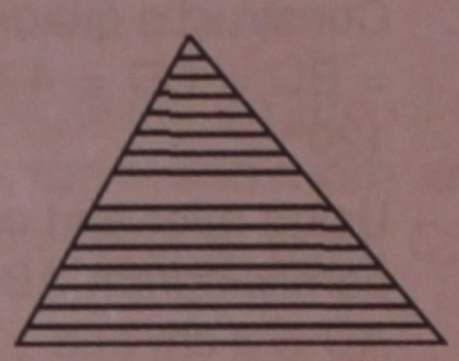
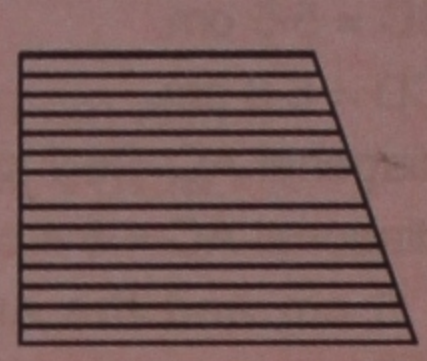
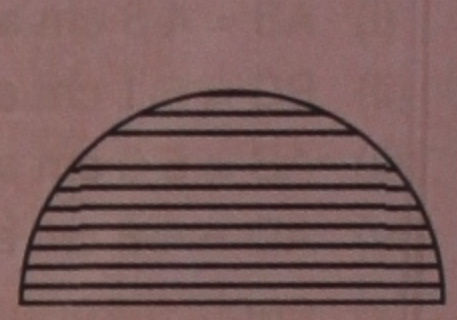
CHAPTER 29

AREA PROPOSITIONS

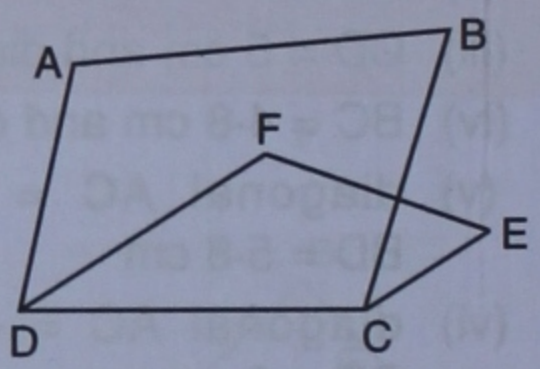
29.1 INTRODUCTION

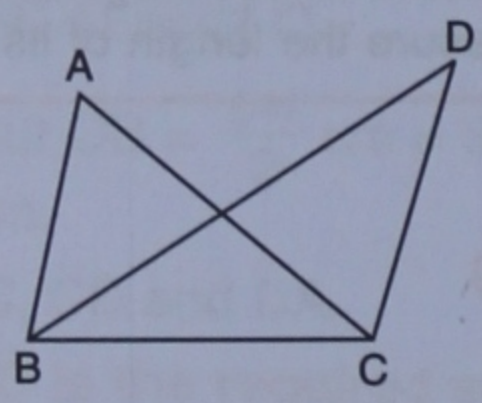
Area	The area of a plane closed figure is the measure of the region (surface) enclosed by its boundary. In each figure, given below, the shaded portion represents the area enclosed : (i)  (ii)  (iii) 
Equal Figures	In geometry, equal figures mean figures having equal areas.

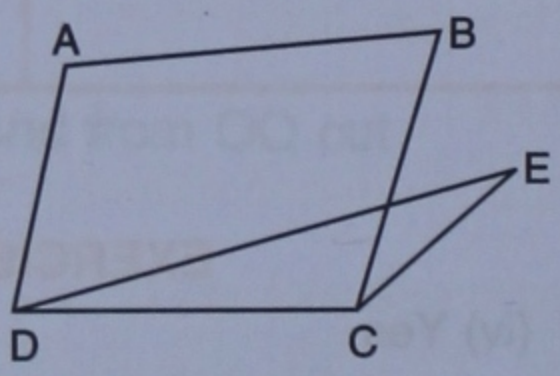
1. Congruent figures are always equal in area.
2. But, figures equal in area are not necessarily congruent.

29.2 FIGURES ON THE SAME BASE

The adjoining figure shows two quadrilaterals ABCD and DCEF with their one side DC common. We say that both of these quadrilaterals are on the same base DC.







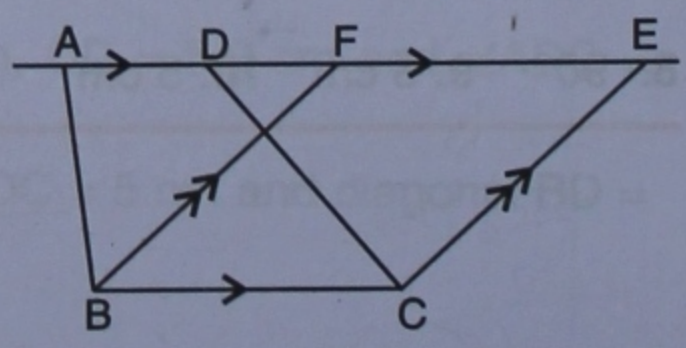
Similarly, triangles ABC and DBC, as shown above, are one same base BC. Also, quadrilateral ABCD and triangle EDC, as shown above, are on same base DC.

29.3 FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

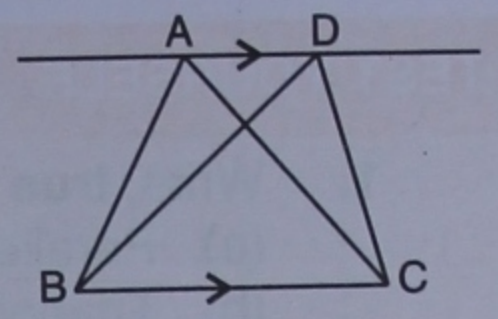
Figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

Examples :

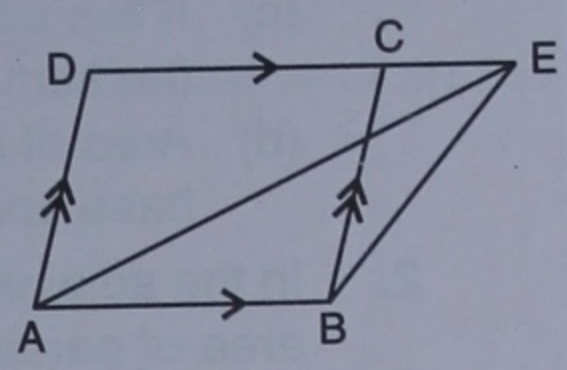
1. In the adjoining figure, trapezium ABCD and parallelogram BCEF are on the same base BC and between the same parallels BC parallel to AE.



2. Triangles ABC and DBC are on the same base BC and between the same parallels AD // BC.



3. Parallelogram ABCD and triangle ABE are on the same base AB and between the same parallels AB // DE.

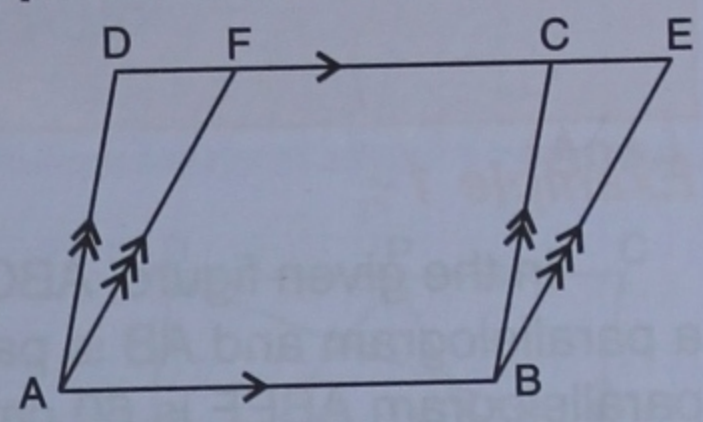


29.4 SOME IMPORTANT PROPOSITIONS (without proof)

Proposition 1 :

Parallelograms on the same base and between the same parallels are equal in area.

The adjoining figure shows two parallelograms ABCD and ABEF on the same base AB and between the same parallels AB // DE;

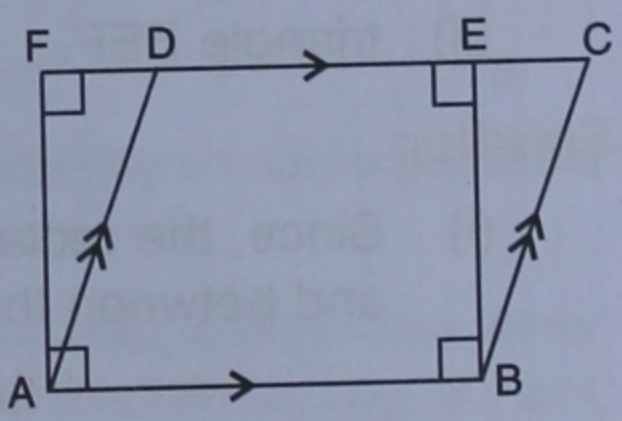


∴ Area of parallelogram ABCD = Area of parallelogram ABEF.

Proposition 2 :

The area of a parallelogram is equal to the area of rectangle on the same base and between the same parallels.

The adjoining figure shows a parallelogram ABCD and a rectangle ABEF on the same base AB and between the same parallels AB // FC;

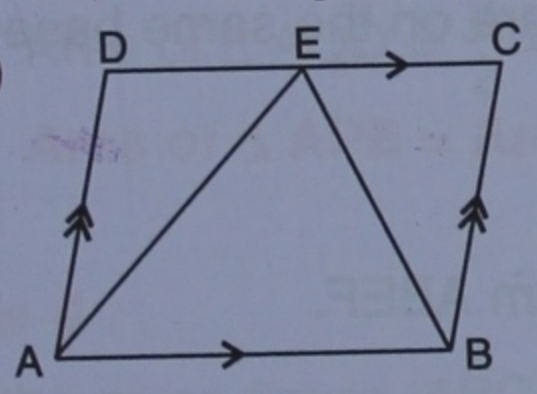


∴ Area of parallelogram ABCD = Area of rectangle ABEF.

Proposition 3 :

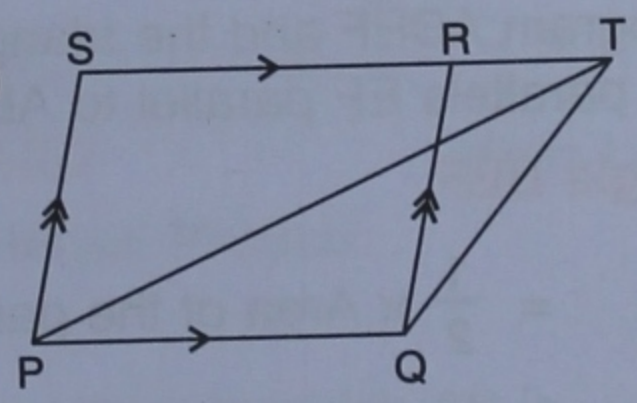
The area of a triangle is half the area of a parallelogram on the same base and between the same parallels.

(i)



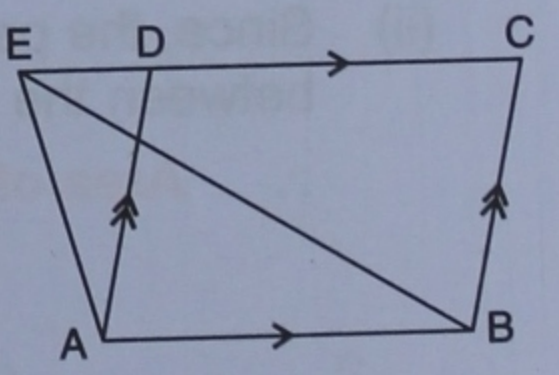
Δ ABE = $\frac{1}{2}$ of //gm. ABCD

(ii)



Δ PQT = $\frac{1}{2}$ of //gm. PQRS

(iii)

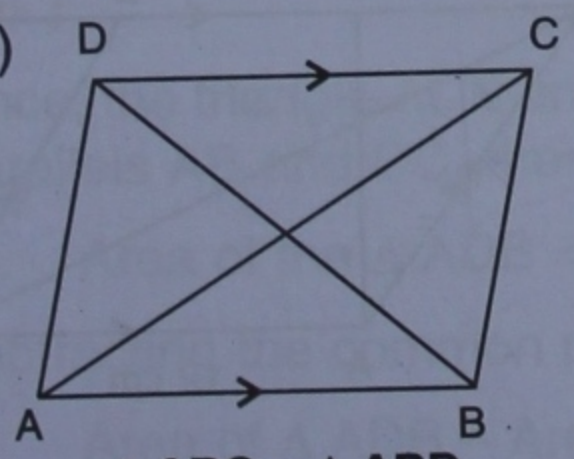


Δ ABE = $\frac{1}{2}$ of //gm. ABCD.

Proposition 4 :

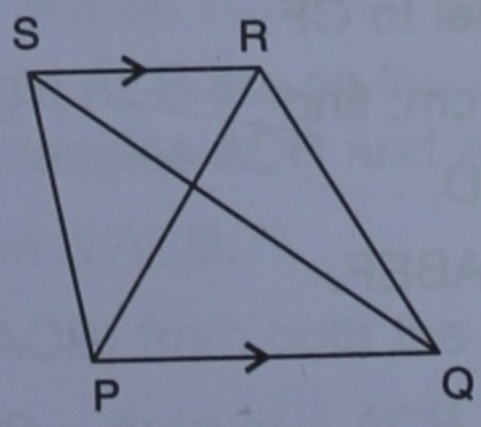
Triangles on the same base and between the same parallels are equal in area

(i)



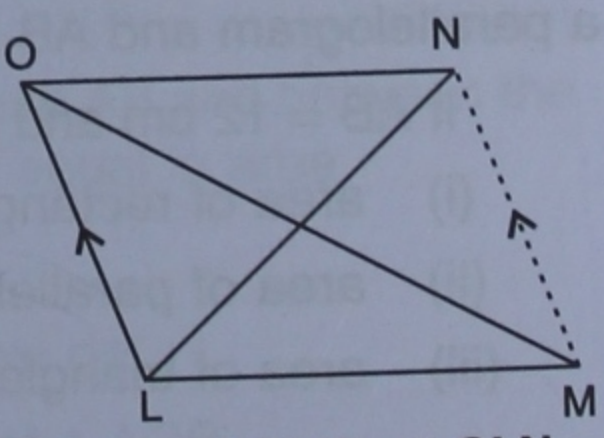
Δ ABC = Δ ABD

(ii)



Δ PQR = Δ PQS

(iii)



Δ OLM = Δ OLN

TEST YOURSELF

1. Write, **true** or **false** :

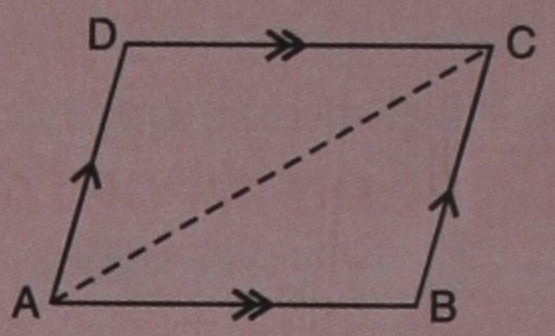
(a) Parallelograms on the same base are equal in area .....

(b) Triangles between the same parallels are equal in area .....

(c) If two parallelgorams are equal in area, they are on the same base and between the same parallels .....

(d) Area of a rectangle is equal to area of the parallelogram if both are on the same base and between the same parallels .....

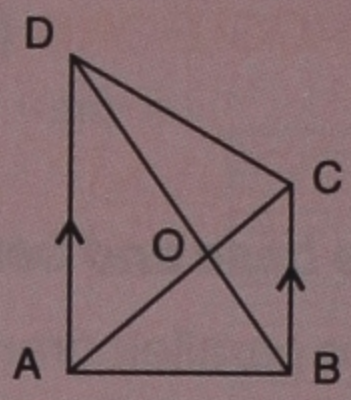
2. In the adjoining figure, if area of triangle ABC = 40 cm^2, the area of parallelogram ABCD = ...



3. In the given figure

(a) Δ ABC = ...

(b) Δ ABC – Δ AOB = ...

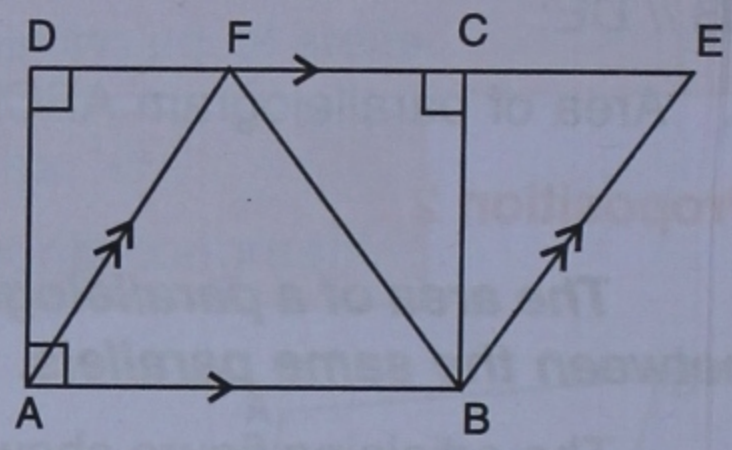


Example 1 :

In the given figure, ABCD is a rectangle, ABEF is a parallelogram and AB is parallel to DE. If the area of parallelogram ABEF is 60 cm^2; find the area of :

(i) rectangle ABCD.

(ii) triangle BEF.



Solution :

(i) Since, the rectangle ABCD and the parallelogram ABEF are on the same base AB and between the same parallels AB parallel to DE.

$\therefore$ **Area of the rectangle ABCD**

= Area of the parallelogram ABEF

= **60 cm^2** **(Ans.)**

(ii) Since, the parallelogram ABEF and the triangle BEF are on the same base EF and between the same parallels EF parallel to AB.

$\therefore$ **Area of triangle BEF**

$= \frac{1}{2} \times$ Area of the parallelogram ABEF.

$= \frac{1}{2} \times 60\ cm^2 =$ **30 cm^2** **(Ans.)**

Example 2 :

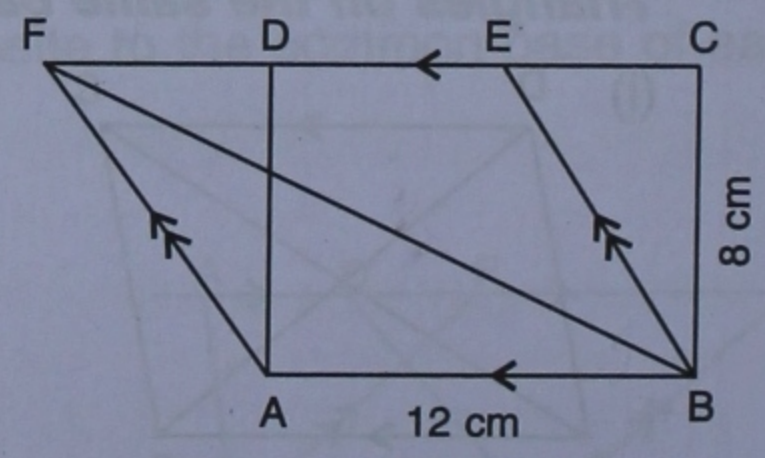
In the given figure, ABCD is a rectangle, ABEF is a parallelogram and AB is parallel to CF.

If AB = 12 cm and BC = 8 cm; find :

(i) area of rectangle ABCD.

(ii) area of parallelogram ABEF.

(iii) area of triangle BEF.



Solution :

(i) **Area of the rectangle ABCD**

= Base × height

= 12 cm × 8 cm = **96 cm²** **(Ans.)**

(ii) Since, the parallelogram ABEF and the rectangle ABCD are on the same base AB and between the same parallels AB parallel to CF.

∴ **Area of the parallelogram ABEF**

= Area of the rectangle ABCD = **96 cm²** **(Ans.)**

(iii) Since, the triangle BEF and the parallelogram ABEF are on the same base EF and between the same parallels EF parallel to AB.

∴ **Area of the triangle BEF**

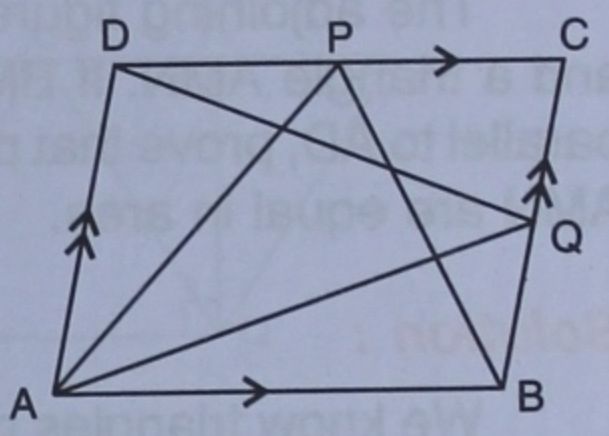
$= \frac{1}{2}$ × Area of the parallelogram ABEF

$= \frac{1}{2} \times 96 \text{ cm}^2 =$ **48 cm²** **(Ans.)**

Example 3 :

Given : A parallelogram ABCD. P is a point on side DC and Q is a point on side BC.

To prove : Δ APB and Δ AQD are equal in area.



Solution :

Δ APB and parallelogram ABCD are on the same base AB and between same parallels AB // DC.

∴ Area of Δ APB = $\frac{1}{2}$ of //gm ABCD ..... I [On the same base and between same parallels, the area of Δ is half the area of //gm]

Similarly, Δ AQD and parallelogram ABCD are on the same base AD and between same parallels AD and BC.

∴ Area of Δ AQD = $\frac{1}{2}$ of //gm ABCD II

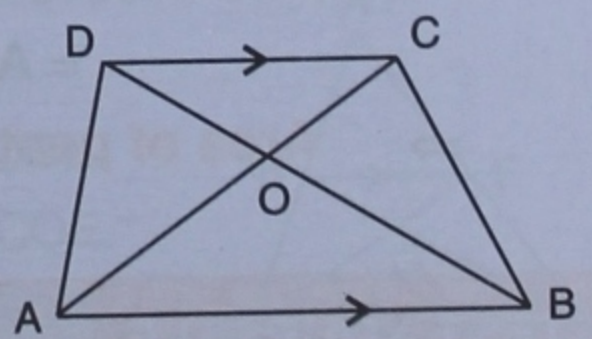
∴ **Area of Δ APB = Area of Δ AQD** [From I and II]

Hence Proved.

Example 4 :

In the given figure, ABCD is a trapezium in which AB is parallel to DC.

If its diagonals AC and BD intersect at point O, prove that the triangles AOD and BOC are equal in area.



Solution :

Since, the triangle ADB and the triangle ACB are on the same base AB and between the same parallels AB and DC, therefore the triangles ADB and ACB are equal in area.

i.e. Area of the Δ ADB = Area of the Δ ACB.

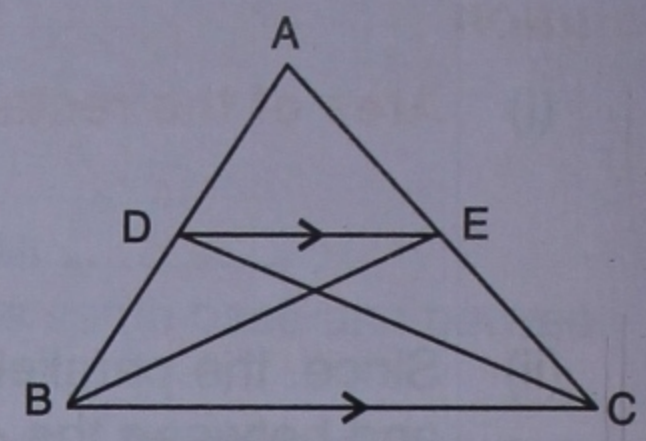
Subtracting the common portion (Δ AOB) from both the sides, we get :

Area of Δ ADB – Area of Δ AOB = Area of Δ ACB – Area of Δ AOB.

⇒ **Area of Δ AOD = Area of Δ BOC.** **Hence Proved**

Example 5 :

The adjoining figure shows a triangle ABC and DE parallel to BC.



Prove that : The triangles ABE and ACD are equal in area.

Solution :

Since, triangles BED and CDE are on the same base DE and between the same parallels DE parallel to BC.

$\therefore$ Area of Δ BED = Area of Δ CDE.

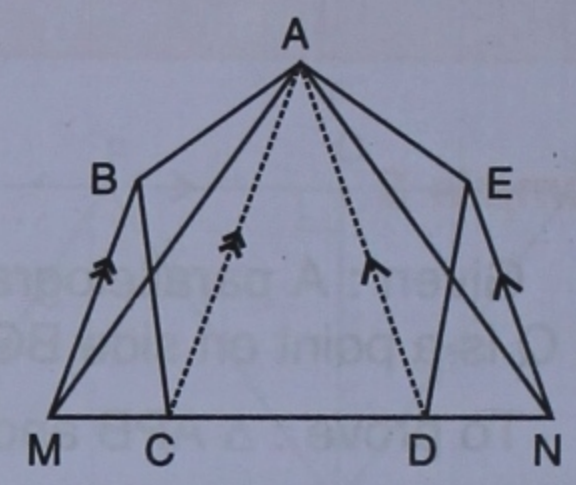
Adding the area of Δ ADE on both the sides, we get :

Area of Δ BED + area of Δ ADE = Area of Δ CDE + Area of Δ ADE

$\Rightarrow$ **Area of Δ ABE = Area of Δ ACD.** **Hence Proved.**

Example 6 :

The adjoining figure shows a pentagon ABCDE and a triangle AMN. If BM is parallel to AC and EN is parallel to AD, prove that pentagon ABCDE and triangle AMN are equal in area.



Solution :

We know triangles on the same base and between the same parallels are equal in area.

Since, triangles ABC and AMC are on the same base AC and between the same parallels (BM // AC), therefore;

$\therefore$ Area of Δ ABC = Area of Δ AMC I

Similarly, triangles AED and AND are on the same base AD and between the same parallels (EN // AD), therefore;

$\therefore$ Area of Δ AED = Area of Δ AND II

On adding I and II, we get :

Area of Δ ABC + Area of Δ AED = Area of Δ AMC + Area of Δ AND

Again, adding area of Δ ACD on both the sides, we get :

Area of Δ ABC + Area of Δ AED + Area of Δ ACD

= Area of Δ AMC + Area of Δ AND + Area of Δ ACD

$\Rightarrow$ **Area of pentagon ABCDE = Area of Δ AMN**

Hence Proved.

TEST YOURSELF

4. In the given figure, DE // BC, then

(a) (i) Δ BDC = ...

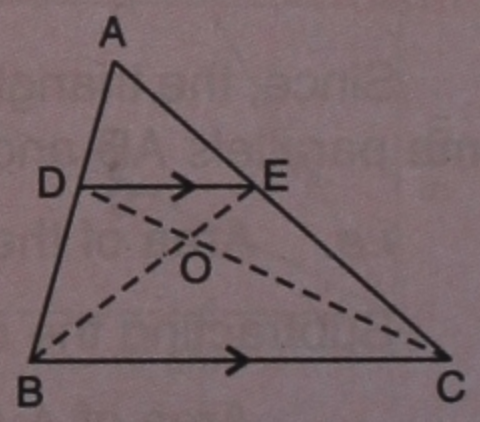
(ii) Δ BDC – Δ BOC = ... – .....

(iii) Δ BOD = ...

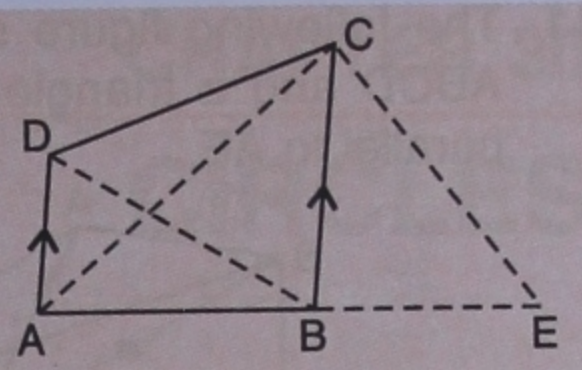
(b) (i) Δ BDE = ...

(ii) Δ BDE + Δ ADE = ... + ...

(iii) Δ ABE = ...



5. In the given figure, ABCD is a quadrilateral, DA is parallel to CB, then

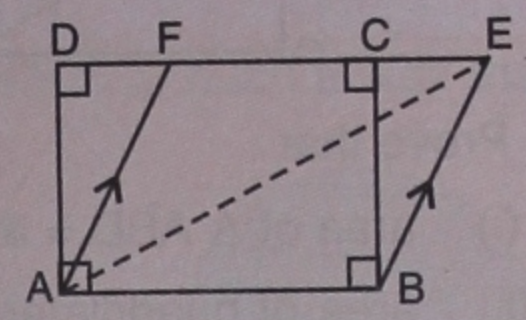


(i) Δ BCD = ...

(ii) Δ BCD + Δ CBE = ... + ...

(iii) Quadrilateral BECD = Δ .....

6. In the given figure, area of Δ ABE = 45 cm^2, then :

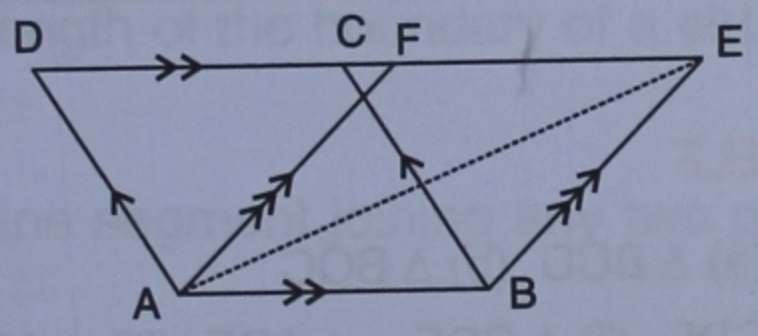


(i) the area of rectangle ABCD = ...

(ii) the area of parallelogram ABEF = ...

EXERCISE 29

1. A rectangle and a parallelogram are on the same base and between the same parallels. If the area of the rectangle is 50 cm^2, what is the area of the parallelogram ? Give reason.
2. A parallelogram and a triangle are on the same base and between the same parallels. If the area of the triangle is 36 cm^2, what is the area of the parallelogram? Give reason.
3. Two triangles are on the same base and between the same parallels. If the area of one triangle is 27 cm^2, what is the area of the other triangle? Give reason.
4. The following figure shows two parallelograms ABCD and ABEF.



If the area of the parallelogram ABCD is 70 cm^2; find the area of :

(i) the parallelogram ABEF

(ii) the triangle AEF.

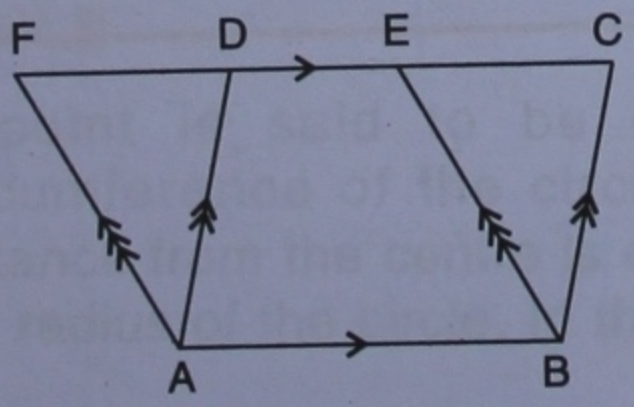
5. In the figure of question no. 4, given above, if the area of the triangle ABE = 43 cm^2; find the area of :

(i) the parallelogram ABEF

(ii) the parallelogram ABCD

(iii) the triangle AEF.

6. Given : Area of //gm ABCD = 60 cm^2.

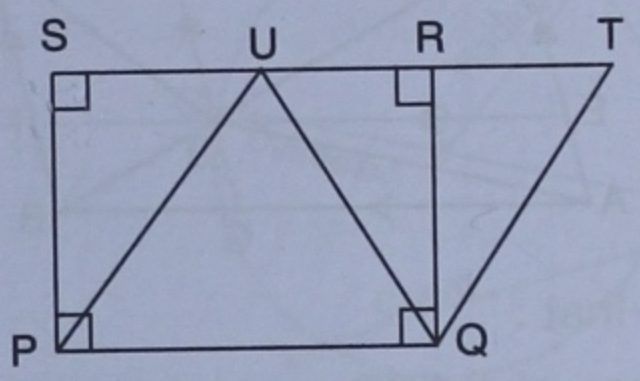


Find, giving reasons,

(i) area of parallelogram ABEF

(ii) area of triangle ABC.

7. Given : Area of Δ PUQ = 25 cm^2.

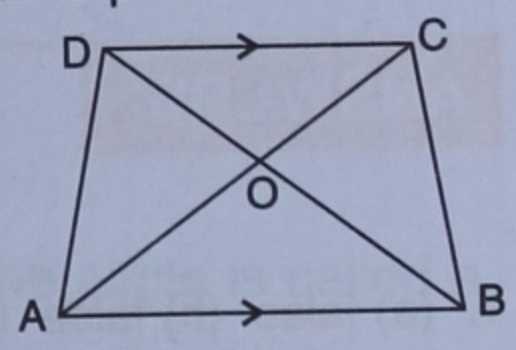


Find :

(i) area of parallelogram PQTU

(ii) area of rectangle PQRS.

8. In trapezium ABCD; AB is parallel to DC. Diagonals AC and BD intersect at O.

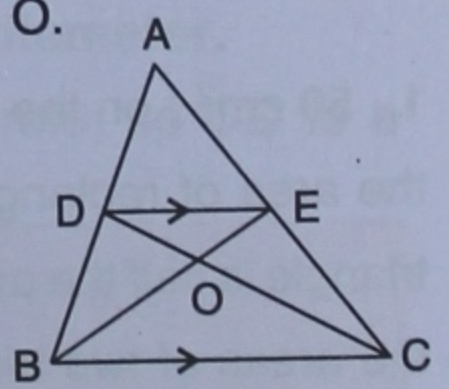


Prove that :

(i) area of Δ ADB = area of Δ ACB.

(ii) area of Δ AOD = area of Δ BOC.

9. Given : In Δ ABC; DE is parallel to BC. BE and CD intersect at point O.



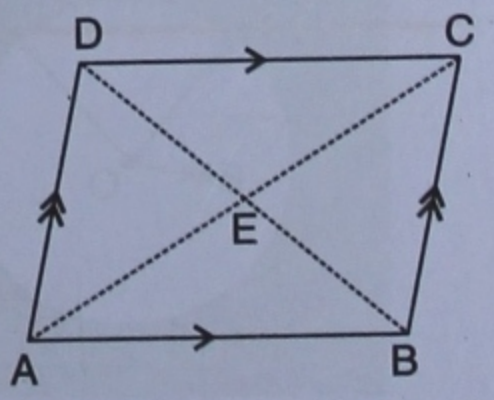
Prove :

(i) Δ BDE = Δ CDE.

(ii) Δ BOD = Δ COE.

(iii) Δ ABE = Δ ACD.

10. The following figure shows a parallelogram ABCD whose diagonals AC and BD intersect each other at point E.

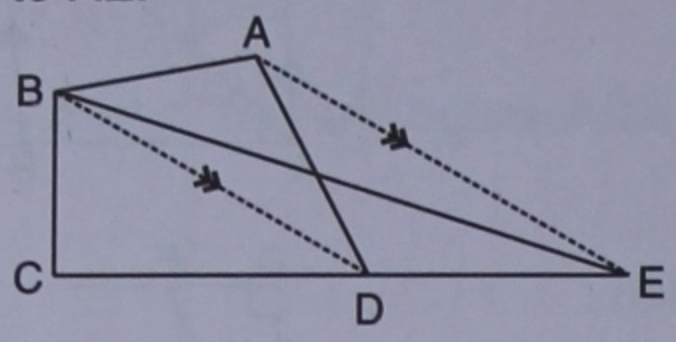


Prove that :

(i) area of Δ AEB = area of Δ CED

(ii) area of Δ AED = area of Δ BEC.

11. The following figure shows a quadrilateral ABCD and a triangle BCE in which BD is parallel to AE.

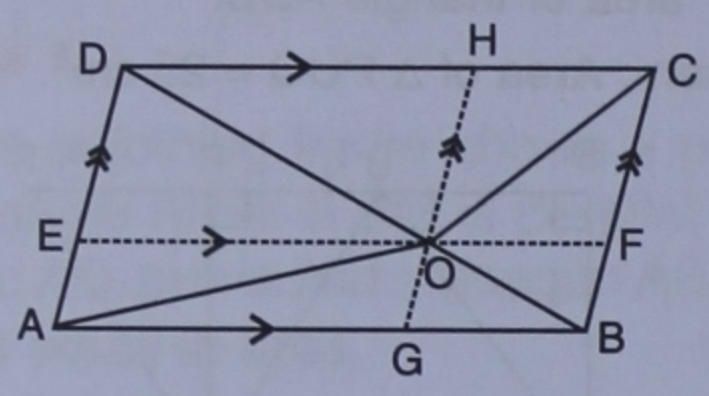


Prove that :

(i) area of Δ ABD = area of Δ EBD.

(ii) area of quadrilateral ABCD
= area of triangle BCE.

12. The following figure shows a parallelogram ABCD and O is any point inside it. EF is parallel to AB and parallel to DC whereas GH is parallel to AD and parallel to BC.



Show that :

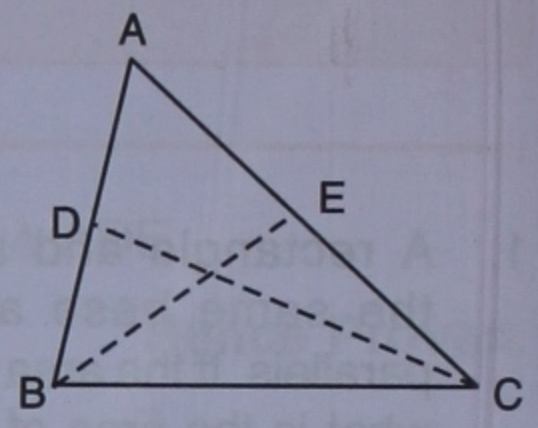
(i) area of Δ AOB
$= \frac{1}{2} \times$ area of parallelogram ABFE

(ii) area of Δ DOC
$= \frac{1}{2} \times$ area of parallelogram DCFE

(iii) area of Δ AOB + area of Δ DOC
$= \frac{1}{2} \times$ area of parallelogram ABCD

(iv) area of Δ AOD + area of Δ BOC
$= \frac{1}{2} \times$ area of parallelogram ABCD.

13. D and E are points on sides AB and AC respectively of Δ ABC such that triangles DBC and EBC are equal in area. State, with reason, whether DE is parallel to BC or note.

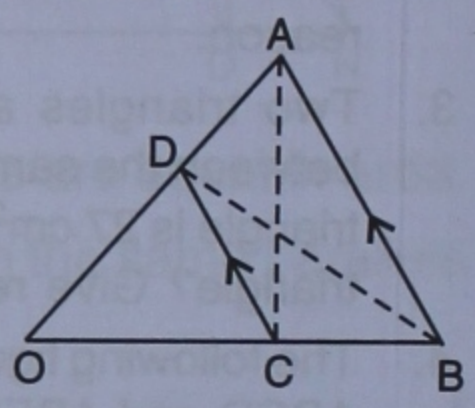


14. In the given figure, OAB is a triangle in which DC // AB. Given : area of Δ CAD = 85 cm^2 and area of Δ ODC = 150 cm^2. Find :

(i) area of Δ DBC

(ii) area of Δ OAC

(iii) area of Δ OBD



ANSWERS

TEST YOURSELF

1. (a) false (b) false (c) false (d) true **2.** 80 cm^2 **3.** (a) Δ BCD (b) Δ BOC
4. (a) (i) Δ BEC (ii) Δ BEC – Δ BOC (iii) Δ COE (b) (i) Δ CDE (ii) Δ CDE + Δ ADE (iii) Δ ADC
5. (i) Δ BCA (ii) Δ BCA + Δ CBE (iii) ACE **6.** (i) 90 cm^2 (ii) 90 cm^2

EXERCISE 29

1. 50 cm^2 ; on the same base and between the same parallels, the area of a parallelogram is equal to the area of rectangle. **2.** 72 cm^2 ; on the same base and between the same parallels, the area of a triangle is half the area of a parallelogram. **3.** 27 cm^2 ; on the same base and between the same parallels, the areas of two triangles are equal. **4.** (i) 70 cm^2 (ii) 35 cm^2 **5.** (i) 86 cm^2 (ii) 86 cm^2 (iii) 43 cm^2 **6.** (i) 60 cm^2 (ii) 30 cm^2 **7.** (i) 50 cm^2 (ii) 50 cm^2 **13.** Yes, DE // BC. **Reason :** Converse of triangles equal in area **14.** (i) 85 cm^2 (ii) 235 cm^2 (iii) 235 cm^2