14 TIME AND WORK

- Relation Between Time and Work

Relation Between Time and Work

Let us find out the relation between time and work in different cases:

1. number of workers is fixed;
2. work to be done is fixed; and
3. time is fixed

Case I: When number of workers is fixed

The amount of work done varies directly with the time spent on doing the work, provided the number of workers remains constant.

or Work done $\propto$ Time taken

or more work will get done in more time

> If a_1 number of articles are made in time t_1 by a certain number of workers and a_2 number of articles are made in time t_2, then
>
> $$a_1 : a_2 :: t_1 : t_2$$

Example 1: A weaver takes 2 days to weave 3 shawls. How many days will he take to weave 15 shawls?

Method I: Using Direct Proportion

Let the weaver take x number of days to weave 15 shawls.

Then $3 : 15 :: 2 : x$

$\Rightarrow$ $3x = 2 \times 15$

$\Rightarrow$ $x = \frac{30}{3} = 10$ days

Thus, the weaver will take 10 days to weave 15 shawls.

Method II: Using Unitary Method

If 3 shawls can be woven in 2 days

1 shawl can be woven in $\frac{2}{3}$ days

and 15 shawls can be woven in $\frac{2}{3} \times 15 = 10$ days

Thus, the weaver will take 10 days to weave 15 shawls.

Case II: When work to be done is fixed

The time taken to do a job varies inversely with the number of workers doing the job, provided the amount of work involved in doing the job remains constant.

or Time taken $\propto \frac{1}{\text{Number of workers}}$

or more workers will do the same job in less time.

> If w_1 number of workers take time t_1 to complete a work and w_2 number of workers take time t_2 to complete the same amount of work, then
>
> $w_1 : w_2 =$ inverse of $t_1 : t_2$
>
> or $w_1 : w_2 :: t_2 : t_1$

Example 2: 3 gardeners take 90 minutes to weed a garden. If the job has to be done in only 15 minutes, how many more gardeners need to be put on the job?

Method I: Using Inverse Proportion

Let x number of gardeners be needed to weed the garden in 15 minutes.

Then $3 : x$ = inverse of $90 : 15$

$\Rightarrow \quad 3 : x :: 15 : 90$

$\Rightarrow \quad 15x = 90 \times 3$

$\Rightarrow \quad x = \frac{270}{15} = 18$ gardeners

Thus, $18 - 3 = 15$ more gardeners need to be put on the job.

Method II: Using Unitary Method

Weeding the garden in 90 minutes takes: 3 gardeners

or Weeding the garden in 1 minute will take: $3 \times 90 = 270$ gardeners

or Weeding the garden in 15 minutes will take: $\frac{270}{15} = 18$ gardeners

Thus, $18 - 3 = 15$ more gardeners need to be put on the job.

Case III: When time is fixed

The number of workers required varies directly with the amount of work to be done when the time in which to do the work is constant.

Number of workers $\propto$ Work to be done

or more workers will do more work, and less work will need a less number of workers.

> If w_1 number of workers can make a_1 number of articles in a certain time and w_2 number of workers can make a_2 number of articles in the same time, then
>
> $$w_1 : w_2 :: a_1 : a_2$$

Example 3: 8 mechanics can assemble 4 TV sets in 2 days. How many mechanics will assemble 3 TV sets in 2 days?

Method I: Using Direct Proportion

Let x number of mechanics take 2 days to assemble 3 TV sets.

Then $8 : x :: 4 : 3$

$\Rightarrow \quad 4x = 3 \times 8$

$\Rightarrow \quad x = \frac{24}{4} = 6$ mechanics

Thus, 6 mechanics will assemble 3 TV sets in 2 days.

Method II: Using Unitary Method

If 4 TV sets can be assembled by 8 mechanics in 2 days,

then 1 TV set can be assembled by $\frac{8}{4}$ or 2 mechanics in 2 days

and 3 TV sets can be assembled by $2 \times 3 = 6$ mechanics in 2 days

Thus in 2 days, 6 mechanics will assemble 3 TV sets.

Example 4: A takes 9 minutes while B takes 12 minutes to make a candle. Working together how long will A and B take to make 35 candles?

In 9 minutes, A can make 1 candle

In 1 minute, A can make $\frac{1}{9}$ candle

In 12 minutes, B can make 1 candle

In 1 minute, B can make $\frac{1}{12}$ candle

In 1 minute A and B can make

$$\frac{1}{9} + \frac{1}{12} = \frac{7}{36} \text{ candle}$$

If $\frac{7}{36}$ candle can be made by A and B in 1 minute, then 1 candle can be made by A and B in

$$1 \times \frac{36}{7} \text{ minutes}$$

and 35 candles can be made by A and B in

$$\frac{36}{7} \times 35 = 180 \text{ minutes}$$

Thus, working together, A and B will take 180 minutes or 3 hours to make 35 candles.

Example 5: A can paint a shed in 5 hours while B takes 6 hours to do the same job. A and B begin painting the shed together, but B is called away on some other work after 2 hours. How long will A take to finish painting the rest of the shed?

In 5 hours A can paint 1 shed

In 1 hour A can paint $\frac{1}{5}$ of the shed

In 6 hours B can paint 1 shed

In 1 hour B can paint $\frac{1}{6}$ of the shed

In 1 hour A and B can paint $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$ of the shed

In 2 hours A and B would have painted

$$\frac{11}{30}\times 2=\frac{11}{15} \text{ of the shed}$$

Part of the shed remaining to be painted $1-\frac{11}{15}=\frac{4}{15}$

To paint 1 shed, A takes 5 hours.

To paint $\frac{4}{15}$ of the shed, A will take $5\times\frac{4}{15}$

$$=1\frac{1}{3} \text{ hours or 1 h 20 minutes}$$

Thus, A will take another 1 h and 20 minutes to finish the job.

Example 6: Carpenters A and B can make an almirah, working together, in 10 hours. B and C take 15 hours while A and C take $7\frac{1}{2}$ hours to do the same job. How long will A, B, and C, working together, take to make an almirah? If after selling the almirah, a profit of Rs 1260 is made, how should it be shared among A, B, and C?

A and B make an almirah in 10 hours.

Thus in 1 hour, A + B make $\frac{1}{10}$ of the almirah

B and C make an almirah in 15 hours.

Thus in 1 hour, B + C make $\frac{1}{15}$ of the almirah.

A and C make an almirah in $\frac{15}{2}$ hours.

Thus in 1 hour, A + C make $\frac{2}{15}$ of the almirah.

Thus in 1 hour, A + B + B + C + A + C make $\left(\frac{1}{10}+\frac{1}{15}+\frac{2}{15}\right)$ of the almirah.

$\Rightarrow$ 2 (A + B + C) make $\frac{3+2+4}{30}$ of the almirah,

or A + B + C make $\frac{9}{30\times 2}=\frac{9}{60}$ of the almirah

Now working together, A + B + C make $\frac{9}{60}$ of the almirah in 1 hour.

So working together, A + B + C make 1 almirah in $\frac{60}{9}=6\frac{2}{3}$ hours.

But the efficiency of A, B, and C is not the same.

In 1 hour A's share of work

= Joint work – (B + C)'s share of work

= $\frac{9}{60}$ of the almirah – $\frac{1}{15}$ of the almirah

= $\frac{5}{60}$ or $\frac{1}{12}$ of the almirah

In 1 hour B's share of work

= Joint work – (A + C)'s share of work

= $\frac{9}{60}$ of the almirah – $\frac{2}{15}$ of the almirah

= $\frac{1}{60}$ of the almirah

In 1 hour C's share of work

= Joint work – (A + B)'s share of work

= $\frac{9}{60}$ of the almirah – $\frac{1}{10}$ of the almirah

= $\frac{3}{60}$ or $\frac{1}{20}$ of the almirah

Thus, the contribution of A, B, and C in the completion of the work is in the ratio $\frac{1}{12}:\frac{1}{60}:\frac{1}{20}$ or 5 : 1 : 3. The profit made should be shared in this ratio as well.

Thus A's share of profit = $\frac{5}{9}\times 1260$ = Rs 700

B's share of profit = $\frac{1}{9}\times 1260$ = Rs 140

C's share of profit = $\frac{3}{9}\times 1260$ = Rs 420

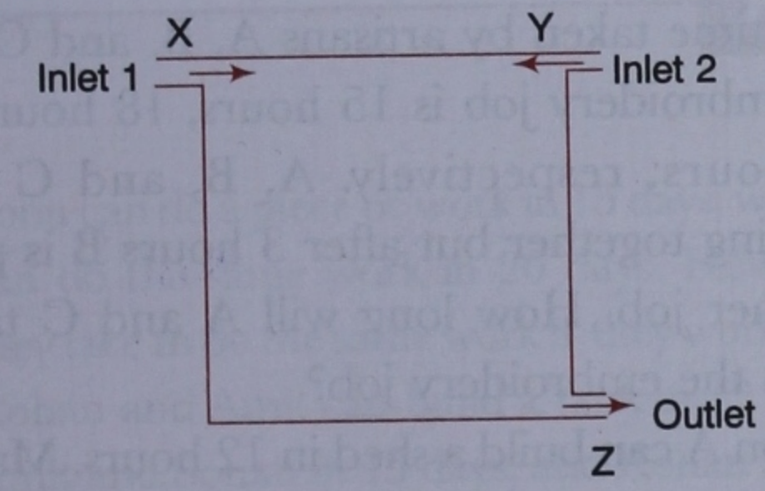
Thus, after working for 6 h and 40 min the almirah will be made, and after selling it A will earn Rs 700, B will earn Rs 140, and C will earn Rs 420 as profit.

Try this!

A can do a piece of work in 10 days which B can do in 12 days. In how many days will they finish the work, both working together?

Problems on Pipes and Cisterns

The cistern shown below has two inlet pipes X and Y and one outlet pipe Z, all of different diametres through which water flows at different rates.



If inlet X takes x minutes to fill the cistern, then in 1 minute $\frac{1}{x}$ part of the cistern will be filled by X alone. If inlet Y takes y minutes to fill the cistern, then in 1 minute $\frac{1}{y}$ part of the cistern will be filled by Y alone.

In 1 minute $\frac{1}{x}+\frac{1}{y}$ part of the cistern will be filled by X and Y together. If outlet Z takes z minutes to empty the cistern, then in 1 minute $\frac{1}{z}$ part of the cistern will be emptied by Z.

In 1 minute $\frac{1}{x}-\frac{1}{z}$ part of the cistern will be filled if both X and Z are open and the diametre of X is greater than the diametre of Z.

In 1 minute $\left(\frac{1}{x}+\frac{1}{y}\right)-\frac{1}{z}$ part of the cistern will be filled if all three, X, Y, and Z, are open.

Example 7: Two inlet pipes can fill up a cistern in 3 hours and 5 hours, respectively. If both pipes start filling the cistern together, how long will it take for the cistern to get filled?

In 1 hour the first pipe will fill up $\frac{1}{3}$ part of the cistern.

In 1 hour the second pipe will fill up $\frac{1}{5}$ part of the cistern.

In 1 hour both pipes will fill up $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$ part of the cistern.

If $\frac{8}{15}$ part of the cistern is filled by both pipes in 1 hour, then 1 whole of the cistern is filled by both pipes in $1\times\frac{15}{8}$ hours $=1\frac{7}{8}$ hours = 1 h 52.5 min.

Example 8: An overhead tank springs a leak that can empty the full tank in 16 hours. If its inlet pipe could fill it in 4 hours before the leak developed, how long would it now take to get filled up, when empty ?

In 1 hour, the inlet pipe fills up $\frac{1}{4}$ part of the tank.

In 1 hour, the leak empties $\frac{1}{16}$ part of the tank.

In 1 hour $\frac{1}{4}-\frac{1}{16}=\frac{3}{16}$ part of the tank gets filled up.

If $\frac{3}{16}$ part of the tank is filled up in 1 hour, the whole tank is filled up in $1\times\frac{16}{3}=5\frac{1}{3}$ hours or 5 h 20 min.

Try this!

Two pipes A and B can fill a tank in 36 minutes and 45 minutes, respectively. A drain pipe C can empty the tank in 30 minutes. First A and B are opened, and after 7 minutes, C is opened. In how much time will the tank become full?

Exercise 14.1

1. At a community feast if 6 women can make 540 *rotis* in an hour, how many women will be needed to make 990 *rotis* in an hour?
2. In a bakery 3 chefs take 8 hours to make 576 pastries. If on Christmas Eve a big order of 1728 pastries has to be delivered in 8 hours, how many more chefs need to be put on the job?
3. 6 workers can plaster the exterior of a building in 5 hours. If the job is to be completed in only 3 h 45 min how many workers will be needed?
4. 28 tea-pluckers can pluck 1260 kg of green leaves in one shift of 8 hours. By how much can the work force be reduced if the same amount of green leaves is to be plucked in 14 hours?

5. A group of workers take 8 hours to load 15 trucks with bricks. How much time would the group take to load $2\frac{1}{2}$ trucks with bricks?

6. 3 plumbers can repair 10 taps in 2 hours. How much time will the 3 plumbers take to repair 12 taps?
7. A carpenter takes 8 hours to make an armchair while his assistant takes 12 hours to do the same job. Working together, how long would they take to make the wooden armchair?
8. Electrician A takes 15 minutes to fit a ceiling fan while electrician B takes 18 minutes to do the same job. Working together how long will they take to fit 22 ceiling fans?
9. Jyoti takes 2 hours to clean up her room. Her elder sister Jalaja takes $1\frac{1}{2}$ hours while their mother takes only an hour to do the same job. If the two daughters help their mother, what fraction of the room will remain to be cleaned after 18 minutes?
10. Gardener A takes $1\frac{1}{2}$ hours to water a garden while gardener B takes only 54 minutes. Both gardeners begin watering the garden together but B leaves after 18 minutes. How much longer will A take to finish watering the garden?
11. The time taken by artisans A, B, and C to do an embroidery job is 15 hours, 18 hours, and 15 hours; respectively. A, B, and C begin working together but after 3 hours B is put on another job. How long will A and C take to finish the embroidery job?
12. Mason A can build a shed in 12 hours. Mason B takes 9 hours while Mason C takes 12 hours to do the same job. If the three masons are put to work together how long will they take to build the shed? How will the payment of Rs 1000 for the job be distributed among the three?
13. A can do a job in 15 minutes, B takes 20 minutes while C takes 30 minutes to do the same job. All three begin working together. A leaves after 2 minutes, leaving B and C to continue working together. B leaves 4 minutes after that, leaving C to finish the job. How much more time will C take to finish the job?
14. A and B take 10 hours to do a job working together. B and C, working together, take 15 hours while A and C take 20 hours to do the same job. How long would each of them take to do the job if they worked alone?

15. Masons A and B take 12 hours to build a stage together. B and C take 15 hours while A and C take 20 hours to do the same job. How long will it take to build the stage if A, B, and C worked together? If Rs 1200 is received for building the stage, how will the money be shared among A, B, and C?
16. A swimming pool has 4 inlet pipes. Each pipe can fill the swimming pool in 15 hours. How long will it take to fill up the swimming pool if all 4 pipes were opened simultaneously?
17. Pipe A can fill up a cistern in 20 minutes while pipe B can fill it up in 25 minutes. How much time will it take for the cistern to get filled up if both the pipes were opened simultaneously?
18. The inlet pipe can fill a tank in 3 hours while the outlet pipe can empty the full tank in 8 hours. How long will it take for the tank to fill up if the inlet and the outlet pipes are opened simultaneously?
19. Pipe A can fill a tank in 50 minutes while pipe B takes an hour to fill it up. Pipe A is opened first and the tank starts filling up. After 17 minutes pipe B is also opened. In how many more minutes will the tank get filled up?
20. A tank has two inlet pipes A and B and an outlet pipe C. A takes 5 hours to fill up the tank, B takes 8 hours to fill it up, and outlet C takes 13 hours 20 minutes to empty the full tank. Pipe A is opened first and the tank starts filling up. After 50 minutes pipe B is also opened. If outlet C is opened after another 80 minutes, how many more minutes will it take to fill up the tank?

Revision Exercise

1. Sonu can do a piece of work in 15 days, while Monu can do the same work in 20 days. How long will they take to do the same work if they work together?
2. Rohan and Amit can build a tank in 12 days while Amit and Sohan in 15 days, and Rohan and Sohan in 20 days. How long would each take to do the same work?
3. Rama can do a piece of work in 25 days and Yogita in 35 days. They began to work together, but Rama left after some days. Yogita finished the remaining work in 20 days. After how many days did Rama leave?
4. Two pipes can separately fill a tank in 5 hours and 4 hours respectively. If both the pipes are working together, in how much time will the tank be filled?
5. A pipe can fill a tank in 18 hours. Due to a small hole in the bottom, it is filled in 26 hours. If the tank is full, how much time will the hole take to empty the tank?