12 RATIO AND PROPORTION

- Ratio
- Proportion
- Direct and Inverse Proportion

Ratio

A ratio represents a relation between two quantities. If a and b are two quantities, their ratio is written as a : b (read as a 'is to' b).

If Rahul weighs 48 kg and Rohan weighs 50 kg, the relation between their weights can be represented in many ways.

Method	Representation	Comparison
Average	$\frac{48+50}{2} = 49$ kg	Rahul weighs 1 kg less than their average weight, while Rohan weighs 1 kg more than their average weight.
Fractions	$\frac{48}{50} = \frac{24}{25}$	Rahul's weight is equal to $\frac{24}{25}^{th}$ part of Rohan's weight.
Decimals	$\frac{48}{50} = 0.96$	Rahul's weight is equal to 0.96^{th} part of Rohan's weight.
Percentage	$\frac{48}{50} \times 100 = 96\%$	Rahul's weight is equal to 96% of Rohan's weight.
Ratio	48 : 50 or 24 : 25	If Rahul weighs 24 kg, then Rohan weighs 25 kg.

If the ratio between the weights of silver and gold in an alloy is given as 8 : 5, it means that for every 5 mg of gold in the alloy, there is 8 mg of silver. It also means that for every 5 kg of gold in the alloy, there is 8 kg of silver. Thus, we conclude that:

1. A ratio does not indicate the exact measure of two quantities but is simply the relation between two measures. **A ratio has no units**.
2. For ratios to be used for comparing two measures, the measures need to be represented in the same units.
3. The first term of the ratio is known as its **antecedent** while the second term is known as its **consequent.** If the antecedent becomes the consequent, then the meaning of the ratio is reversed. The ratio of silver and gold in an alloy being 5 : 8 would mean that for every 8 mg of gold, the alloy would have 5 mg silver, or there would be more gold than silver.

Methods of Comparison

Example 1: Out of 45 girls in a class, 18 girls join a choir. Compare the number of girls in the choir to the number of girls in the class using a fraction, a decimal, a percentage, and a ratio.

Fraction

Fraction of girls in the choir = $\frac{18}{45} = \frac{2}{5}$

Thus, $\frac{2}{5}$ of all girls are members of the choir.

Fraction of girls not in the choir = $1 - \frac{18}{45} = 1 - \frac{2}{5} = \frac{3}{5}$

Thus, $\frac{3}{5}$ of all girls are not in the choir.

Decimals

Girls in the choir = $\frac{2}{5} = 0.4$.

Thus, 0.4 of all girls join the choir.

Girls not in the choir = 1 – 0.4 = 0.6.
Thus, 0.6 of all girls are not in the choir.

Percentage

Percentage of girls in the choir = $\frac{2}{5} \times 100 = 40\%$.
Thus 40% of the girls are in the choir.
Percentage of girls not in the choir
$= 100 - 40 = 60\%$.
Thus, 60% of the girls are not in the choir.

Ratio

$\frac{2}{5}$ = 2 : 5. That means 2 out of every 5 girls in the class join the choir.

$\frac{3}{5}$ = 3 : 5. That means 3 out of every 5 girls do not join the choir.

If we divide the above ratios, we get $\frac{2}{3}$ = 2 : 3, or for every 2 girls who joined the choir, 3 girls in the class did not join the choir.

Example 2: Arrange the following ratios in ascending order.

6 : 7, 11 : 13, 5 : 4, 3 : 4, 17 : 19

Converting the given ratios into fractions we have

$$\frac{6}{7}, \frac{11}{13}, \frac{5}{4}, \frac{3}{4}, \frac{17}{19}$$

Converting the fractions into decimals, correct up to 2 places, we have 0.86, 0.85, 1.25, 0.75, 0.89
Now 0.75 < 0.85 < 0.86 < 0.89 < 1.25
Thus 3 : 4 < 11 : 13 < 6 : 7 < 17 : 19 < 5 : 4

Example 3: Reduce the ratio between 7 min 12 s and 8 min 6 s to its simplest form.

7 min 12 s = 432 s
8 min 6 s = 486 s
Thus ratio = 432 : 486
Dividing both terms by their HCF we have

$$\frac{432}{54} : \frac{486}{54} = 8 : 9$$

Example 4: The length and breadth of a parallelogram are in the ratio 11 : 9. If the perimeter of the parallelogram is 50 cm, find its length and breadth.

The number of parts the perimeter has to be divided into = 11 + 9 = 20, of which length is 11 parts and breadth is 9 parts.

Given: The sum of the length and breadth
Perimeter of the parallelogram = 50 cm
Here, 2 (l + b) = 50 cm

$$= \frac{\text{Perimeter}}{2} = \frac{50}{2} = 25 \text{ cm}$$

Thus, length = $\frac{11}{20} \times 25 = 13.75$ cm

and breadth = $\frac{9}{20} \times 25 = 11.25$ cm

CHECK: 2(13.75 + 11.25) = 2 × 25 = 50 cm

Example 5: The weights of Anksh and his sister Anju are in the ratio 11 : 7. Find their weights if the sum of their weights is given as 93.6 kg.

11 : 7 can be written as $\frac{11}{7}$

or $\frac{\text{Anksh's weight}}{\text{Anju's weight}} = \frac{11}{7}$

Now 11 + 7 = 18, but Anksh's weight + Anju's weight is given as 93.6 kg.

If 18 parts = 93.6 kg, one part = $\frac{93.6}{18}$ = 5.2 kg

Thus converting the ratios into quantities (with units),

we have $\frac{11 \times 5.2 \text{ kg}}{7 \times 5.2 \text{ kg}} = \frac{57.2 \text{ kg}}{36.4 \text{ kg}}$

Thus, the weights of Anksh and Anju are 57.2 kg and 36.4 kg, respectively.
CHECK: 57.2 + 36.4 = 93.6 kg

Example 6: The weights of a father and a mother are in the ratio 6 : 4. The weights of the mother and the son are in the ratio 6 : 5. If the three family members weigh 160 kg in all, how much does each weigh?

Given ratios are father : mother = 6 : 4 and mother : son = 6 : 5

Mother's weight, being the common term, is expressed as 4 and 6 in the two ratios.

Find the equivalent ratios such that the mother's weight is represented by the same number.

LCM of 4 and 6 = 12

$$6 : 4 = 6 \times 3 : 4 \times 3 = 18 : 12$$
$$6 : 5 = 6 \times 2 : 5 \times 2 = 12 : 10$$

Now that the mother's weight is represented by the same number, the ratios 6 : 4 and 6 : 5 can be written as 18 : 12 and 12 : 10

or the weights of the father : mother : son

$$= 18 : 12 : 10$$

Now 18 + 12 + 10 = 40

But given weights of father + mother + son = 160 kg

$$\text{Thus father's weight} = \frac{18}{40} \times 160 = 72 \text{ kg}$$

$$\text{mother's weight} = \frac{12}{40} \times 160 = 48 \text{ kg}$$

$$\text{son's weight} = \frac{10}{40} \times 160 = 40 \text{ kg}$$

Try this!

1. Two numbers are in the ratio 8:3 and their sum is 132. Find the numbers.
2. A bag contains 50 paise, 25 paise, and 10 paise coins in the ratio 5:7: 4, amounting to Rs 200. Find the number of coins of each type.

Proportional Parts

Proportional division involves dividing a quantity into parts that are in ratio to each other. Such parts are known as proportional parts.

Example 7: Divide Rs 1940 between A, B, and C such that A's share is $1\frac{3}{4}$ times B's share and C's share is $\frac{5}{7}$ of B's share.

Let B's share be represented by x

$\Rightarrow$ A's share $= 1\frac{3}{4} \times x = \frac{7x}{4}$ and C's share $= \frac{5x}{7}$

Given that $\frac{7x}{4} + x + \frac{5x}{7} = 1940$

$$\Rightarrow \frac{49x + 28x + 20x}{28} = 1940$$

$$\Rightarrow 97x = 54320 \Rightarrow x = 560$$

Thus, B's share = Rs 560

$$\text{A's share} = \frac{7 \times 560}{4} = \text{Rs } 980 \text{ and}$$

$$\text{C's share} = \frac{5 \times 560}{7} = \text{Rs } 400$$

Example 8: Divide Rs 8800 between A, B, and C such that A's share is $\frac{3}{8}$ of the sum of B's and C's shares and B's share is $\frac{5}{17}$ of the sum of A's and C's shares.

Let (B + C)'s share = 1,

Then A's share = $\frac{3}{8}$ of $1 = \frac{3}{8}$

$\Rightarrow$ A's share : (B + C)'s share = $\frac{3}{8} : 1 = 3 : 8$

Now A's share + (B + C)'s share = Rs 8800

If out of every 3 + 8 = 11 parts that A, B, and C get, A gets 3 parts,

then out of Rs 8800, A's share = $\frac{3}{11} \times 8800$

$$= \text{Rs } 2400$$

Let (A + C)'s share = 1,

then B's share = $\frac{5}{17}$ of $1 = \frac{5}{17}$

$\Rightarrow$ B's share : (A + C)'s share = $\frac{5}{17} : 1 = 5 : 17$

Now B's share + (A + C)'s share = Rs 8800

If out of every 5 + 17 = 22 parts that A, B, and C get, B gets 5 parts,

then out of Rs 8800, B's share = $\frac{5}{22} \times 8800$

$$= \text{Rs } 2000$$

Thus C's share = 8800 – (2400 + 2000)

$$= \text{Rs } 4400$$

CHECK: A's share = $\frac{3}{8}$ of (2000 + 4400)

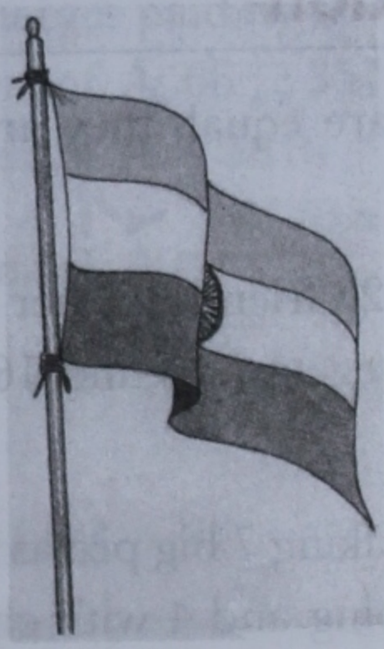
$= \frac{3}{8} \times 6400 = \text{Rs } 2400$

B's share = $\frac{5}{17}$ of (2400 + 4400)

$= \frac{5}{17} \times 6800 = \text{Rs } 2000$

Exercise 12.1

1. Find the ratios of the following.
   (i) 975 and 650
   (ii) 620 and 868
   (iii) 675 g and 600 g
   (iv) Re 1.02 and Re 1.87
   (v) 6 min 45 s and 8 min 6 s
   (vi) 68 boys and 153 girls
   (vii) 8 ℓ and 48 mℓ
   (viii) 1 km 836 m and 170 m
   (ix) 42 min and 48 min
   (x) 33 s and 3 min 33 s
2. Compare the following ratios.
   (i) 5 : 8 ☐ 7 : 9
   (ii) 2.18 : 4.22 ☐ 17.03 : 34.95
   (iii) $\frac{2}{3} : \frac{5}{7}$ ☐ $\frac{6}{7} : \frac{8}{9}$
   (iv) $6\frac{1}{5} : 7\frac{2}{3}$ ☐ $11\frac{2}{3} : 12\frac{3}{4}$
3. Convert the following ratios into fractions, decimals, and percentages.
   (i) 1 : 2 (ii) 7 : 8
   (iii) 3 : 5 (iv) 5 : 3
   (v) 3 : 4 (vi) 9 : 16
4. What is the ratio of the perimetres of two equilateral triangles with lengths measuring 4 cm and $4\frac{2}{3}$ cm, respectively?
5. Mr Singh saves 0.45 part of his salary every month. Find the ratio of his savings and expenditure.
6. Tulips and orchids are used in a 3 : 2 ratio to make some bouquets. If a bouquet has 69 tulips, how many orchids are there in the bouquet?
7. Milk and water are used to make a cold drink in a 4 : 7 ratio. If 38 ℓ of milk is used in all, how many litres of water are used to make the cold drink?
8. A huge Indian Tricolour flag needs to be painted with its length and breadth being in a 3 : 2 ratio and all three colour bands being of the same thickness. If the length of the flag is to be 18 m, how thick will the bottom green band be?



9. Distribute Rs 2164.50 between A and B in a 17 : 20 ratio.
10. Apples, pineapples, and grapes are used in a 11 : 9 : 4 ratio to make a fruit salad. If 1 kg 672 g of apples have been used, find how many grams of grapes have been used to make the fruit salad.
11. Rs 15,12,500 received from the sales of a cassette is to be shared between the music company and the singer in a 91 : 9 ratio. How much money will the singer receive?
12. Find $x : y : z$, given that $x : y = 12 : 13$ and $y : z = 3 : 2$.

13. The length and breadth of a rectangle are in a 15 : 11 ratio. If the length is 56 cm more than the breadth, find the perimetre of the rectangle.
14. If A's and B's heights are in the ratio 4 : 5 and B's and C's heights are in the ratio 3 : 4, who is the tallest among the three?
15. A's and B's earnings are in the ratio 3 : 7 while B's and C's earnings are in the ratio 3 : 2. If the combined earnings of A, B, and C are Rs 28,864, find how much each of them earns.
16. Divide Rs 3153 between A, B, and C such that A's share is 2 times B's share and C's share is 3 times B's share.
17. Divide Rs 2240 between A, B, and C such that A's share is $\frac{11}{13}$ of B's share and B's share is $\frac{5}{8}$ of C's share.
18. Divide Rs 8100 between A, B, and C such that A's share is $\frac{2}{7}$ of the sum of B's and C's shares and C's share is $1\frac{1}{13}$ times the sum of A's and B's shares.
19. Atul and Arun decide to share expenses in a 5 : 7 ratio while Arun and Ajay decide to share expenses in a 4 : 3 ratio. How will a total expense of Rs 1035 be shared by the three?
20. Prabhu and Sohail share expenses in a 13 : 10 ratio while Saif and Prabhu share expenses in a 10 : 11 ratio. On a picnic Prabhu spends Rs 650, Sohail Rs 710, and Saif spends Rs 555. How will the three friends see to it that the expense sharing ratios are followed?

Proportion

When two ratios are equal, they are said to be in proportion.

Priya is expecting 28 friends for her birthday party, 12 of whom are vegetarians and 16 of whom are non-vegetarians.

Priya's mother is baking 7 big pizzas for her friends, 3 with cheese topping and 4 with chicken topping. Each big pizza is to be cut into 8 pieces.

Vegetarians = 12
Non-vegetarians = 16
Cheese pizza pieces = 24
Chicken pizza pieces = 32

So each friend can have 2 pieces of pizza.

The ratio between Priya's vegetarian and non-vegetarian friends is 12 : 16.

The ratio between cheese pizza pieces and chicken pizza pieces is 24 : 32

Now 12 : 16 = 24 : 32 = 3 : 4

Thus the two ratios, being equal, are in proportion.

The proportion is written as 12 : 16 :: 24 : 32
where product of extremes = 12 × 32
= product of means = 16 × 24 = 384

The last term of the proportion or the term extreme to the right is called the **fourth proportional**.

In a set of three terms, when the ratio between the first and the second term is equal to the ratio between the second and the third term, the proportion is known as a **continued proportion**.

17 : 102 = 102 : 612.

Thus 17, 102, and 612 are in continued proportion where the last term or 612 is called the **third proportional** and the middle term, or 102, is called the **mean proportional**.

Remember

If x, y, and z are in continued proportion

$x \times z = y^2$

$\Rightarrow y = \sqrt{xz}$

$\Rightarrow$ mean proportional

$= \sqrt{\text{first proportional} \times \text{third proportional}}$

Example 9: A map is drawn to a 1 : 50,00,000 scale. If a river measures 11.72 cm in length on the map, what is the actual length of the river?

Ratio of length on map and actual length
= 1 : 50,00,000

Ratio of length of river on map and actual length of river = 11.72 cm : x cm
As the map is drawn to scale,
1 : 50,00,000 = 11.72 : x
$\Rightarrow$ 1 : 50,00,000 : : 11.72 : x
$\Rightarrow$ $1 \times x = 11.72 \times 50,00,000$
$\Rightarrow$ x = 5,86,00,000 cm = 586 km
Thus, the river is 586 km long.

Example 10: Find the mean proportional between 19 and 8,94,691.

Let the mean proportional be represented by x such that 19, x, and 8,94,691 are in continued proportion.
$\Rightarrow$ 19 : x : : x : 8,94,691
$\Rightarrow$ $x^2 = 8,94,691 \times 19 = 1,69,99,129$
$\Rightarrow$ $x = \sqrt{16999129}$
$\Rightarrow$ $x = 4123$

Thus, 19, 4123, and 894691 are in continued proportion, Hence, the mean proportional between 19 and 894691 is 4123.

Try this!
Find the third proportional of 16 and 36.

	4123
4	16 99 91 29
	16
81	99
	81
822	1891
	1644
8243	24729
	24729
	×

Example 11: The maintenance charges paid by shops A and B is in proportion to their floor area. Shop A pays Rs 3570 per month while shop B pays Rs 5610. If the floor area of shop B is 66 sq.m., find the floor area of shop B.

Given, ratio of floor area is proportional to ratio of maintenance charges.
Or floor area of A: floor area of B:: charges paid by A: charges paid by B.
Or floor area of A: 66 : : 3570 : 5610
Or floor area of A $\times$ 5610 = 3570 $\times$ 66
$\Rightarrow$ floor area of A = $\frac{3570 \times 66}{5610}$ = 42

Thus, the floor area of shop A = 42 sq. m.

Try this!
Find mean proportional between 0.08 and 0.18.

Exercise 12.2

1. Find the fourth proportional to the following numbers.
   (i) 11, 143, 13 (ii) 168, 42, 16
   (iii) 2, $\frac{5}{7}$, $\frac{14}{15}$ (iv) $\frac{1}{5}$, $\frac{3}{4}$, $\frac{3}{5}$
   (v) 4, 3.5, 4.8 (vi) 0.25, 7.9, 2.5

2. Find the third proportional to the following numbers.
   (i) 12, 18 (ii) 54, 18
   (iii) $\frac{3}{5}$, 3 (iv) $\frac{4}{5}$, $\frac{4}{15}$
   (v) 3.6, 7.2 (vi) 0.125, 0.25

3. Find the mean proportional between the following numbers.
   (i) 12 and 108 (ii) 13 and 325
   (iii) 10 and 153760 (iv) 28 and 202300
   (v) $\frac{1}{2}$ and $\frac{1}{18}$ (vi) 0.25 and 0.01
4. Find the value of x in the following proportions.
   (i) $33 : x :: 27 : 108$
   (ii) $x : 54 :: 77 : 22$
   (iii) $\frac{5}{8} : \frac{2}{3} :: x : \frac{4}{5}$
   (iv) $1\frac{2}{5} : 2\frac{1}{7} :: x : 1\frac{1}{14}$
   (v) $x : 1.1 :: 0.1 : 11$
   (vi) $3.8 : x :: 1.9 : 2.5$
5. A father and son contributed money at a family function in proportion to their present ages. If the father, who is 63, contributed Rs 11,907, find how much his 36-year-old son contributed.
6. In an architect's blueprint of an office complex drawn to a scale of 1 : 64, the length of a conference hall measures 37.5 cm. What would be the actual length of the conference hall?
7. If the distance between towns A and B is 584 km, what would be the distance between A and B shown on a map drawn to a 2 : 80,00,000 scale?
8. For every 5 diyas that Mr Gupta's son lights on Diwali, his daughter lights 3. If Mr Gupta has bought diyas worth Rs 95.40 for his daughter, how much should he spend on diyas for his son?
9. At a shopping festival, free gifts are being offered proportional to purchases made. If gifts worth Rs 2 are offered per Rs 10 spent, how much money would one have to spend to get gifts worth Rs 150?
10. In an examination, the ratio between those who passed with distinction and those who passed is the same as the ratio between those who passed and those who appeared for the examination. If only 4 candidates passed with distinction out of the 1156 candidates who appeared, find the number of candidates who passed the exam.

Direct and Inverse Proportion

Priya goes to the market to buy some chocolates for her friends.

1. Suppose Priya needs to buy exactly 2 chocolates for each of her 28 friends, or 56 chocolates in all.
   If she buys chocolates costing Rs 6 each, she pays Rs 336.
   If she buys chocolates costing Rs 9 each, she pays Rs 504.
   If she buys chocolates costing Rs 3 each, she pays Rs 168.

Thus, the amount Priya pays depends on the price of each chocolate, if the number of chocolates she buys is **constant**.

The more expensive the chocolates, the more Priya has to pay.

The cheaper the chocolates and Priya has to pay less.

$6 : 9 :: 336 : 504$ and $9 : 3 :: 504 : 168$

The ratio between the amount Priya pays is said to be **directly proportional** to the price of each chocolate.

The amount Priya pays is said to **vary directly** with the price of each chocolate.

This is written as Amount paid $\propto$ Price per unit, where $\propto$ is the symbol for variation.

Now this holds provided the quantity bought is kept constant.

Observe that $\frac{\text{Amount paid}}{\text{Price per unit}} = \frac{336}{6} = \frac{504}{9}$

$= \frac{168}{3} = 56 =$ constant

In direct proportion, this is known as the **constant of proportionality**.

2. Suppsose Priya wants to spend exactly Rs 336 on the chocolates she buys for each friend.
If each chocolate costs Rs 6, Priya will be able to buy 56 chocolates.
If each chocolate costs Rs 8, Priya will be able to buy 42 chocolates.
If each chocolate costs Rs 4, Priya will be able to buy 84 chocolates.

Thus if the amount Priya can spend is constant, then the number of chocolates she can buy will depend on the price of each chocolate.
The more expensive the chocolates, the less Priya can buy.
The cheaper the chocolates, the more she can buy.

$6 : 8$ = inverse of $56 : 42$
and $8 : 4$ = inverse of $42 : 84$
or $6 : 8 :: 42 : 56$
and $8 : 4 :: 84 : 42$

The ratios between the number of chocolates bought is said to be **inversely proportional** to the price of each chocolate.

The number of chocolates bought is said to **vary inversely** with the price of each chocolate.

This is written as:

$$\text{Number of chocolates} \propto \frac{1}{\text{Price of each chocolate}}$$

Now this holds provided the amount Priya can spend is kept constant.

Observe that Number of Chocolates × Price per unit
$= 6 \times 56 = 8 \times 42 = 4 \times 84 = 336$ = constant.

In inverse proportion, this is known as the **constant of proportionality**.

Example 11: Which of the following relations between variables x and y are direct proportions and which are inverse proportions?

(i)

x	14	28	42	40	30	6	1
y	7	14	21	20	15	3	0.5

In the above table, as the value of x increases, the value of y also increases and as the value of x decreases, the value of y also decreases.

Method I

Write the direct proportions for at least two sets of values.

$14 : 28 = 7 : 14$ or $42 : 40 = 21 : 20$
or $30 : 1 = 15 : 0.5$

CHECK: $14 \times 14 = 7 \times 28 = 196$
$42 \times 20 = 21 \times 40 = 840$
$30 \times 0.5 = 15 \times 1 = 15$

Thus, the values of x and y are directly proportional to each other.

Method II

Find $\frac{x}{y} = \frac{14}{7} = \frac{28}{14} = \frac{42}{21} = \frac{40}{20} = \frac{30}{15}$

$= \frac{6}{3} = \frac{1}{0.5} = 2$

Thus $x \propto y$, the constant of proportionality being 2 or $x = 2y$.

(ii)

x	5	7	2	12	1
y	11	13	8	18	7

In this table too, the value of y increases or decreases with every increase or decrease in the value of x.

Method I

$5 : 7$ and $11 : 13$ $\quad$ $2 : 1$ and $8 : 7$
$5 \times 13 \neq 11 \times 7$ $\quad$ $2 \times 7 \neq 8 \times 1$

Thus, the values of x and y are not in direct proportion to each other.

Method II

$$\frac{y}{x} = \frac{11}{5} \neq \frac{13}{7} \neq \frac{8}{2} \neq \frac{18}{12} \neq \frac{7}{1}$$

Thus, x does not vary directly with y.

(iii)

x	8	16	4	5	20
y	10	5	20	16	4

In the above table, as the value of x increases, the value of y decreases and as the value of x decreases, the value of y increases.

Method I

Write the inverse proportions for at least two sets of values

$8 : 16$ = inverse of $10 : 5$

$\Rightarrow$ $8 : 16 :: 5 : 10$

or $20 : 5$ = inverse of $4 : 16$

$\Rightarrow$ $20 : 5 :: 16 : 4$

or $4 : 5$ = inverse of $20 : 16$

$\Rightarrow$ $4 : 5 :: 16 : 20$

CHECK: $8 \times 10 = 16 \times 5 = 80$

$20 \times 4 = 16 \times 5 = 80$

$4 \times 20 = 5 \times 16 = 80$

Thus, the values of x and y are inversely proportional to each other.

Method II:

$xy = 8 \times 10 = 16 \times 5 = 4 \times 20 = 5 \times 16 = 20 \times 4 = 80$

Thus $x \propto \frac{1}{y}$, the constant of proportionality being 80 or $x = \frac{80}{y}$.

(iv)

x	25	30	15	55	20
y	35	20	45	5	40

In this table too, the value of y increases when the value of x decreases and decreases when the value of x increases.

Method I

$25 : 30$ and inverse of $35 : 20$

$55 : 20$ and inverse of $5 : 40$

$\Rightarrow$ $25 : 30$ and $20 : 35$ | $55 : 20$ and $40 : 5$

But $25 \times 35 \neq 30 \times 20$ | $55 \times 5 \neq 20 \times 40$

Thus, the values of x and y are not in inverse proportion to each other.

Method II

$xy = 25 \times 35 \neq 30 \times 20 \neq 15 \times 45 \neq 55 \times 5 \neq 20 \times 40$

Thus, x does not vary inversely with y.

Example 12: Find the value of x if 3.6, 18, 5, x are in direct proportion and inverse proportion.

If 3.6, 18, 5, x are in direct proportion,

$3.6 : 18 :: 5 : x$

$\Rightarrow$ $3.6\,x = 18 \times 5$

$\Rightarrow$ $x = \frac{90}{3.6} = 25$

If 3.6, 18, 5, x are in inverse proportion,

$3.6 : 18$ = inverse of $5 : x$

$\Rightarrow$ $3.6 : 18 :: x : 5$

$18\,x = 5 \times 3.6$

$\Rightarrow$ $x = \frac{18}{18} = 1$

Example 13: If a certain sum of money earns Rs 16,200 as interest at 9% p.a. over a period of time, how much interest will it earn at 11% p.a. over the same period of time?

$$I = \frac{PRT}{100}$$

$I \propto R$ when $\frac{PT}{100}$ is constant (in this case the principal and time period are unchanged.)

Thus, a change in the rate of interest will be directly proportional to the change in interest earned.

or $R_1 : R_2 :: I_1 : I_2$

or $9 : 11 :: 16200 : I_2$

$\Rightarrow$ $9 \times I_2 = 16200 \times 11$

$\Rightarrow$ $I_2 = \frac{178200}{9}$ = Rs 19,800

Thus, at 11% over the same period of time, the sum will earn Rs 19,800 as interest.

Example 14: A school hostel with 40 children has enough provisions to last 8 days. If 20 more children move into the hostel, how many days will the provisions last now?

More children $\Rightarrow$ More provisions needed $\Rightarrow$ Provisions will finish sooner

Thus, number of days that provisions will last

$$\propto \frac{1}{\text{Number of children}}$$

provided that the amount of provisions each child requires is constant.

or if n = number of children and d = number of days that provisions will last,

$n_1 : n_2$ = inverse of $d_1 : d_2$

or $40 : 60$ = inverse of $8 : d_2$

$\Rightarrow \quad 40 : 60 :: d_2 : 8$

$\Rightarrow \quad 60 \times d_2 = 40 \times 8$

$\Rightarrow \quad d_2 = \frac{320}{60} = 5\frac{1}{3}$

Thus, with 60 children in the hostel, the provisions will last only $5\frac{1}{3}$ days.

Try this!

If x varies inversely as square of y and when y = 3, then x = 4. Find x, when y = 6.

Exercise 12.3

1. Which of the following sets of variables x and y are direct proportions and which are inverse proportions. Find the constant of proportionality in each.

(i)

x	10	12	3	7	30
y	20	24	6	14	60

(ii)

x	14	10	18	15	7.5
y	5.6	4	7.2	6	3

(iii)

x	15	30	25	13.5	9
y	45	22.5	27	50	75

(iv)

x	8	9	6	12	16
y	72	64	96	48	36

(v)

x	0.7	2	3.5	4	0.2
y	80	28	16	14	280

(vi)

x	0.24	0.42	0.45	0.18	0.06
y	8	14	15	6	2

2. Given $x \propto y$, write the missing values of x and y in the following tables.

(i)

x		5		3		11
y	130	65	91		104	

(ii)

x	22.5		20		27.5	
y	9	10		7		5

3. Given $x \propto \frac{1}{y}$, write the missing values of x and y in the following tables.

(i)

x	90		180		45	
y	2	3		12		6

(ii)

x		0.5	0.1		0.05	
y	5	7		10		2

4. Find the values of x if the following ratios are in direct proportion.
   (i) $15 : 12$ and $x : 36$
   (ii) $12 : x$ and $30 : 15$
   (iii) $3.4 : 22.1$ and $x : 7.15$
   (iv) $0.2 : x$ and $1.9 : 0.019$
   (v) $\frac{2}{3} : \frac{4}{15}$ and $x : 2$
   (vi) $x : \frac{2}{5}$ and $1\frac{1}{2} : 1\frac{4}{5}$

5. Find the value of x if the following ratios are in inverse proportion.
   (i) $33 : 11$ and $x : 24$
   (ii) $x : 13.6$ and $15.3 : 18$
   (iii) $2.6 : x$ and $7.6 : 15.2$
   (iv) $5.8 : 8.12$ and $x : 1$
   (v) $\frac{4}{7} : 2$ and $x : \frac{1}{8}$
   (vi) $\frac{1}{3} : \frac{1}{5}$ and $x : \frac{5}{6}$

6. Which of the following are correct statements?
   (i) Perimeter of a triangle ∝ length of a side.
   (ii) Perimeter of an equilateral triangle ∝ length of a side
   (iii) Principal ∝ Interest
   (iv) Circumference of a circle ∝ length of the radius
   (v) Area of a rectangle ∝ measure of its breadth
   (vi) S.P ∝ Profit
   (vii) Loss ∝ Loss percentage
7. If 9 diaries cost Rs 401.40, how much would 13 diaries cost?
8. If 24 pears cost Rs 163.20, how much would 18 pears cost?
9. If 185 books can be packed in 5 cartons, how many books can be packed in 3 cartons?
10. If Prakash buys 18 mangoes spending Rs 81, how many more mangoes does Parul buy spending Rs 121.50?
11. The interest earned on a certain amount of money is Rs 18562.50 over a period of time at 12.5% p.a. How much interest will the same amount of money earn at $13\frac{1}{3}\%$ p.a. over the same period of time?
12. The interest earned on Rs 35000 over a period of time is Rs 1400. How much interest will be earned by Rs 45000 at the same rate of interest over double the period of time?
13. A chef stores enough food to feed 15 guests for 5 days. If he wishes to finish off the food in only 3 days, how many more guests can he feed?
14. A summer camp with 60 children has enough provisions to last 10 days. If 20 more children join the camp, how long will the provisions last?
15. In a fort under siege there is enough food to feed the 90 inmates for 7 days. 20 inmates quietly slip away and escape. How many days will the food now last for the rest of the inmates?

Revision Exercise

1. Find the ratios of the following :
   (i) 800 g and 6 kg (ii) 560 mℓ and 28 mℓ
   (iii) 72 sec and 4 min. (iv) 50 paise and Rs 8.
2. Convert the following ratios into fractions, decimals and percentage.
   (i) 6 : 18 (ii) 9 : 14
   (iii) 5 : 9 (iv) 13 : 15
3. Find the value of x in the following proportions :
   (i) $x : 4 = 48 : 96$. (ii) $18 : x = 36 : 72$.
   (iii) $\frac{3}{10} : \frac{1}{8} = x : \frac{5}{9}$ (iv) $x : 2.2 = 0.2 : 22$.
4. The cost of 89 metres of cloth is Rs. 1646.50. What length of this cloth can be purchased for Rs 1776.00?
5. 80 men can build a factory in 16 months. How many months will 105 men require to build it under the same working conditions?

Mental maths

1. If 2 music CDs cost Rs 140, then 5 similar CDs cost _____

2. Find the value of x when $\frac{x}{5} = \frac{12}{15}$