# CHAPTER 30

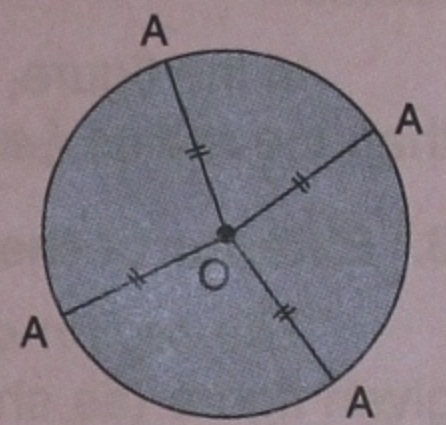
# CIRCLES

## 30.1 INTRODUCTION

**Circle**

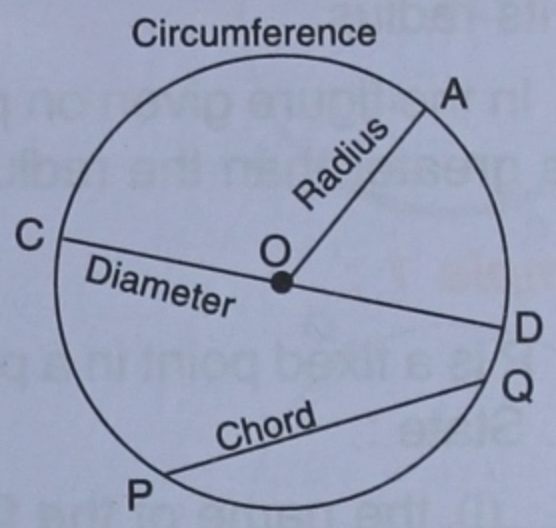
A circle is a plane figure obtained when a point moves in such a way that it is always at same distance from a fixed point.

The adjoining figure shows a fixed point O and a moving point A. The point A moves in such a way that it is always at the same (constant) distance from the fixed point O.




Clearly, the figure obtained (shown by the shaded region) is a circle.

## 30.2 SOME IMPORTANT TERMS




**1. Centre :**

The fixed point is called the **centre**.

In the given figure **the fixed point O is the centre.**

**2. Radius :**

The fixed (constant) distance of the moving point from the centre is called **radius.**

In the given figure, the fixed distance is OA, so **radius = OA.**

**3. Circumference :**

The length of the boundary of a circle is called its **circumference.**

**4. Chord :**

The line segment joining any two points on the circumference of the circle is called a **chord.**

In the given figure, PQ and CD are the chords of the given circle.

**5. Diameter :**

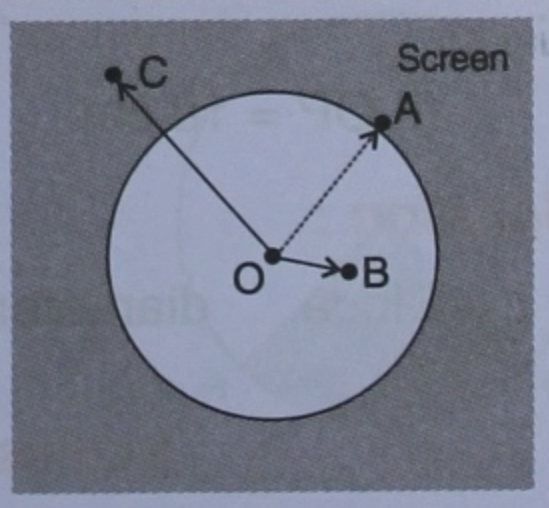
The chord of the circle, which passes through the centre, is called its **diameter.**

In the given figure, the chord CD passes through the centre of the circle, so **CD is a diameter of the circle.**

1. Diameter is the largest chord of the circle.
2. Diameter = $2 \times$ Radius and Radius = $\frac{\text{Diameter}}{2}$.

## 30.3 INTERIOR AND EXTERIOR OF A CIRCLE

1. A point is said to be on the circumference of the circle if its distance from the centre is equal to the radius of the circle. In the given



figure, the point A lies on the circumference of the circle; therefore, OA = r (the radius of the circle).

**2. Interior of the circle :**

*The region inside the circle is called interior of the circle.* The unshaded portion of the given circle (as shown above) represents the interior of the given circle.

The distance of every point in the interior of a circle from its centre is always less than its radius.

In the figure, given above, the point B lies in the interior of the circle, therefore OB is less than the radius *i.e.* **OB < r.**

**3. Exterior of the circle :**

*The region outside the circle is called exterior of the circle.* The shaded portion of the given figure (as shown on previous page) represents the exterior of the given circle.

The distance of every point in the exterior of a circle from its centre is always greater than its radius.

In the figure given on previous page, the point C lies in the exterior of the circle, therefore OC is greater than the radius *i.e.* **OC > r**.

***Example 1 :***

P is a fixed point in a plane and a point Q moves in the same plane such the PQ is always 8 cm. State :

(i) the name of the figure formed.

(ii) the value of the radius of the figure formed.

(iii) the value of the diameter of the figure formed.

Can a chord of length 20 cm be drawn in this circle ? Give reason.

***Solution :***

(i) **The figure formed is a circle.** **(Ans.)**

(ii) **The value of the radius**

= The distance between the fixed point P and the moving point Q

= PQ = **8 cm** **(Ans.)**

(iii) **The diameter of the figure** (circle) **formed**

= 2 × Radius = 2 × 8 cm = **16 cm** **(Ans.)**

Since, the diameter is the largest chord of the circle and the diameter of the circle formed is 16 cm, therefore **a chord of length 20 cm cannot be drawn in this circle.** **(Ans.)**

***Example 2 :***

The diameter of a circle is 24 cm. Find its radius.

If O is the centre of the circle, state, giving reasons, the position of points P, Q and R, if:

(i) OP = 18 cm (ii) OQ = 12 cm (iii) OR = 9 cm.

***Solution :***

Since, diameter = 2 × radius

$$\textbf{Radius} = \frac{\text{Diameter}}{2} = \frac{24\text{ cm}}{2} = \textbf{12 cm}$$ **(Ans.)**

(i) Since, OP = 18 cm and radius = 12 cm

⇒ The distance of the point P from the centre O is greater than the radius.

⇒ **The point P lies in the exterior of the given circle.** **(Ans.)**

(ii) Since, OQ = 12 cm

⇒ The distance of the point Q from the centre O is equal to the radius.

⇒ **The point Q lies on the circumference of the given circle.** **(Ans.)**

(iii) Since, OR = 9 cm

⇒ The distance of the point R from the centre O is less than the radius.

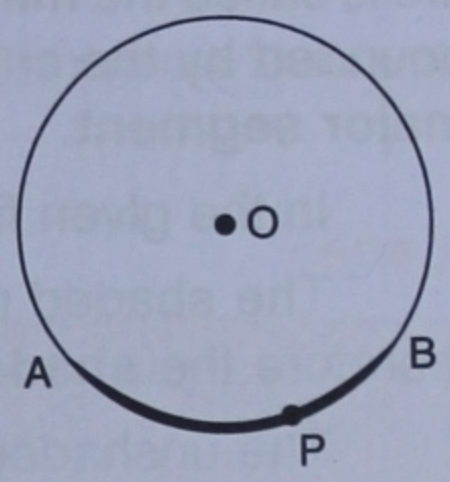
⇒ **The point R lies in the interior of the given circle.** **(Ans.)**

## 30.4 ARCS AND TYPES OF ARCS

**1. Arc :**

A part of the circumference of a circle is called its **arc.**

In the adjoining figure, APB is an arc of the given circle.



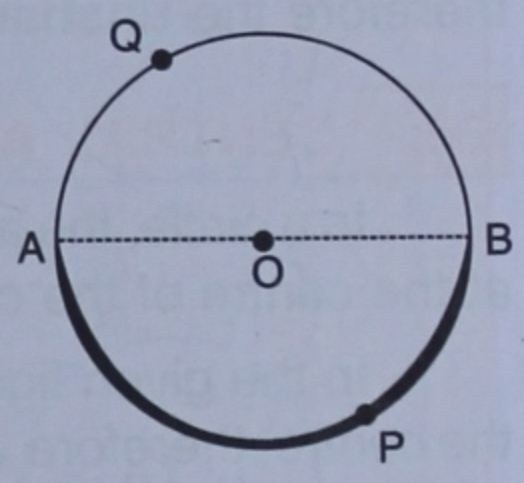

**2. Semi-circle :**

A diameter of a circle divides the circumference of the circle into two equal arcs.

In other words, a diameter of a circle bisects the circumference of the circle.

Each of the two arcs, so obtained, is called a **semi-circle.**

In the given figure, arc APB is a semi-circle and arc AQB is also a semi-circle.



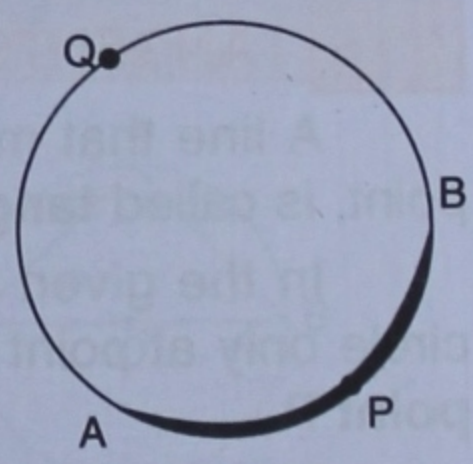

**3. Major and minor arcs :**

Let the circumference of circle be divided into two unequal parts.

The part of the circumference, which is greater than the semi-circle is called **major arc** and the part of the circumference, which is smaller than the semi-circle, is called minor arc.

In the given figure, the circumference of the circle is divided into two unequal parts APB and AQB.

The arc AQB is greater than the semi-circle and the arc APB is smaller than the semi-circle.




∴ Arc AQB = Major arc

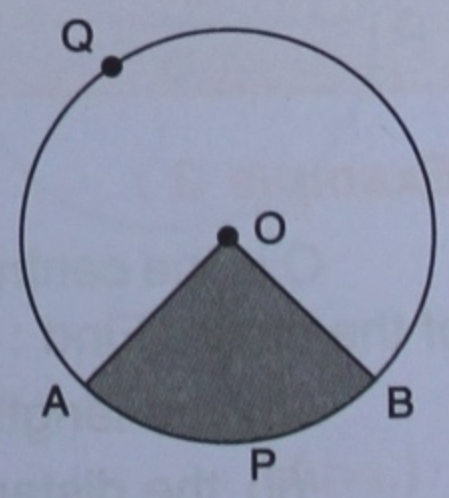
and, arc APB = Minor arc

## 30.5 SECTORS

*The part of a circle, bounded by two radii and an arc, is called the sector of the circle.*

The shaded portion of the given figure shows a sector as it is bounded by two radii OA and OB; and a minor arc APB.

Similarly, the unshaded portion of the given figure also shows a sector as it is also bounded by two radii OA and OB; a major arc AQB.



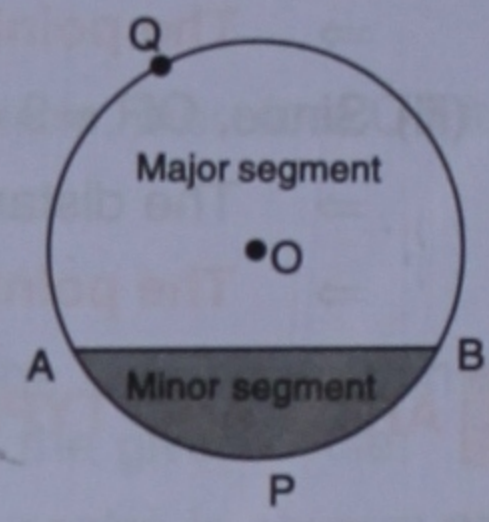
The part of the circle bounded by two radii and a minor arc is called the minor sector and the part of the circle bounded by two radii and a major arc is called the major sector.

In the circle, given above :

**Minor arc** = the shaded portion of the circle.

and, **major arc** = the unshaded portion of the circle.

## 30.6 SEGMENTS




A chord of a circle divides the given circle into two parts and each part so obtained is called a **segment**.

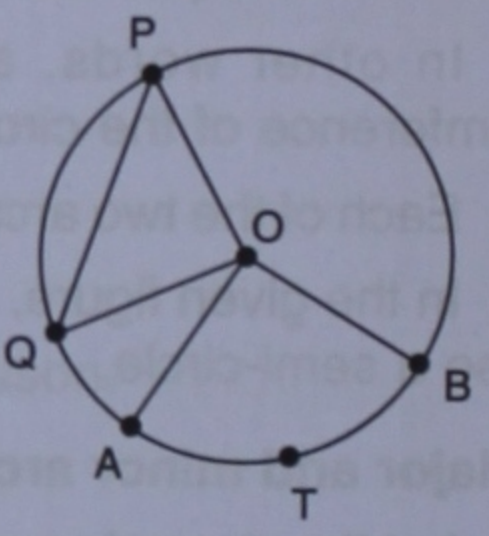
The segment bounded by the chord and the minor arc is called the **minor segment**, whereas the segment bounded by the chord and the major arc is called the **major segment**.

In the given figure, chord AB divides the circle into two unequal parts.

The shaded portion of the circle is bounded by the chord AB and the minor arc APB, therefore the **shaded portion** represents the **minor segment.**

The unshaded portion of the circle is bounded by the chord AB and the major arc AQB, therefore the **unshaded portion** represents the **major arc.**

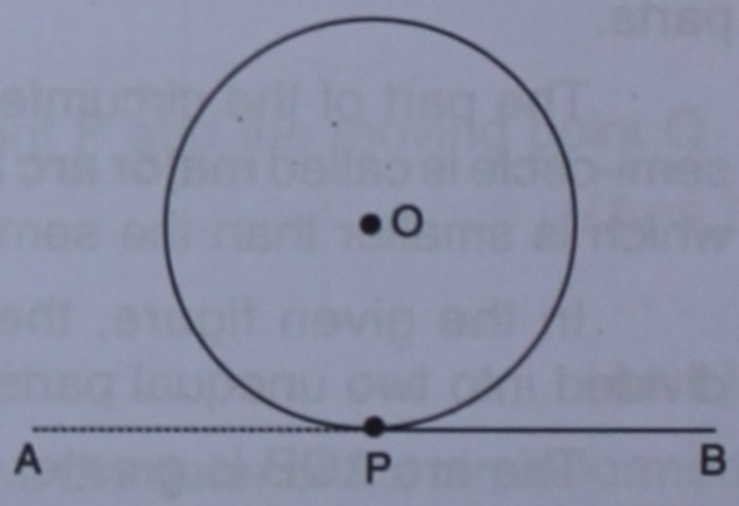
## 30.7 CENTRAL ANGLE




In a circle, the angle subtended by an arc (or a chord) at the centre of the circle, is called **central angle**.

In the given figure, arc ATB subtends angle AOB at the centre, therefore **∠AOB is the central angle.** Similarly, chord PQ subtends angle POQ at the centre, therefore **∠POQ is the central angle.**

## 30.8 TANGENTS

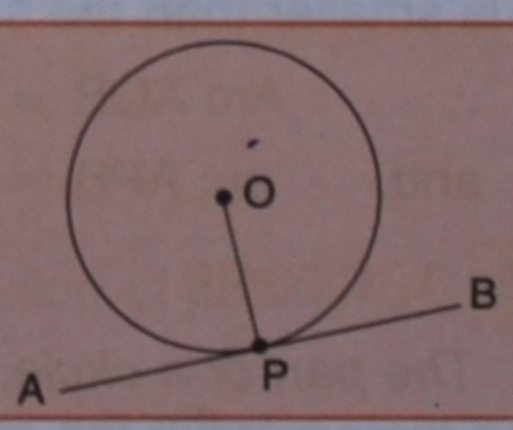



A line that meets (touches) the circle only at one point, is called **tangent of the circle.**

In the given figure, line segment AB touches the circle only at point P, so **AB is tangent to the circle at point P.**

The point P at which the tangent touches the circle is called the **point of contact.**

At the point of contact, the angle between the tangent and the radius is always 90°, *i.e.* the tangent and the radius are perpendicular to each other.




***Example 3 :***

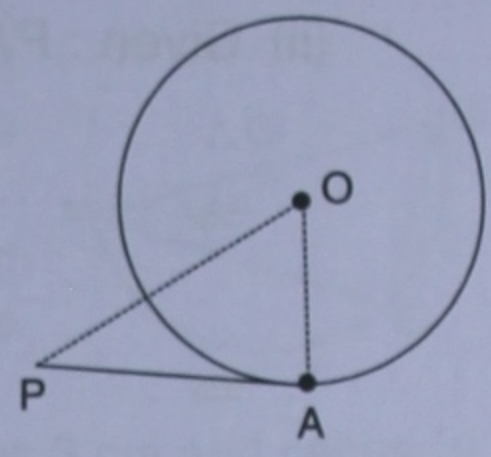
O is the centre of a circle with radius 8 cm. P is a point outside the circle and PA is tangent of the circle. Find :

(i) the length of tangent PA, if OP = 10 cm

(ii) the distance between O and P, if the length of the tangent PA is 15 cm.

***Solution :***

According to the given informations, the figure will be as shown alongside. Since, at the point of contact the angle between the tangent and the radius is 90°, therefore ∠OAP = 90°.




(i) Given : Radius OA = 8 cm and OP = 10 cm

$\therefore \quad PA^2 + OA^2 = OP^2$ [Using Pythagoras Theorem]

$\Rightarrow \quad PA^2 + 8^2 = 10^2$

$\Rightarrow \quad PA^2 = 100 - 64 = 36 = 6^2 \Rightarrow$ **PA = 6 cm** **(Ans.)**

(ii) Given : Radius OA = 8 cm and PA = 15 cm

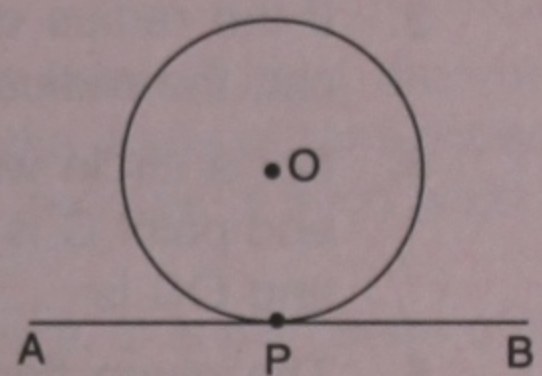
$\therefore \quad OP^2 - OA^2 = PA^2$ [Using Pythagoras Theorem]

$\Rightarrow \quad OP^2 - 8^2 = 15^2$

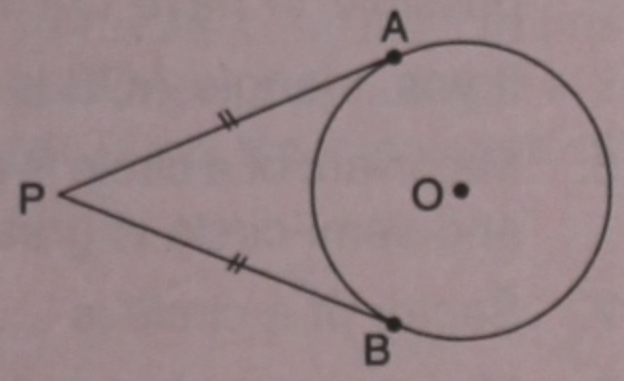
$\Rightarrow \quad OP^2 = 64 + 225 = 289 = 17^2 \Rightarrow$ **OP = 17 cm** **(Ans.)**

**Remember :**

1. Through any point on the circumference of a circle, one and only one tangent can be drawn.

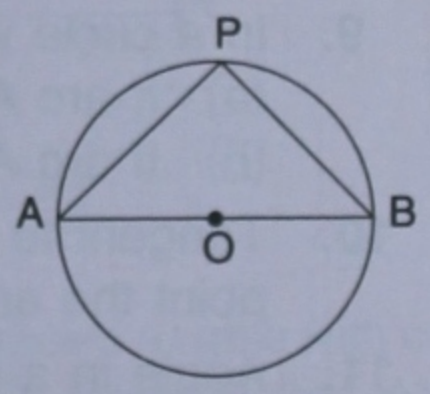



2. Through any point, outside a circle, at the most two tangents can be drawn and both of these tangents are always equal to each other.
   In the adjoining figure, the point P lies outside the circle and two tangents PA and PB are drawn to the circle. Thus, **PA = PB.**




## 30.9 ANGLE IN A SEMI-CIRCLE

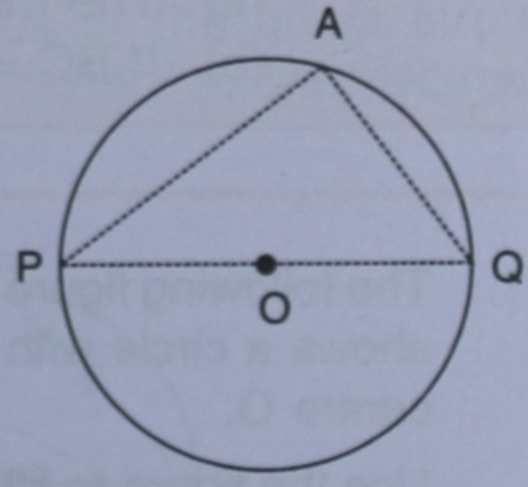
If AB is a diameter of a circle and P is any point on the circumference of the circle, the **angle APB** is called the **angle of semi-circle.**




The measure of the angle of semi-circle is 90° and so **∠APB = 90°.**

***Example 4 :***

The given figure shows a circle with centre O. PQ is a diameter and A is a point on the circumference of the given circle.




(i) Write the measure of angle PAQ.

(ii) If PA = 16 cm and AQ = 12 cm; find the length of PQ.

(iii) If PQ = 34 cm and AQ = 16 cm; find the length of PA.

***Solution :***

(i) Since, the angle of the semi-circle is 90°

$\therefore \quad$ **∠PAQ = 90°** **(Ans.)**

(ii) Given : PA = 16 cm and AQ = 12 cm

$\therefore \quad PQ^2 = PA^2 + AQ^2$ [Using Pythagoras Theorem]

$\Rightarrow \quad PQ^2 = 16^2 + 12^2$

$= 256 + 144 = 400 = 20^2$

$\Rightarrow \quad$ **PQ = 20 cm** **(Ans.)**

(iii) Given : PQ = 34 cm and AQ = 16 cm

$\therefore \quad PA^2 + AQ^2 = PQ^2$ [Using Pythogras Theorem]

$\Rightarrow \quad PA^2 + 16^2 = 34^2$

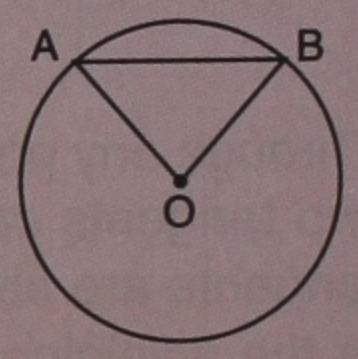
$\Rightarrow \quad PA^2 = 1156 - 256 = 900 = 30^2$

$\Rightarrow \quad$ **PA = 30 cm** **(Ans.)**

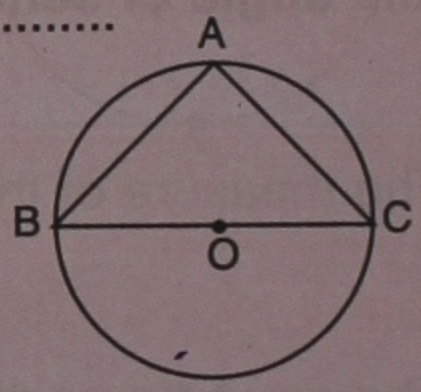
## TEST YOURSELF

1. Diameter is the ..... chord of every circle.
2. Chord passing through the centre of the circle is called its .....
3. If the radius of a circle is 8 cm, its diameter is ..... and if its diameter is 8 cm, the radius is .....
4. For a circle with centre O, point A is outside the circle, point B is inside the circle and point C is on the circumference of the circle, then the relation between OA, OB and OC is ..... > ..... > .....
5. The given figure shows a circle with centre O. Is it possible to have OA = AB ? .....

   If yes, triangle AOB is ..... .




6. Major-arc of a circle is greater than its ..... circle and semi-circle is greater than .....
7. Sector of a circle is .....
8. Minor segment of a circle is bounded by .....
9. In a circle with centre O, AB and CD are two arcs such that $\angle AOB = 50°$
   (a) If arc AB = arc CD, then $\angle COD$ = .....
   (b) If arc AB = 2 × arc CD, then $\angle COD$ = .....
10. Tangent to a circle is the line that touches ..... at one point only and at this point the angle between the radius and the tangent is .....
11. Angle in a semi-circle is .....
12. The given figure shows a circle with centre O and BC is a diameter :
    (a) The measure of angle BAC = .....
    (b) If BC = 10 cm and AB = 8 cm; AC = .....




## EXERCISE 30

1. The following figure shows a circle with centre O.

   Use the figure to fill the blanks in each of the following :

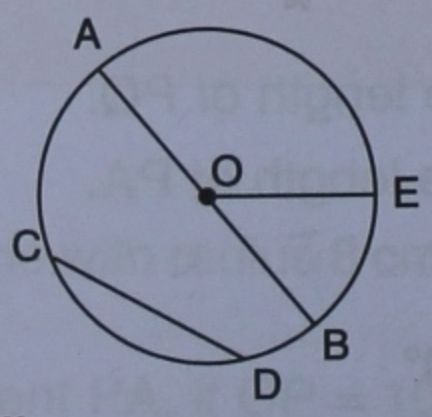
   (i) AB = .....

   (ii) Radius = .....

   (iii) Chords = ..... and .....

   (iv) Diameter = .....

   (v) AB = 2 × .....




2. M is a fixed point in a plane and a point P moves in the same plane such that PM = 10 cm. State :

   (i) the name of the figure formed

(ii) the length of the radius of the circle

(iii) the length of the diameter of the circle.

Can a chord of length 16 cm be drawn in this circle ? Give reason.

3. The radius of a circle is 6 cm. Find its diameter. If O is the centre of the circle; state, giving reasons, the positions of points A, B and C; if:

(i) OA = 4·8 cm (ii) OB = 7·5 cm

(iii) OC = 6 cm

4. Fill in the blanks :

(i) An arc is the part of the .....

(ii) Diameter of a circle bisects .....

(iii) The part of the circumference greater than the semi-circle is called .....

(iv) Sector of a circle is its region bounded by .....

(v) The segment of a circle is the region bounded by .....

(vi) A tangent of a circle meets the circle at .....

(vii) The number of tangents that can be drawn through a point on its circumference = .....

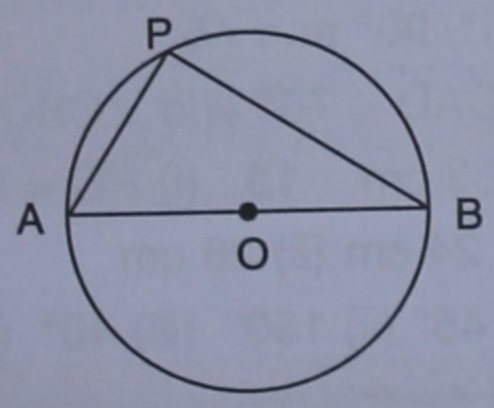
(viii) The number of tangents that can be drawn through a point outside the circle is .....

5. O is the centre of a circle with diameter 30 cm. P is a point outside the circle and PA is tangent of the circle. Find :

(i) the length of tangent PA; if OP = 39 cm.

(ii) the distance between O and P, if the length of the tangent PA is 20 cm

6. The following figure shows a circle with centre O and a diameter AB.




(i) Name the angle APB.

(ii) State the measure of angle APB.

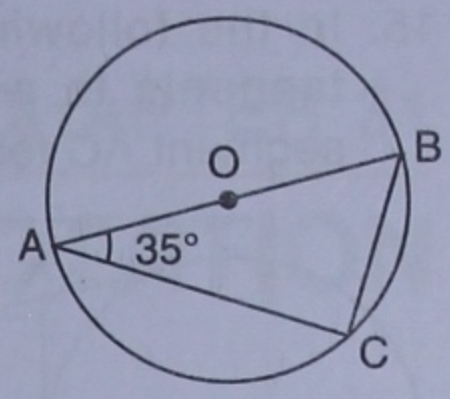
(iii) If AP = 12 cm and OA = 10 cm; find the lengths of AB and BP.

(iv) If 4AP = 3PB = 12 cm; find the radius of the circle.

7. Find the length of the tangent to a circle with radius 5 cm, from a point at a distance of 13 cm from its centre.

8. The following figure shows a circle with diameter AB and centre at point O.

If angle CAB = 35°; find the measure of angle ABC.




9. Draw a circle with radius 3 cm and centre at point O. Draw two radii OA and OB of the circle drawn such that ∠AOB = 60°. Join A and B and measure the length of AB.

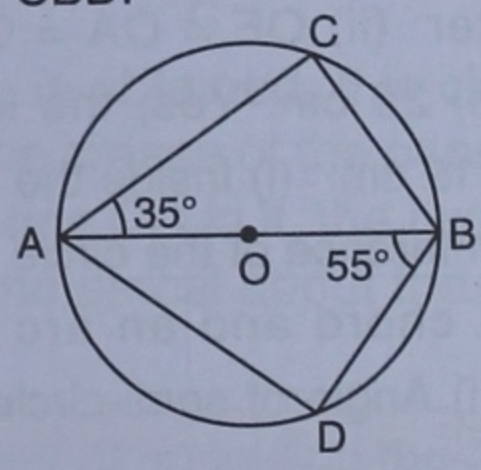
10. Draw a circle with radius 4 cm. Draw a chord AB of length 6 cm. Shade its minor segment by horizontal lines and major segment by vertical lines.

11. Draw a circle with centre at point P. Draw its radii PA and PB such that angle APB = 90°. Shade the minor sector of the circle by horizontal lines and its major sector by vertical lines.

12. In a circle with centre O and diameter AB; angle APB is the angle of semi-circle. If PA = PB; find the measure of each angle of the triangle APB.

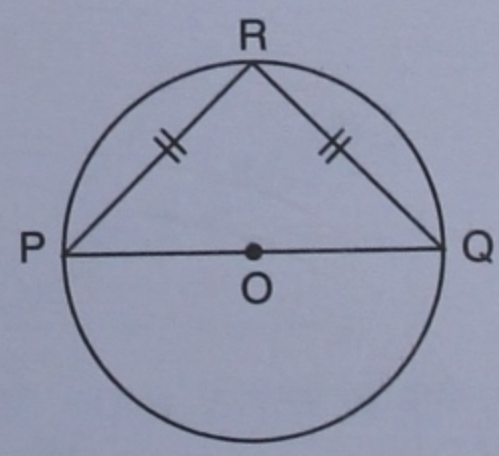
13. In the following figure, O is the centre of the circle and AB is a diameter.

C and D are points on the circumference of the circle such that ∠CAB = 35° and ∠ABD = 55°. Find the measures of angles CAD and CBD.

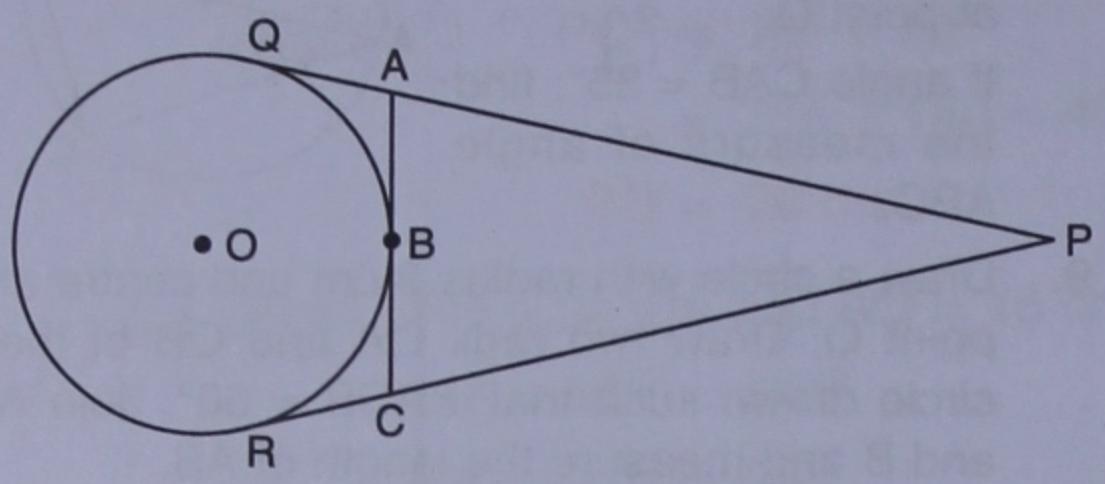



14. The following figure shows a circle with centre O and diameter PQ. Point R lies on the circumference of the circle such that PR = QR and PR = 4 cm.

Calculate the radius of the given circle and state its value correct to two decimal places.



15. In the following figure, PQ and PR are tangents to a circle with centre O. Line segment AC touches the circle at point B.




(i) State the relation between tangents PQ and PR.

Also, show that :

(ii) PQ = PA + AB

(iii) PR = PC + CB

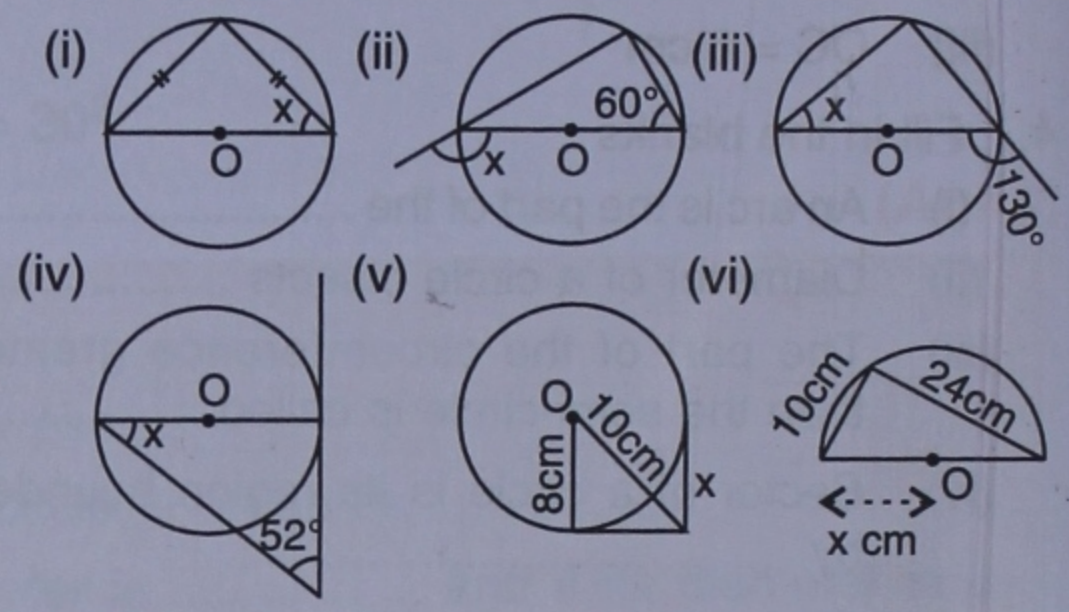
(iv) PQ + PR = Perimeter of Δ PAC.

16. From an exterior point P, the tangent PA is drawn to a circle with centre O.

(i) If OP = 20 cm and tangent PA = 16 cm, find the diameter of the circle.

(ii) If diameter of the circle is 20 cm and tangent PA = 24 cm; find the length of OP.

17. In each figure, given below, O is centre of the circle. Use the given informations to find the value of x :




## ANSWERS

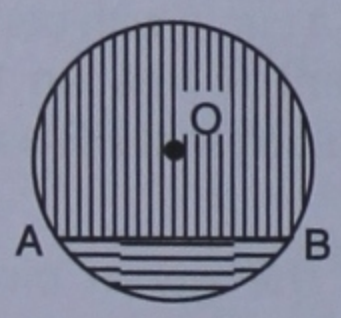
### TEST YOURSELF

**1.** largest **2.** diameter **3.** 16 cm, 4 cm **4.** OA > OC > OB **5.** yes, equilateral triangle **6.** semi, minor-arc **7.** portion of the circle bounded by its two radii and an arc **8.** a chord and a minor arc of the circle **9.** (a) 50° (b) 25° **10.** the circle, 90° **11.** 90° **12.** (a) 90° (b) $\sqrt{10^2 - 8^2} = 6$ cm

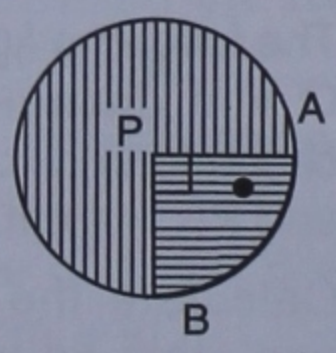
### EXERCISE 30

**1.** (i) Diameter (ii) OE = OA = OB (iii) CD and AB (iv) AB (v) OE = OA = OB **2.** (i) Circle (ii) 10 cm (iii) 20 cm. Yes, the length of the chord is smaller than the length of the diameter **3.** Diameter = 12 cm. (i) Inside the circle (ii) Outside the circle (iii) On the circumference of the circle **4.** (i) the circumference of the circle (ii) the circumference of the circle (iii) major-arc (iv) two radii and an arc (v) a chord and an arc (vi) one point only (vii) only one (viii) two **5.** (i) 36 cm (ii) 25 cm **6.** (i) Angle of semi-circle (ii) 90° (iii) AB = 20 cm and BP = 16 cm (iv) 2·5 cm **7.** 12 cm **8.** 55° **9.** 3 cm

**10.**




**11.**




**12.** 45°, 90° and 45°

**13.** ∠CAD = 70° and ∠CBD = 110°

**14.** 2·83 cm **15.** (i) PQ = PR

**16.** (i) 24 cm (ii) 26 cm

**17.** (i) 45° (ii) 150° (iii) 40° (iv) 38° (v) 6 cm (vi) 13 cm