CHAPTER 27

QUADRILATERAL

27.1 REVIEW

Quadrilateral

A quadrilateral is a closed polygon with four sides.

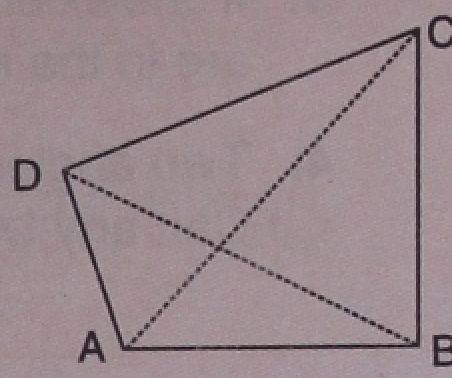
The adjoining figure shows a quadrilateral ABCD which has :

(i) **four sides :** AB, BC, CD and DA

(ii) **four vertices :** A, B, C and D

(iii) **four angles :** ∠ABC, ∠BCD, ∠CDA and ∠DAB

(iv) **two diagonals :** AC and BD



The sum of angles of a quadrilateral = 4 right angles = 360°.

Example 1 :

The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find all its angles.

Solution :

Since, $3 + 4 + 5 + 6 = 18$ and sum of the angles of a quadrilateral is 360°.

∴ **First angle** $= \frac{3}{18} \times 360° = 60°$, **second angle** $= \frac{4}{18} \times 360° = 80°$,

third angle $= \frac{5}{18} \times 360° = 100°$ and, **fourth angle** $= \frac{6}{18} \times 360° = 120°$ **(Ans.)**

Alternative method :

Let the angles of the quadrilateral be 3x, 4x, 5x and 6x.

∴ $3x + 4x + 5x + 6x = 360°$ ⇒ $18x = 360°$ and $x = 20°$

∴ **First angle** $= 3x = 3 \times 20° = 60°$, **second angle** $= 4x = 4 \times 20° = 80°$,

third angle $= 5x = 5 \times 20° = 100°$ and **fourth angle** $= 6x = 6 \times 20° = 120°$ **(Ans.)**

Example 2 :

Three angles of a quadrilateral are in the ratio 4 : 6 : 3. If the fourth angle is 100°; find the other three angles of the quadrilateral.

Solution :

Let the three angles be 4x, 6x and 3x

∴ $4x + 6x + 3x + 100° = 360°$

⇒ $13x = 360° - 100° = 260°$ and, $x = \frac{260°}{13} = 20°$

∴ **The other three angles** are : 4x, 6x and 3x

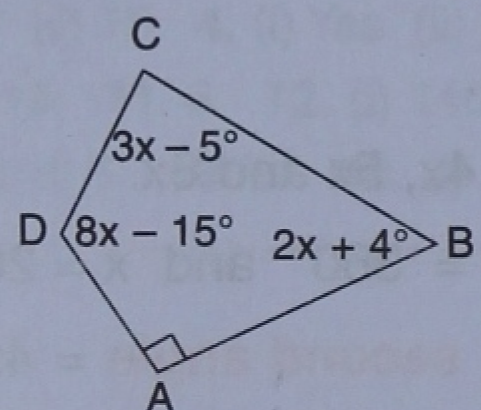
$= 4 \times 20°, 6 \times 20°$ and $3 \times 20° = 80°, 120°$ and $60°$ **(Ans.)**

TEST YOURSELF

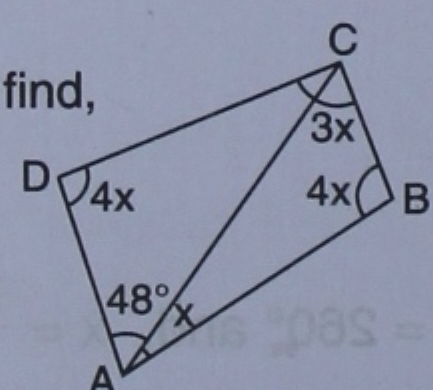
1. In quadrilateral ABCD, $\angle A : \angle B : \angle C : \angle D = 6 : 4 : 5 : 3$. $\angle A$ = ... = ... and $\angle D$ = ... = ... The special name of this quadrilateral is ....
2. In quadrilateral ABCD, $\angle A = 100°$, $\angle B = 70°$ and $\angle C : \angle D = 8 : 11$; the angle $\angle D$ = ...
3. If one exterior angle of a quadrilateral is 140° and the other three angles of the quadrilateral are in the ratio 8 : 15 : 9, the largest angle of the quadrialteral ....
4. Two angles of a quadrialteral are equal and the two other angles are separately equal. Can any two sides of the given quadrialteral be parallel ? ....

EXERCISE 27 (A)

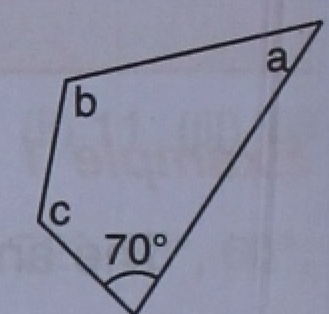
1. Two angles of a quadrilateral are 89° and 113°. If the other two angles are equal, find the equal angles.
2. Two angles of a quadrilateral are 68° and 76°. If the other two angles are in the ratio 5 : 7, find the measure of each of them.
3. Angles of a quadrilateral are $(4x)°$, $5(x + 2)°$, $(7x - 20)°$ and $6(x + 3)°$. Find :
   (i) the value of x.
   (ii) each angle of the quadrilateral.
4. Use the information given in the following figure to find :
   (i) x.
   (ii) $\angle B$ and $\angle C$.



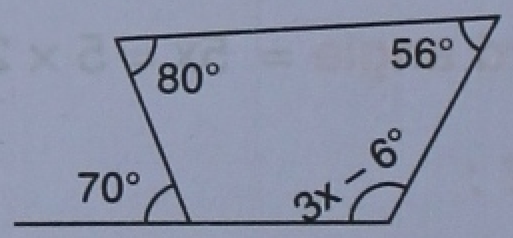
5. In quadrilateral ABCD, side AB is parallel to side DC. If $\angle A : \angle D = 1 : 2$ and $\angle C : \angle B = 4 : 5$.
   (i) Calculate each angle of the quadrilateral.
   (ii) Assign a special name to quadrilateral ABCD.
6. From the following figure find,
   (i) x
   (ii) $\angle ABC$
   (iii) $\angle ACD$



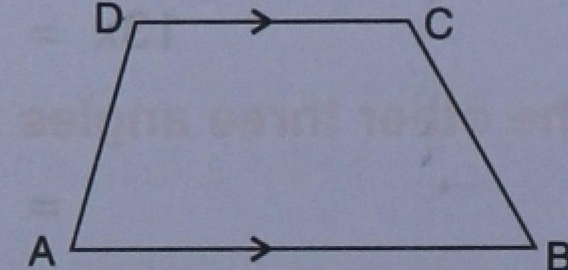
7. Given : In quadrilateral ABCD, $\angle C = 64°$, $\angle D = \angle C - 8°$; $\angle A = 5(a + 2)°$ and $\angle B = 2(2a + 7)°$. Calculate $\angle A$.
8. In the given figure :
   $\angle b = 2a + 15°$ and
   $\angle c = 3a + 5°$,
   find the values of b and c.



9. Three angles of a quadrilateral are equal. If the fourth angle is 69°, find the measure of equal angles.
10. In quadrilateral PQRS, $\angle P : \angle Q : \angle R : \angle S = 3 : 4 : 6 : 7$. Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other.
    (i) Is PS also parallel to QR ?
    (ii) Assign a special name to quadrilateral PQRS.
11. Use the information given in the following figure to find the value of x.



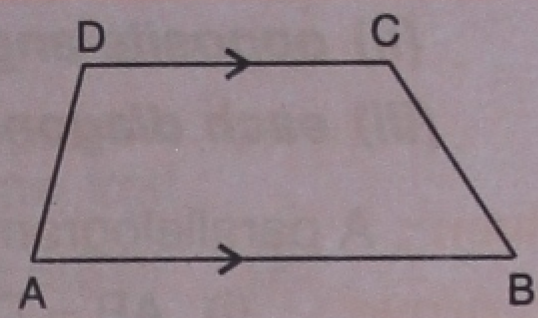
12. The following figure shows a quadrilateral in which sides AB and DC are parallel.
    If $\angle A : \angle D = 4 : 5$, $\angle B = (3x - 15)°$ and $\angle C = (4x + 20)°$, find each angle of the quadrilateral ABCD.



27.2 SPECIAL TYPES OF QUADRILATERALS

1. Trapezium :

A trapezium is a quadrilateral in which one pair of opposite sides are parallel but other two sides of it are non-parallel.

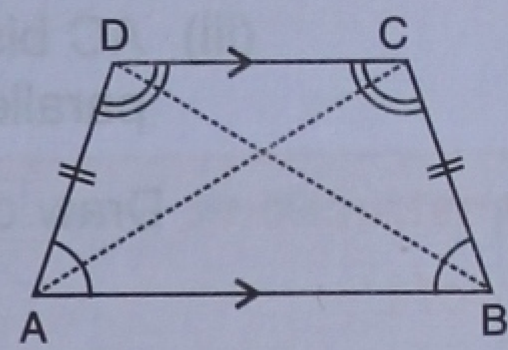


If the non-parallel sides of a trapezium are equal, it is called an **isosceles trapezium.**

In an isosceles trapezium ABCD, side AB // side DC and non-parallel sides AD and BC are equal. Also,

(i) $\angle A = \angle B$ and $\angle C = \angle D$

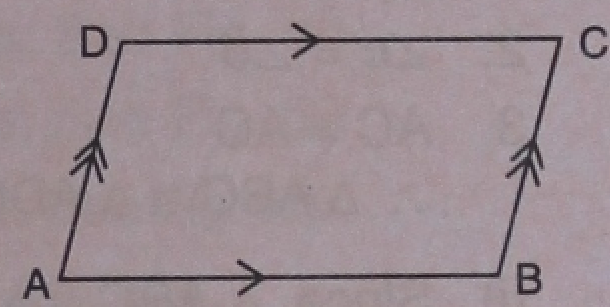
and, (ii) diagonal AC = diagonal BD.



2. Parallelogram :

A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.

The given figure shows a parallelogram ABCD as AB is parallel to DC and AD is parallel to BC.



Theorem 5

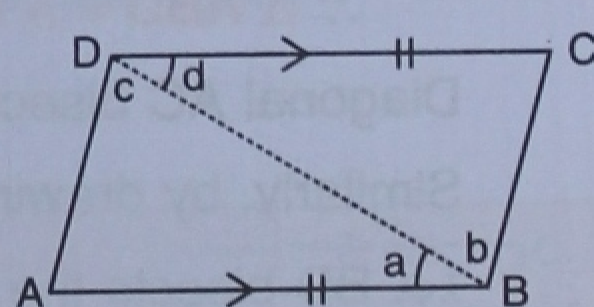
If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.

Given : A quadrilateral ABCD in which AB = DC and AB is parallel to DC.

To prove : ABCD is a parallelogram.

Construction : Join B and D.

Proof :



Statement	***Reason***
In Δ ABD and Δ CDB;	
1. AB = DC	Given
2. BD = BD	Common (identity).
3. $\angle a = \angle d$	Alternate angles, since BD cuts parallel sides AB and DC
$\therefore$ $\Delta ABD \cong \Delta CDB$	S.A.S.
$\Rightarrow \angle c = \angle b$	Corresponding parts of congruent triangles are congruent.
But these are alternate angles,	
$\therefore$ AD is parallel to BC.	When alternate angles are equal, the lines are parallel.

Since, both the pairs of opposite sides of the quadrilateral ABCD are parallel, **it is a parallelogram.**

Hence Proved.

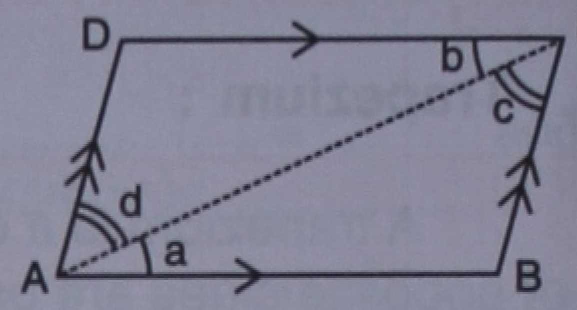
Theorem 6

In a parallelogram :

(i) opposite sides are equal,

(ii) opposite angles are equal and

(iii) each diagonal bisects the parallelogram.



Given : A parallelogram ABCD.

To prove : (i) AB = DC and AD = BC

(ii) ∠A = ∠C and ∠B = ∠D

(iii) AC bisects the parallelogram *i.e.* Δ ABC = Δ ADC and also, BD bisects the parallelogram *i.e.* Δ ABD = Δ BCD.

Construction : Draw diagonal AC

Proof :

Statement	***Reason***
In Δ ABC and Δ ADC,	
1. ∠a = ∠b	Alternate angles, as AC cuts parallel sides AB and DC
2. ∠c = ∠d	Alternate angles, as AC cuts parallel sides AD and BC.
3. AC = AC	Common.
∴ Δ ABC ≅ Δ ADC	A.S.A.
(i) Since, Δ ABC ≅ Δ ADC **AB = DC** and **AD = BC**	Corresponding parts of congruent triangles are congruent.
(ii) Since, Δ ABC ≅ Δ ADC ∴ **∠B = ∠D**	Corresponding parts of congruent triangles are congruent.
and **∠A = ∠C**	From 1 and 2; ∠a + ∠d = ∠b + ∠c *i.e.* ∠A = ∠C
(iii) Since, Δ ABC ≅ Δ ADC ∴ **Δ ABC = Δ ADC**	Congruent triangles are equal.

Diagonal AC bisects parallelogram ABCD.

Similarly, by drawing the diagonal BD, it can be proved that **Δ ABD = Δ BCD**.

i.e. BD bisects the parallelgoram ABCD.

Hence Proved.

Theorem 7

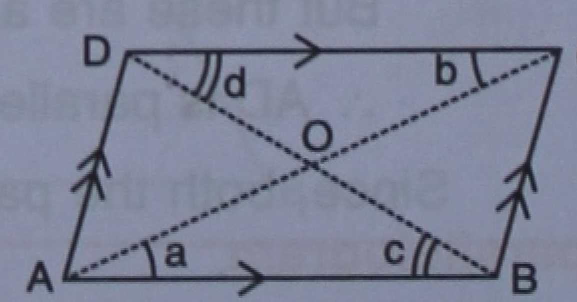
The diagonals of a parallelogram bisect each other.

Given : A parallelogram ABCD whose diagonals intersect each other at O.

To prove : Diagonals bisect each other. *i.e.* OA = OC and OB = OD.

Proof :

Statement	***Reason***
In Δ AOB and Δ COD,	
1. ∠a = ∠b	Alternate angles.
2. ∠c = ∠d	Alternate angles.



3. AB = DC	Opposite sides of a parallelogram are equal.
∴ Δ AOB ≅ Δ COD	A.S.A.
∴ OA = OC and OB = OD	Corresponding parts of congruent triangles are congruent.

Hence Proved.

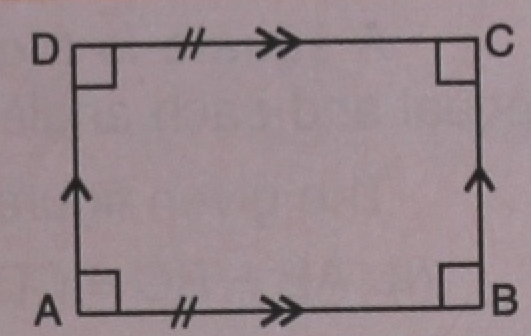
In order to prove that given quadrilateral is a parallelogram; show that :

(i) opposite sides are parallel or, (ii) opposite sides are equal or,
(iii) opposite angles are equal or, (iv) diagonals bisect each other or,
(iv) a pair of opposite sides are equal and parallel.

2. Rectangle :

A rectangle is a quadrilateral in which :

(i) opposite sides are equal *i.e.* AB = DC and AD = BC
(ii) opposite sides are parallel *i.e.* AB // DC and AD // BC and
(iii) each angle is 90° *i.e.* ∠A = ∠B = ∠C = ∠D = 90°.



1. A quadrilateral, with each angle 90° is called a **rectangle.** See Fig. (a)
2. A parallelogram, with any angle 90° is called a **rectangle.** See Fig. (b)
3. A parallelogram, with equal diagonals is called a **rectangle.** See Fig. (c)

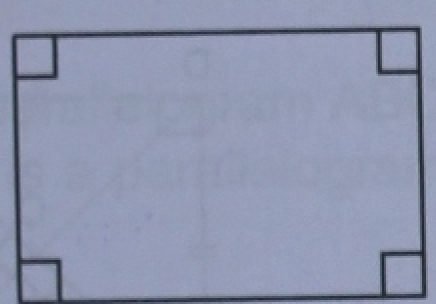


Fig. (a)

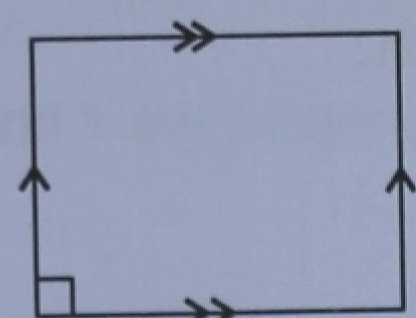


Fig. (b)

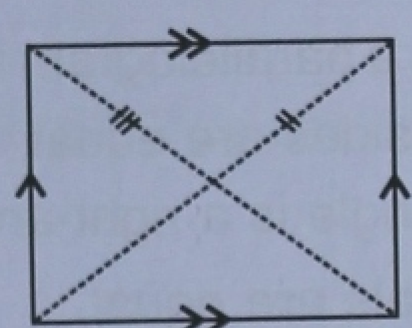
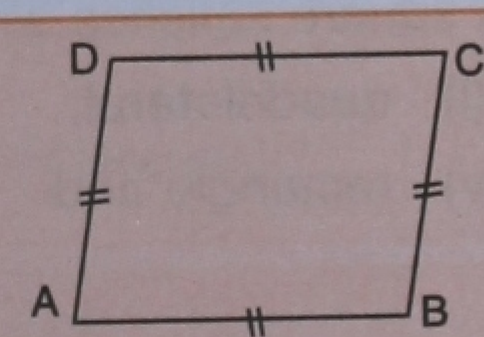


Fig. (c)

1. In order to prove that given quadrilateral is a rectangle show that each of its angles is 90°
2. In order to prove that the given parallelogram is a rectangle; show that :
   (i) any angle of it is 90° or (ii) its diagonals are equal.

3. Rhombus :

A rhombus is a quadrilateral in which all the sides are equal.
The given figure shows a rhombus
ABCD, so : AB = BC = CD = DA



1. A quadrilateral, with all the sides equal is called a **rhombus.** See. Fig. (a)
2. A parallelogram, with any pair of adjacent sides equal, is called a rhombus. See Fig. (b)
3. A parallelogran whose diagonals interesect at 90°, is a rhombus. See Fig.(c).

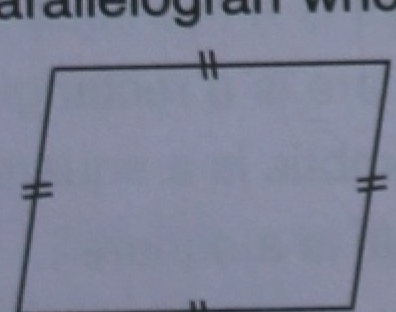


Fig. (a)

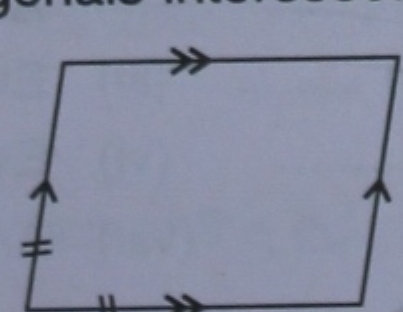


Fig. (b)

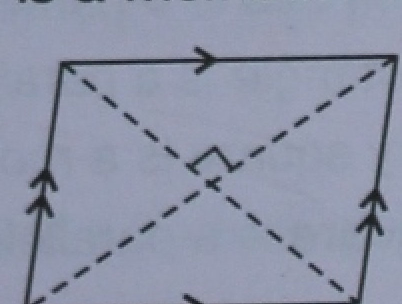


Fig. (c)

1. In order to prove that given quadrilateral is a rhombus, show that :
   (i) its all sides are equal or
   (ii) its diagonals bisect each other at 90°.
2. In order to prove that given parallelogram is a rhombus, show that :
   (i) diagonals intersect each other at 90°
   (ii) any air of adjacent sides is equal.

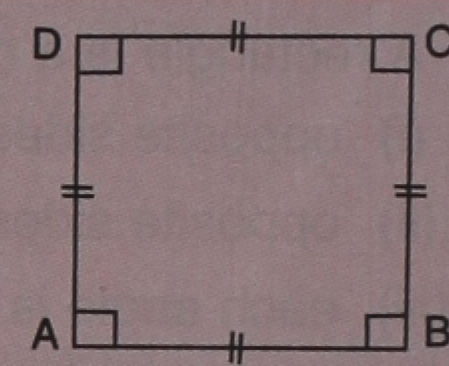
Since, in parallelogram, opposite sides are already equal, so if the adjacent sides are also equal, it is a rhombus.

4. Square :

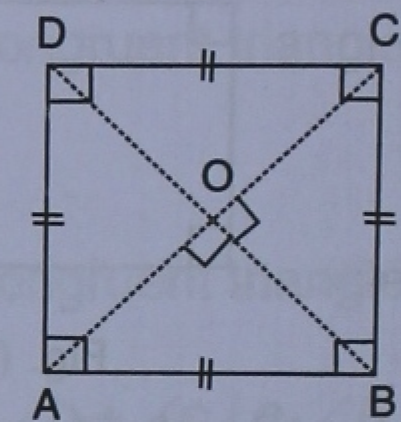
A square is a quadrilateral in which all the sides are equal and each angle is 90°.

The given figure shows a square ABCD, so,

(i) AB = BC = CD = DA and (ii) ∠A = ∠B = ∠C = ∠D = 90°.



1. A square is quadrilateral in when :
   (i) all the sides are equal. (ii) each angle is 90°.
   (iii) diagonals bisect each other at right angle. (iv) diagonals are equal.
2. A square is a parallelogram in which :
   (i) all the sides are equal,
   (ii) each angle is a right-angle,
   (iii) diagonals are equal,
   (iv) diagonals intersect at right-angle.



3. A square is a rectangle in which :
   (i) adjacent sides are equal and (ii) diagonals intersect at right-angle.
4. A square is a rhombus in which :
   (i) each angle is 90° (ii) diagonals are equal.

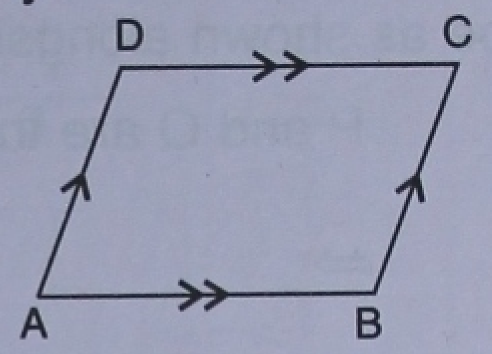
A square satisfies all the properties of a :
(i) quadrilateral, (ii) trapezium, (iii) parallelogram,
(iv) rectangle and (v) rhombus.

TEST YOURSELF

5. Three angles of a quadrilateral are 73°, 84° and 102°.
   Therefore, the fourth angle = 360° – (......... + ... + ...) = 360° – .... = ...
6. State, **true** or **false** :
   (i) A rhombus is a parallelogram .... (ii) A rectangle is a rhombus ....
   (iii) A rectangle is a parallelogram .... (iv) Every square is a rectangle ....
   (v) Every square is a rhombus .... (vi) Every rhombus is a square ....
   (vii) A square is a parallelogram .... (viii) A rectangle is a square ....

Example 3 :

Prove that consecutive angles of a parallelogram are supplementary.



Solution :

Consider a parallelogram ABCD in which AB//DC and AD//BC.

To prove : $\angle A + \angle B = 180°$

Proof :

We know the opposite angles of a parallelogram are equal, therefore $\angle A = \angle C$ and $\angle B = \angle D$.

Also, as the sum of interior angles of a quadrilateral = 360°

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle A + \angle B + \angle A + \angle B = 360°$ [$\because \angle C = \angle A$ and $\angle D = \angle B$]

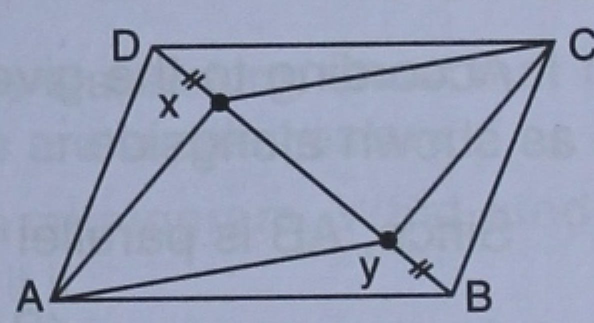
$\Rightarrow 2\angle A + 2\angle B = 360°$

i.e. $\angle A + \angle B = \frac{360°}{2} = 180°$ **Hence proved.**

Similary, we can prove that :
$\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$.

Example 4 :

In a parallelogram ABCD, X and Y are points on diagonal BD such that DX = BY. Prove that AXCY is a parallelogram.



Solution :

According to the given statement, the figure will be as shown alongside.

In Δ ADX and Δ CBY

	AD = BC	[Opposite sides of a parallelgram are equal]
	DX = BY	[Given]
	$\angle ADX = \angle CBY$	[Alternate angles as AD//BC and BD is transversal]
$\therefore$	$\Delta ADX \cong \Delta CBY$	[By S.A.S.]
$\Rightarrow$	AX = CY	[Corresponding parts of congruent triangles]
Similarly,	$\Delta CDX \cong \Delta ABY$	[By S.A.S.]
$\Rightarrow$	CX = AY	
	AX = CY and CX = AY	

$\Rightarrow$ **AXCY is a parallelogram.**

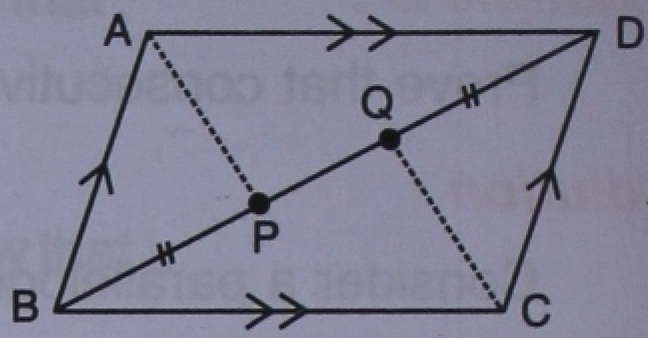
Hence proved.

Whenever the opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram.

Example 5 :

P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP.

Proof :



According to the given statement, the figure will be as shown alongside.

P and Q are the points of trisection of the diagonal BD.

$\Rightarrow$ $BP = PQ = QD = \frac{1}{3} BD$

In Δ APD and Δ BQC : $BQ = \frac{2}{3} BD$ and $PD = \frac{2}{3} BD \Rightarrow BQ = PD$

$\therefore$ BQ = PD [Proved above]

AD = BC [Opposite sides of the parallelogram ABCD]

and, $\angle ADB = \angle CBD$ [Alternate angles]

$\therefore$ $\Delta APD \cong \Delta BQC$ [By S.A.S.]

$\Rightarrow$ $\angle APD = \angle CQB$ [Corresponding parts of congruent triangles]

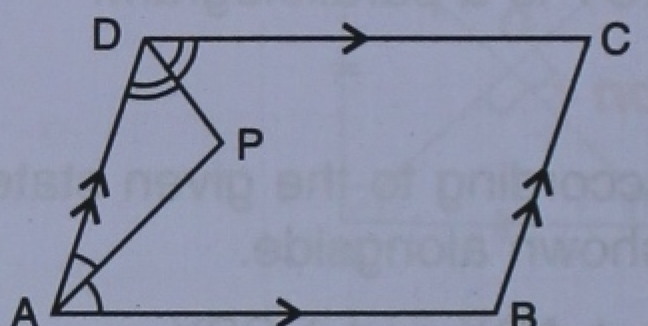
But these are alternate angles and whenever the alternate angles are equal, the lines are parallel.

$\therefore$ **CQ is parallel to AP.** **Hence proved.**

Example 6 :

In parallelogram ABCD, the bisectors of adjacent angles A and D intersect each other at point P. Prove that $\angle APD = 90°$.

Solution :



According to the given statement, the figure will be as shown alongside :

Since, AB is parallel to DC and AD is transversal,

$\therefore$ $\angle BAD + \angle ADC = 180°$ I

Since, AP bisects angle BAD

$\therefore$ $\angle PAD = \frac{1}{2} \angle BAD$

And, DP bisects angle ADC

$\Rightarrow$ $\angle PDA = \frac{1}{2} \angle ADC$

$\therefore$ $\angle PAD + \angle PDA = \frac{1}{2} \angle BAD + \frac{1}{2} \angle ADC$

$= \frac{1}{2} (\angle BAD + \angle ADC)$

$= \frac{1}{2} \times 180° = 90°$ [From equation I]

In triangle APD,

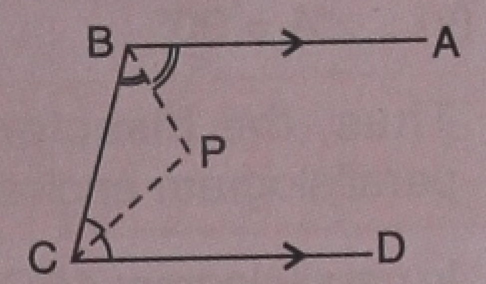
$\angle PAD + \angle PDA + \angle APD = 180°$ [Sum of angles of a Δ = 180°]

$\Rightarrow$ $90° + \angle APD = 180°$

$\Rightarrow$ $\angle APD = 180° - 90° = 90°$ **Hence proved.**

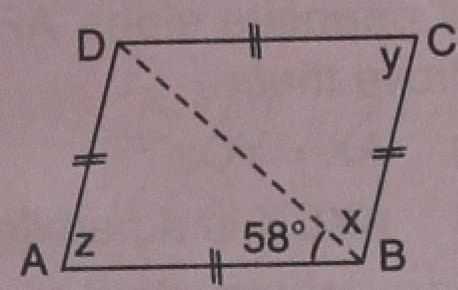
TEST YOURSELF

7. When opposite angles of a quadrilateral are equal, it is a ....
8. When any angle of a parallelogram is 90°, it is a ....
9. When any pair of adjacent sides of a parallelogram are equal, it is a ....
10. When diagonals of a .... are equal, it is a square.
11. Adjacent sides of a rectangle are equal, if it is a ....
12. When each angle of a quadrilateral is 90°, it is a ....
13. If the diagonals of quadrilateral bisect each other at 90°; the quadrilateral is ....
14. AB // DC, CP bisects angle BCD and BP bisects angle ABC; then



   (i) ∠PBC + ∠PCB = ...
   (ii) ∠BPC = ...

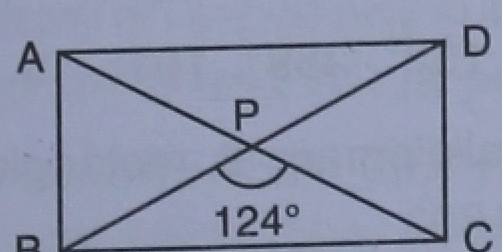
15. In rhombus ABCD, angle ABD = 58°, then



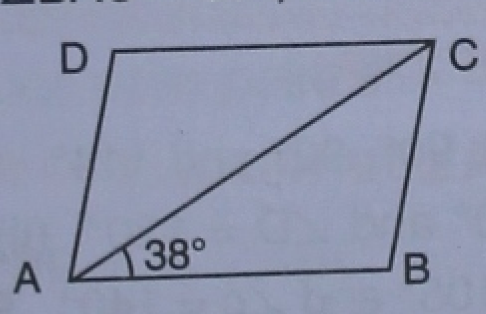
   x = ...
   y = ...
   z = ...

EXERCISE 27(B)

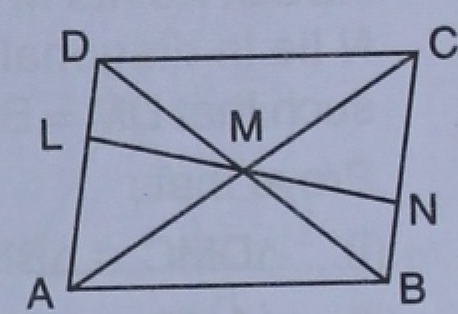
1. In parallelogram ABCD, ∠A = 3 times ∠B. Find all the angles of the parallelogram. In the same parallelogram, if AB = 5x − 7 and CD = 3x + 1; find the length of CD.
2. In parallelogram PQRS, ∠Q = (4x − 5)° and ∠S = (3x + 10)°. Calculate : ∠Q and ∠R.
3. In rhombus ABCD :
   (i) if ∠A = 74°; find ∠B and ∠C.
   (ii) if AD = 7·5 cm; find BC and CD.
4. In square PQRS :
   (i) if PQ = 3x − 7 and QR = x + 3; find PS.
   (ii) if PR = 5x and QS = 9x − 8. Find QS.
5. ABCD is a rectangle. If ∠BPC = 124°. Calculate :
   (i) ∠BAP (ii) ∠ADP.



6. ABCD is a rhombus. If ∠BAC = 38°, find :
   (i) ∠ACB
   (ii) ∠DAC
   (iii) ∠ADC.

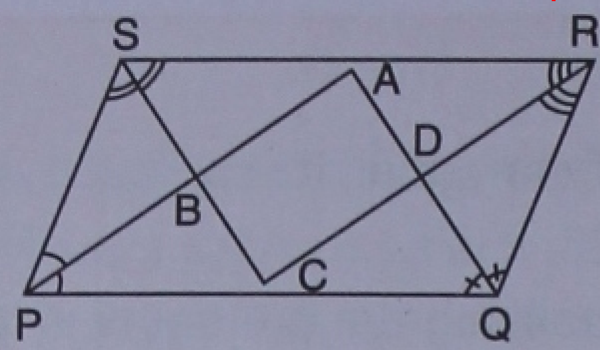


7. ABCD is a rhombus. If ∠BCA = 35°, find ∠ADC.
8. PQRS is a parallelogram whose diagonals intersect at M.
   If ∠PMS = 54°, ∠QSR = 25° and ∠SQR = 30°; find :
   (i) ∠RPS (ii) ∠PRS (iii) ∠PSR.
9. Given : Parallelogram ABCD in which diagonals AC and BD intersect at M.
   Prove : M is mid-point of LN.



10. In an isosceles-trapezium, show that the opposite angles are supplementary.
11. ABCD is a parallelogram. What kind of quadrilateral is it if :
    (i) AC = BD and AC is perpendicular to BD ?
    (ii) AC is perpendicular to BD but is not equal to it ?
    (iii) AC = BD but AC is not perpendicular to BD ?
12. Prove that the diagonals of a parallelogram bisect each other.
13. If the diagonals of a parallelogram are of equal lengths, the parallelogram is a rectangle. Prove it.
14. In parallelogram ABCD, E is the mid-point of AD and F is the mid-point of BC. Prove that BFDE is a parallelogram.
15. In parallelogram ABCD, E is the mid-point of side AB and CE bisects angle BCD. Prove that :
    (i) AE = AD
    (ii) DE bisects and ∠ADC and
    (iii) Angle DEC is a right angle.

16. In the alongside diagram, the bisectors of interior angles of the parallelogram PQRS enclose a quadrilateral ABCD.



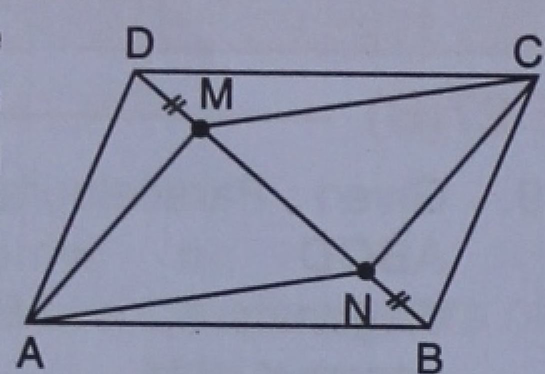
Show that :

(i) ∠PSB + ∠SPB = 90° (ii) ∠PBS = 90°
(iii) ∠ABC = 90° (iv) ∠ADC = 90°
(v) ∠A = 90° (vi) ABCD is a rectangle

Thus, the bisectors of the angles of a parallelogram enclose a rectangle.

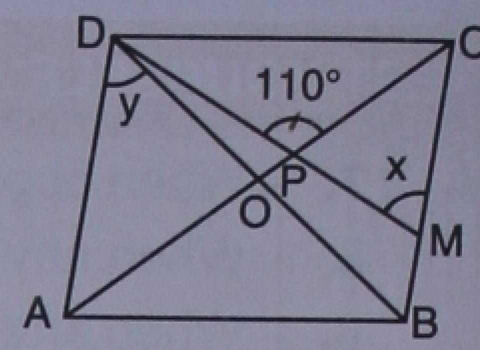
17. In parallelogram ABCD, X and Y are mid-points of opposite sides AB and DC respectively. Prove that :
(i) AX = YC. (ii) AX is parallel to YC
(iii) AXCY is a parallelogram.

18. The given figure shows parallelogram ABCD. Points M and N lie in diagonal BD such that DM = BN.

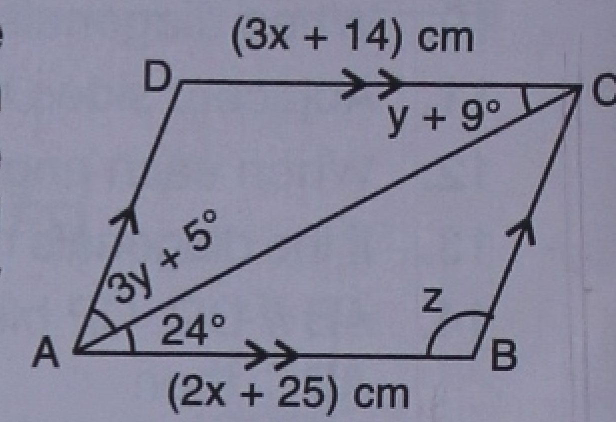


Prove that :
(i) ΔDMC ≅ ΔBNA and so CM = AN.
(ii) ΔAMD ≅ ΔCNB and so AM = CN.
(iii) ANCM is a parallelogram.

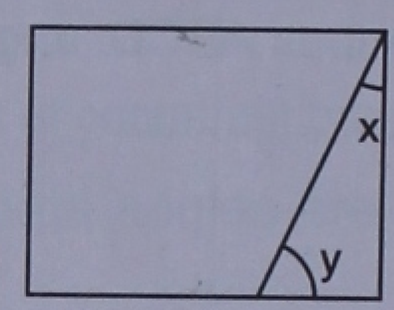
19. The given figure shows a rhombus ABCD in which angle BCD = 80°. Find angles x and y.



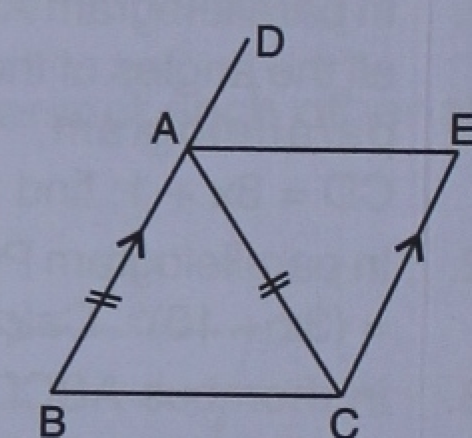
20. Use the information given in the alongside diagram to find the values of x, y and z.



21. The following figure is a rectangle in which x : y = 3 : 7; find the values of x and y.



22. In the given figure, AB // EC, AB = AC and AE bisects ∠DAC. Prove that :
(i) ∠EAC = ∠ACB
(ii) ABCE is a parallelogram.



ANSWERS

TEST YOURSELF

1. $\frac{6}{18} \times 360°$; 120°, $\frac{3}{18} \times 360°$, 60°, trapezium **2.** $\frac{11}{19}$ (360° − 100° − 70°) = 110° **3.** $\frac{15}{32} \times (360° - 40°) = 150°$ **4.** yes **5.** 73°, 84°, 102°, 259°, 101° **6.** (i) true (ii) false (iii) true (iv) true (v) true (vi) fasle (vii) true (viii) false **7.** parallelogram **8.** rectangle **9.** rhombus **10.** rhombus **11.** square **12.** rectangle **13.** rhombus **14.** (i) $\frac{180°}{2}$ = 90° (ii) 180° − 90° = 90° **15.** x = 58°, y = 180° − 2 × 58° = 64°, z = y = 64°

EXERCISE 27(A)

1. 79° each **2.** 90° and 126° **3.** (i) x = 16° (ii) 64°, 90°, 92° and 114° **4.** (i) x = 22° (ii) ∠B = 48° and ∠C = 61° **5.** (i) ∠A = 60°, ∠B = 100°, ∠C = 80° and ∠D = 120° (ii) Trapezium **6.** (i) x = 26° (ii) ∠ABC = 104° (iii) ∠ACD = 28° **7.** 130° **8.** ∠b = 105° and ∠c = 140° **9.** 97° each **10.** ∠P = 54°, ∠Q = 72°, ∠R = 108° and ∠S = 126° (i) No (ii) Trapezium **11.** 40° **12.** ∠A = 80°, ∠B = 60° ∠C = 120° and ∠D = 100°

EXERCISE 27(B)

1. 135°, 45°, 135° and 45°, CD = 13 units **2.** 55° and 125° **3.** (i) 106° and 74° (ii) BC = CD = 7·5 cm **4.** (i) 8 (ii) 10 **5.** (i) 62° (ii) 28° **6.** (i) 38° (ii) 38° (iii) 104° **7.** 110° **8.** (i) 96° (ii) 29° (iii) 55° **11.** (i) Square (ii) Rhombus (iii) Rectangle **19.** x = 70°, y = 50° **20.** x = 11, y = 15° and z = 106° **21.** x = 27° and y = 63°