UNIT – 5
MENSURATION

CHAPTER 32

PERIMETER AND AREA OF PLANE FIGURES

32.1 REVIEW

Perimeter	The perimeter of a plane figure is the length of its boundary.
Area	The area of a plane figure is the amount of surface enclosed by its sides (boundary).

Common units of perimeter are metre (m), centimetre (cm), decimetre (dm), etc.

1. $1 \text{ m} = 100 \text{ cm}$ and $1 \text{ cm} = \frac{1}{100} \text{ m} = 0{\cdot}01 \text{ m}$
2. $1 \text{ m} = 10 \text{ dm}$ and $1 \text{ dm} = \frac{1}{10} \text{ m} = 0{\cdot}1 \text{ m}$
3. $1 \text{ dm} = 10 \text{ cm}$ and $1 \text{ cm} = \frac{1}{10} \text{ dm} = 0{\cdot}1 \text{ dm}$
4. $1 \text{ m} = 1000 \text{ mm}$ and $1 \text{ mm} = \frac{1}{1000} \text{ m} = 0{\cdot}001 \text{ m}$ and so on.

Common units of area are square metre (sq. m or m^2), square centimetre (sq. cm or cm^2), square millimetre (sq. mm or mm^2), etc.

1. $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10{,}000 \text{ cm}^2$ and $1 \text{ cm}^2 = \frac{1}{100 \times 100} \text{ m}^2 = 0{\cdot}0001 \text{ m}^2$
2. $1 \text{ m}^2 = 10 \times 10 \text{ dm}^2 = 100 \text{ dm}^2$ and $1 \text{ dm}^2 = \frac{1}{10 \times 10} \text{ m}^2 = 0{\cdot}01 \text{ m}^2$
3. $1 \text{ dm}^2 = 10 \times 10 \text{ cm}^2 = 100 \text{ cm}^2$ and $1 \text{ cm}^2 = \frac{1}{100} \text{ dm}^2 = 0{\cdot}01 \text{ dm}^2$
4. $1 \text{ cm}^2 = 10 \times 10 \text{ mm}^2 = 100 \text{ mm}^2$ and $1 \text{ mm}^2 = \frac{1}{100} \text{ cm}^2 = 0{\cdot}01 \text{ cm}^2$ and so on.

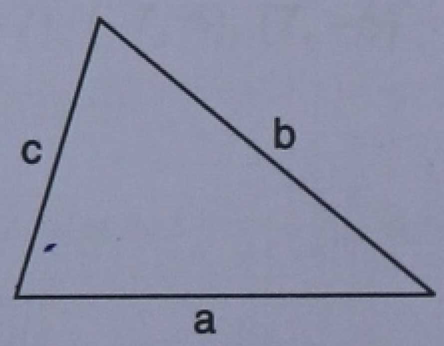
32.2 PERIMETER AND AREA OF TRIANGLES

1. If a, b and c are the three sides of a triangle; then its

   (i) perimeter = a + b + c

   (ii) area = $\sqrt{s(s-a)(s-b)(s-c)}$

   Where, s = semi-perimeter of the triangle = $\frac{a+b+c}{2}$

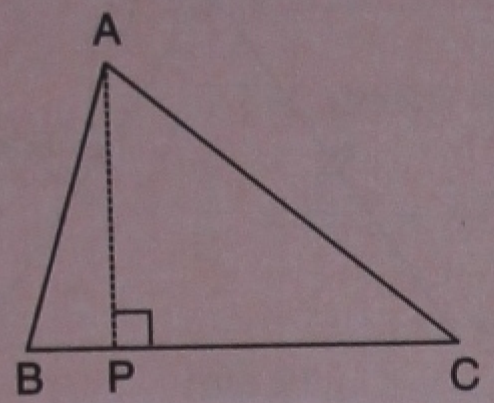


2. If one side (base) and the corresponding height (altitude) of the triangle are known, its

$$\boxed{\text{area} = \frac{1}{2} \text{ base} \times \text{height}}$$

Any side of the triangle can be taken as its base and the corresponding height means : the length of perpendicular to this side from the opposite vertex.

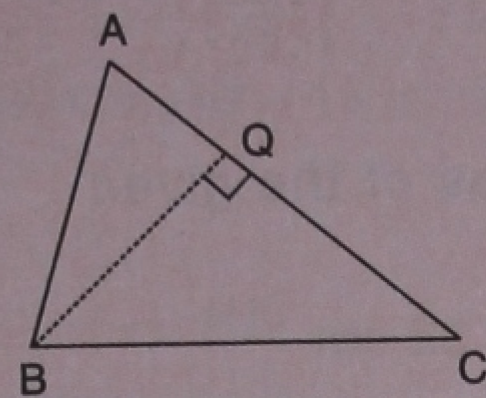
(i)



If BC is taken as base

area = $\frac{1}{2} \times BC \times AP$

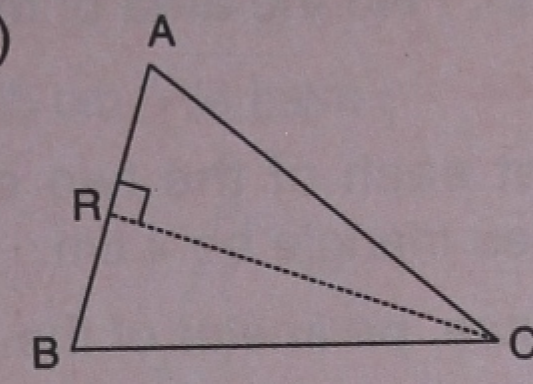
(ii)



If AC is taken as base

area = $\frac{1}{2} \times AC \times BQ$

(iii)



If AB is taken as base

area = $\frac{1}{2} \times AB \times CR$

Also, area = $\frac{1}{2} \times$ base $\times$ height $\Rightarrow$ (i) base = $\frac{2 \times \text{area}}{\text{height}}$

(ii) height = $\frac{2 \times \text{area}}{\text{base}}$

Example 1 :

Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm. Also, find the length of altitude corresponding to the largest side of the triangle.

Solution :

Let a = 9 cm, b = 12 cm and c = 15 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{9+12+15}{2} \text{ cm} = \frac{36}{2} \text{ cm} = 18 \text{ cm}$$

$$\therefore \quad \textbf{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)}$$

$$= \sqrt{18 \times 9 \times 6 \times 3} = \sqrt{2916} = \mathbf{54\ cm^2} \qquad \textbf{(Ans.)}$$

Also, area of triangle = $\frac{1}{2}$ base $\times$ corresponding altitude

$$\therefore \quad 54 = \frac{1}{2} \times 15 \times h \qquad \text{[Taking largest side as the base]}$$

$$\Rightarrow \quad h = \frac{54 \times 2}{15} \text{ cm} = \mathbf{7{\cdot}2\ cm} \qquad \textbf{(Ans.)}$$

Example 2 :

Find the area of an equilateral triangle, whose one side is a cm.

Solution :

$$s = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2} \qquad \text{[Sides of an equilateral triangle are equal]}$$

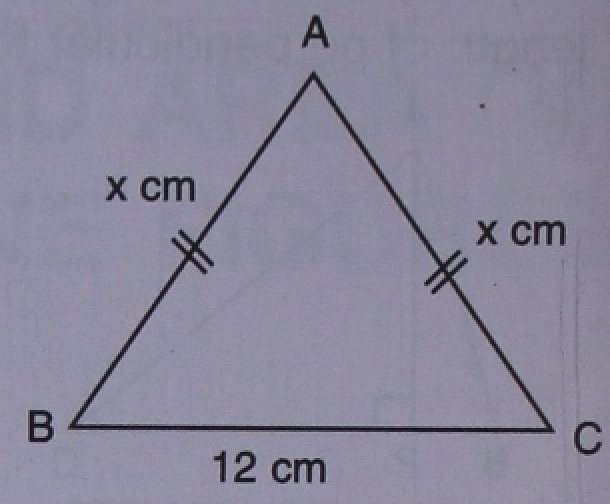
$$\therefore \quad \textbf{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{a \times a}{2 \times 2}\sqrt{3} = \mathbf{\frac{\sqrt{3}}{4}\ a^2 cm^2} \qquad \textbf{(Ans.)}$$

Example 3 :

The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Find the area of the triangle.



Solution :

Let each of the two equal sides of the given isosceles triangle be x cm.

Since, perimeter of the triangle is 32 cm

$\Rightarrow \quad x + x + 12 = 32 \Rightarrow x = 10$

Hence, the sides of the given isosceles triangle are 10 cm, 10 cm and 12 cm

Let $\quad a = 10$ cm, $b = 10$ cm and $c = 12$ cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{10+10+12}{2} \text{ cm} = 16 \text{ cm}$$

$$\text{Area of the } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

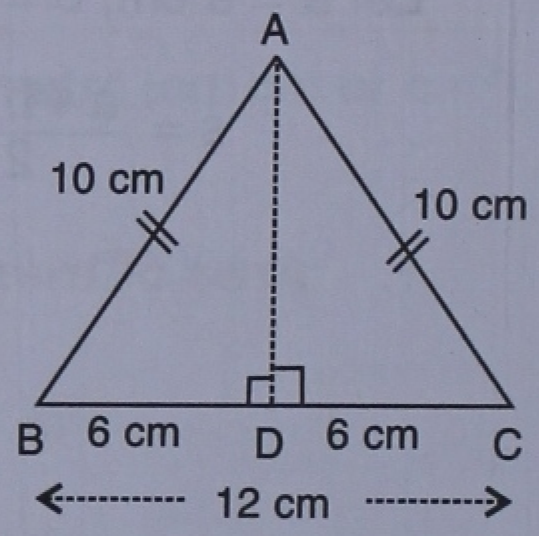
$$= \sqrt{16(16-10)(16-10)(16-12)} \text{ cm}^2$$

$$= \sqrt{16 \times 6 \times 6 \times 4} \text{ cm}^2 = 4 \times 6 \times 2 \text{ cm}^2 = 48 \text{ cm}^2$$ **(Ans.)**

Alternative method :

Draw AD perpendicular to base BC of the given triangle.

Since, the perpendicular from the vertex of an isosceles triangle to its base bisects the base, therefore



$$BD = CD = \frac{BC}{2} = \frac{12}{2} \text{ cm} = 6 \text{ cm.}$$

In right-angled triangle ABD,

$$AD^2 + BD^2 = AB^2$$

$$\Rightarrow \quad AD^2 + 6^2 = 10^2$$

$$\Rightarrow \quad AD^2 = 100 - 36 = 64 \text{ and } AD = \sqrt{64} \text{ cm} = 8 \text{ cm}$$

Now the base BC of the given triangle is 12 cm and its height AD is 8 cm

$$\therefore \quad \text{Area of the } \Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm} = 48 \text{ cm}^2$$ **(Ans.)**

TEST YOURSELF

1. In case of a triangle; if :
   (a) base = 6 m and height = 4 m; area = ...
   (b) area = $67{\cdot}2$ m^2 and height = 12 m; base = ...
   (c) area = $15{\cdot}36$ cm^2 and base = $4{\cdot}8$ cm; height = ...
2. If each side of an equilateral triangle is 8 cm;
   its area = ...

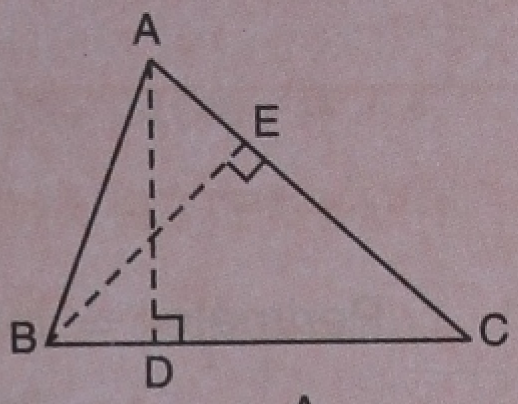
3. The area of an equilateral triangle, with side a cm is numerically equal to its perimeter, then .... = ... ⇒ .... = ... ⇒ a ....

4. In the given figure, BC = 20 cm, AD = 15 cm and AC = 25 cm. We have :

$\frac{1}{2} \times BC \times AD =$ ....,

.... = ... and

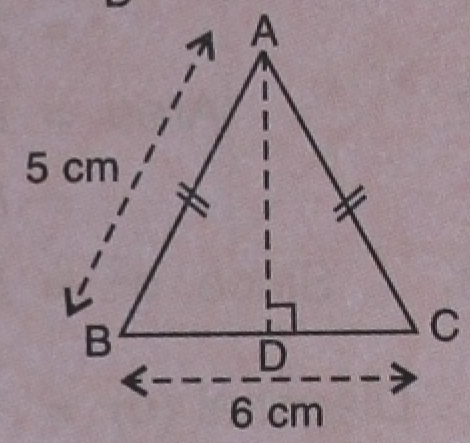
BE = ... = ...



5. In the given figure

(a) BD = ... and AD = ...

(b) area of Δ ABC = ... = ...



EXERCISE 32 (A)

1. Find the area of of a triangle; whose sides are :
   (i) 10 cm, 24 cm and 26 cm
   (ii) 18 mm, 24 mm and 30 mm
   (iii) 21 m, 28 m and 35 m

2. Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm; find :
   (i) area of the triangle
   (ii) height of the triangle corresponding to 8 cm side.

3. The sides of a triangle are 16 cm, 12 cm and 20 cm. Find :
   (i) area of the triangle
   (ii) height of the triangle, corresponding to the largest side
   (iii) height of the triangle, corresponding to the smallest side.

4. Two sides of a triangle are 6·4 m and 4·8 m. If height of the triangle corresponding to 4·8 m side is 6 m; find :
   (i) area of the triangle;
   (ii) height of the triangle corresponding to 6·4 m side

5. The base and the height of a triangle are in the ratio 4 : 5. If the area of the triangle is 40 m^2; find its base and height.

> Let base be 4x m and height be 5x m.
>
> $\therefore \frac{1}{2} \times 4x \times 5x = 40.$

6. The base and the height of a triangle are in the ratio 5 : 3. If the area of the triangle is 67·5 m^2; find its base and height.

7. The area of an equilateral triangle is $144\sqrt{3}$ cm^2; find its perimeter.

> Area of an equilateral triangle
>
> $= \frac{\sqrt{3}}{4} \times (\text{side})^2$ and its perimeter = 3 × side

8. The area of an equilateral triangle is numerically equal to its perimeter. Find its perimeter correct to 2 decimal places.

9. A field is in the shape of a quadrilateral ABCD in which side AB = 18 m, side AD = 24 m, side BC = 40 m, DC = 50 m and angle A = 90°. Find the area of the field.

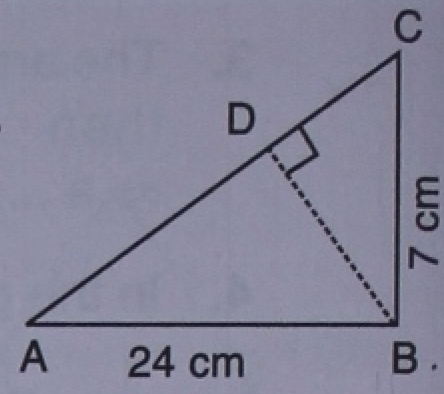
10. The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm. Find its area.

11. One of the equal sides of an isosceles triangle is 13 cm and its perimeter is 50 cm. Find the area of the triangle.

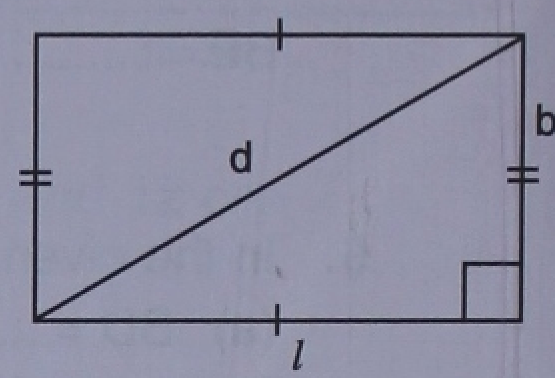
12. The altitude and the base of a triangular field are in the ratio 6 : 5. If its cost is ₹ 49,57,200 at the rate of ₹ 36,720 per hectare and 1 hectare = 10,000 sq. m, find (in metre) dimensions of the field.

> **Area of the given triangular field**
>
> $= \frac{\text{Cost of the field}}{\text{Rate}} = \frac{49,57,200}{36,720}$ hectare
>
> = 135 hectare
>
> = 135 × 10,000 sq. m = 1350000 m^2.

13. Find the area of the right-angled triangle with hypotenuse 40 cm and one of the other two sides 24 cm.
14. Use the information given in the adjoining figure to find :
    (i) the length of AC.
    (ii) the are a of Δ ABC.
    (iii) the length of BD, correct to one decimal place.



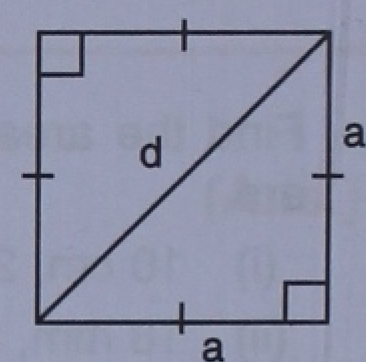
32.3 PERIMETER AND AREA OF RECTANGLES



1. Perimeter = length of boundary
   $= 2l + 2b = 2(l + b)$
2. Area = length × breadth
   $= l \times b$
3. Since, $d^2 = l^2 + b^2$ [Applying Pythagoras Theorem]

$\therefore$ Diagonal (d) $= \sqrt{l^2 + b^2}$

32.4 PERIMETER AND AREA OF SQUARES



1. Perimeter = $4a = 4 \times$ side
2. Area = $a \times a = a^2 = (\text{side})^2$
3. Diagonal (d) $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = \text{side}\sqrt{2}$

Example 4 :

The perimeter of a rectangle is 28 cm and its length is 8 cm. Find its :

(i) breadth (ii) area (iii) diagonal

Solution :

(i) Since, perimeter $= 2(l + b)$

$\Rightarrow$ $28 = 2(8 + b)$ $\Rightarrow$ **b = 6 cm** **(Ans.)**

(ii) **Area** $= l \times b = 8 \times 6 \text{ cm}^2 =$ **48 cm²** **(Ans.)**

(iii) **Diagonal (d)** $= \sqrt{l^2 + b^2} = \sqrt{8^2 + 6^2} =$ **10 cm** **(Ans.)**

Example 5 :

The area of a rectangle is 5·4 m². If its breadth is 1·5 m; find its :

(i) length (ii) perimeter

Solution :

(i) **Area** $= l \times b$

$\Rightarrow$ $5{\cdot}4 = l \times 1{\cdot}5$ or $l = \frac{5{\cdot}4}{1{\cdot}5}$ m = **3·6 m** **(Ans.)**

(ii) **Perimeter** $= 2(l + b) = 2\,(3{\cdot}6 + 1{\cdot}5)$ m = **10·2 cm** **(Ans.)**

Example 6 :

The perimeter of a square is 28 cm. Find its :

(i) one side (ii) area (iii) diagonal

Solution :

(i) Perimeter = 4 × side ⇒ 28 = 4 × side

$\Rightarrow$ side $= \frac{28}{4} = 7$ cm **(Ans.)**

(ii) Area = $(\text{side})^2 = 7^2 \text{ cm}^2 = 49 \text{ cm}^2$ **(Ans.)**

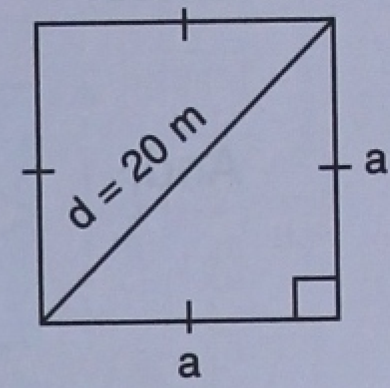
(iii) Diagonal (d) = side $\sqrt{2} = 7 \times 1.414$ $[\sqrt{2} = 1.414]$

$= 9.898$ cm **(Ans.)**

Example 7 :

The diagonal of square is 20 m. Find its :

(i) area (ii) length of one side (iii) perimeter



Solution :

(i) If each side of the square is a m;

Then, $d^2 = a^2 + a^2$ [Pythagoras Theorem]

$\Rightarrow (20)^2 = 2a^2$

$\Rightarrow a^2 = \frac{400}{2} = 200$

$\therefore$ (i) Area $= a^2 = 200 \text{ m}^2$ **(Ans.)**

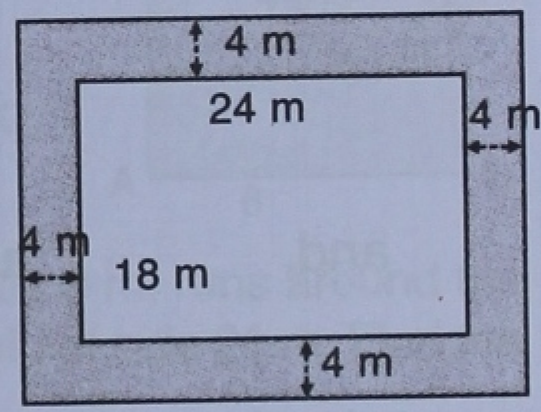
(ii) Since, $a^2 = 200 \Rightarrow a = \sqrt{200}$ m = 14·1 m **(Ans.)**

(iii) Perimeter = 4a = 4 × 14·1 m = 56·4 m **(Ans.)**

Example 8 :

A path of uniform width 4 m runs around the outside of a rectangular field 24 m by 18 m. Find the area of the path.

Solution :



According to the given information, the figure will be as shown alongside in which the shaded portion is the area of the path.

Clearly :

Length of the field excluding path = 24 m

and, width of the field excluding path = 18 m

$\therefore$ **Area of the field excluding path** = 24 m × 18 m = **432 m²**

Now, length of the field including path = (24 + 2 × 4) m = 32 m

and, width of the field including path = (18 + 2 × 4) m = 26 m

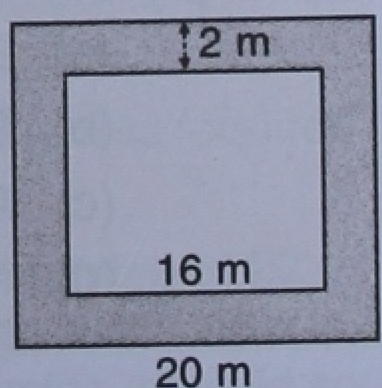
$\therefore$ **Area of the field including path** = 32 m × 26 m = **832 m²**

$\therefore$ Area of the path $= 832 \text{ m}^2 - 432 \text{ m}^2 = 400 \text{ m}^2$ **(Ans.)**

Example 9 :

A path of uniform width 2 m runs around the inside of a square field of side 20 m. Find the area of the path.

Solution :



According to the given information, the figure will be as shown alongside :

Clearly :

area of the field including path = 20 m × 20 m = 400 m²

Each side of the square field excluding path = (20 – 2 × 2) m = 16 m

∴ Area of the field excluding path = 16 m × 16 m = 256 m^2

∴ **Area of the path** = 400 m^2 – 256 m^2 = **144 m^2** **(Ans.)**

Example 10 :

A rectangular hall is 5·25 m long and 3·78 m wide. Its floor is to be covered with square tiles, each of side 21 cm. Find the cost of tiles required at the rate of ₹ 5 per tile.

Solution :

Since, floor area of hall = 5·25 m × 3·78 m

= 525 × 378 cm^2

And, area of each tile = 21 × 21 cm^2

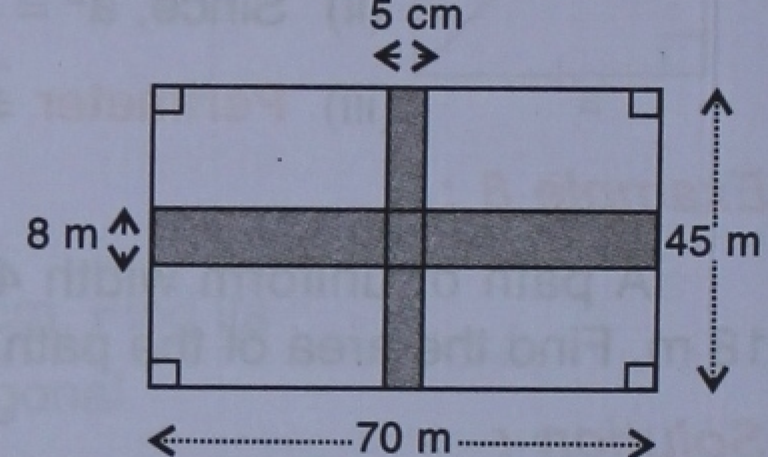
$$\text{Number of tiles required} = \frac{\text{Floor area of hall}}{\text{Area of each tile}} = \frac{525 \times 378}{21 \times 21} = 450$$

Since, cost of each tile = ₹ 5

∴ **The cost of all the tiles required** = 450 × ₹ 5 = **₹ 2,250** **(Ans.)**

Example 11 :

The adjoining figure shows a rectangular field 70 m long and 45 m wide. The shaded portion shows two mutually perpendicular roads, one of width 5 m and the other with width 8 cm. Find the cost of levelling the roads at the rate of ₹ 180 per sq. m.



Solution :

Area of road along the length = 70 m × 8 m = 560 m^2,

area of road along the width = 45 m × 5 m = 225 m^2

and, area common to both the roads = 5 m × 8 m = 40 m^2

∴ Actual area of the two roads = 560 m^2 + 225 m^2 – 40 m^2

= 745 m^2

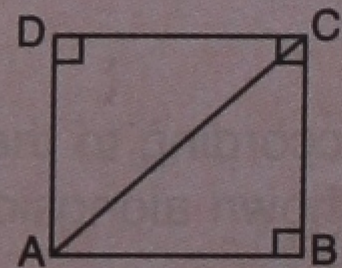
⇒ **The cost of levelling the roads** = Area of roads × Rate of levelling

= 745 × ₹ 180 = **₹ 1,34,100** **(Ans.)**

TEST YOURSELF

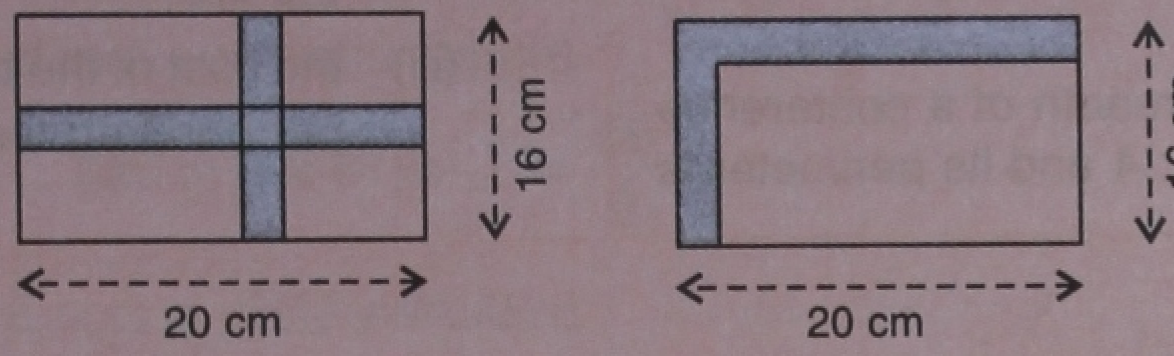
6. If area of a square = its perimeter (numerically)
⇒ (its side)^2 = ... ⇒ **Its side** = ... unit

7. If AC = 10 cm and AB = 8 cm

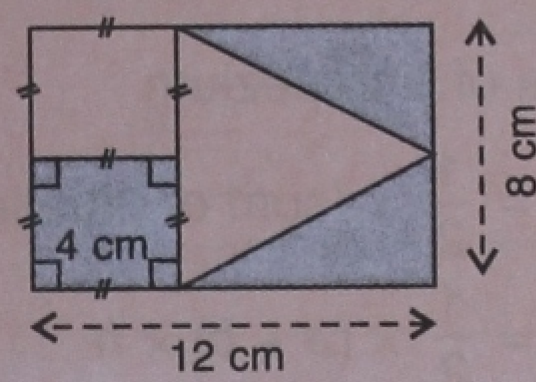


(a) BC = ...
(b) Perimeter of ABCD = ... = ...
(c) Area of Δ ABC = ... = ...
(d) Area of ABCD = ... = ...

8. In both the figures, widths of shaded portions are same, then in both the figures areas of
(a) shaded portions are .... (b) unshaded portions are ....

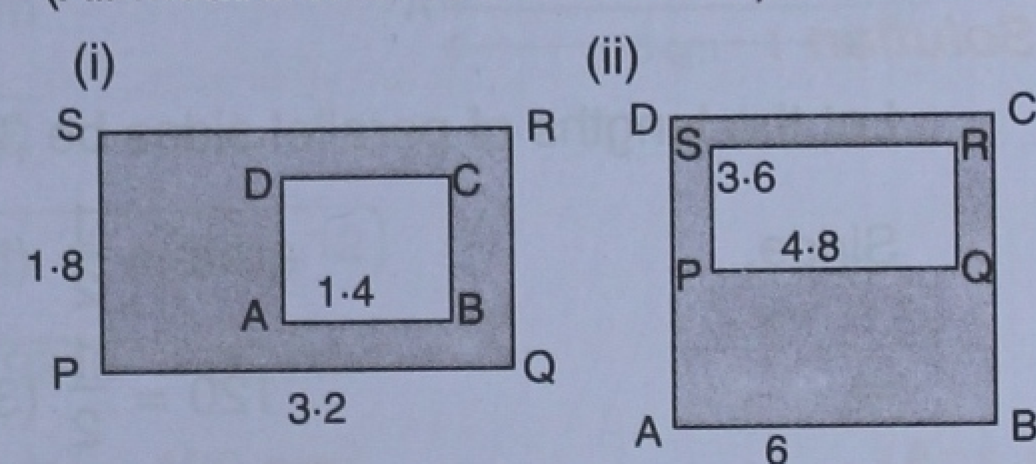


9. Area of shaded portion
= .... − .... − ....
= .... − .... − ....
= 48 cm^2



EXERCISE 32 (B)

1. Find the length and perimeter of a rectangle, whose area = 120 cm^2 and breadth = 8 cm.
2. The perimeter of a rectangle is 46 m and its length is 15 m. Find its :
   (i) breadth (ii) area (iii) diagonal
3. The diagonal of a rectangle is 34 cm. If its breadth is 16 cm; find its :
   (i) length (ii) area
4. The area of a small rectangular plot is 84 m^2. If the difference between its length and the breadth is 5 m; find its perimeter.
5. The perimeter of a square is 36 cm; find its area.
6. Find the perimeter of a square whose area is 1·69 m^2.
7. The diagonal of a square is 12 cm long; find its area and length of one side.
8. The diagonal of a square is 15 m; find the length of its one side and perimeter.
9. The area of a square is 169 cm^2. Find its :
   (i) one side (ii) perimeter
10. The length of a rectangle is 16 cm and its perimeter is equal to the perimeter of a square with side 12·5 cm. Find the area of the rectangle.
11. The perimeter of a square is numerically equal to its area. Find its area.
12. Each side of a rectangle is doubled. Find the ratio between :
    (i) perimeters of the original rectangle and the resulting rectangle
    (ii) areas of the original rectangle and the resulting rectangle
13. In each of the following cases ABCD is a square and PQRS is a rectangle. Find, in each case, the area of the shaded portion.
    (All measurements are in metre).



14. A path of uniform width, 3 m, runs around the outside of a square field of side 21 m. Find the area of the path.
15. A path of uniform width, 2·5 m, runs around the inside of a rectangular field 30 m by 27 m. Find the area of the path.
16. The length of a hall is 18 m and its width is 13·5 m. Find the least number of square tiles, each of side 25 cm, required to cover the floor of the hall,
    (i) without leaving any margin.
    (ii) leaving a margin of width 1·5 m all around.

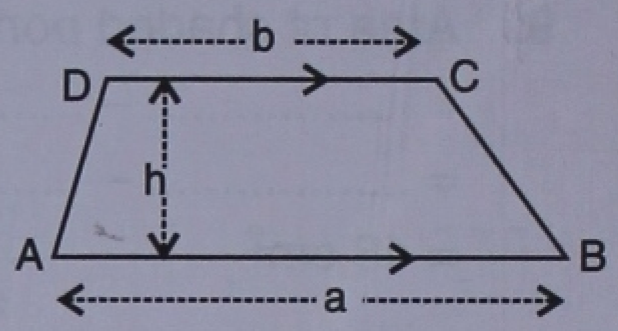
    In each case, find the cost of the tiles required at the rate of ₹ 6 per tile.
17. A rectangular field is 30 m in length and 22 m in width. Two mutually perpendicular roads, each 2·5 m wide, are drawn inside the field so that one road is parallel to the length of the

field and the other road is parallel to its width. Calculate the area of the crossroads.

18. The length and the breadth of a rectangular field are in the ratio 5 : 4 and its area is 3380 m^2. Find the cost of fencing it at the rate of ₹ 75 per m .

19. The length and the breadth of a conference hall are in the ratio 7 : 4 and its perimeter is 110 m. Find :
   (i) area of the floor of the hall.
   (ii) number of tiles, each a rectangle of size 25 cm × 20 cm, required for flooring of the hall.
   (iii) the cost of the tiles at the rate of ₹ 1,400 per hundred tiles.

32.5 TRAPEZIUM



Area of a trapezium

$= \frac{1}{2} \times$ (sum of parallel sides) × height

$= \frac{1}{2} \times (a + b) \times h$

1. Here, a and b are the parallel sides of the trapezium and h is the height.
2. Height of the trapezium means the distance between its parallel sides.

Example 12 :

The lengths of parallel sides of a trapezium are in the ratio 3 : 5 and the distance between them is 10 cm. If the area of the trapezium is 120 cm^2; find the lengths of its parallel sides.

Solution :

Let the lengths of parallel sides be 3x cm and 5x cm

Since, area $= \frac{1}{2}$ (the sum of parallel sides) × height

$\Rightarrow$ $120 = \frac{1}{2}(3x + 5x) \times 10$ *i.e.* $x = \frac{120}{40} = 3$

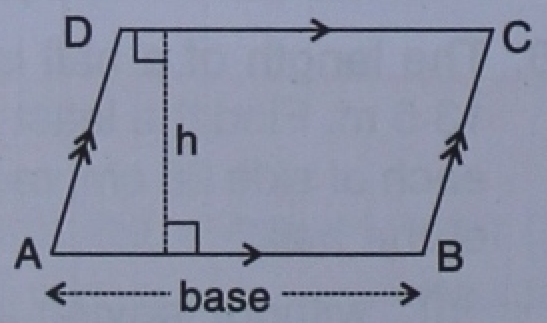
∴ **Lengths of parallel sides** = 3x cm and 5x cm

= 3 × 3 cm and 5 × 3 cm = **9 cm and 15 cm** **(Ans.)**

32.6 PARALLELOGRAM

Area = Base × corresponding height

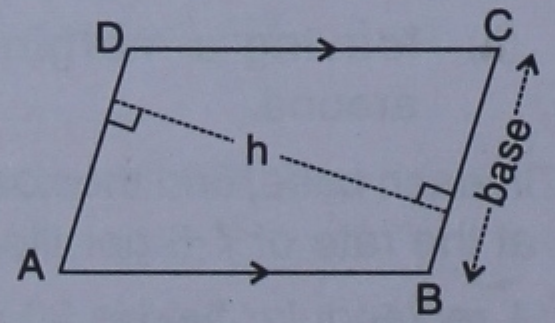
e.g. (i)



For parallelogram ABCD, if side AB is taken as base, the corresponding height, is the distance between parallel sides AB and DC.

∴ **Area = AB × h**

(ii)



And, if the side BC is taken as base, the corresponding height is the distance between parallel sides BC and AD.

∴ **Area = BC × h**

Example 13 :

A parallelogram has sides of 12 cm and 8 cm. If the distance between the 12 cm sides is 5 cm; find the distance between 8 cm sides.

Solution :

According to the question; if base = 12 cm; height = 5 cm

$\therefore$ Area = base × height = $12 \times 5\ cm^2 = 60\ cm^2$.

Now, if base = 8 cm; then to find height ?

Area = base × height

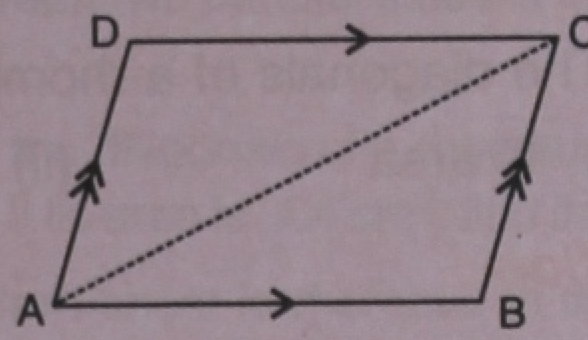
$\Rightarrow$ $60 = 8 \times h \Rightarrow$ **$h = \dfrac{60}{8} = 7{\cdot}5$ cm** **(Ans.)**

Each diagonal bisects a parallelogram;

i.e. $\Delta\ ABC = \Delta\ ADC = \frac{1}{2}$ (// gm ABCD)

Similarly, if diagonal BD is drawn :

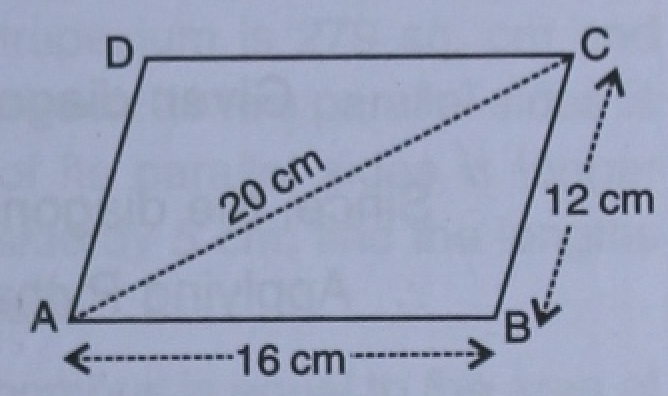
$\Delta\ ABD = \Delta\ BCD = \frac{1}{2}$ (// gm ABCD).



Example 14 :

In parallelogram ABCD; AB = 16 cm, BC = 12 cm and diagonal AC = 20 cm. Find the area of the parallelogram.

Solution :



For triangle ABC,

AB = 16 cm, AC = 20 cm and BC = 12 cm

$\therefore \quad s = \dfrac{a+b+c}{2} = \dfrac{16+20+12}{2} = 24$ cm

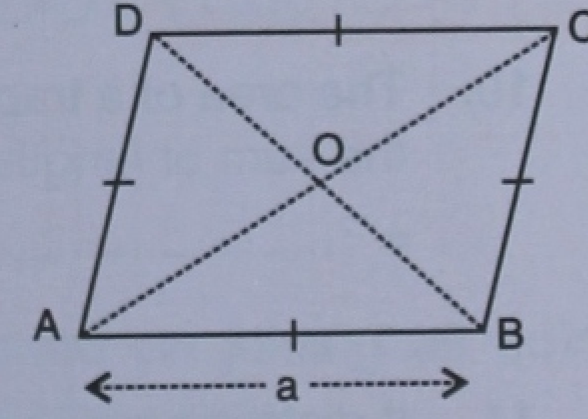
$\therefore \quad$ Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24\ (24\ -\ 16)(24\ -\ 20)(24\ -\ 12)}$

$= \sqrt{24 \times 8 \times 4 \times 12} = \sqrt{9216} = 96\ cm^2$

$\therefore$ **Area of parallelogram ABCD** $= 2 \times 96\ cm^2 =$ **$192\ cm^2$** **(Ans.)**

32.7 RHOMBUS



A rhombus is a parallelogram whose all the sides are equal.

The adjoining figure shows a rhombus ABCD in which AB = BC = CD = DA = a (say).

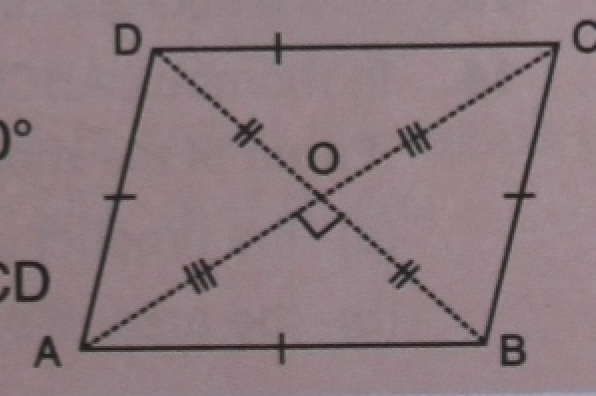
1. Perimeter of a rhombus = 4 × side = 4a
2. Area of a rhombus $= \dfrac{1}{2} \times$ product of its diagonals

$= \dfrac{1}{2} \times AC \times BD$

1. Diagonals of a rhombus bisect each other at right angle.

$\therefore$ $OA = OC = \frac{1}{2}\ AC$, $OB = OD = \frac{1}{2}\ BD$ and $\angle AOB = 90°$

2. $\Delta\ AOB = \Delta\ BOC = \Delta\ COD = \Delta\ DOA = \frac{1}{4} \times$ rhombus ABCD



3. Since, a rhombus is a parallelogram :
   (i) each diagonal bisects it

   i.e. $\Delta\,ABC = \Delta\,ADC = \frac{1}{2} \times$ rhombus ABCD

   and, $\Delta\,ABD = \Delta\,BCD = \frac{1}{2} \times$ rhombus ABCD

   (ii) area of rhombus = base × height

Example 15 :

The diagonals of a rhombus are 16 cm and 12 cm; find :

(i) its area (ii) length of its side (iii) its perimeter.

Solution :

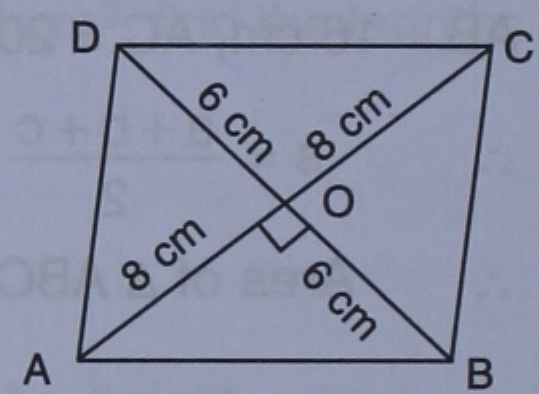
(i) **Area of rhombus** $= \frac{1}{2} \times$ product of its diagonals

$= \frac{1}{2} \times 16 \times 12\ cm^2 =$ **96 cm²** **(Ans.)**

(ii) Given diagonal AC = 16 cm; then $OA = OC = \frac{16}{2}$ cm = 8 cm

Given diagonal BD = 12 cm, then $OB = OD = \frac{12}{2}$ cm = 6 cm

Since, the diagonals of a rhombus bisect at 90°,



∴ Applying Pythagoras Theorem in Δ AOB; we get :

$$(AB^2) = OA^2 + OB^2$$
$$= 8^2 + 6^2 = 100$$

∴ $AB = \sqrt{100} = 10$ cm

∴ **Length of its side = 10 cm** **(Ans.)**

(iii) **Perimeter of rhombus** = 4 × side = 4 × 10 = **40 cm** **(Ans.)**

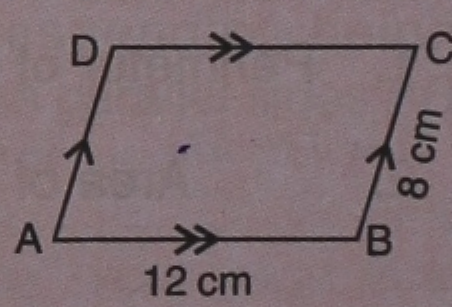
TEST YOURSELF

10. The area of a trapezium is 30 cm² and the distance between its parallel sides is 5 cm; the sum of lengths of its parallel sides

= ....

....

11. If distance between parallel sides AB and DC is 6 cm and the distance between parallel sides AD and BC is x cm. Then .... = ...
⇒ x = ... cm = ... cm



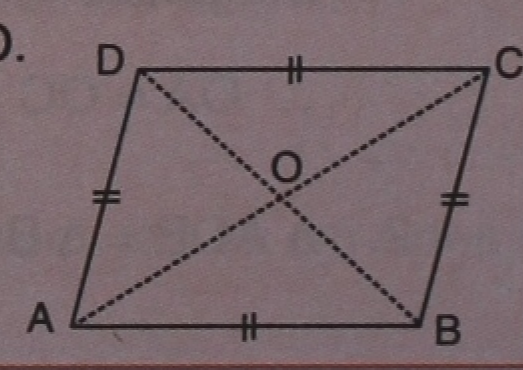
12. Each diagonal of a parallelgoram divides the .... into .... equal triangles.

13. The diagonals of a parallelogram divide it into .... equal ....

14. In rhombus ABCD, its diagonals intersect each other at point O. If area of the rhombus is 60 cm²;

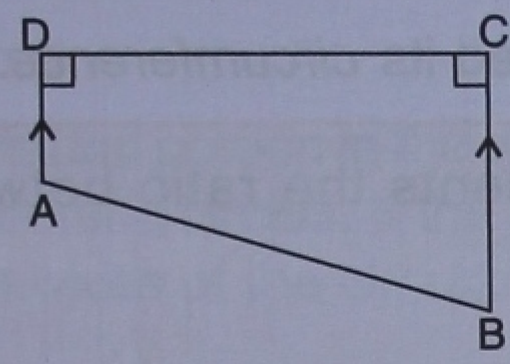
(a) the product of its diagonals = ...

(b) the area of Δ AOB = ...



EXERCISE 32 (C)

1. The following figure shows the cross-section ABCD of a swimming pool which is trapezium in shape.



If the width DC, of the swimming pool is 6·4 m, depth (AD) at the shallow end is 80 cm and depth (BC) at deepest end is 2·4 m, find its area of the cross-section.

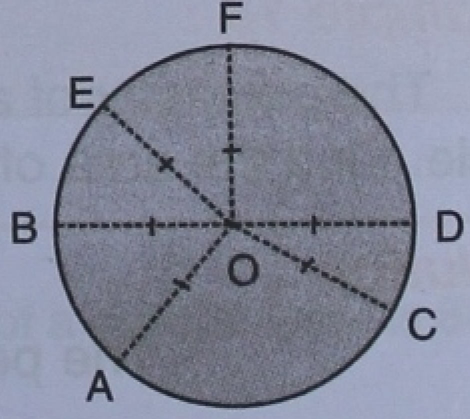
2. The parallel sides of a trapezium are in the ratio 3 : 4. If the distance between the parallel sides is 9 dm and its area is 126 dm^2, find the lengths of its parallel sides.
3. The two parallel sides and the distance between them are in the ratio 3 : 4 : 2. If the area of the trapezium is 175 cm^2; find its height.
4. A parallelogram has sides of 15 cm and 12 cm. If the distance between the 15 cm sides is 6 cm, find the distance between 12 cm sides.
5. A parallelogram has sides of 20 cm and 30 cm. If the distance between its shorter sides is 15 cm; find the distance between the longer sides.
6. The adjacent sides of a parallelogram are 21 cm and 28 cm. If its one diagonal is 35 cm; find the area of the parallelogram.
7. The diagonals of a rhombus are 18 cm and 24 cm. Find ;
   (i) its area (ii) length of its sides
   (iii) its perimeter
8. The perimeter of a rhombus is 40 cm. If one diagonal is 16 cm, find :
   (i) its another diagonal (ii) area
9. Each side of a rhombus is 18 cm. If the distance between two parallel sides is 12 cm, find its area.
10. The length of the diagonals of a rhombus is in the ratio 4 : 3. If its area is 384 cm^2, find its side.
11. A thin metal iron-sheet is rhombus in shape, with each side 10 m. If one of its diaginals is 16 m, find the cost of painting its both sides at the rate of ₹ 6 per m^2.

    Also, find the distance between the opposite sides of this rhombus.
12. The area of a trapezium is 279 sq. cm and the distance between its two parallel sides is 18 cm. If one of its parallel sides is longer than the other side by 5 cm, find the lengths of its parallel sides.
13. The area of a rhombus is equal to the area of a triangle. If base of Δ is 24 cm, its corresponding altitude is 16 cm and one of the diagonals of the rhombus is 19·2 cm. Find its other diagonal.
14. Find the area of the trapezium ABCD in which AB//DC, AB = 18 cm, $\angle B = \angle C = 90°$, CD = 12 cm and AD = 10 cm.

32.8 CIRCLE

A ***circle*** is the surface enclosed by a closed curve, obtained on joining all points that are at the same distance from a fixed point and lie in the same plane.

The adjoining figure shows a fixed point O and points A, B, C, D, ..., etc., that are at the same distance from fixed point O and lie in the same plane. The surface obtained on drawing a closed curve through points A, B, C, D, ..., etc., is the ***circle*** shown by shaded portion.



The *fixed point* is called the *centre* and the *constant distance* is called the *radius* of the circle. Thus, point O is the centre and radius = OA = OB = OC = OD and so on.

The radius of a circle is in general represented by *r*.

1. **Diameter :** A straight line, joining any two points on the circumference of the circle and passing through the centre, is called diameter of the circle. Thus, BOD is a diameter.

$$\boxed{\text{Diameter} = 2 \times \text{radius} \quad i.e. \quad d = 2r.}$$

2. **Circumference :** The *perimeter* of the circle is called its *circumference.*

The Greek letter π (pronounced as Pie) represents the ratio between the circumference and diameter of a circle.

i.e. $\dfrac{\text{circumference of the circle}}{\text{its diameter (d)}} = \pi$

$\Rightarrow$ **circumference of the circle** $= \pi \times d$

$= \mathbf{2\pi r}$ [As, d = 2r]

3. **Area of a circle** $= \pi r^2$; where $\pi = 3\frac{1}{7} = \frac{22}{7}$ and r = radius of the circle.

Example 16 :

Find radius and area of a circle whose circumference is 132 cm.

Solution :

Given circumference = 132

$\Rightarrow$ $2\pi r = 132$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21$ cm **(Ans.)**

$\therefore$ Area $= \pi r^2 = \frac{22}{7} \times (21)^2 = 1386$ cm^2 **(Ans.)**

Example 17 :

Find circumference of the circle, whose area is 24·64 m^2.

Solution :

Given : $\pi r^2 = 24{\cdot}64$

$\Rightarrow$ $\frac{22}{7} r^2 = 24{\cdot}64$

$\Rightarrow$ $r^2 = 24{\cdot}64 \times \frac{7}{22} = 7{\cdot}84 \Rightarrow r = 2{\cdot}8$ m

$\therefore$ Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 2{\cdot}8$ m $= 17{\cdot}6$ m **(Ans.)**

Example 18 :

The perimeter of a square, whose each side is 22 cm, is the same as circumference of a circle. Find the area of the circle.

Solution :

The perimeter of the square = 4 × its side

= 4 × 22 cm = 88 cm

$\Rightarrow$ The circumference of the circle = 88 cm

i.e., $2\pi r = 88$ cm [r = radius of the circle]

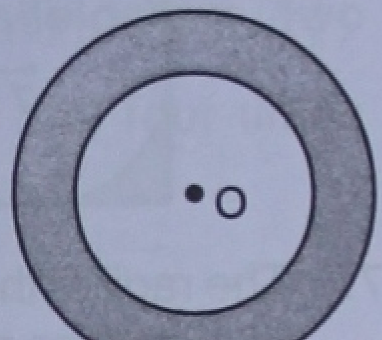
$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88$ cm *i.e.* $r = 14$ cm

$\therefore$ **Area of the circle** $= \pi r^2$

$= \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm} = $ **616 cm²** **(Ans.)**

Example 19 :

The shaded portion in the adjoining figure shows a circular path enclosed by two concentric circles. If the inner circumference of the path is 176 m and the uniform width of the circular path is 3·5 m; find the area of the path.



Solution :

Let the radius of the inner circle be *r* m

$\therefore \quad 2\pi r = 176 \Rightarrow 2 \times \frac{22}{7} \times r = 176$ m

$\Rightarrow \quad r = 176 \times \frac{7}{2 \times 22}$ m $= 28$ m

Since, the width of the path = 3·5 m

the radius of the outer circle (R) = r + 3·5 m = 28 m + 3·5 m = 31·5 m

The area of the circular path = Area of the outer circle – Area of the inner circle

$= \pi R^2 - \pi r^2$

$= (\frac{22}{7} \times 31{\cdot}5 \times 31{\cdot}5 - \frac{22}{7} \times 28 \times 28)\text{m}^2$

$= (3118{\cdot}5 - 2464)\text{m}^2 = $ **654·5 m²** **(Ans.)**

Whenever the value of π is not given, take : $\pi = \frac{22}{7}$.

TEST YOURSELF

15. The radius of a circle is doubled, then its :

(a) diameter is ....

(b) circumference is ....

(c) area is made ....

16. The area of a circle is numerically equal to its circumference ⇒ .... = ...
⇒ r = ... unit.

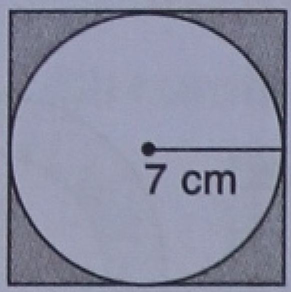
17. Sum of circumferences of two circles with radii 8 cm and 15 cm is equal to the circumference of the circle with radius r, then .... = ... + ...
⇒ r = ... + ... ⇒ r = ... cm.

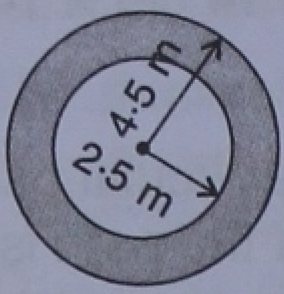
EXERCISE 32 (D)

1. Find the radius and area of a circle, whose circumference is :
   (i) 132 cm (ii) 22 m
2. Find the radius and circumference of a circle, whose area is :
   (i) 154 cm² (ii) 6·16 m²
3. The circumference of a circular table is 88 m. Find its area.
4. The area of a circle is 1386 sq. cm, find its circumference.
5. Find the area of a flat circular ring formed by two concentric circles (circles with

same centre) whose radii are 9 cm and 5 cm.

6. Find the area of the shaded portion in each of the following diagrams :

(i) 

(ii) 

7. The radii of the inner and outer circumferences of a circular-running-track are 63 m and 70 m respectively. Find :
   (i) the area of the track
   (ii) the difference between the lengths of the two circumferences of the track

8. A circular field of radius 105 m has a circular path of uniform width of 5 m along and inside its boundary. Find the area of the path.

9. There is a path of uniform width 7 m round and outside a circular garden of diameter 210 m. Find the area of the path.

10. A wire, when bent in the form of a square, encloses an area of 484 cm^2. Find :
    (i) one side of the square
    (ii) length of the wire
    (iii) the largest area enclosed, if the same wire is bent to form a circle.

11. A wire, when bent in the form of a square, encloses an area of 196 cm^2. If the same wire is bent to form a circle, find the area of the circle.

12. The radius of a circular wheel is 42 cm. Find the distance travelled by it in :
    (i) 1 revolution (ii) 50 revolutions
    (iii) 200 revolutions

The distance travelled by a wheel in one revolution is equal to its circumference.

13. The diameter of wheel is 0·70 m. Find the distance covered by it in 500 revolutions.

    If the wheel takes 5 minutes to make 500 revolutions; find its speed in :
    (i) m/s (ii) km/hr

14. A bicycle wheel, diameter 56 cm, is making 45 revolutions in every 10 seconds. At what speed, in kilometre per hour, is the bicycle travelling ?

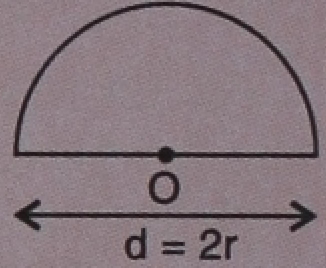
15. A roller has a diameter of 1·4 m. Find :
    (i) its circumference
    (ii) the number of revolutions it makes while travelling 61·6 m.

16. Find the area of the circle, length of whose circumference is equal to the sum of the lengths of the circumferences with radii 15 cm and 13 cm.

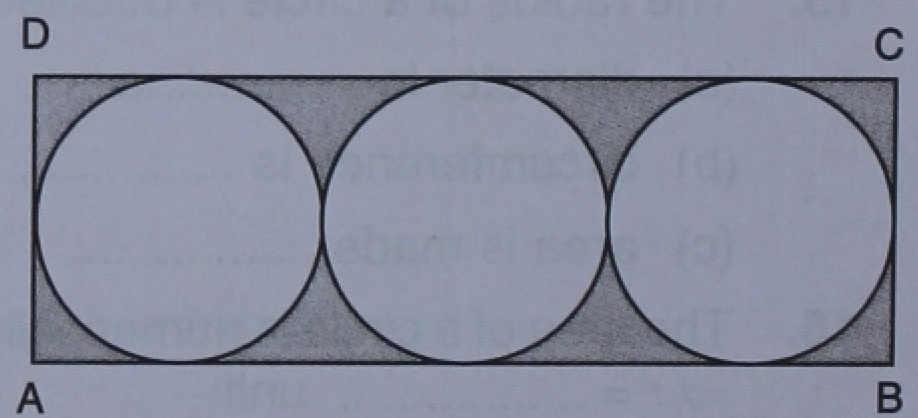
17. A piece of wire of length 108 cm is bent to form a semicircular arc bounded by its diameter. Find its radius and area enclosed.

$\frac{2\pi r}{2} + d = 108$

Area enclosed $= \frac{\pi r^2}{2}$



18. In the following figure, a rectangle ABCD encloses three circles. If BC = 14 cm, find the area of the shaded portion. (Take $\pi = 3\frac{1}{7}$)



ANSWERS

TEST YOURSELF

1. (a) $\frac{1}{2} \times 6 \times 4$ m^2 = 12 m^2 (b) $\frac{2 \times 67{\cdot}2}{12}$ m = 11·2 m (c) $\frac{2 \times 15{\cdot}36}{4{\cdot}8}$ cm = 6·4 cm **2.** $\frac{\sqrt{3}}{4} \times 8^2$ cm^2 = $16\sqrt{3}$ cm^2 **3.** $\frac{\sqrt{3}}{4}a^2 = 3a \Rightarrow a = 3 \times \frac{4}{\sqrt{3}} \Rightarrow a = 4\sqrt{3}$ unit **4.** $\frac{1}{2} \times$ AC $\times$ BE; $\frac{1}{2} \times 20 \times 15 = \frac{1}{2} \times 25 \times$ BE; $\frac{20 \times 15}{25}$ cm = 12 cm **5.** (a) 3 cm and 4 cm (b) $\frac{1}{2} \times 6 \times 4$ cm^2 = 12 cm^2 **6.** 4 × its side; 4 unit **7.** (a) $\sqrt{10^2 - 8^2}$ = 6 cm (b) 2(8 + 6) cm = 28 cm (c) $\frac{1}{2} \times 8 \times 6$ cm^2 = 24 cm^2

(d) $2 \times 24\ cm^2 = 48\ cm^2$ **8.** (a) equal (b) equal **9.** : $12 \times 8\ cm^2 - \frac{1}{2} \times 8 \times 8\ cm^2 - 4 \times 4\ cm^2$; $96\ cm^2 - 32\ cm^2 - 16\ cm^2$ **10.** $\frac{1}{2}$(sum of parallel sides) $\times 5 = 30 \Rightarrow$ sum of parallel sides $= \frac{30 \times 2}{5}$ $= 12$ cm **11.** 12 cm × 6 cm = 8 cm × x cm; $x = \frac{12 \times 6}{8}$ cm = 9 cm **12.** parallelogram, two **13.** four, triangles **14.** (a) 120 (b) 15 cm^2 **15.** (a) doubled (b) doubled (c) four times **16.** $\pi r^2 = 2\pi r \Rightarrow r = 2$ unit **17.** $2\pi r = 2\pi \times 8 + 2\pi \times 15 \Rightarrow r = 8 + 15 \Rightarrow r = 23$ cm

EXERCISE 32(A)

1. (i) 120 cm^2 (ii) 216 mm^2 (iii) 294 m^2 **2.** (i) 12 cm^2 (ii) 3 cm **3.** (i) 96 cm^2 (ii) 9·6 cm (iii) 16 cm **4.** (i) 14·4 m^2 (ii) 4·5 m **5.** 8 m and 10 m **6.** 15 m and 9 m **7.** 72 cm **8.** $12\sqrt{3}$ unit = 20·78 unit **9.** 816 m^2 **10.** 384 cm^2 **11.** 60 cm^2 **12.** 1800 m and 1500 m **13.** 384 cm^2 **14.** (i) 25 cm (ii) 84 cm^2 (iii) 6·7 cm

EXERCISE 32(B)

1. 15 cm and 46 cm **2.** (i) 8 m (ii) 120 m^2 (iii) 17 m **3.** (i) 30 cm (ii) 480 cm^2 **4.** 38 m **5.** 81 cm^2 **6.** 5·2 m **7.** 72 cm^2 and 8·49 cm **8.** 10·6 m and 42·4 m **9.** (i) 13 cm (ii) 52 cm **10.** 144 cm^2 **11.** 16 sq. units **12.** (i) 1 : 2 (ii) 1 : 4 **13.** (i) 3·8 m^2 (ii) 18·72 sq. m **14.** 288 m^2 **15.** 260 m^2 **16.** (i) 3888 and ₹ 23,328 (ii) 2520 and ₹ 15,120 **17.** 123·75 m^2 **18.** ₹ 17,550 **19.** (i) 700 m^2 (ii) 14,000 (iii) ₹ 1,96,000

EXERCISE 32(C)

1. 10·24 sq m **2.** 12 dm and 16 dm **3.** 10 cm **4.** 7·5 cm **5.** 10 cm **6.** 588 cm^2 **7.** (i) 216 cm^2 (ii) 15 cm (iii) 60 cm **8.** (i) 12 cm (ii) 96 cm^2 **9.** 216 cm^2 **10.** 20 cm **11.** ₹ 1,152; 9·6 m **12.** 13 cm and 18 cm **13.** 20 cm **14.** 120 cm^2

EXERCISE 32(D)

1. (i) 21 cm and 1386 cm^2 (ii) 3·5 m and 38·5 m^2 **2.** (i) 7 cm and 44 cm (ii) 1·4 m and 8·8 m **3.** 616 m^2 **4.** 132 cm **5.** 176 cm^2 **6.** (i) 42 cm^2 (ii) 44 m^2 **7.** (i) 2926 m^2 (ii) 44 m **8.** $3221\frac{3}{7}$ m^2 **9.** 4774 m^2 **10.** (i) 22 cm (ii) 88 cm (iii) 616 cm^2 **11.** 249·45 cm^2 **12.** (i) 2·64 m (ii) 132 m (iii) 528 m **13.** 1100 m (i) $3\frac{2}{3}$ m/s (ii) 13·2 kmh^{-1} **14.** 28·512 kmh^{-1} **15.** (i) 4·4 m (ii) 14 **16.** 2464 cm^2 **17.** 21 cm and 693 cm^2 **18.** 126 cm^2