VENN-DIAGRAMS

38.1 REVIEW

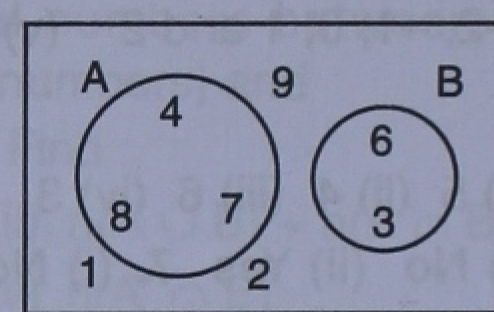
Venn-diagram is the most commonly used pictorial representation of sets. This idea was first developed by John Venn, an English mathematician, that is why the figures (geometrical figures) used in this type of representation are called *Venn-diagrams.*

In fact, in a Venn-diagram, a closed curve (figure) represents a set and the interior points within this closed curve represent the elements of the set.

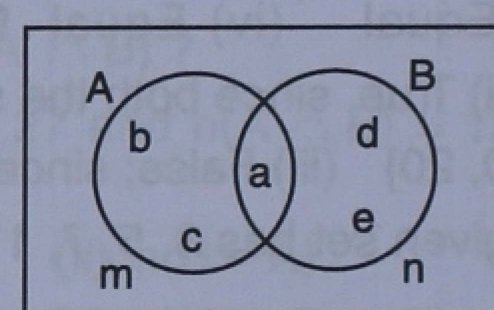
In a Venn-diagram, the universal set is represented by a rectangle and all other sets under consideration by circles or ovals within the rectangle.

38.2 USE OF VENN-DIAGRAMS TO SHOW THE RELATIONSHIP BETWEEN THE SETS

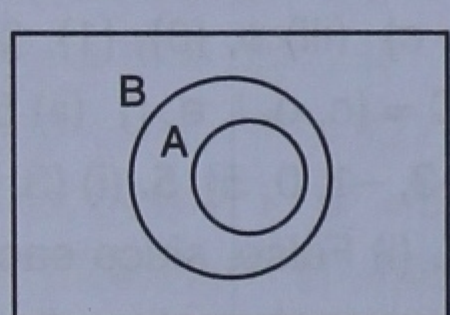
1. The diagram shows **two disjoint sets A and B.**



2. The diagram shows **A and B are joint or overlapping sets.**

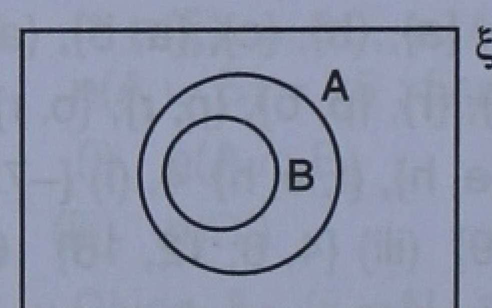


3. (i)



$A \subset B$

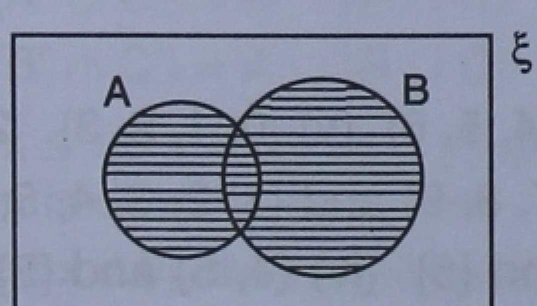
(ii)



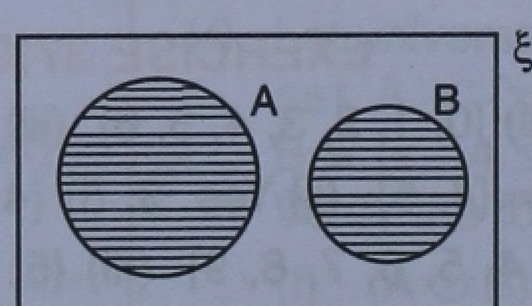
$B \subset A$

4. The **shaded portions** in the following figures show $A \cup B$.

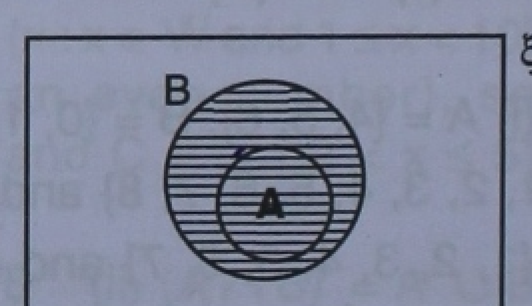
(i)



(ii)

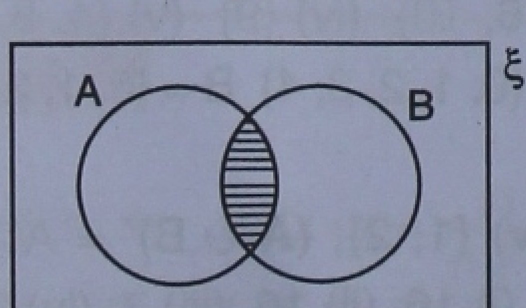


(iii)

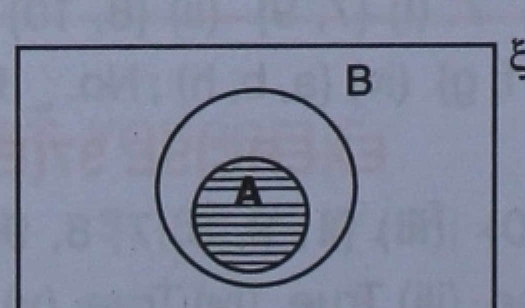


5. The **shaded portions** in the following figures show $A \cap B$.

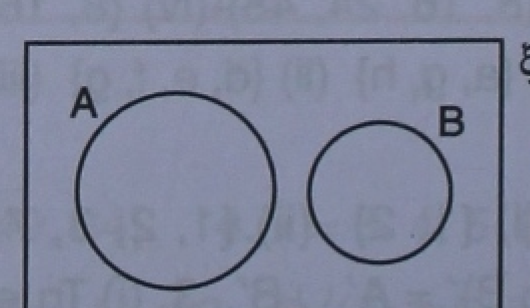
(i)



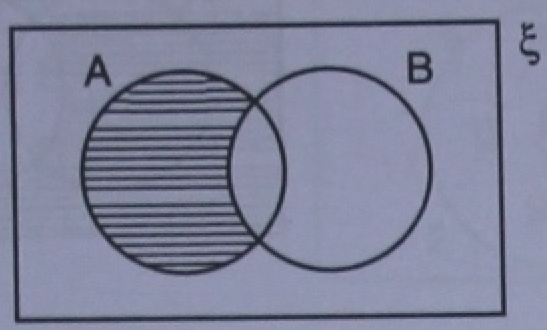
(ii)

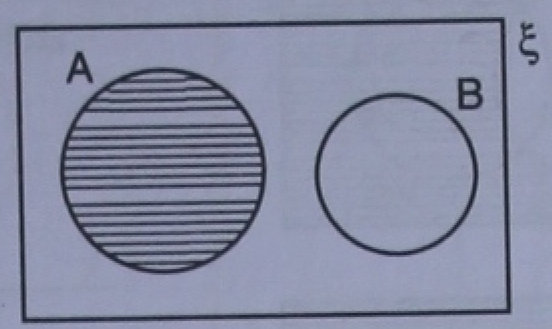


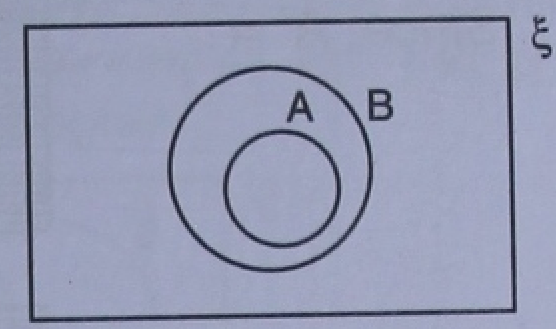
(iii)



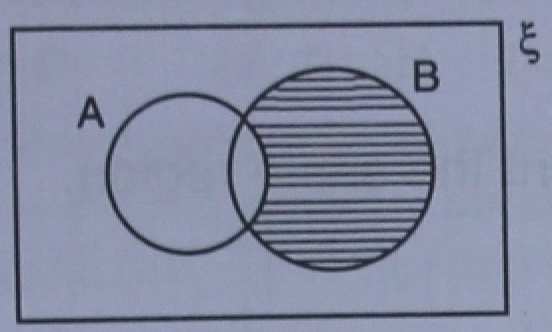
6. The **shaded portions** in the following figures show **A – B**.

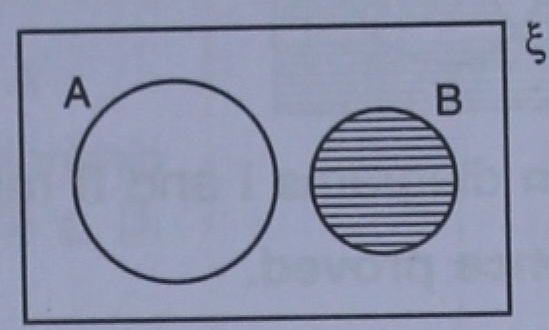
(i)


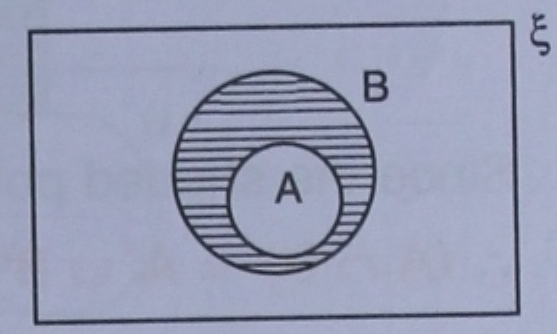
(ii)


(iii)


7. The **shaded portions** in the following figures show **B – A**.

(i)


(ii)


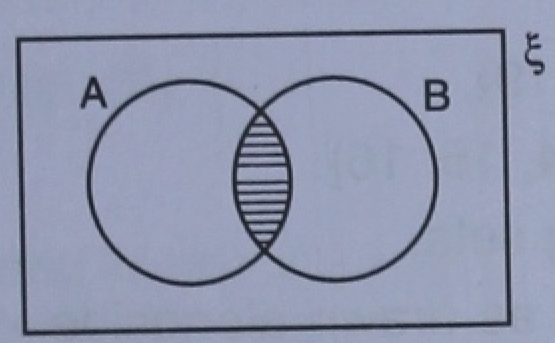
(iii)


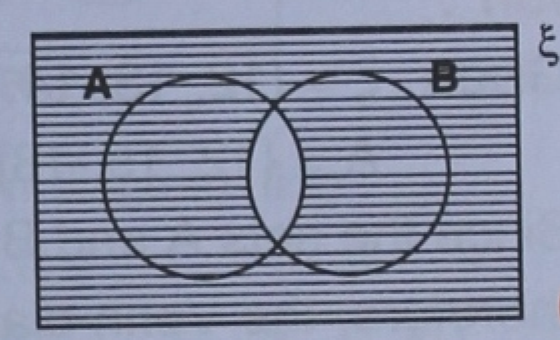
Example 1 :

For two overlapping sets A and B, draw Venn-diagrams to represent the following sets :

(i) $(A \cap B)'$ (ii) $(A \cup B)'$ (iii) $A' \cap B$

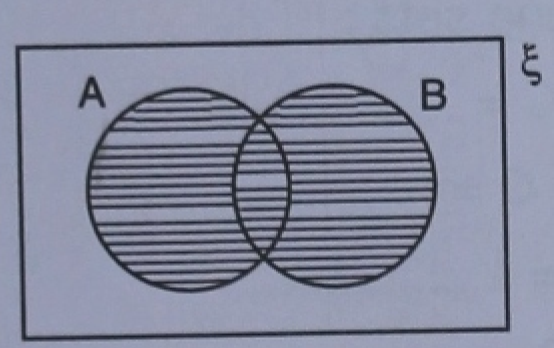
Solution :

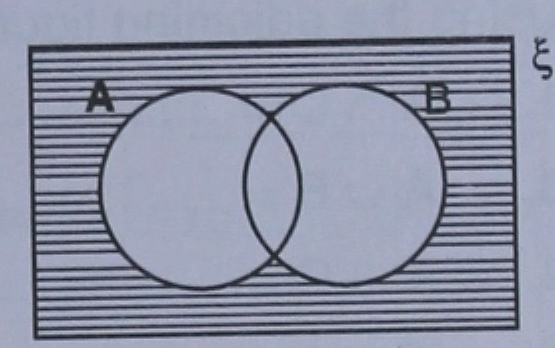
(i) Since, $A \cap B =$


$\therefore$ **$(A \cap B)' =$**


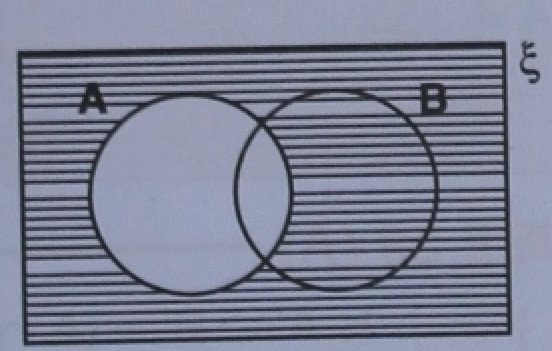
(Ans.)

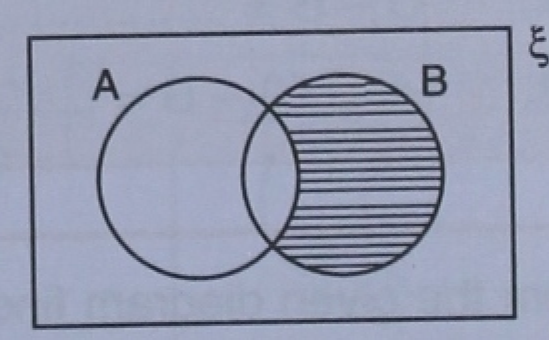
[The region of universal set, which is not in $A \cap B$]

(ii) Since, $A \cup B =$


$\therefore$ **$(A \cup B)' =$**


(Ans.)

(iii) Since, $A' =$


$\therefore$ **$A' \cap B =$**


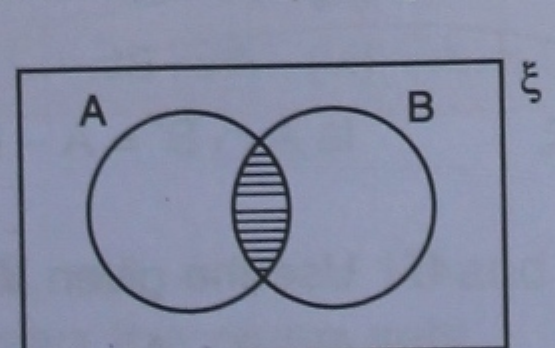
(Ans.)

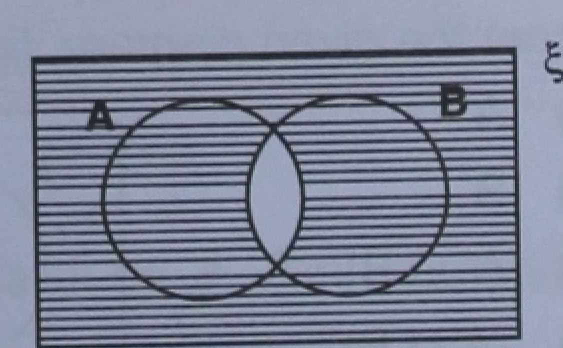
Example 2 :

Use Venn-diagrams to prove that : $(A \cap B)' = A' \cup B'$.

Solution :

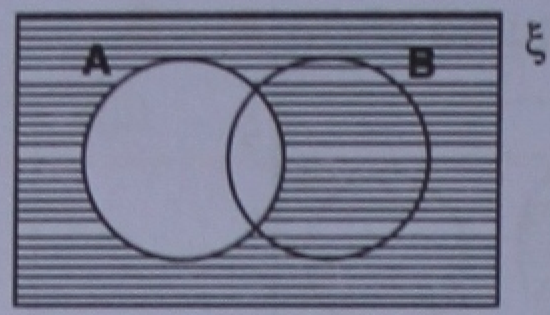
Consider A and B as two overlapping sets.

Since, $A \cap B =$


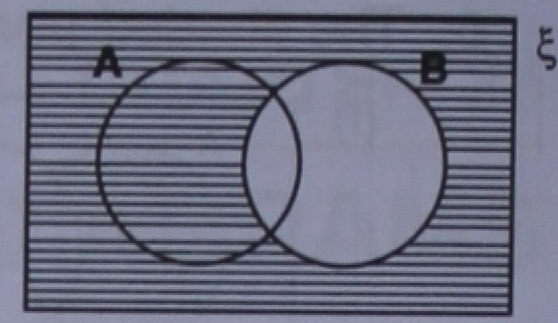
$\therefore$ **$(A \cap B)' =$**


....I

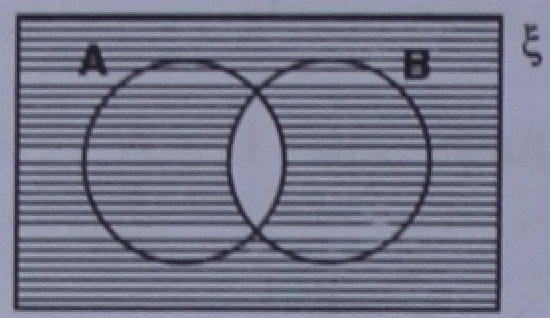
Since, $A' =$



and $B' =$



$\therefore$ **$A' \cup B' =$**



.... II

Since the shaded portions in diagrams I and II represent the same region,

$\therefore$ **$(A \cap B)' = A' \cup B'$** **Hence proved.**

Example 3 :

Given : $\xi = \{x : x \in N, 12 \le x < 20\}$,

$A = \{x : x$ is divisible by $3\}$ and $B = \{12, 14, 15, 16\}$.

Draw a Venn-diagram to show the relationship between the given sets.

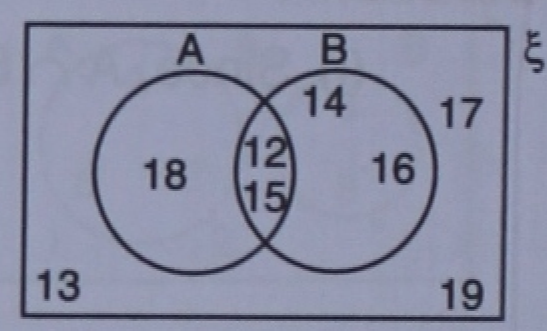
Solution :

Given : $\xi = \{12, 13, 14, 15, 16, 17, 18, 19\}$,

$A = \{12, 15, 18\}$ and $B = \{12, 14, 15, 16\}$.

Since the sets A and B are overlapping sets,

therefore, the Venn-diagram should be as drawn alongside.



TEST YOURSELF

Using the adjoining figure, find the following sets :

1. $A \cap C =$ .... **2.** $B \cup C =$ ....

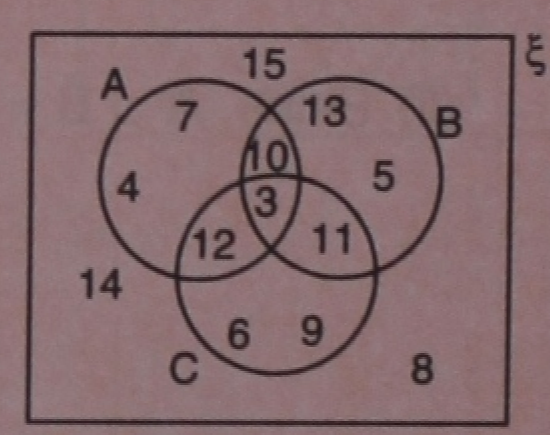
3. $A \cup B =$ .... and $(A \cup B) \cap C =$ ....

4. $A \cup C =$ .... and $(A \cup C)' =$ ....

5. $C - A =$ .... **6.** $B - C =$ ....

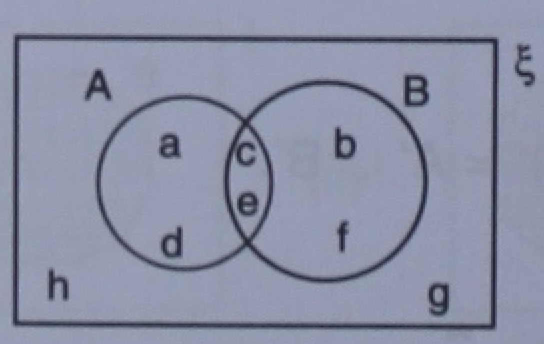
7. $C - B =$ .... and $(C - B) \cup A =$ ....

8. $(A \cup C) - B =$ ....

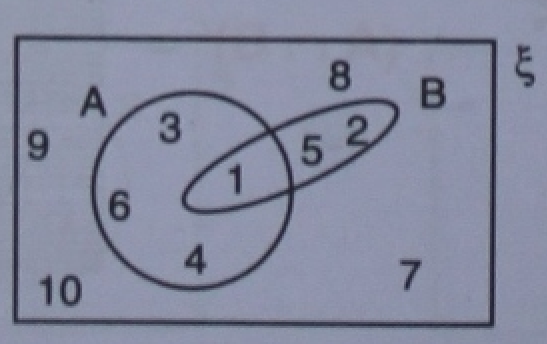


EXERCISE 38 (A)

1. From the given diagram find :
   (i) $A \cup B$
   (ii) $A' \cap B$
   (iii) $A - B$
   (iv) $B - A$
   (v) $(A \cup B)'$



2. From the given diagram, find :
   (i) A'
   (ii) B'
   (iii) $A' \cup B'$
   (iv) $(A \cap B)'$

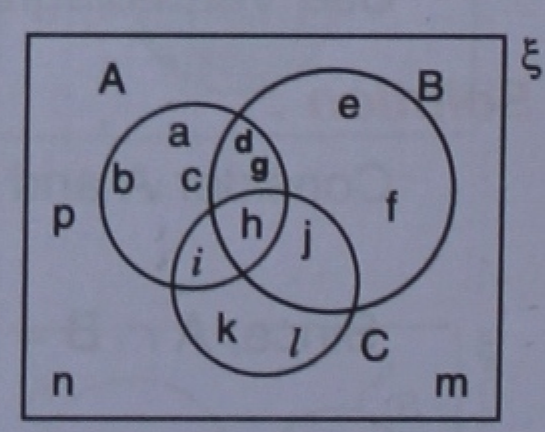


Is $A' \cup B' = (A \cap B)'$?

Also, verify if $A' \cap B' = (A \cup B)'$

3. Use the given diagram to find :
   (i) $A \cup (B \cap C)$
   (ii) $B - (A - C)$
   (iii) $A - B$
   (iv) $A \cap B'$

   Is $A \cap B' = A - B$?

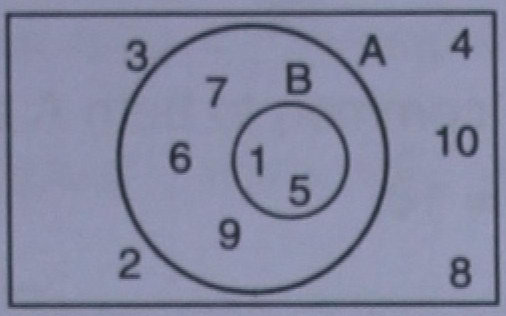


4. Use the given Venn-diagram to find :
   (i) $B - A$ (ii) A

(iii) B'

(iv) $A \cap B$

(v) $A \cup B$



5. Draw a Venn-diagram to show the relationship between two overlapping sets A and B. Now shade the region representing :

(i) $A \cap B$ (ii) $A \cup B$ (iii) $B - A$

6. Draw a Venn-diagram to show the relationship between sets A and B ; such that $A \subseteq B$. Now shade the region representing :

(i) $A \cup B$ (ii) $B' \cap A$ (iii) $A \cap B$

(iv) $(A \cup B)'$

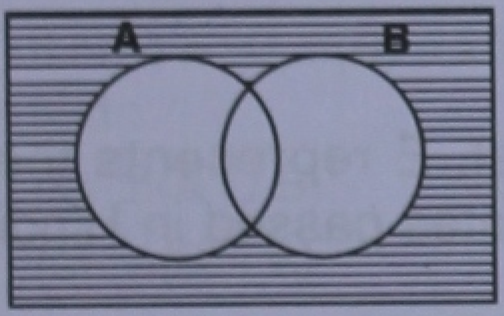
7. Two sets A and B are such that $A \cap B = \varnothing$. Draw a Venn-diagram to show the relationship between A and B. Shade the region representing :

(i) $A \cup B$ (ii) $(A \cup B)'$ (iii) $B - A$

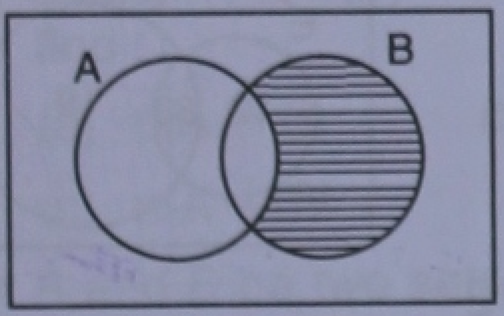
(iv) $B \cap A'$

8. State the sets represented by the shaded portion of the following Venn-diagrams :

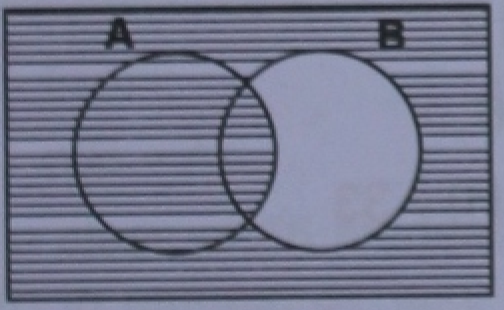
(i)



(ii)

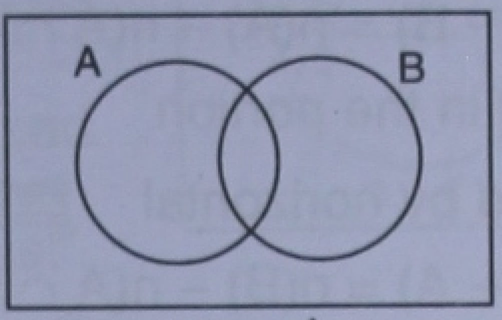


(iii)



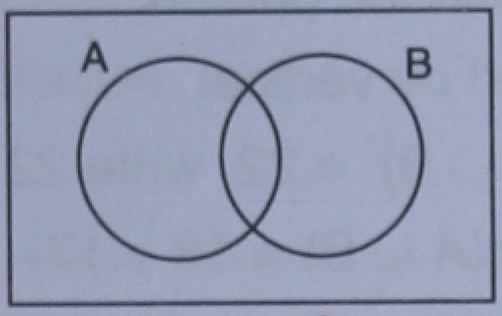
9. In each of the given diagrams, shade the region which represents the set given underneath the diagram :

(i)



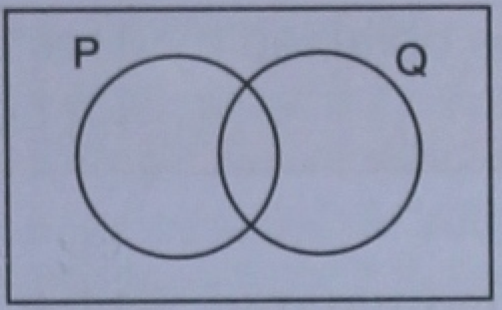
$(B - A)'$

(ii)



$(A \cap B)'$

(iii)



$(P \cup Q)'$

10. From the given diagram, find :

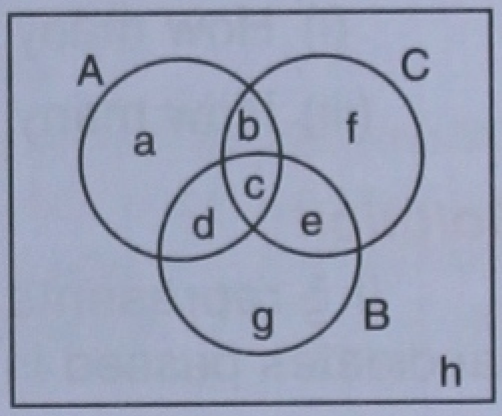
(i) $(A \cup B) - C$

(ii) $B - (A \cap C)$

(iii) $(B \cap C) \cup A$

Verify : $A - (B \cap C)$

$= (A - B) \cup (A - C)$



11. Using the given diagram, express the following sets in terms of set A and B.

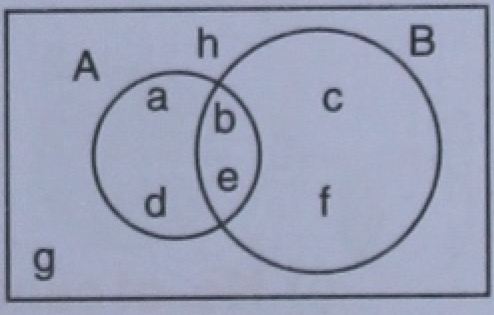
(i) {a, d}

(ii) {a, d, c, f}

(iii) {a, d, c, f, g, h}

(iv) {a, d, g, h}

(v) {g, h}



While using Venn-diagrams, remember,

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. If A and B are disjoint sets, then $n(A \cap B) = 0$ and therefore, $n(A \cup B) = n(A) + n(B)$
3. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Example 4 :

If $n(A) = 35$, $n(B) = 42$, $n(A \cap B) = 17$ and $n(A \cup B)' = 22$, draw a Venn-diagram to show the relationship between the given sets.

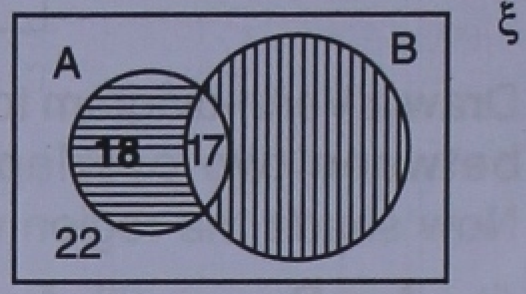
Use the Venn-diagram drawn to find : (i) $n(A \cup B) - n(B)$ (ii) $n(\xi)$

Solution :

Since, $n(A \cap B) = 17$, write 17 in the portion common to both A and B.

Since, $n(A - B) = n(A) - n(A \cap B) = 35 - 17 = 18$,

$\therefore$ Write 18 in the portion represented by horizontal lines.



Since, $n(B - A) = n(B) - n(A \cap B) = 42 - 17 = 25$,

$\therefore$ Write 25 in the portion. represented by vertical lines.

Since, $n(A \cup B)' = 22$, write 22 in the portion outside the sets A and B.

(i) Now, $n(A \cup B) = 18 + 17 + 25 = 60$, $\therefore$ **$n(A \cup B) - n(B)$** $= 60 - 42 =$ **18** **(Ans.)**

(ii) **$n(\xi)$** $= n(A \cup B) + n(A \cup B)' = 60 + 22 =$ **82** **(Ans.)**

$n(\xi)$ can also be obtained by directly adding all the numbers in the Venn-diagram, such as $n(\xi) = 18 + 17 + 25 + 22 = 82$.

Example 5 :

Out of 80 candidates, which appeared in a combined test in English and Hindi, 64 passed at least in one subject. If 45 passed in English and 52 passed in Hindi, use a Venn-diagram to find :

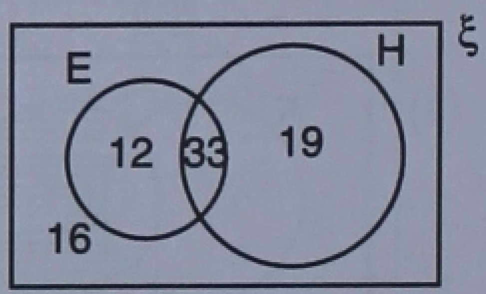
(i) How many passed in both the subjects ? (ii) How many passed in Hindi only ?

(iii) How many did not pass in English ?

Solution :

If ξ represents the set of candidates appeared in the test, E represents the set of candidates passed in English and H represents the set of candidates passed in Hindi, then

$$n(\xi) = 80,\ n(E \cup H) = 64,\ n(E) = 45 \text{ and } n(H) = 52$$



Now, $n(E \cup H) = n(E) + n(H) - n(E \cap H)$

$\Rightarrow \quad 64 = 45 + 52 - n(E \cap H)$

$\Rightarrow \quad n(E \cap H) = 97 - 64 = 33$

$n(E - H) = n(E) - n(E \cap H) = 45 - 33 = 12$

$n(H - E) = n(H) - n(E \cap H) = 52 - 33 = 19$

$n(E \cup H)' = n(\xi) - n(E \cup H) = 80 - 64 = 16$

(i) **No. of candidates passed in both the subjects** $= n(E \cap H) =$ **33** **(Ans.)**

(ii) **No. of candidates passed in Hindi only** $=$ **19** **(Ans.)**

(iii) **No. of candidates, who did not pass in English** $= n(\xi) - n(E) = 80 - 45 =$ **35** **(Ans.)**

Example 6 :

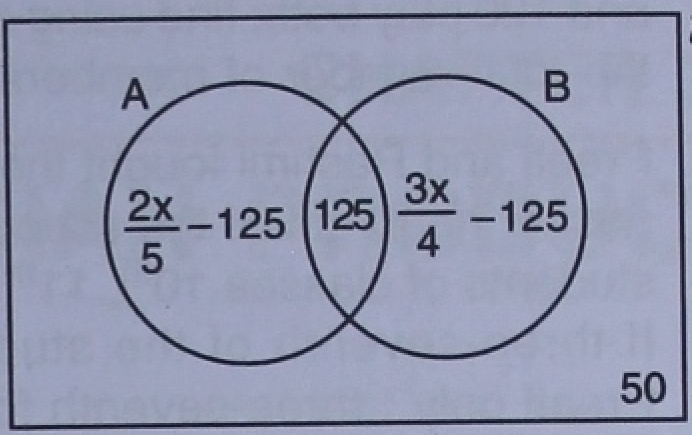
In a particular colony, two-fifths of the families read the magazine "The India Today" and three-fourths of the families read the magazine "Stardust". If fifty families read none of these two magazines and 125 families read both, use a Venn-diagram to find the number of families in the colony.

Solution :

Let there be x families in the colony.

If ξ be the set of families in the colony, then $n(\xi) = x$. If A be the set of families reading

"The India Today", then n(A) = $\frac{2x}{5}$ and if B be the set of families reading "Stardust", then n(B) = $\frac{3x}{4}$. Since 125 families read both the magazines, therefore, $\frac{2x}{5}$ −125 read only "The India Today" and $\frac{3x}{4}$ − 125 read only "Stardust".



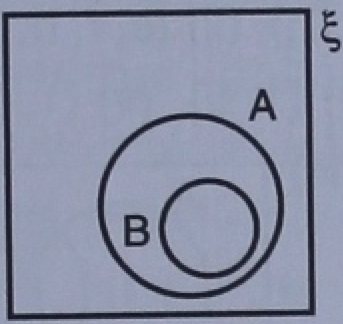
It is clear from the Venn-diagram that $\frac{2x}{5} - 125 + 125 + \frac{3x}{4} - 125 + 50 = x \Rightarrow x = 500$

∴ **The number of families in the colony = 500** **(Ans.)**

EXERCISE 38 (B)

1. A and B are two sets such that n(A ∪ B) = 25, n(A′ ∩ B) = 8 and n(A ∩ B′) = 9. Use venn-diagram to find :
   (i) n(A ∩ B) (ii) n(B – A)

2. Use the given figure to find :
   (i) n(A′)
   (ii) n(B′)
   (iii) n(A – B)
   (iv) n(B′ ∩ A)



   Given n (ξ) = 52, n(A) = 43 and n(B) = 27.

3. (i) If n(A) = 20, n(B) = 25 and n(A ∩ B) = 10, find :
   (a) n(A ∪ B) (b) n(A – B)
   (c) n(B – A)
   (ii) If n(A) = 40, n(B) = 50 and n(A ∪ B) = 75, find :
   (a) n(A ∩ B) (b) n(B – A)
   (c) n(A – B)

4. Let A and B be two sets such that n(A ∩ B) = 10, n(A ∪ B) = 60 and n(A – B) = 20. Find :
   (i) n(A) (ii) n(B)
   (iii) n(B – A)

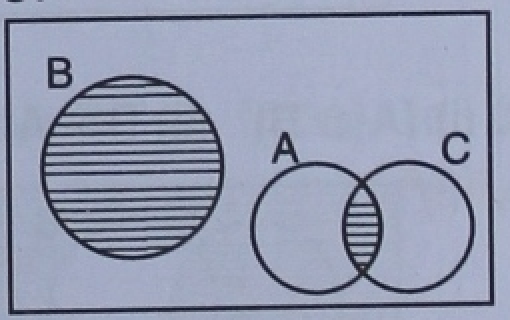
5. In a class of 65 students, 35 students take part in games, 37 take part in dances and 12 take part in none of these two. Draw a Venn-diagram to find ; how many take part :
   (i) in both
   (ii) in games only
   (iii) in dances only

6. Fourty-three persons went to a canteen, which sells soup and tea. If 18 persons took soup only ; 8 took tea only and 5 took nothing. Use a Venn-diagram to find :
   (i) how many took both '
   (ii) how many took 'soup' '
   (iii) how many took tea

7. Eighty-nine students of class VIII appeared for a combined test in Maths and Physics. If 62 students passed in both ; 4 failed in Maths and Physics and 7 failed only in Maths. Use a Venn-diagram to find ; how many :
   (i) failed in Physics only
   (ii) passed in Maths
   (iii) passed in Physics

8. If ζ is the set of boys in your school and B is the set of boys who play badminton. Draw a Venn-diagram showing that some of boys do not play badminton. If n(ζ) = 40 and n(B′) = 17 ; find :
   (i) how many do not play badminton;
   (ii) how many play badminton

9. Express the shaded part of the diagram given below in terms of union and intersection of the sets A, B and C.



   If n(ζ) = 200, n(A) = 80, n(B) = 75, n(C) = 50 and n(A ∩ C) = 30 ; find :
   (i) n(A ∪ C) (ii) n(A ∪ C)′
   (iii) n(A ∪ B)′ (iv) n(A ∪ B)′ – n(C)

10. A and B are two overlapping sets such that n(A ∩ B) = x + 4, n(A – B) = 4x – 8 and n(B) = 3x + 8. Find x, if n(A ∪ B) = 70. Also, find n(A′ ∩ B).

11. In a club, three-tenth of its members play cards only and four-tenth play carom-board only. If 30 members play none of these games

and 120 play both; find using a Venn-diagram, the total number of members in the club.

12. Preeti and Rashmi fought the election for the post of head girl of the school, for which the students of classes 10^{th}, 11^{th} and 12^{th} voted. If three-seventh of the students voted for Preeti only ; three-seventh for Rashmi ; fifty for both and 50 students did not use their votes, find :

(i) the total number of students in classes 10^{th}, 11^{th} and 12^{th}.

(ii) the number of students, who voted for Preeti and

(iii) the number of students, who voted for Rashmi only.

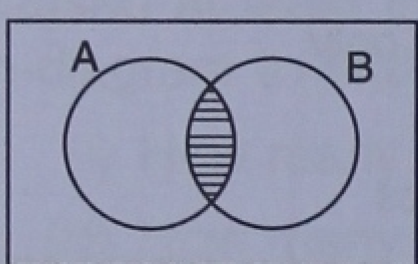
ANSWERS

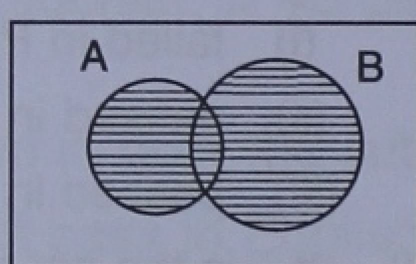
TEST YOURSELF

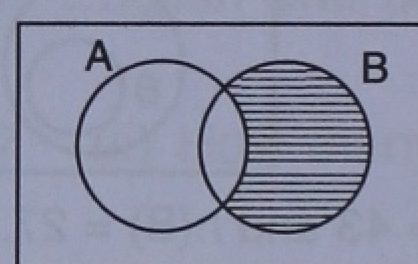
1. {3, 12} **2.** {3, 12, 11, 6, 9, 5, 13, 10} **3.** {3, 4, 7, 10, 12, 5, 11, 13} and {3, 11, 12}
4. {3, 4, 7, 10, 12, 6, 9, 11} and {8, 14, 15, 5, 13} **5.** {6, 9, 11} **6.** {5, 10, 13}
7. {6, 9, 12} and {6, 9, 12, 3, 10, 4, 7} **8.** {4, 7, 12, 6 , 9}

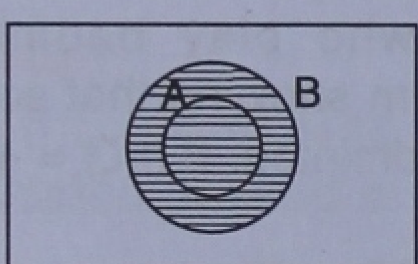
EXERCISE 38(A)

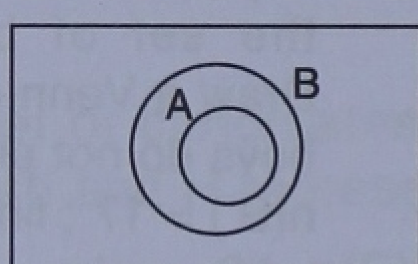
1. (i) $A \cup B$ = {a, b, c, d, e, f} (ii) $A' \cap B$ = {b, f} (iii) $A - B$ = {a, d} (iv) $B - A$ = {b, f} (v) $(A \cup B)'$ = {h, g}
2. (i) A' = {2, 5, 7, 8, 9, 10} (ii) B' = {3, 4, 6, 7, 8, 9, 10} (iii) $A' \cup B'$ = {2, 3, 4, 5, 6, 7, 8, 9, 10} (iv) {2, 3, 4, 5, 6, 7, 8, 9, 10}; Yes **3.** (i) {a, b, c, d, g, h, i, j} (ii) {e, f, h, j} (iii) {a, b, c, i} (iv) {a, b, c, i}; yes
4. (i) ϕ (ii) {1, 5, 6, 7, 9} (iii) {2, 3, 4, 6, 7, 8, 9, 10} (iv) {1, 5} (v) {1, 5, 6, 7, 9}

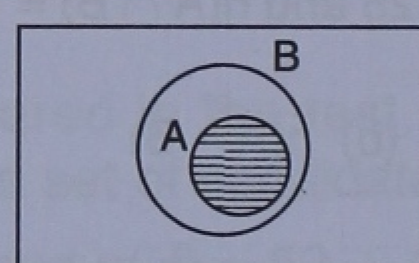
5. (i) 

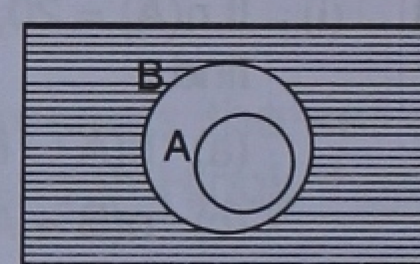
(ii) 

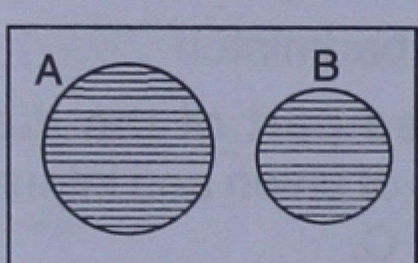
(iii) 

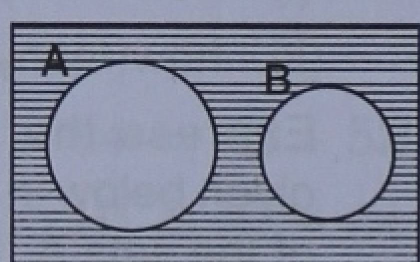
6. (i) 

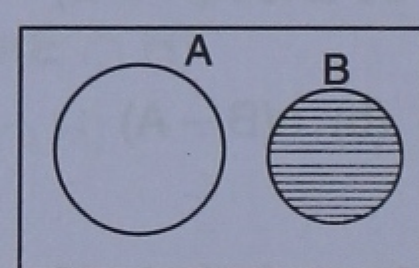
(ii) 

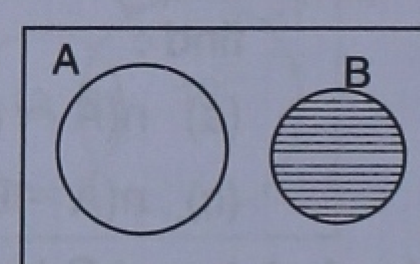
(iii) 

(iv) 

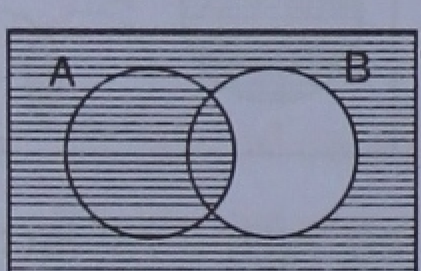
7. (i) 

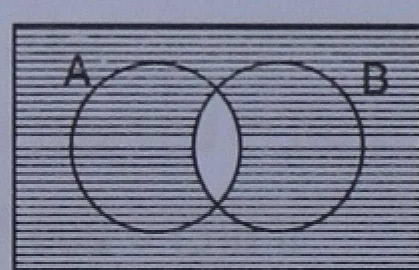
(ii) 

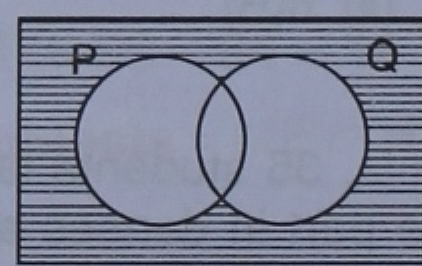
(iii) 

(iv) 

8. (i) $(A \cup B)'$ (ii) $B - A$ or $A' \cap B$ (iii) $(B - A)'$

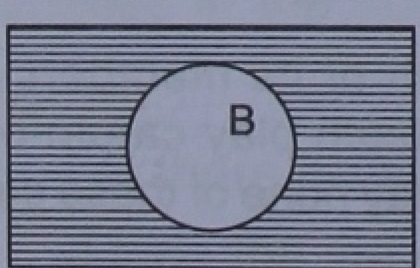
9. (i) 

(ii) 

(iii) 

10. (i) {a, d, g} (ii) {d, e, g} (iii) {a, b, c, d, e} **11.** (i) $A - B$ (ii) $(A \cup B) - (A \cap B)$ or $(A - B) \cup (B - A)$ (iii) $(A \cap B)'$ (iv) B' (v) $(A \cup B)'$

EXERCISE 38(B)

1. (i) 8 (ii) 8 **2.** (i) 9 (ii) 25 (iii) 16 (iv) 16 **3.** (i) (a) 35 (b) 10 (c) 15 (ii) (a) 15 (b) 35 (c) 25
4. (i) 30 (ii) 40 (iii) 30 **5.** (i) 19 (ii) 16 (iii) 18 **6.** (i) 12 (ii) 30 (iii) 20 **7.** (i) 16 (ii) 78 (iii) 69
8. 

(i) 17 (ii) 23 **9.** $B \cup (A \cap C)$ (i) 100 (ii) 100 (iii) 45 (iv) –5
10. x = 10 ; 24 **11.** 500 **12.** (i) 350 (ii) 200 (iii) 100