ORDERED PAIR ; CARTESIAN PRODUCT

39.1 ORDERED PAIR

An ordered pair is the pair of two objects which occur in a particular order (i.e. in which the order of objects is important).

e.g. consider two objects (numbers) '6' and '8'. If written as (6, 8), they form one ordered pair and if written as (8, 6), they form another ordered pair.

Since, the order of writing the objects in both the pairs is different, therefore, (6, 8) and (8, 6) are two different ordered pairs.

1. In an ordered pair (a, b), ***a*** is called the **first component** and ***b*** is called the **second component** of the ordered pair.
   Conversely, if first and second components of an ordered pair are ***x*** and ***y*** respectively, then the ordered pair is (x, y).
2. The components of ordered pairs may be the same e.g. (3, 3), (a, a), (p, p), etc.
3. It is important to know that {a, b} and (a, b) are not the same.
   Reason : {a, b} represents a set whose elements are a and b, whereas (a, b) represents an ordered pair, whose components are a and b.
   Moreover, {a, b} = {b, a} as the change in order of writing the elements does not change the set, but (a, b) ≠ (b, a) as by changing the order of writing the components, the ordered pair is changed.

39.2 EQUALITY OF ORDERED PAIRS

Two ordered pairs are said to be equal, if they have the same (equal) first components and the same second components.

e.g. (7, 10) = (7, 10) ; (5, – 3) = (5, –3) ; Also, if (x, y) = (5, 7), then x = 5 and y = 7.

Example 1 :

Use the elements of set A = {a, b, c} to form all possible ordered pairs.

Solution :

The possible ordered pairs are : (a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b) and (c, c). **(Ans.)**

Example 2 :

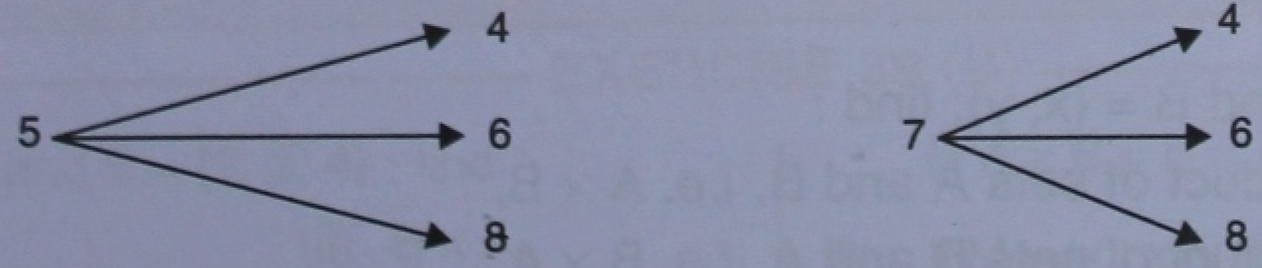
Given A = {5, 7} and B = {4, 6, 8}, form all ordered pairs, so that in each ordered pair,

(i) the first component is from A and the second component is from B.

(ii) the first component is from B and the second component is from A.

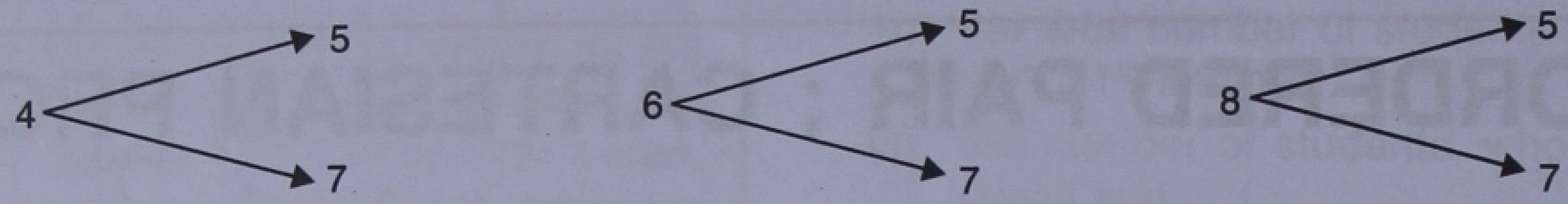
Solution :

(i) Associate each element of set A with each element of set B. This can be done in the following manner :



Thus, **the required ordered pairs are : (5, 4), (5, 6), (5, 8), (7, 4), (7, 6) and (7, 8)** **(Ans.)**

(ii) Associate each element of set B with each element of set A, *i.e.*



∴ **The required ordered pairs are : (4, 5), (4, 7), (6, 5), (6, 7), (8, 5) and (8, 7)** **(Ans.)**

TEST YOURSELF

1. If (x – 3, 5) = (4, 2 – y), then x – 3 = ... and 5 = ...
⇒ x = ... and y = ...

2. If (3x – 2, 5 – y) = (4, – 1) ⇒ .... = ... and ....
⇒ .... = ... and ....
⇒ x = ... and y = ...

EXERCISE 39 (A)

1. Use elements of set P = {2, 3} to form all possible ordered pairs.
2. Use the elements of set A = {x, y, z} to form all possible ordered pairs.
3. Given A = {5, 6, 7} and B = {3, 4}. Form all possible ordered pairs, so that in each ordered pair;
   (i) first component is from set A and second component is from set B.
   (ii) first component is from B and second is from A;
   (iii) both the components are from A ;
   (iv) both the components are from B.

   In each case, write the total number of ordered pairs formed.
4. State, **true** or **false** :
   (i) {x, y} is an ordered pair, whose first component is x and second component is y.
   (ii) (x, y) is an ordered pair, whose components are x and y.
   (iii) (a, b) is a set, whose elements are a and b
   (iv) {5, 7} = {7, 5}
   (v) (5, 7) = (7, 5)
   (vi) If {x, y} = {3, 5} ; then x = 3 and y = 5
   (vii) If (x, y) = (3, 5) ; then x = 3 and y = 5
   (viii) Ordered pairs (a, 3) and (5, x) are equal means, a = 5 and x = 3.
5. Given (2a – 3, 3b + 1) = (7, 7); find a and b.
6. Given (3x, –5) = (x – 2, y + 3); find x and y.
7. Given $(a - 2, \frac{b}{3}) = (0, 0)$; find a and b.
8. If the ordered pairs (a – 3, a + 2b) and (3a – 1, 3) are equal, find the values of a and b.

39.3 CARTESIAN PRODUCT

If A and B are two non-empty sets, then their cartesian product is the set of all possible ordered pairs such that the first components of all the ordered pairs are from set A and the second components of all the ordered pairs are from set B.

i.e. cartesian product of sets A and B = set of all possible ordered pairs (x, y) such that x ∈ A and y ∈ B.

The cartesian product of sets A and B is represented by **A × B** and is read as **A cross B**.

Example 3 :

Given A = {b, c, d} and B = {x, y}; find :

(i) the cartesian product of sets A and B, *i.e.* A × B.
(ii) the cartesian product of sets B and A, *i.e.* B × A.
(iii) A × A. (iv) B × B.

Solution :

(i) $A \times B$ = Set of all possible ordered pairs such that their first components are from A and second componets are from B.

= { (b, x), (b, y), (c, x), (c, y), (d, x), (d, y) } **(Ans.)**

(ii) $B \times A$ = Set of all possible ordered pairs (x, y) such that $x \in B$ and $y \in A$.

= { (x, b), (x, c), (x, d), (y, b), (y, c), (y, d) } **(Ans.)**

(iii) $A \times A$ = { (x, y) : x and y both belong to A }

= { (b, b), (b, c), (b, d), (c, b), (c, c), (c, d), (d, b), (d, c), (d, d) } **(Ans.)**

(iv) $B \times B$ = { (x, x), (x, y), (y, x), (y, y) } **(Ans.)**

1. The cartesian product $A \times B$ is not the same as the cartesian product $B \times A$
   i.e. $A \times B \neq B \times A$.
   Hence, the cartesian product of two unequal sets is not commutative.
2. The product sets $A \times B$ and $B \times A$ have an equal number of ordered pairs.
   i.e. $n(A \times B) = n(B \times A)$
   $= n(A) \times n(B)$
3. If $A \times B = B \times A$, then $A = B$. Conversely, if $A = B$, then $A \times B = B \times A$.
4. Since empty set $\varnothing$ contains no element in it, therefore for any set A, the cartesian product $A \times \varnothing$ also contains no element, *i.e.* $n(A \times \varnothing) = O = n(\varnothing \times A)$.

Example 4 :

Given A = {a, b, c} and B = {x, y}, show that the product sets $A \times B$ and $B \times A$ are equivalent sets.

Solution :

$A \times B$ = { (a, x), (a, y), (b, x), (b, y), (c, x), (c, y) }

$B \times A$ = { (x, a), (x, b), (x, c), (y, a), (y, b), (y, c) }

Since the number of ordered pairs in $A \times B$ is 6, *i.e.* $n(A \times B) = 6$.

Also, the number of ordered pairs in $B \times A$ is 6, *i.e.* $n(B \times A) = 6$.

$\therefore$ **$A \times B$ and $B \times A$ are equivalent sets.** **(Ans.)**

[Two sets are said to be equivalent if they contain an equal number of elements].

TEST YOURSELF

If A = {5, 6, 7} and B = {6, 8}, then

3. $A \times B$ = ...
4. $B \times A$ = ...
5. $n(A \times B)$ = ...
6. $n(B \times A)$ = ...
7. $n(A) \times n(B)$ = ...
8. Is $A \times B = B \times A$? ....
9. Is $n(A \times B) = n(B \times A) = n(A) \times n(B)$? .... Is this result always true ?.................
10. Are $A \times B$ and $B \times A$ equivalent sets ? ....

EXERCISE 39 (B)

1. If A = {1, 2} and B = {2, 3, 4} ; find :
   (i) $A \times B$ (ii) $B \times A$
   (iii) $A \times A$ (iv) $B \times B$
   Is $A \times B = B \times A$?
2. If $M = \{x : x \in N, 1 < x \leq 4\}$ and $N = \{y : y \in W, y < 3\}$; find :

(i) $M \times N$ (ii) $N \times M$
(iii) $n(M \times M)$ (iv) $n(N \times N)$
Is $M \times N = N \times M$?
Is $n(M \times N) = n(N \times M)$?

3. State, **true** or **false** :
   (i) If $n(A) = 4$ and $n(B) = 5$, then $n(A \times B) = 20$.
   (ii) If $n(A) = m$ and $n(B) = n$; then $A \times B = mn$.
   (iii) If $A = \{x, y\}$ and $B = \{y, x\}$; then $A \times B = \{ (x, y), (y, x) \}$
   (iv) $A \times B$ and $B \times A$ are equal sets.
   (v) If $A = B$; then $A \times B = B \times A$.

4. Given $A = \{5, 6, 7\}$ and $B = \{6, 8, 10\}$; find :
   (i) $A \cup B$ (ii) $(A \cup B) \times B$
   (iii) $B \cap A$ (iv) $A \times (B \cap A)$

5. If $A = \{x : x \in W, 3 \le x < 6\}$, $B = \{3, 5, 7\}$ and $C = \{2, 4\}$; find :
   (i) $A - B$ (ii) $(A - B) \times C$
   (iii) $B - C$ (iv) $A \times (B - C)$

6. If $A = \{5, 6, 7, 8\}$ and $B = \{6, 8, 10\}$; find :
   (i) $A \cup B$ (ii) $A \cap B$
   (iii) $(A \cup B) \times (A \cap B)$

7. If $P = \{a, b, c\}$ and $Q = \{b, c, d\}$; find $(P \cap Q) \times P$.

8. If $A = \{5, 7\}$, $B = \{7, 9\}$ and $C = \{7, 9, 11\}$, find :
   (i) $A \times (B \cup C)$
   (ii) $(A \times B) \cup (A \times C)$
   Is $A \times (B \cup C) = (A \times B) \cup (A \times C)$?

9. Given $A = \{x \in W : 7 < x \le 10\}$; find : $A \times A$.

10. Given $M = \{0, 1, 2\}$ and $N = \{1, 2, 3\}$; find :
   (i) $(N - M) \times (N \cap M)$
   (ii) $(M \cup N) \times (M - N)$

ANSWERS

TEST YOURSELF

1. $x - 3 = 4$ and $5 = 2 - y \Rightarrow x = 7$ and $y = -3$ **2.** $3x - 2 = 4$ and $5 - y = -1 \Rightarrow 3x = 6$ and $6 = y \Rightarrow x = 2$ and $y = 6$ **3.** $A \times B = \{(5, 6), (5, 8), (6, 6), (6, 8), (7, 6), (7, 8)\}$ **4.** $\{(6, 5), (6, 6), (6, 7), (8, 5), (8, 6), (8, 7)\}$ **5.** 6 **6.** 6 **7.** $3 \times 2 = 6$ **8.** No **9.** yes; yes **10.** yes

EXERCISE 39(A)

1. (2, 2) (2, 3), (3, 3), (3, 2) **2.** (x, x), (x, y), (x, z), (y, x), (y, y), (y, z), (z, x), (z, y), (z, z) **3.** (i) (5, 3), (5, 4), (6, 3), (6, 4), (7, 3), (7, 4) (ii) (3, 5), (3, 6), (3, 7), (4, 5), (4, 6), (4, 7) (iii) (5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7) (iv) (3, 3), (3, 4), (4, 3), (4, 4); 6, 6, 9 and 4 **4.** (i) False (ii) True (iii) False (iv) True (v) False (vi) False (vii) True (viii) True **5.** a = 5 and b = 2 **6.** x = –1 and y = –8 **7.** a = 2 and b = 0 **8.** a = –1 and b = 2

EXERCISE 39(B)

1. (i) {(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)} (ii) {(2, 1), (2, 2), (3, 1), (3, 2), (4, 1), (4, 2)} (iii) {(1, 1), (1, 2), (2, 1), (2, 2)} (iv) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)} No. **2.** (i) {(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0) (4, 1), (4, 2)} (ii) {(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)} (iii) 9 (iv) 9 No; Yes **3.** (i) True (ii) False (iii) False (iv) False (v) True **4.** (i) {5, 6, 7, 8, 10} (ii) {(5, 6), (5, 8), (5, 10), (6, 6), (6, 8), (6, 10), (7, 6), (7, 8), (7, 10), (8, 6), (8, 8), (8, 10), (10, 6), (10, 8), (10, 10)} (iii) {6} (iv) {(5, 6), (6, 6), (7, 6)} **5.** (i) {4} (ii) {(4, 2), (4, 4)} (iii) {3, 5, 7} (iv) {(3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7)} **6.** (i) {5, 6, 7, 8, 10} (ii) {6, 8} (iii) {(5, 6), (5, 8), (6, 6), (6, 8), (7, 6), (7, 8), (8, 6), (8, 8), (10, 6), (10, 8)} **7.** {(b, a), (b, b), (b, c), (c, a), (c, b), (c, c)} **8.** (i) {(5, 7), (5, 9), (5, 11), (7, 7), (7, 9), (7, 11)} (ii) {(5, 7), (5, 9), (5, 11), (7, 7), (7, 9), (7, 11)} ; Yes **9.** {(8, 8), (8, 9), (8, 10), (9, 8), (9, 9), (9, 10), (10, 8), (10, 9), (10, 10)} **10.** (i) {(3, 1), (3, 2)} (ii) {(0, 0), (1, 0), (2, 0), (3, 0)}