## CONSTRUCTIONS

## (Using ruler and compasses only)

### 28.1 CONSTRUCTION OF AN ANGLE

## 1. To construct an angle equal to given angle.

Let the given angle be $\angle A B C$ as shown alongside and we have to construct another angle (say, $\angle D E F$ ) equal to $\angle A B C$.

## Steps :

1. Draw a line segment $E F$ of any suitable size.
2. With $B$ as centre, draw an arc of any suitable radius which cuts $A B$ at point $P$ and $B C$ at point $Q$.
3. With $E$ as centre and the same radius as taken in step 2, draw an arc which cuts EF at point R.
4. With $R$ as centre and radius equal to $P Q$, draw an arc which cuts the previous arc at point S .
5. Join E and S , and produce upto point D . $\angle D E F$ so obtained is equal to $\angle A B C$.
6. To draw the bisector of a given angle.

Let the given angle be $\angle A B C$ whose bisector is to be drawn.

## Steps :

1. With $B$ as centre, draw an arc of any suitable radius which cuts $A B$ at point $D$ and $B C$ at point $E$.
2. Taking $D$ and $E$ as centres, draw arcs of equal radii and let these arcs cut each other at point O .


The radii of these arcs must be more than half the distance between points D and E .
3. Join $B O$ and produce upto point $P$.
$\therefore B P$ is the required bisector of $\angle A B C$.
Thus, $\angle A B P=\angle P B C=\frac{1}{2} \angle A B C$.
3. Construction of angles of $60^{\circ}, 30^{\circ}, 90^{\circ}$ and $45^{\circ}$.

## 1. Construction of angle of $60^{\circ}$ :

## Steps :

1. Draw a line segment $B C$ of any suitable length.
2. With $B$ as centre, draw an arc of any suitable radius which cuts BC at point D .

3. With $D$ as centre and the same radius as taken in step 2, draw one more arc which cuts the previous arc at point E .
4. Join $B E$ and produce upto any point $A$.
$\therefore \angle A B C$ so obtained is of $60^{\circ}$ i.e. $\angle A B C=60^{\circ}$.

## 2. Construction of angle of $30^{\circ}$ :

## Steps :

1. Draw angle $A B C=60^{\circ}$.
2. Draw $B P$, the bisector of $\angle A B C$.

$$
\therefore \angle \mathrm{PBC}=\frac{1}{2} \angle \mathrm{ABC}=\frac{1}{2} \times 60^{\circ}=30^{\circ}
$$



## 3. Construction of angle of $90^{\circ}$ :

## Steps :

1. Draw a line segment $B C$ of any suitable length.
2. Taking $B$ as centre, draw an arc of any suitable radius, which cuts $B C$ at point $D$.
3. With D as centre and the same radius, as taken in step 2, draw an arc which cuts previous arc at point $E$.
4. With $E$ as centre and the same radius, draw one
 more arc which cuts the first arc at point $F$.
5. With $E$ and $F$ as centres and radii equal to more than half the distance between $E$ and $F$, draw arcs which cut each other at point $P$.
6. Join BP and produce upto any point A.
$\therefore \angle A B C$ so obtained is of $90^{\circ}$ i.e. $\angle A B C=90^{\circ}$.
Since, $\angle A B C=90^{\circ} \Rightarrow A B$ and $B C$ are perpendicular to each other.

## 4. Construction of angle of $45^{\circ}$ :

## Steps :

1. Draw a line segment $B C$ of any suitable length.
2. Construct angle $O B C=90^{\circ}$.
3. Draw $B A$, the bisector of angle $O B C$.
$\therefore \angle A B C$ so obtained is the angle of $45^{\circ}$.


Since, $B A$ is bisector of angle $O B C, \angle A B C=\angle A B O=\frac{90^{\circ}}{2}=45^{\circ}$.

## Example 1:

Given below are the two angles $x$ and $y$. Construct an angle ABC such that :

(ii) $\angle A B C=2 x+y$

(i)




## Steps :

## As shown above :

1. Draw line segment $B C$ of any suitable length.
2. With $B$ as centre, draw an arc of any suitable radius. With the same radius, draw arcs with the vertices of given angles as centres. Let these arcs cut arms of the angle $x$ at points $P$ and $Q$, and arms of the angle $y$ at points $R$ and $S$.
3. From the arc, with centre $B$, cut $D E=P Q=x$ and $E F=S R=y$.
4. Join $B F$ and produce upto point $A$.

Thus, $\angle A B C=x+y$
(ii)



Proceed in exactly the same way as in part (i) taking $D E=P Q=x, E F=P Q=x$ and $\mathrm{FG}=\mathrm{RS}=\mathrm{y}$.
Thus, $\angle A B C=x+x+y=2 x+y$

## EXERCISE 28 (A)

1. Given below are the angles $x$ and $y$.


Without measuring these angles, construct :
(i) $\angle A B C=x+y$
(ii) $\angle A B C=2 x+y$
(iii) $\angle A B C=x+2 y$
2. Given below are the angles $x, y$ and $z$.


Without measuring these angles construct : (i) $\angle A B C=x+y+z$
(ii) $\angle A B C=2 x+y+z$
(iii) $\angle A B C=x+2 y+z$
3. Draw a line segment $B C=4 \mathrm{~cm}$. Construct angle $A B C=60^{\circ}$.
4. Construct angle $A B C=45^{\circ}$ in which $B C=5$ cm and $A B=4.6 \mathrm{~cm}$.
5. Construct angle $A B C=90^{\circ}$. Draw $B P$, the bisector of angle $A B C$. State, the measure of angle PBC.
6. Draw angle $A B C$ of any suitable measure.
(i) Draw BP , the bisector of angle ABC .
(ii) Draw BR, the bisector of angle PBC and draw $B Q$, the bisector of angle $A B P$.
(iii) Are the angles $A B Q, Q B P$, PBR and RBC equal ?
(iv) Are the angles ABR and QBC equal ?

## 4. Construction of bisector of a line segment.

## Steps :

1. Draw the given line segment and represent it by BC.
2. At $B$, construct angle PBC of any suitable measure and at $C$, construct angle QCB equal to angle PBC.
i.e. $\angle \mathrm{PBC}=\angle \mathrm{QCB}$.
3. From $B P$, cut $B R$ of any suitable length and from $C Q$, cut $C S=B R$.
4. Join R and S.

5. Let $R S$ cut the given line segment $B C$ at point $O$.

Thus, $R S$ is a bisector of $B C$ such that $O B=O C=\frac{1}{2} B C$.

## Alternative method :

## Steps :

1. Draw BC.
2. With $B$ as centre, and radius equal to more than half of $B C$, draw arcs on both the sides of $B C$.
3. With C as centre and with the same radius as taken in step 2, draw arcs on both the sides of BC.
4. Let the arcs intersect each other at points $P$ and $Q$.
5. Join P and Q.

6. The line $P Q$ cuts the given line segment $B C$ at point $O$.

Thus, $P Q$ is a bisector of $B C$ such that $O B=O C=\frac{1}{2} B C$.
5. Construction of perpendicular bisector of a line segment.

## Steps :

1. Draw the given line segment and represent it by BC.
2. Now proceed in exactly the same way as in alternative method of construction 4 , given above.
In this construction, the line PQ bisects the given line segment $B C$ and is perpendicular to it.
i.e. $\mathrm{OB}=\mathrm{OC}$ and $\angle \mathrm{POC}=90^{\circ}$.


Hence, $P Q$ is perpendicular bisector of $B C$.
6. Construction of perpendicular to a line.

1. To construct the perpendicular to a line at a given point in it :

Let $P$ be the given point in the given line $A B$.


## Steps :

1. With $P$ as centre, draw an arc with a suitable radius which cuts $A B$ at points $C$ and $D$.
2. Taking $C$ and $D$ as centres, draw arcs of equal radii which cut each other at point $O$.


The radius must be more than half the distance between C and D .
3. Join P and O

Then, $O P$ is the required perpendicular.
So, $\angle \mathrm{OPA}=\angle \mathrm{OPB}=90^{\circ}$
2. To construct the perpendicular to a line from an external point :

Let $P$ be the given external point of line $A B$.

## Steps :



1. With $P$ as centre, draw an arc of a suitable radius which cuts $A B$ as points $C$ and $D$.
2. With $C$ and $D$ as centres, draw arcs of equal radii and let these arcs intersect each other at point $Q$.

The radius of these arcs must be more than half of CD and both the arcs must be drawn on the other side.

3. Join P and Q.
4. Let $P Q$ cut $A B$ at point $O$.

Thus, OP is the required perpendicular.
Clearly, $\angle A O P=\angle B O P=90^{\circ}$
EXERCISE 28 (B)

1. Draw a line segment $A B$ of length 5.3 cm . Using two different methods bisect $A B$.
2. Draw a line segment $P Q=4.8 \mathrm{~cm}$.

Construct the perpendicular bisector of PQ .
3. In each of the following, draw a perpendicular through point $P$ to the line segment $A B$ :
(i)
. $P$

(ii)
(iii)

4. Draw a line segment $A B=5.5 \mathrm{~cm}$. Mark a point $P$, such that $P A=6 \mathrm{~cm}$ and $P B=4.8 \mathrm{~cm}$. From the point $P$, draw a perpendicular to $A B$.
5. Draw a line segment $A B=6.2 \mathrm{~cm}$. Mark a point $P$ in $A B$ such that $B P=4 \mathrm{~cm}$. Through point $P$ draw a perpendicular to $A B$.

## 7. Constructions of Parallel Lines.

## 1. To construct a line parallel to a given line and passing through a given point :

Let the given line be $A B$ and the given point be $P$.
First Method : (By drawing alternate angles)

## Steps :

1. Take any point $Q$ in line $A B$ and join it with the given point $P$.
2. At point $P$, construct $\angle \mathrm{CPQ}=\angle \mathrm{PQB}$.
3. Produce CP upto any point D.

Thus, CPD is the required parallel line.


Alternative method : (By drawing corresponding angles)

## Steps:

1. Join QP and produce it to any point R.
2. At $P$, construct $\angle R P D=\angle P Q B$.
3. Produce DP upto any point C.

Thus, CPD is the required parallel line.

2. To construct a line parallel to a given line at a given distance from it :

Steps:

1. At any point $P$ in line $A B$, draw $P Q$ perpendicular to $A B$.
2. With $P$ as centre and radius equal to 3.6 cm , draw an arc which cuts PQ at point R.
3. At point R, draw RD perpendicular to $P Q$.
4. Produce DR upto any point $C$.

Then, $C D$ is the required parallel line.


## EXERCISE 28 (C)

1. Draw a line $A B=6 \mathrm{~cm}$. Mark a point $P$ any where outside the line $A B$. Through the point $P$, construct a line parallel to $A B$.
2. Draw a line $M N=5.8 \mathrm{~cm}$. Locate a point $A$ which is 4.5 cm from M and 5 cm from N . Through A draw a line parallel to line MN.
3. Draw a straight line $A B=6.5 \mathrm{~cm}$. Draw another line which is parallel to $A B$ at a distance of 2.8 cm from it.
4. Construct an angle $\mathrm{PQR}=80^{\circ}$. Draw a line parallel to $P Q$ at a distane of 3 cm from it and
another line parallel to QR at a distance of 3.5 cm from it. Mark the point of intersection of these parallel lines as $A$.
5. Draw an angle $\mathrm{ABC}=60^{\circ}$. Draw the bisector of it. Also draw a line parallel to BC a distance of 2.5 cm from it.
Let this parallel line meet $A B$ at point $P$ and angle bisectors at point $Q$. Measure the length of $B P$ and $P Q$. Is $B P=P Q$ ?
6. Construct an angle $A B C=90^{\circ}$. Locate a point $P$ which is 2.5 cm from $A B$ and 3.2 cm from BC.

## 8. Construction of Scalene Triangles.

1. When lengths of its three sides are given :

Let the lengths of three sides be $4.5 \mathrm{~cm}, 4 \mathrm{~cm}$ and 5.2 cm .

## Steps :

1. Draw $A B=4.5 \mathrm{~cm}$.
2. With $A$ as centre and radius $=4 \mathrm{~cm}$, draw an arc.
3. With $B$ as centre and radius $=5.2 \mathrm{~cm}$, draw one more arc which cuts the former arc at point C .
4. Join $A C$ and $B C$.
 Thus, $A B C$ is the required triangle.
5. When lengths of two sides and the included angle are given :

Let the two sides be 5 cm and 4.2 cm , and the included angle be $75^{\circ}$.

## Steps :

1. $\operatorname{Draw} A B=5 \mathrm{~cm}$.
2. At $A$, draw line $A P$, so that angle $\angle P A B=75^{\circ}$.
3. From $A P$, cut $A C=4.2 \mathrm{~cm}$ and then join $B$ and $C$. Thus, $A B C$ is the required triangle.


## 3. When two angles and the included side are given :

Let the two angles be $60^{\circ}$ and $45^{\circ}$, and the included side be 5.4 cm .

## Steps :

1. Draw $A B=5.4 \mathrm{~cm}$.
2. At $A$, draw $A P$ so that $\angle P A B=60^{\circ}$.
3. At $B$, draw $B Q$ so that $\angle Q B A=45^{\circ}$.
4. Let $A P$ and $B Q$ intersect at point $C$.


Thus, $A B C$ is the required triangle.
4. When the base, one base angle and the sum of lengths of other two sides are given :
Let the base be 4.8 cm , one base angle be $60^{\circ}$ and the sum of other two sides be 9.2 cm .

## Steps :

1. Draw $A B=4.8 \mathrm{~cm}$.
2. At A, draw $\mathrm{AP}=9.2 \mathrm{~cm}$ such that $\angle \mathrm{PAB}=60^{\circ}$.
3. Join P and B.
4. Draw the perpendicular bisector of BP which meets AP at point $C$. Join $C$ and $B$.
Thus, $A B C$ is the required triangle.

5. When the base, one base angle and the difference between the lengths of other two sides are given :
Let the base be 4.5 cm , one base angle be $30^{\circ}$ and the difference between the lengths of other two sides be 1.2 cm .

## Steps :

1. Draw $A B=4.5 \mathrm{~cm}$.
2. At $A$, draw $A P$ such that $\angle P A B=30^{\circ}$.
3. From $A P$, cut $A Q=1.2 \mathrm{~cm}$.
4. Join $B$ and $Q$, and draw the perpendicular bisector of $B Q$ which meets AP at point C.
5. Join C and B


Thus, $A B C$ is the required triangle.
6. When the perimeter (the sum of lengths of all the three sides) and both the base angles are given :

Let the perimeter be 9.6 cm and both the base angles be $60^{\circ}$ and $90^{\circ}$.
Steps :

1. Draw $A B=9.6 \mathrm{~cm}$.
2. At $A$, draw $A P$ so that $\angle P A B=60^{\circ}$.
3. At $B$, draw $B Q$ so that $\angle A B Q=90^{\circ}$.

4. Draw the bisectors of angles $P A B$ and $A B Q$. Let these bisectors meet at point $C$. Join $A C$ and BC.
5. Draw the perpendicular bisector of $A C$ which meets $A B$ at point $D$. Also, draw the perpendicular bisector of $B C$ which meets $A B$ at point $E$.
6. Join DC and EC.

Thus, CDE is the required triangle.

## EXERCISE 28(D)

1. Construct a triangle, when :
(i) the lengths of its three sides be 6 cm , 5.2 cm and 4.6 cm .
(ii) the lengths of two sides be 6.3 cm and 4 cm , and the included angle be $45^{\circ}$.
(iii) two angles be $75^{\circ}$ and $60^{\circ}$; and the included side be 5 cm .
(iv) the base be 4 cm , one base angle be $60^{\circ}$ and the sum of other two sides be 8.4 cm .
(v) the base be 5.1 cm , one base angle be $45^{\circ}$ and the difference between other two sides be 1.5 cm .
(vi) the perimeter of the triangle be 10.8 cm and both the base angles be $60^{\circ}$ and $75^{\circ}$.
2. Construct a triangle $A B C$, when :
(i) $A B=6.3 \mathrm{~cm}, \mathrm{BC}=7.5 \mathrm{~cm}$ and $\mathrm{CA}=6 \mathrm{~cm}$.
(ii) $\mathrm{BC}=6 \mathrm{~cm}, \mathrm{CA}=4.8 \mathrm{~cm}$ and $\angle \mathrm{C}=60^{\circ}$.
(iii) $\mathrm{CA}=5.5 \mathrm{~cm}, \angle \mathrm{C}=45^{\circ}$ and $\angle \mathrm{A}=75^{\circ}$.
(iv) $\mathrm{BC}=5.7 \mathrm{~cm}, \angle \mathrm{~B}=45^{\circ}$ and $\mathrm{AB}+\mathrm{AC}=8.2$ cm .
(v) $\mathrm{AB}=5.0 \mathrm{~cm}, \angle \mathrm{~A}=60^{\circ}$ and $\mathrm{AC}-\mathrm{BC}=$ 1.5 cm .
(vi) $\mathrm{AB}+\mathrm{BC}+\mathrm{CA}=11.6 \mathrm{~cm}, \angle \mathrm{~A}=60^{\circ}$ and $\angle B=45^{\circ}$.
3. Construct a $\triangle A B C$ with $A B=6.3 \mathrm{~cm}, \angle B=60^{\circ}$ and $A C+B C=10.0 \mathrm{~cm}$. Measure the length of $A C$.
4. Construct a triangle $A B C$ with $A B=7 \mathrm{~cm}, B C$ $=6.5 \mathrm{~cm}$ and $\angle C A B=60^{\circ}$. Measure the length of AC.
5. Using ruler and compasses only, construct a triangle $A B C$, having $A B=4.7 \mathrm{~cm}, A C=$ 3.4 cm and $\angle \mathrm{BAC}=75^{\circ}$.

Draw the perpendicular bisector of $B C$ and the bisector of angle BAC. If the perpendicular bisector and the angle bisector meet at a point $M$, measure $\angle B M C$.
6. Using ruler and compasses only, construct a triangle $A B C$ having $\angle C=135^{\circ}$ and $\angle B=30^{\circ}$, $B C=5 \mathrm{~cm}$. Bisect angles $B$ and $C$ and measure the distance of $A$ from the point where the bisectors meet.
7. Using ruler and compasses only, construct a triangle $A B C$ from the following data :
$A B+B C+A C=12 \mathrm{~cm}, \angle B=45^{\circ}$ and $\angle C=$ $60^{\circ}$. Measure BC.
8. Construct a triangle $A B C$, given : $B C=7 \mathrm{~cm}$, $A B-A C=1 \mathrm{~cm}$ and $\angle A B C=45^{\circ}$. Measure the lengths of $A B$ and $A C$.

## 9. Construction of Equilateral Triangles.

## When altitude is given :

Let the altitude of the required triangle be 3.6 cm .

## Steps:

1. Draw a line $P Q$ of any suitable length.
2. In $P Q$, mark a point $O$.
3. At $O$, draw a line $O R$ perpendicular to $P Q$ and from $O R$ cut $O A=3.6 \mathrm{~cm}$.
4. At $A$, draw $A B$ so that $\angle O A B=30^{\circ}$ and $A C$ so that $\angle O A C$
 $=30^{\circ}$. $B$ and $C$ being the points on $P Q$.
Thus, $A B C$ is the required triangle.

## Alternative method :

## Steps :

1. Draw a line $P Q$ of any suitable length.
2. Through point $P$, draw $P M$ perpendicular to $P Q$ and from PM cut PR $=3.6 \mathrm{~cm}$.
3. Through point $R$, draw $R S$ perpendicular to $P M$. Clearly, RS is parallel to PQ at a distance of 3.6 cm from it.

4. Mark a point $A$ in $R S$.
5. At $A$, draw a line $A B$ so that $\angle R A B=60^{\circ}$ and a line $A C$ so that $\angle S A C=60^{\circ}$. $B$ and $C$ being the points on PQ.
Thus, $A B C$ is the required triangle.

## 10. Construction of Isosceles Triangles.

## 1. When base and one base angle are given :

Let the base be 5.2 cm and one base angle be $45^{\circ}$.
Students know that the two base angles of an isosceles triangle are equal.

## Steps:

1. Draw $A B=5.2 \mathrm{~cm}$
2. At $A$ and $B$ both, construct angles $=45^{\circ}$ each.

Let the two lines making $45^{\circ}$ angles meet at $C$. Thus, $A B C$ is the required triangle.

2. When base and altitude (height) are given :

Let the base be 5.4 cm and height be 3.0 cm .

## Steps:

1. Draw $A B=5.4 \mathrm{~cm}$
2. Draw $O P$, the perpendicular bisector of $A B$, which meets $A B$ at point $O$.
3. From OP, cut $O C=3.0 \mathrm{~cm}$.
4. Join AC and BC.

Thus, $A B C$ is the required triangle.


## 11. Construction of right-angled triangles.

## 1. When lengths of one side and hypotenuse are given :

Let one side be 3.5 cm and hypotenuse be 5.5 cm .

## Steps :

1. Draw $A B=3.5 \mathrm{~cm}$
2. At $A$, draw $A P$ so that $A P \perp A B$ i.e. $\angle P A B=90^{\circ}$.
3. With $B$ as centre and radius $=5.5 \mathrm{~cm}$, draw an arc which cuts AP at point $C$. Join $C$ and $B$.


Thus, $A B C$ is the required triangle.

## 2. When triangle is isosceles and its hypotenuse is given :

Let the hypotenuse of the required right-angled isoscele triangle be 5.8 cm .

## Steps :

1. Draw $A B=5.8 \mathrm{~cm}$
2. At $A$ and $B$, construct angles of $45^{\circ}$ each.

Let these two lines, making $45^{\circ}$ angles with $A B$, intersect each other at point $C$.
Thus, $A B C$ is the required triangle.


## EXERCISE 28 (E)

1. Construct an equilateral triangle, whose :
(i) one side is 3.8 cm .
(ii) altitude is 3 cm .
2. Construct an isosceles triangle, if :
(i) its base $=4.3 \mathrm{~cm}$ and one base angle $=75^{\circ}$.
(ii) its base $=5.8 \mathrm{~cm}$ and altitude $=4 \mathrm{~cm}$.
3. Construct a right-angled triangle, whose one side is 3 cm and the length of hypotenuse is 5 cm . Measure the length of its other side.
4. Construct an isosceles right-angled triangle, whose :
(i) hypotenuse is $=6 \mathrm{~cm}$.
(ii) one side is 3.5 cm .
5. Construct a triangle ABC ; if :
(i) $\mathrm{AB}=\mathrm{BC}=\mathrm{CA}=4.2 \mathrm{~cm}$.
(ii) $\mathrm{AB}=\mathrm{BC}=4 \mathrm{~cm}$ and $\mathrm{AC}=4.5 \mathrm{~cm}$.
(iii) $\angle A=90^{\circ}, A B=4 \mathrm{~cm}$ and $B C=6 \mathrm{~cm}$.
(iv) $\mathrm{AB}=\mathrm{BC}=\mathrm{CA}$ and altitude $\mathrm{AD}=4.5 \mathrm{~cm}$
6. Construct an isosceles triangle PQR; if :
(i) base $\mathrm{QR}=5.6 \mathrm{~cm}$ and base $\angle \mathrm{PQR}=75^{\circ}$.
(ii) base $\mathrm{PQ}=6.4 \mathrm{~cm}$ and altitude $=4 \mathrm{~cm}$.
7. Construct a right-angled triangle XYZ , if :
(i) $\angle Z=90^{\circ}, Y Z=5 \mathrm{~cm}$ and $X Y=7.2 \mathrm{~cm}$.
(ii) $\angle X=90^{\circ}, X Y=3.8 \mathrm{~cm}$ and $Y Z=6 \mathrm{~cm}$.
(iii) $\angle Y=90^{\circ}, X Y=Y Z$ and $X Z=6.4 \mathrm{~cm}$.
8. Using ruler and compasses only, draw a triangle $A B C$, such that $A B=A C=4.8 \mathrm{~cm}$ and $\angle B=45^{\circ}$. Measure the length of $B C$.

## 12. Circumcircle and Incircle of a triangle.

## 1. Circumcircle of a triangle :

The circle, which passes through all the three vertices of a triangle, is called circumcircle of the triangle. Its centre is called circumcentre and radius is called circumradius.

## To construct the circumcircle of a given triangle :

## Steps :

1. Construct the triangle $A B C$ with given measures.
2. Draw perpendicular bisectors of any two sides of the triangle.
Here the perpendicular bisectors of sides $A B$ and $B C$ are drawn.

Let these bisectors intersect each other at point O .
3. With O as centre and OA or OB or OC as radius (since, $O A=O B=O C$ ), draw a circle. This circle will pass through all the three vertices of the triangle.


In this construction, O is the circumcentre and $\mathrm{OA}=\mathrm{OB}$ $=O C=$ circumradius.

## 2. Incircle of a triangle :

A circle, which touches all the three sides of a triangle, is called the incircle of the triangle. Its centre is called the incentre.

## To construct the incircle of a given triangle :

## Steps :

1. Construct the $\triangle A B C$ with given measurements.
2. Draw bisectors of any two angles of the triangle.

Here the bisectors of angles $B$ and $C$ are drawn which intersect each other at point I .
3. From I draw IP, the perpendicular to BC.
4. With I as centre and IP as radius, draw a circle which will touch all the three sides of the triangle.
In this construction, I is the incentre and IP is the radius
 of the incircle.

## EXERCISE 28 (F)

1. Construct a triangle $A B C$, having given $A B=$ $6 \mathrm{~cm}, A C=7 \mathrm{~cm}$ and $\angle C=30^{\circ}$. Draw the circumcircle of the triangle. Measure its radius.
2. Using ruler and compasses only, draw a $\triangle A B C$ such that $A B=4.5 \mathrm{~cm}, A C=5.4 \mathrm{~cm}$ and $\angle A=90^{\circ}$. Draw the circumcircle of the triangle and measure its radius.
3. Construct a triangle $A B C$ in which base $B C=$ $6.0 \mathrm{~cm}, \angle B=60^{\circ}$ and altitude (height) is 4.5 cm .

In the same figure, construct a circle which
passes through A, B and C. Name the circle drawn and measure its radius.
4. Draw an equilateral triangle of side 5 cm . Draw its circumcircle.
5. Draw a triangle with sides $4.5 \mathrm{~cm}, 5 \mathrm{~cm}$ and 6 cm . Draw the incircle of this triangle.
6. Draw an equilateral triangle of side 4.5 cm . Draw a circle which touches all its sides.
7. Using ruler and compasses only, construct a triangle PQR such that $\angle \mathrm{P}=120^{\circ}, \mathrm{PQ}=5 \mathrm{~cm}$ and $\mathrm{PR}=6 \mathrm{~cm}$.

In the same figure, find a point which is equidistant from its sides. Name this point. With this point as centre draw a circle touching all the sides of the triangle.
8. Construct a triangle $A B C$ such that: $B C=$
$6 \mathrm{~cm}, \angle B=60^{\circ}$ and $\angle C=45^{\circ}$.
In the same figure, find a point which is equidistant from the vertices of the triangle. Name this point. Draw the circumcircle of the triangle.

## 13. Construction of quadrilateral $A B C D$.

## 1. When four sides and one angle are given :

Let $A B=4 \mathrm{~cm}, B C=3.6 \mathrm{~cm}, C D=3.5 \mathrm{~cm}, A D=3 \mathrm{~cm}$ and $Đ A=60^{\circ}$.

## Steps :

1. Draw $A B=4 \mathrm{~cm}$.
2. At $A$, construct angle $P A B=60^{\circ}$ and from $A P$ cut $A D=3 \mathrm{~cm}$.
3. Taking D as centre, draw an arc of radius 3.5 cm (=CD) and taking B as centre draw one more arc of radius $3.6 \mathrm{~cm}(=B C)$ which cuts the previous arc at point C .
4. Join CD and CB.

Then, $A B C D$ is the required quadrilateral.

2. When three consecutive sides and two included angles are given :

Let $A B=3.8 \mathrm{~cm}, B C=4.2 \mathrm{~cm}, C D=4.0 \mathrm{~cm}, \angle B=60^{\circ}$ and $\angle C=75^{\circ}$.

## Steps :

1. Draw $B C=4.2 \mathrm{~cm}$.
2. At $B$, construct angle $P B C=60^{\circ}$ and at $C$, construct angle $\mathrm{QCB}=75^{\circ}$.
3. From $B P$ cut $A B=3.8 \mathrm{~cm}$ and from $C Q$ cut $C D=4.0 \mathrm{~cm}$.
4. Join A and D.

Then, $A B C D$ is the required quadrilateral.

3. When four sides and one diagonal are given :

Let $A B=4 \mathrm{~cm}, B C=3.8 \mathrm{~cm}, C D=3.5 \mathrm{~cm}, A D=4.2 \mathrm{~cm}$ and diagonal $A C=5.5 \mathrm{~cm}$.

## Steps :

1. Draw $A B=4 \mathrm{~cm}$.
2. Taking $B$ as centre, draw an arc of radius 3.8 cm (= BC) and taking A as centre, draw one more arc of radius 5.5 cm (= diagonal AC ). Let the two arcs intersect at point C .
3. Taking $C$ as centre, draw an arc of radius 3.5 cm (= CD) and taking A as centre, draw one more arc of radius 4.2 cm (= AD). Let the two arcs intersect at point $D$.


Then, $A B C D$ is the required quadrilateral.

## 14. Construction of parallelogram ABCD.

1. When two consecutive sides and the included angles are given :

Let $B C=4.2 \mathrm{~cm}, C D=3.6 \mathrm{~cm}$ and $Đ C=60^{\circ}$.
Students know that the opposite sides of a parallelogram are always equal.
$\therefore B C=4.2 \mathrm{~cm}=A D$ and $C D=3.6 \mathrm{~cm}=A B$.

## Steps :

1. Draw $B C=4.2 \mathrm{~cm}$.
2. At $C$, construct angle $P C B=60^{\circ}$ and from $C P$ cut $C D=$ 3.6 cm .
3. Taking $D$ as centre, draw an arc of radius 4.2 cm (= AD) and taking $B$ as centre draw one more arc of radius $3.6 \mathrm{~cm}(=A B)$. Let the two arcs intersect at point $A$.
4. Join $A B$ and $A D$.


Then, $A B C D$ is the required parallelogram.
2. When two consecutive sides and one diagonal are given :

Let $A B=4.5 \mathrm{~cm}, B C=3.7 \mathrm{~cm}$ and diagonal $A C=6.2$.

## Steps :

1. Draw $A B=4.5 \mathrm{~cm}$.
2. Taking $B$ as centre, draw an arc of radius 3.7 cm (= $B C$ ) and taking $A$ as centre draw one more arc of radius 6.2 cm (= diagonal AC). Let the two arcs intersect at point C. Join B and C.
3. Taking $C$ as centre, draw an arc of radius 4.5 cm (= $A B$ ) and taking $A$ as centre draw one more arc of radius $3.7 \mathrm{~cm}(=B C)$. Let the two arcs intersect at point D.

4. Join AD and CD.

Then, $A B C D$ is the required parallelogram.
3. When both the diagonals and the angle between them are given :

Let the diagonal $A C=5.5 \mathrm{~cm}$, diagonal $\mathrm{BD}=6.0 \mathrm{~cm}$ and the angle between them $=60^{\circ}$.
Students know that the diagonals of a parallelogram bisect each other.

## Steps:

1. Draw $A C=5.5 \mathrm{~cm}$.
2. Locate the mid-point $O$ of $A C$ by drawing its perpendicular bisector.
3. Through $O$, construct a line POQ so that angle $P O C=60^{\circ}$.
4. From $P Q$, cut $O D=\frac{B D}{2}=\frac{6.0}{2}=3.0 \mathrm{~cm}$ and also $O B=\frac{6.0}{2}=3.0 \mathrm{~cm}$.
5. Join $A B, B C, C D$ and $D A$.

Then, $A B C D$ is the required parallelogram.

15. Construction of rectangle $A B C D$.

1. When two adjacent sides are given :

Let $A B=5.0 \mathrm{~cm}$ and $B C=3.5 \mathrm{~cm}$.
The opposite sides of a rectangle are equal and each angle of it is $90^{\circ}$.
$\therefore A B=5.0 \mathrm{~cm}=D C, B C=3.5 \mathrm{~cm}=A D$ and $\angle A=\angle B=\angle C=\angle D=90^{\circ}$.

## Steps :

1. Draw $A B=5.0 \mathrm{~cm}$.
2. At $B$, construct angle $P B A=90^{\circ}$. From $B P$ cut $B C=3.5 \mathrm{~cm}$.
3. Taking $C$ as centre, draw an arc of radius 5.0 cm (= $A B$ ) and taking $A$ as centre, draw one more arc of radius 3.5 $\mathrm{cm}(=B C)$. Let these two arcs intersect at point $D$.
4. Join AD and CD.

Then, $A B C D$ is the required rectangle.


## 2. When one side and one diagonal are given :

Let $A B=4.8 \mathrm{~cm}$ and diagonal $A C=6.2 \mathrm{~cm}$.

## Steps:

1. Draw $A B=4.8 \mathrm{~cm}$.
2. At $B$, construct angle $P B A=90^{\circ}$.
3. Taking $A$ as centre, draw an arc of radius 6.2 cm (= AC) which cuts $B P$ at point $C$.
4. Taking $C$ as centre, draw an arc of radius $4.8 \mathrm{~cm}(=A B)$ and taking $A$ as centre draw another arc of radius equal to $B C$. Let these two arcs intersect at point $D$.

5. Join AD and CD.

Then, $A B C D$ is the required rectangle.
3. When one diagonal and the angle between the two diagonals are given :

Let diagonal $A C=6.4 \mathrm{~cm}$ and the angle between the two diagonals be $60^{\circ}$.
The diagonals of a rectangle are equal i.e. $\mathrm{AC}=\mathrm{BD}=6.4 \mathrm{~cm}$.

## Steps :

1. Draw $A C=6.4 \mathrm{~cm}$.
2. Draw the perpendicular bisector of $A C$ to locate the mid-point of AC. Let the perpendicular bisector intersect $A C$ at point $O$.
Therefore, O is the mid-point of $A C$.
3. Through O , construct a line POQ so that angle $\mathrm{POC}=$ $60^{\circ}$.
4. From OP cut $O D$ equal to $O C$ (i.e. 3.2 cm ) and from $O Q$ cut $O B$ equal to $O A$ (i.e. 3.2 cm ).
5. Join $A B, B C, C D$ and $D A$.

Then, $A B C D$ is the required rectangle.


## 16. Construction of rhombus $A B C D$.

## 1. When one side and one angle are given :

Let the side $A B=4.5 \mathrm{~cm}$ and angle $A=60^{\circ}$.
Students know that the sides of a rhombus are equal i.e. $A B=B C=C D=A D=4.5 \mathrm{~cm}$

## Steps :

1. Draw $A B=4.5 \mathrm{~cm}$.
2. At $A$, construct angle $P A B=60^{\circ}$
3. From $A P$, cut $A D=4.5 \mathrm{~m}$.
4. Taking $D$ as centre, draw an arc of radius 4.5 cm (= $A B$ ) and taking $B$ as centre draw one more arc of radius $4.5 \mathrm{~cm}(=A B)$. Let the two arcs intersect at point C.
5. Join $B C$ and $D C$.


Then, $A B C D$ is the required rhombus.
2. When one side and one diagonal are given :

Let the side $A B=5 \mathrm{~cm}$ and the diagonal $A C=7 \mathrm{~cm}$.

## Steps :

1. Draw $A B=5 \mathrm{~cm}$.
2. Taking $A$ as centre, draw an arc of radius 7 cm (= $A C$ ) and taking $B$ as centre, draw one more arc of radius $5 \mathrm{~cm}(=A B)$. Let the two arcs intersect at point $C$.
3. Taking $C$ as centre, draw an arc of radius 5 cm (= $A B$ ) and taking $A$ as centre, draw one more arc of radius $5 \mathrm{~cm}(=A B)$. Let the two arcs intersect at point $D$.

4. Join $B C, C D$ and $D A$.

Then, $A B C D$ is the required rhombus.
3. When both the diagonals are given :

Let the diagonal $A C=4.8 \mathrm{~cm}$ and the diagonal $B D=5.4 \mathrm{~cm}$.
The diagonals of a rhombus bisect each other at $90^{\circ}$.

## Steps :

1. Draw $A C=4.8 \mathrm{~cm}$.
2. Draw the perpendicular bisector of $A C$. Let $P Q$ be the perpendicular bisector of $A C$ which bisects $A C$ at point O.
3. From $O P$, cut $O D=\frac{B D}{2}=\frac{5.4 \mathrm{~cm}}{2}=2.7 \mathrm{~cm}$ and from $O Q$, cut $O B=\frac{B D}{2}=2.7 \mathrm{~cm}$.
4. Join $A B, B C, C D$ and $D A$.


Then, $A B C D$ is the required rhombus.
17. Construction of square $A B C D$.

## 1. When one side is given :

Let side $A B=4.5 \mathrm{~cm}$.
Students know that sides of a square are equal i.e. $A B=B C=C D=A D=4.5 \mathrm{~cm}$ and each angle of the square is $90^{\circ}$.

## Steps :

1. Draw $A B=4.5 \mathrm{~cm}$.
2. At $A$, construct angle $P A B=90^{\circ}$
3. From $A P$, cut $A D=4.5 \mathrm{~cm}$.
4. Taking D as centre, draw an arc of radius 4.5 cm and taking $B$ as centre, draw one more arc of radius 4.5 cm . Let the two arcs intersect at point $C$.
5. Join BC and DC.

Then, $A B C D$ is the required square.

## 2. When a diagonal is given :

Let diagonal $A C=6.4 \mathrm{~cm}$.
In a square, the diagonals bisect each other at $90^{\circ}$. Also, the diagonals of a square are equal i.e. diagonal $A C=$ diagonal $B D=6.4 \mathrm{~cm}$.

## Steps :

1. Draw $A C=6.4 \mathrm{~cm}$.
2. Construct $P O Q$, the perpendicular bisector of $A C$ which intersects $A C$ at point $O$.
3. From OP , cut $\mathrm{OB}=\frac{6.4}{2} \mathrm{~cm}=3.2 \mathrm{~cm}$ and from OQ cut $O D=3.2 \mathrm{~cm}$.
4. Join $A B, B C, C D$ and $D A$.

Then, $A B C D$ is the required square.


## EXERCISE 28 (G)

Students are advised to draw a rough free-hand sketch in each case, before starting the actual construction.

1. Construct a quadrilateral $A B C D$; if :
(i) $\mathrm{AB}=4.3 \mathrm{~cm}, \mathrm{BC}=5.4 \mathrm{~cm}, \mathrm{CD}=5 \mathrm{~cm}, \mathrm{DA}$ $=4.8 \mathrm{~cm}$ and angle $A B C=75^{\circ}$.
(ii) $\mathrm{AB}=6 \mathrm{~cm}, \mathrm{CD}=4.5 \mathrm{~cm}, \mathrm{BC}=\mathrm{AD}=5 \mathrm{~cm}$ and $\angle B C D=60^{\circ}$.
(iii) $\mathrm{AB}=8 \mathrm{~cm}, \mathrm{BC}=5.4 \mathrm{~cm}, \mathrm{AD}=6 \mathrm{~cm}$, $\angle A=60^{\circ}$ and $\angle B=75^{\circ}$.
(iv) $A B=5 \mathrm{~cm}, \mathrm{BC}=6.5 \mathrm{~cm}, C D=4.8 \mathrm{~cm}$, $\angle B=75^{\circ}$ and $\angle C=120^{\circ}$.
(v) $\mathrm{AB}=6 \mathrm{~cm}=\mathrm{AC}, \mathrm{BC}=4 \mathrm{~cm}, \mathrm{CD}=5 \mathrm{~cm}$ and $A D=4.5 \mathrm{~cm}$.
(vi) $\mathrm{AB}=\mathrm{AD}=5 \mathrm{~cm}, \mathrm{BD}=7 \mathrm{~cm}$ and $\mathrm{BC}=\mathrm{DC}$ $=5.5 \mathrm{~cm}$.
2. Construct a parallelogram $A B C D$, if :
(i) $\mathrm{AB}=3.6 \mathrm{~cm}, \mathrm{BC}=4.5 \mathrm{~cm}$ and $\angle \mathrm{ABC}=$ $120^{\circ}$.
(ii) $\mathrm{BC}=4.5 \mathrm{~cm}, \mathrm{CD}=5.2 \mathrm{~cm}$ and $\angle \mathrm{ADC}=$ $75^{\circ}$.
(iii) $\mathrm{AD}=4 \mathrm{~cm}, \mathrm{DC}=5 \mathrm{~cm}$ and diagonal $\mathrm{BD}=$ 7 cm .
(iv) $\mathrm{AB}=5.8 \mathrm{~cm}, \mathrm{AD}=4.6 \mathrm{~cm}$ and diagonal $A C=7.5 \mathrm{~cm}$.
(v) diagonal $\mathrm{AC}=6.4 \mathrm{~cm}$, diagonal $\mathrm{BD}=5.6$ cm and angle between the diagonals is $75^{\circ}$.
(vi) lengths of diagonals AC and BD are 6.3 cm and 7.0 cm respectively, and the angle between them is $45^{\circ}$.
(vii) lengths of diagonals $A C$ and $B D$ are 5.4 cm and 6.7 cm respectively, and the angle between them is $60^{\circ}$.
3. Construct a rectangle $A B C D$; if :
(i) $\mathrm{AB}=4.5 \mathrm{~cm}$ and $\mathrm{BC}=5.5 \mathrm{~cm}$.
(ii) $\mathrm{BC}=6.1 \mathrm{~cm}$ and $\mathrm{CD}=6.8 \mathrm{~cm}$.
(iii) $\mathrm{AB}=5.0 \mathrm{~cm}$ and diagonal $\mathrm{AC}=6.7 \mathrm{~cm}$.
(iv) $\mathrm{AD}=4.8 \mathrm{~cm}$ and diagonal $\mathrm{AC}=6.4 \mathrm{~cm}$.
(v) each diagonal is 6 cm and the angle between them is $45^{\circ}$.
(vi) each diagonal is 5.5 cm and the angle between them is $60^{\circ}$.
4. Construct a rhombus $A B C D$, if :
(i) $A B=4 \mathrm{~cm}$ and $\angle B=120^{\circ}$.
(ii) $\mathrm{BC}=4.7 \mathrm{~cm}$ and $\angle \mathrm{B}=75^{\circ}$.
(iii) $\mathrm{CD}=5 \mathrm{~cm}$ and diagonal $\mathrm{BD}=8.5 \mathrm{~cm}$.
(iv) $\mathrm{BC}=4.8 \mathrm{~cm}$ and diagonal $\mathrm{AC}=7 \mathrm{~cm}$.
(v) diagonal $A C=6 \mathrm{~cm}$ and diagonal $B D=5.8 \mathrm{~cm}$
(vi) diagonal $A C=4.9 \mathrm{~cm}$ and diagonal $B D=6 \mathrm{~cm}$.
(vii) diagonal $\mathrm{AC}=6.6 \mathrm{~cm}$ and diagonal $\mathrm{BD}=$ 5.3 cm .
5. Construct a square, if :
(i) its one side is 3.8 cm .
(ii) its each side is 4.3 cm .
(iii) one diagonal is 6.2 cm .
(iv) each diagonal is 5.7 cm .
6. Construct a quadrilateral $A B C D$ in which; $\angle A=120^{\circ}, \angle B=60^{\circ}, A B=4 \mathrm{~cm}, B C=4.5$ cm and $\mathrm{CD}=5 \mathrm{~cm}$.
7. Construct a quadrilateral $A B C D$, such that $A B$ $=B C=C D=4.4 \mathrm{~cm}, \angle B=90^{\circ}$ and $\angle C=$ $120^{\circ}$.
8. Using ruler and compasses only, construct a parellelogram $A B C D$, in which: $A B=6 \mathrm{~cm}$, $A D=3 \mathrm{~cm}$ and $\angle D A B=60^{\circ}$.
In the same figure draw the bisector of angle $D A B$ and let it meet DC at point $P$. Measure angle APB.
9. Draw a parallelogram $A B C D$, with $A B=6 \mathrm{~cm}$, $A D=4.8 \mathrm{~cm}$ and $\angle D A B=45^{\circ}$.
Draw the perpendicular bisector of side $A D$ and let it meet $A D$ at point $P$. Also, draw the diagonals $A C$ and $B D$, and let them intersect at point O . Join O and P . Measure OP.
10. Using ruler and compasses only, construct a rhombus whose diagonals are 8 cm and 6 cm . Measure the length of its one side.

## ANSWERS

EXERCISE 28(A)
5. $\angle \mathrm{PBC}=45^{\circ}$ 6. (iii) Yes (iv) Yes

EXERCISE 28(C)
5. Yes

EXERCISE 28(D)
3. 5.6 cm
4. $5 \cdot 7 \mathrm{~cm}$
5. $106^{\circ}$
6. 9.4 cm
7.4 .6 cm

## EXERCISE 28(E)

8. $A B=6.1 \mathrm{~cm} ; A C=5.1 \mathrm{~cm}$
9. $4 \mathrm{~cm} \quad 8.6 .8 \mathrm{~cm}$

## EXERCISE 28(F)

1. $5.8 \mathrm{~cm} 2.3 .5 \mathrm{~cm} \quad$ 3. Circumcircle; $3.3 \mathrm{~cm} \quad$ 7. Incentre 8. Circumcentre

EXERCISE 28(G)
8. $90^{\circ}$
9. 3 cm
10.5 cm

