $\angle B = 90°$ and $\triangle ABC$ is right-angled.

Solved Examples

EXAMPLE 1 **Calculate the length of the unknown side in each of the following triangles.**



Solution (i) $AC^2 = AB^2 + BC^2$, by Pythagoras' theorem

$\Rightarrow \quad AC^2 = (12 \text{ cm})^2 + (5 \text{ cm})^2 = 169 \text{ cm}^2 \quad \Rightarrow \quad AC = 13$ cm.

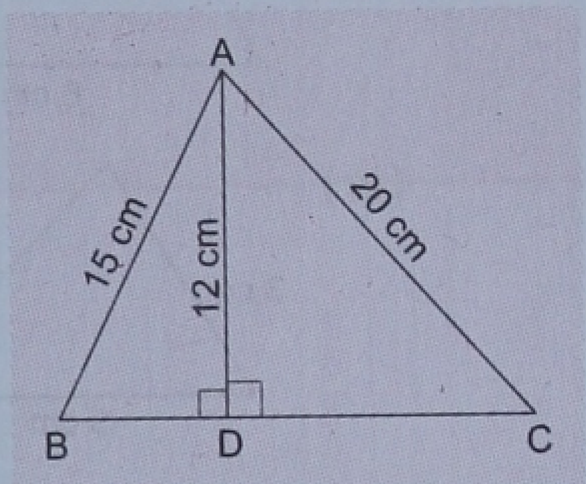
(ii) By Pythagoras' theorem, $QR^2 = PQ^2 + PR^2$

$\Rightarrow \quad PQ^2 = QR^2 - PR^2 = (17 \text{ cm})^2 - (15 \text{ cm})^2 = 64 \text{ cm}^2 \quad \Rightarrow \quad PQ = 8$ cm.

(iii) $YZ^2 = XY^2 + ZX^2$, by Pythagoras' theorem

$\Rightarrow \quad XY^2 = YZ^2 - ZX^2 = (5 \text{ cm})^2 - (3 \text{ cm})^2 = 16 \text{ cm}^2 \quad \Rightarrow \quad XY = 4$ cm.

EXAMPLE 2 **Calculate the length of BC in the adjoining figure. Is $\triangle ABC$ a right-angled triangle?**



Solution In $\triangle ABD$, $AB^2 = BD^2 + AD^2$, by Pythagoras' theorem

$\Rightarrow \quad BD = \sqrt{AB^2 - AD^2} = \sqrt{15^2 - 12^2}$ cm

$= \sqrt{225 - 144}$ cm $= \sqrt{81}$ cm $= 9$ cm.

In $\triangle ACD$, $AC^2 = AD^2 + DC^2$, by Pythagoras' theorem

$\Rightarrow \quad DC = \sqrt{AC^2 - AD^2} = \sqrt{20^2 - 12^2}$ cm $= \sqrt{400 - 144}$ cm $= \sqrt{256}$ cm $= 16$ cm.

$\therefore \quad BC = BD + DC = (9 + 16)$ cm $= 25$ cm.

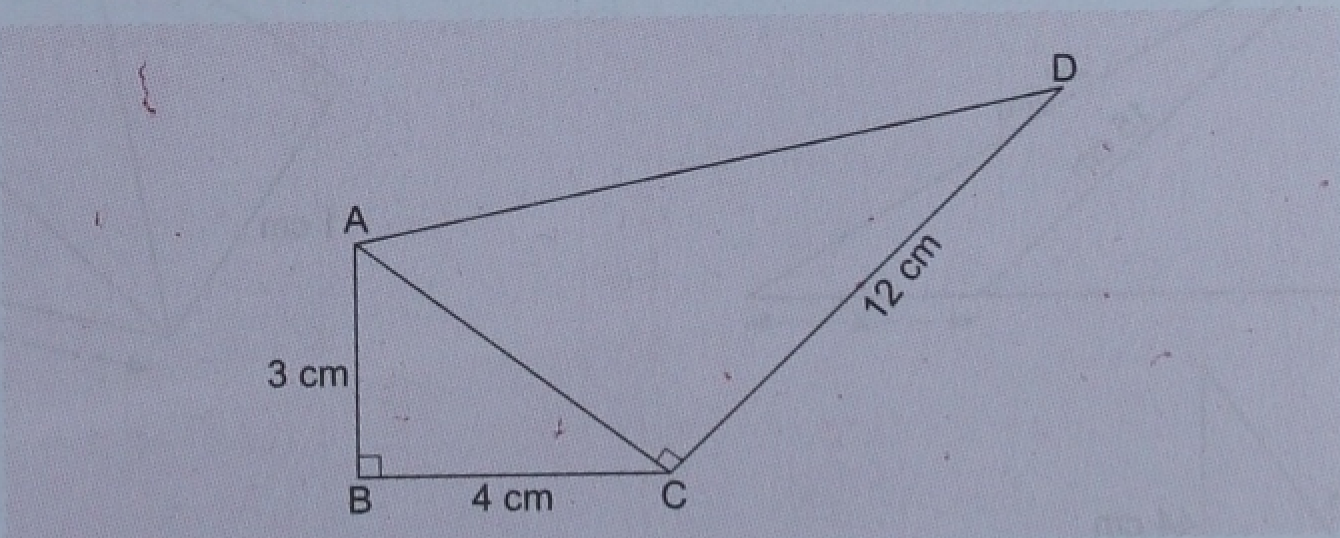
We observe that $(15 \text{ cm})^2 + (20 \text{ cm})^2 = (25 \text{ cm})^2$,

$\{\because\ 15^2 + 20^2 = 225 + 400 = 625 = 25^2\}$

$\therefore$ in $\triangle ABC$, $AB^2 + AC^2 = BC^2$.

So, $\triangle ABC$ is a right-angled triangle in which BC is the hypotenuse, that is, $\angle A = 90°$.

EXAMPLE 3 **Find AD in the following figure.**



Solution In the right angled $\triangle ABC$, $AC^2 = AB^2 + BC^2 = (3 \text{ cm})^2 + (4 \text{ cm})^2 = 25 \text{ cm}^2$

$\Rightarrow \quad AC = \sqrt{25}$ cm $= 5$ cm.

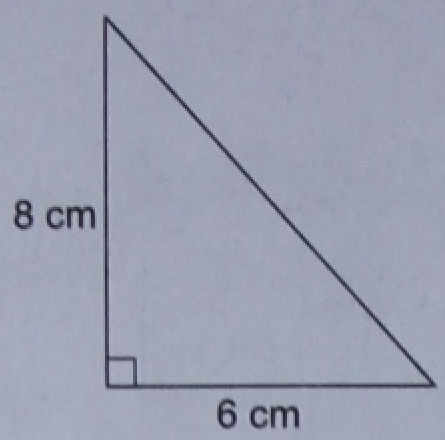
In the right angled $\triangle ACD$, $AD^2 = AC^2 + CD^2 = (5 \text{ cm})^2 + (12 \text{ cm})^2 = 169 \text{ cm}^2$

$\Rightarrow \quad AD = \sqrt{169}$ cm $= 13$ cm.

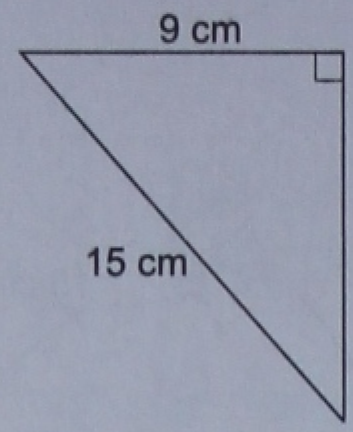
EXERCISE 2D

1. Calculate the length of the unknown side in each of the following right-angled triangles.

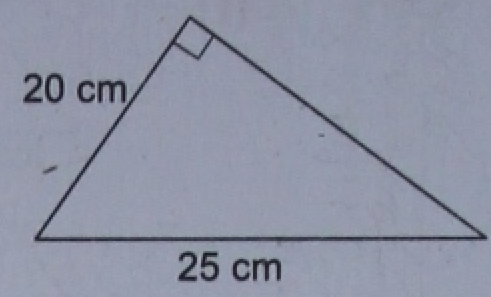
(i)



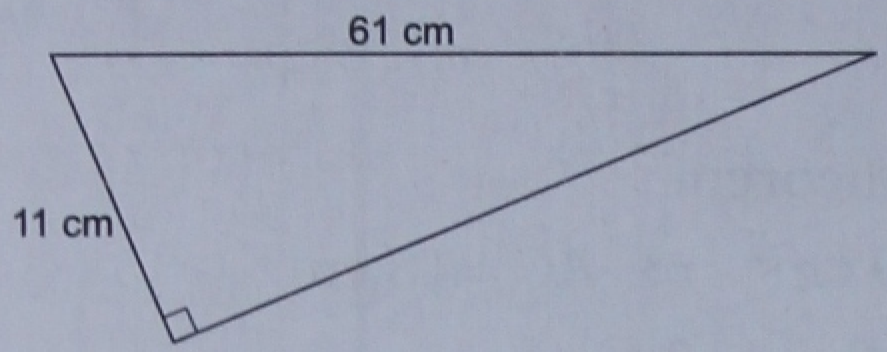
(ii)



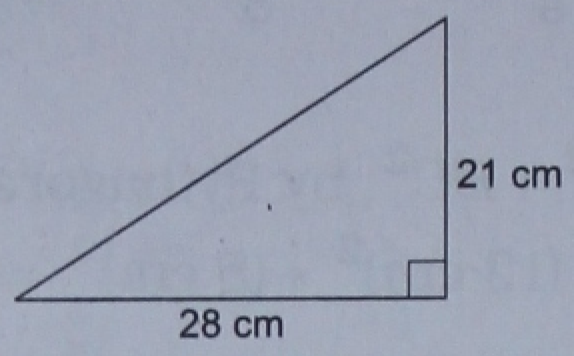
(iii)



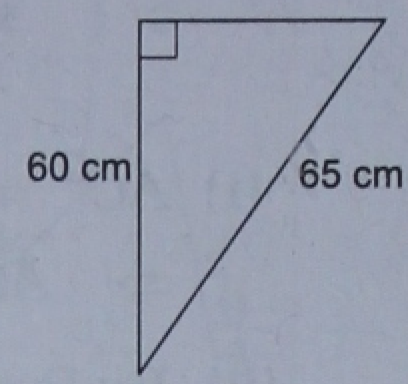
(iv)



(v)

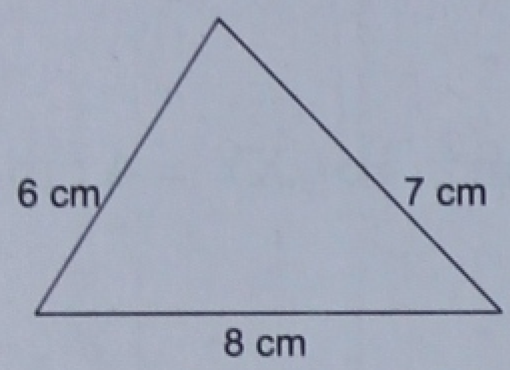


(vi)

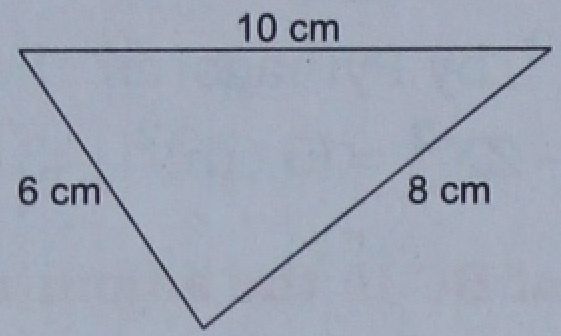


2. Which of the following triangles are right-angled?

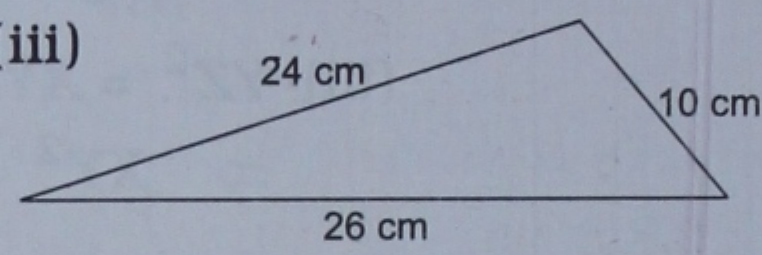
(i)



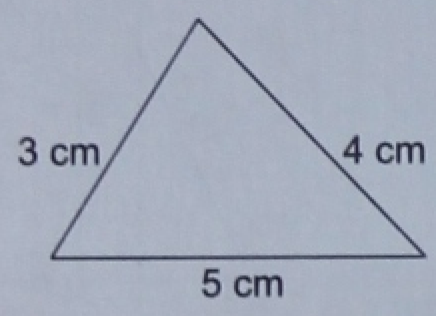
(ii)



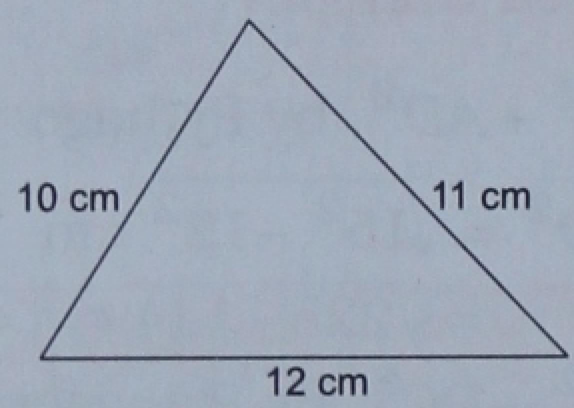
(iii)



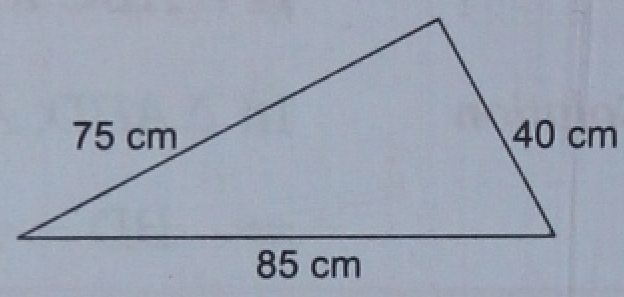
(iv)



(v)

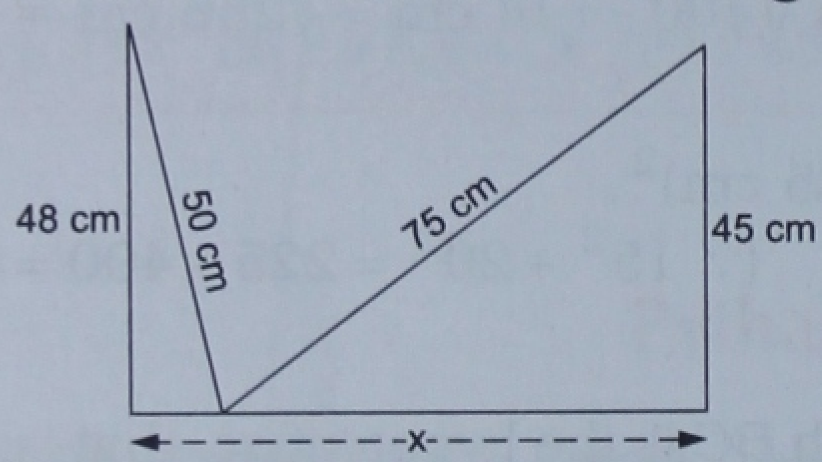


(vi)

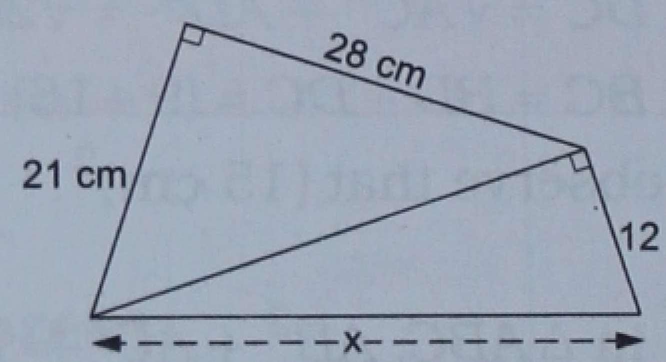


3. Calculate x in each of the following figures.

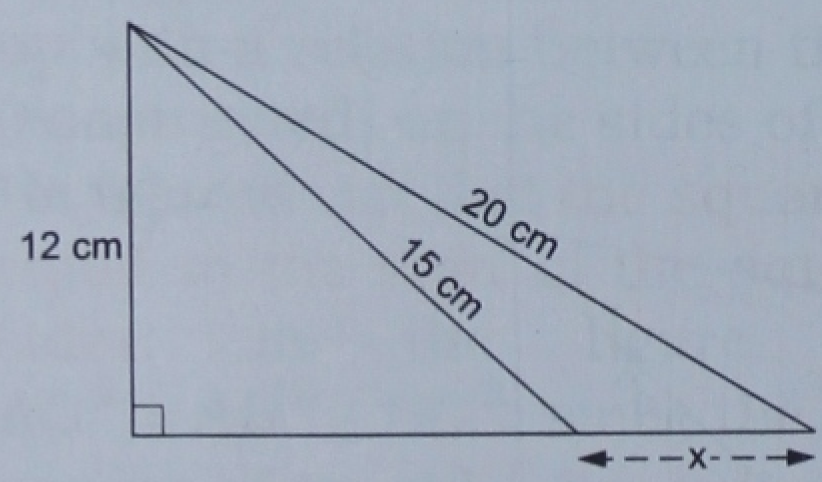
(i)



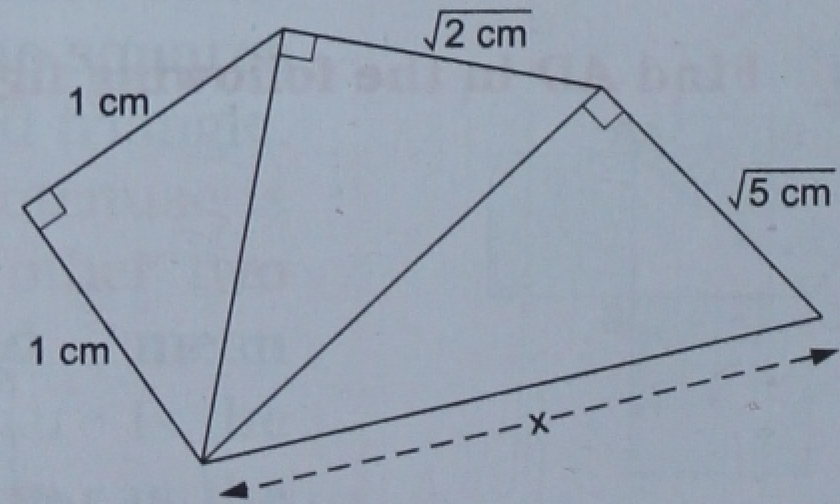
(ii)



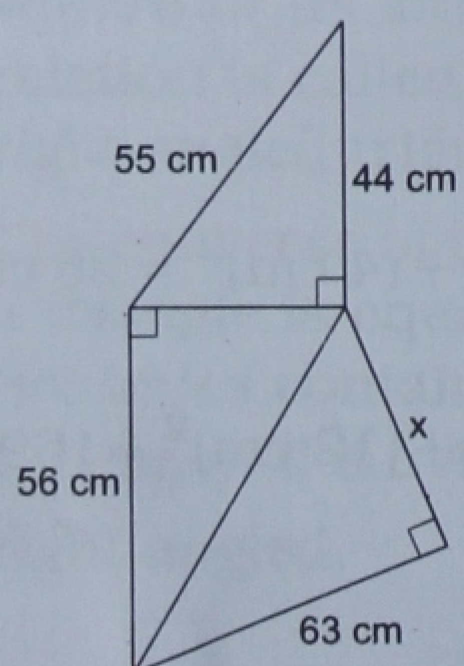
(iii)



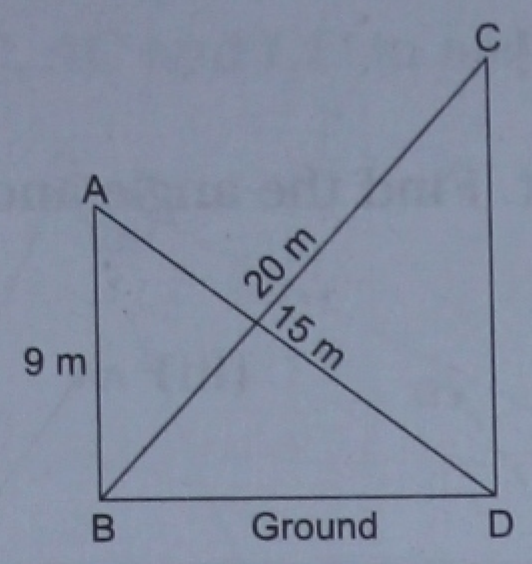
(iv)



(v)

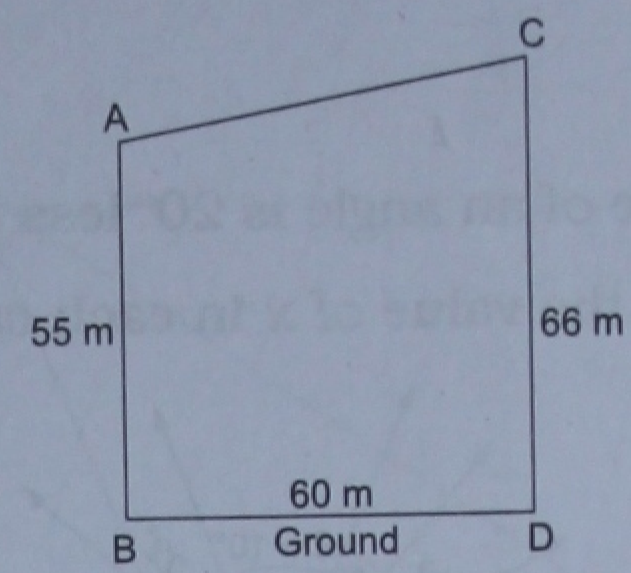


4.



Two poles AB and CD stand vertically on flat ground. Find BD and CD.

5.



AB and CD are vertical poles fixed to the ground. Find the length of the wire AC.

6. In $\triangle ABC$, $AB = 26$ cm $BC = 28$ cm and the altitude $AD = 24$ cm. Calculate AC.

7. In a quadrilateral $ABCD$, $\angle BAD = 90°$, $AB = 7$ cm, $BC = 15$ cm, $CD = 20$ cm and $DA = 24$ cm. Prove that $\angle BCD = 90°$.

ANSWERS

1. (i) 10 cm (ii) 12 cm (iii) 15 cm (iv) 60 cm (v) 35 cm (vi) 25 cm

2. (ii), (iii), (iv), (vi)

3. (i) 74 cm (ii) 37 cm (iii) 7 cm (iv) 3 cm (v) 16 cm

4. $BD = 12$ cm, $CD = 16$ m **5.** 61 m **6.** 30 cm

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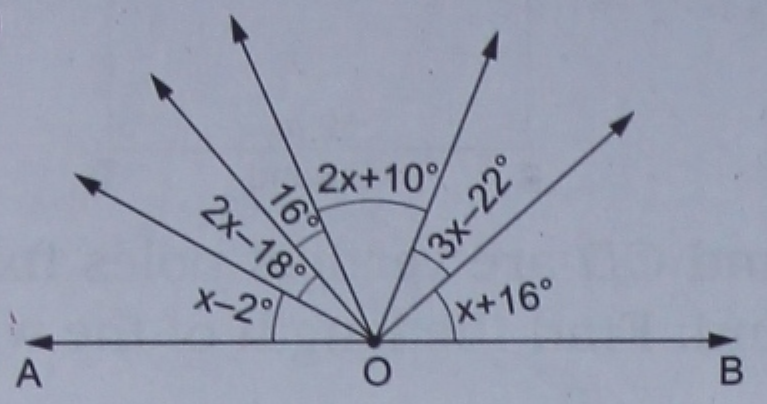
Revision Exercise 1

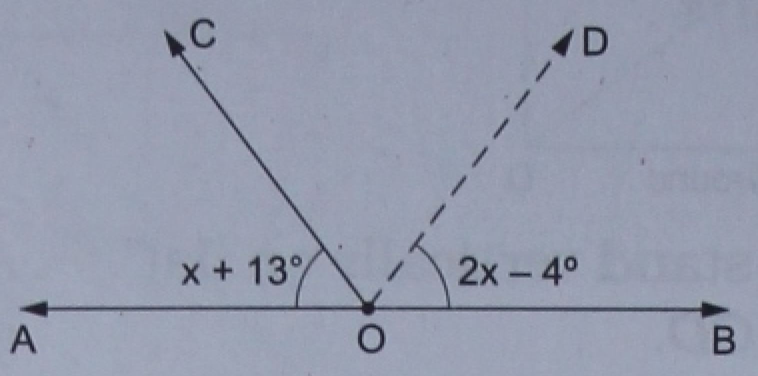
1. Twice of an angle is 20° less than thrice its supplement. Find the angle and its supplement.

2. Find the value of x in each case.

(i)

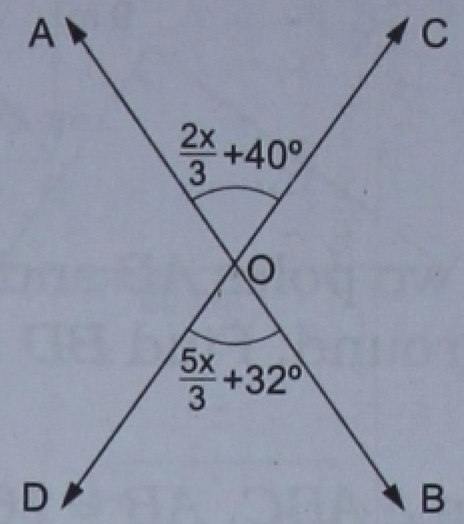


(ii)



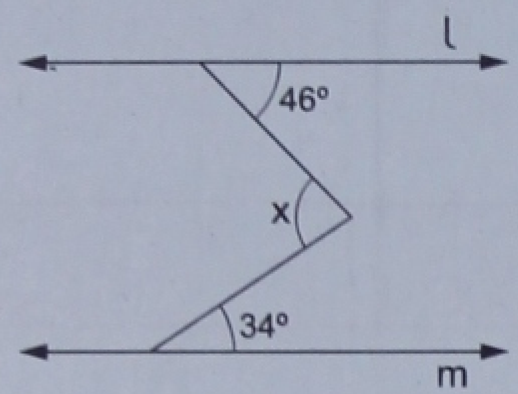
(OD bisects ∠BOC)

(iii)

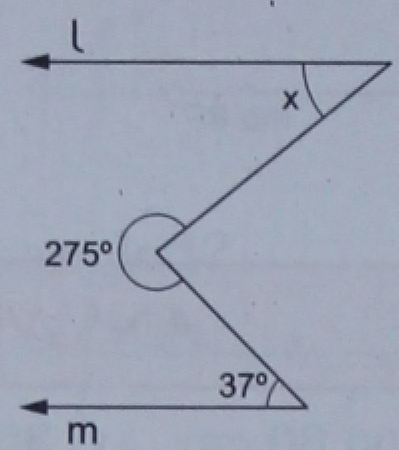


3. If $l \parallel m$, find x.

(i)

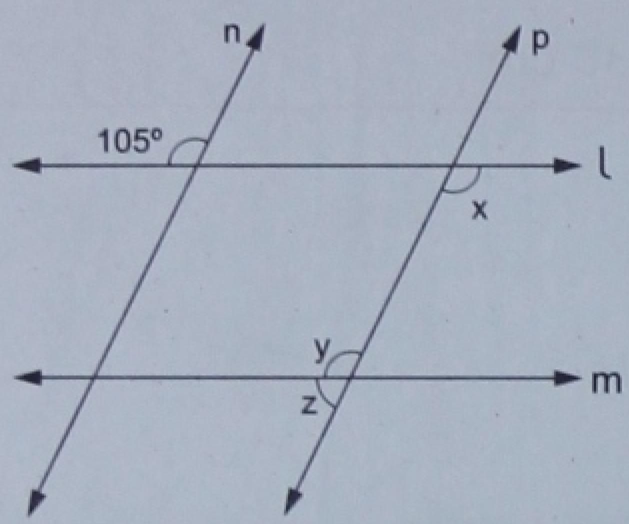


(ii)

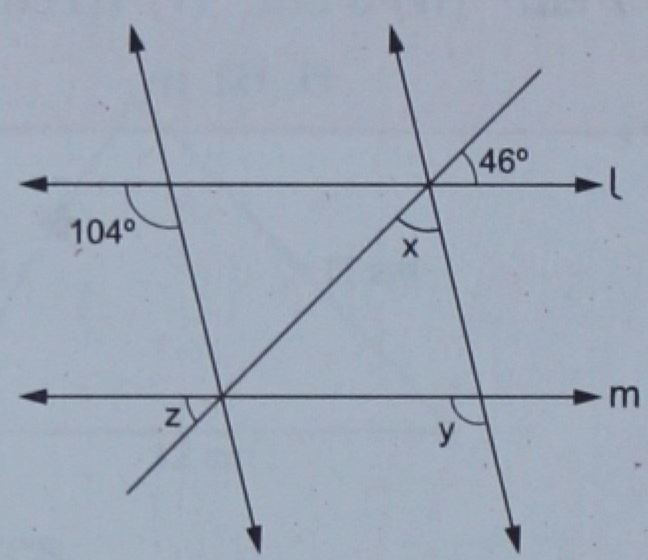


4. If $l \parallel m$ and $n \parallel p$, find x, y and z.

(i)

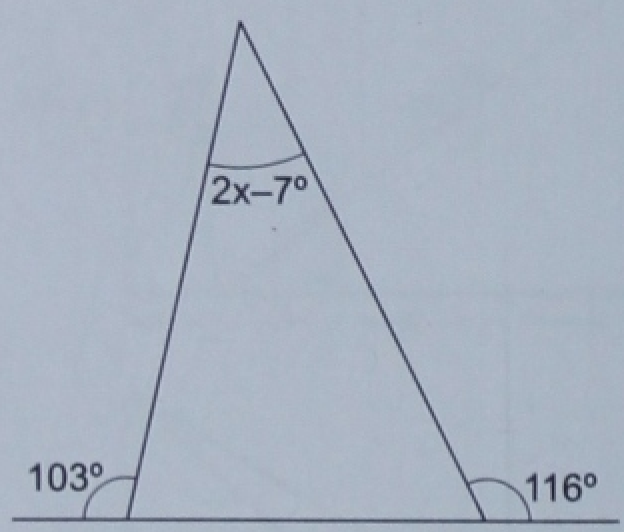


(ii)

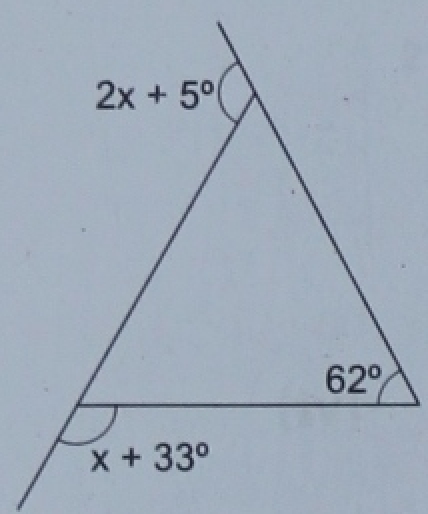


5. Find x in the following figures.

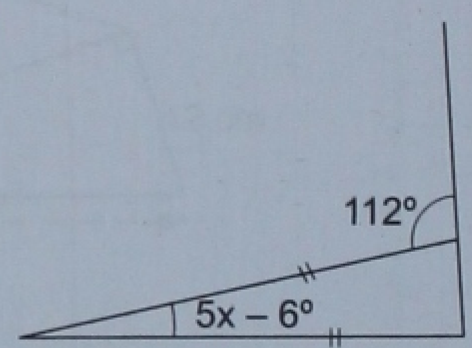
(i)



(ii)

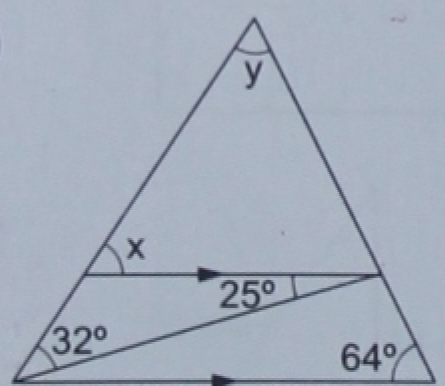


(iii)

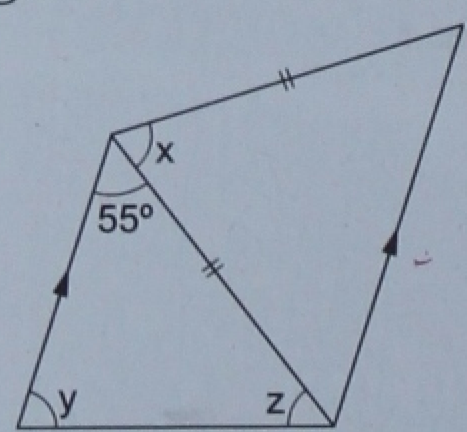


6. Calculate the size of the labelled angles.

(i)

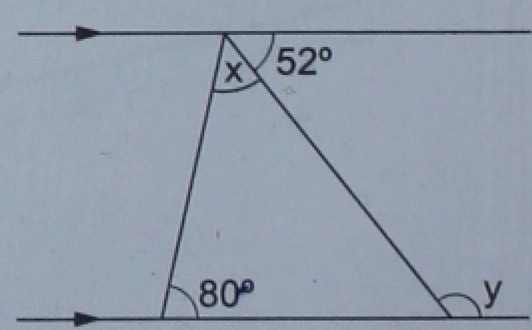


(ii)



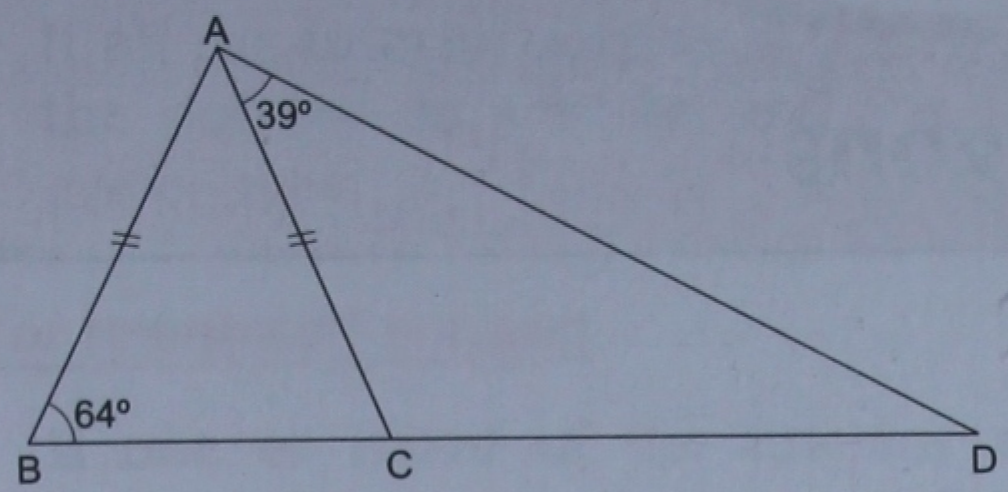
y:z = 3:2

(iii)

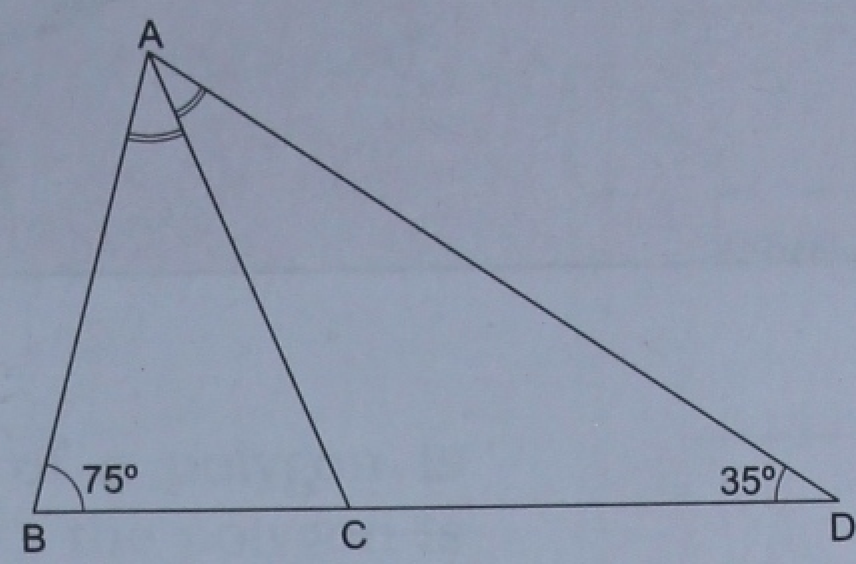


7. Arrange BC, AC and CD in ascending order.

(i)

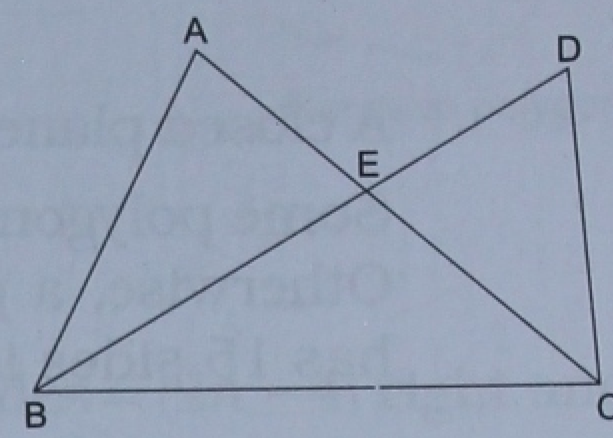


(ii)



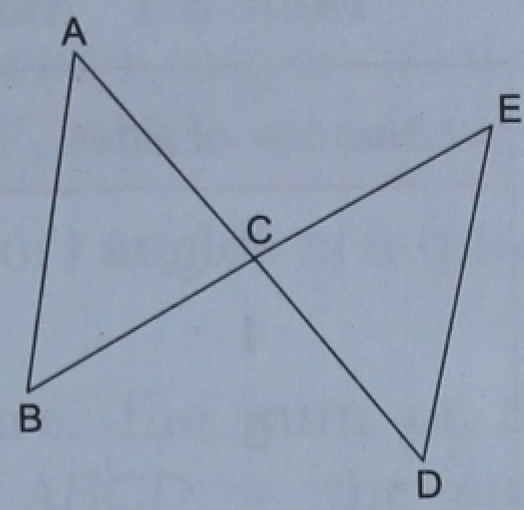
8. In the adjoining figure, $\angle BAC = \angle BDC$ and $\angle ABC = \angle BCD$.

Prove that (i) $\triangle ABC \cong \triangle DCB$ and (ii) $\triangle ABE \cong \triangle DCE$.



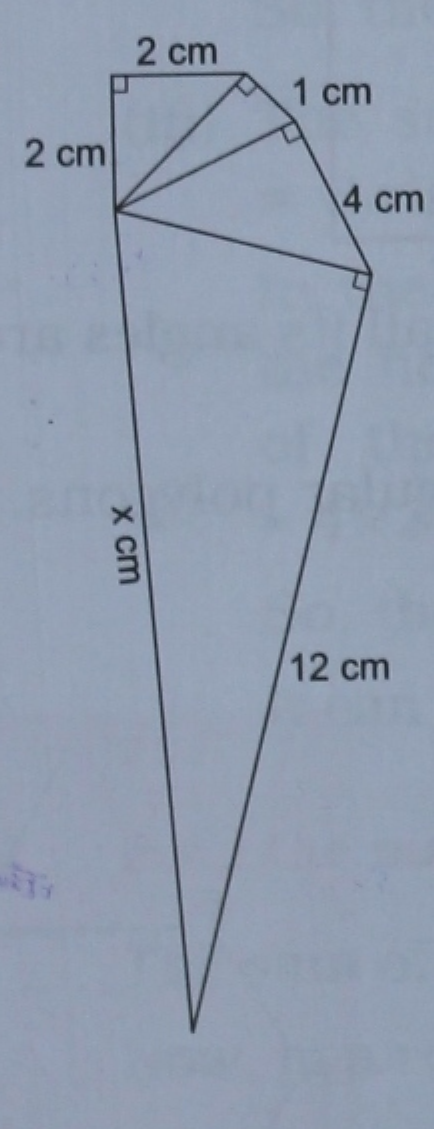
9. In the adjoining figure, $AC = CE$ and $BC = CD$.

Prove that $AB = DE$.

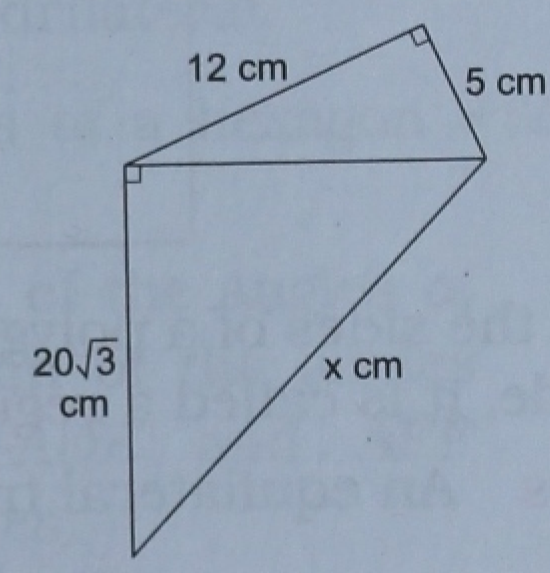


10. Calculate x in each of the following figures.

(i)



(ii)



ANSWERS

1. 104°, 76° **2.** (i) 20° (ii) 35° (iii) 8° **3.** (i) 80° (ii) 48°

4. (i) $x = 105°$, $y = 105°$, $z = 75°$ (ii) $x = 58°$, $y = 104°$, $z = 46°$ **5.** (i) 23° (ii) 68° (iii) 10°

6. (i) $x = 57°$, $y = 59°$ (ii) $x = 70°$, $y = 75°$, $z = 50°$ (iii) $x = 48°$, $y = 128°$

7. (i) $CD > AC > BC$ (ii) $BC < AC = CD$ **10.** (i) 13 (ii) 37

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