

Chapter 25

THEOREMS ON AREA

AREA

The **area** of a plane closed figure is the measure of the region (surface) enclosed by its boundary.

In this section, we shall compare the area of two known figures which have certain relationship between them.

Equal figures

Two plane figures are called **equal** if and only if they have equal areas.

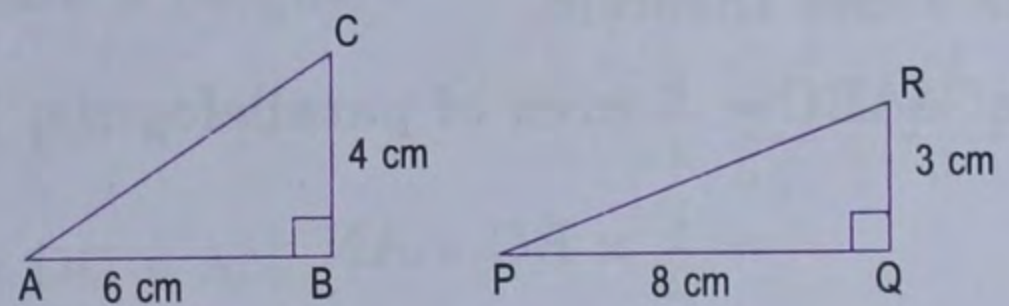
Difference between congruent and equal figures

Since congruent figures are exact duplicate of each other, so they have equal areas. Thus, congruent figures are equal figures. However, two equal figures may not be congruent.

For example, consider the adjoining right angled triangles ABC and PQR.

$$\begin{aligned} \text{Area of } \triangle ABC &= \left(\frac{1}{2} \times 6 \times 4 \right) \text{ cm}^2 \\ &= 12 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle PQR &= \left(\frac{1}{2} \times 8 \times 3 \right) \text{ cm}^2 \\ &= 12 \text{ cm}^2 \end{aligned}$$



$$\therefore \text{Area of } \triangle ABC = \text{Area of } \triangle PQR$$

\Rightarrow $\triangle ABC$ and $\triangle PQR$ are equal figures. Clearly, $\triangle ABC$ and $\triangle PQR$ are not congruent.

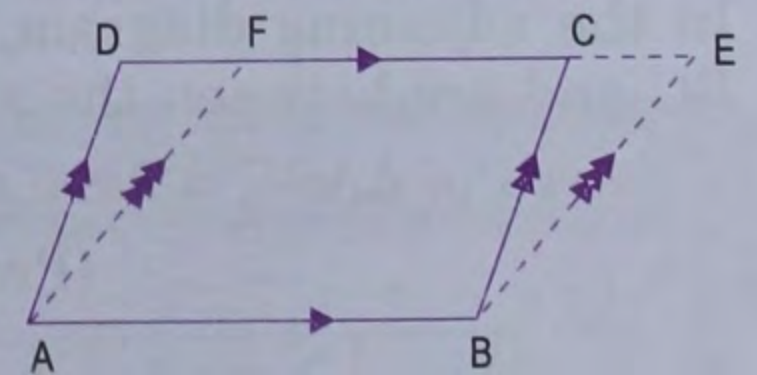
Theorem 1

Parallelograms on the same base and between the same parallel lines are equal in area.

In the adjoining diagram, parallelograms ABCD and ABEF have the same base AB and are between the same parallel lines AB and DE.

$$\therefore \text{Area of parallelogram ABCD} = \text{Area of parallelogram ABEF}$$

Note. Parallelograms ABCD and ABEF are equal figures but these are not congruent figures.

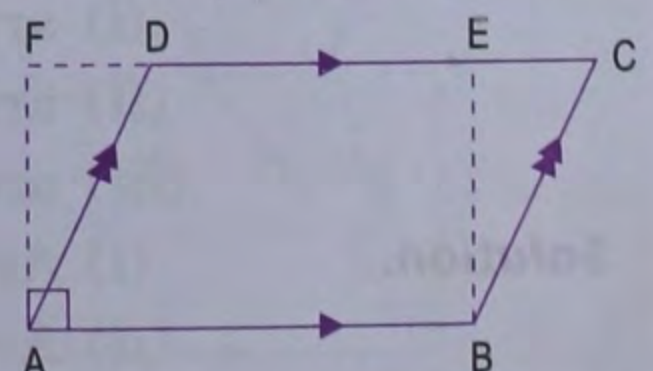


Theorem 2

The area of a parallelogram is equal to the area of a rectangle on the same base and between the same parallel lines.

In the adjoining diagram, parallelogram ABCD and rectangle ABEF have the same base AB and are between the same parallel lines AB and FC.

$$\therefore \text{Area of parallelogram ABCD} = \text{Area of rectangle ABEF.}$$



Corollary

Area of a parallelogram = base \times height

By the above theorem,

area of parallelogram ABCD = area of rectangle ABEF

But area of rectangle = length \times breadth = AB \times BE

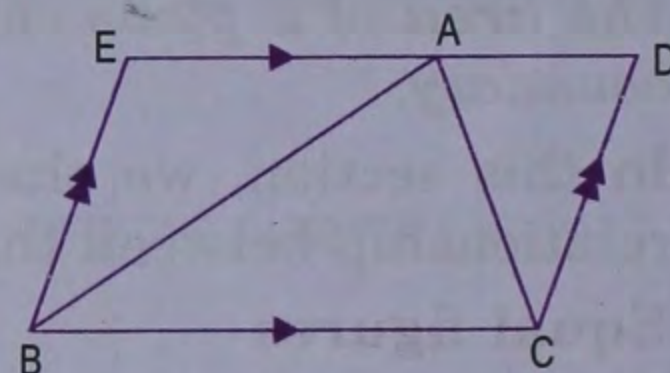
\therefore Area of parallelogram ABCD = AB \times BE = base \times height.

Theorem 3

Area of a triangle is half that of a parallelogram on the same base and between the same parallels.

In the adjoining diagram, $\triangle ABC$ and parallelogram BCDE have the same base BC and are between the same parallel lines BC and ED.

\therefore Area of $\triangle ABC = \frac{1}{2}$ area of parallelogram BCDE

**Corollary**

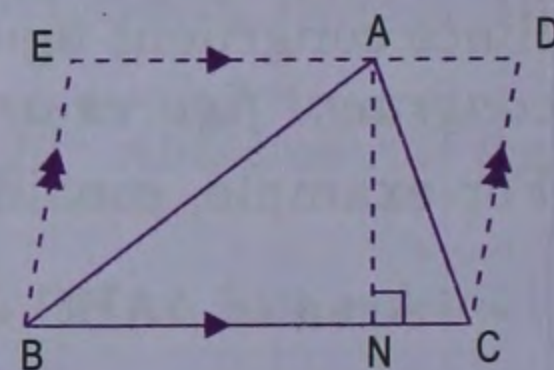
Area of a triangle = $\frac{1}{2}$ base \times height

In the adjoining diagram, note that

base of $\triangle ABC = BC =$ base of parallelogram ABDE and height of $\triangle ABC = AN =$ height of parallelogram BCDE

By the above theorem,

$$\begin{aligned} \text{area of } \triangle ABC &= \frac{1}{2} \text{ area of parallelogram BCDE} \\ &= \frac{1}{2} \times BC \times AN \\ &= \frac{1}{2} \text{ base} \times \text{height.} \end{aligned}$$

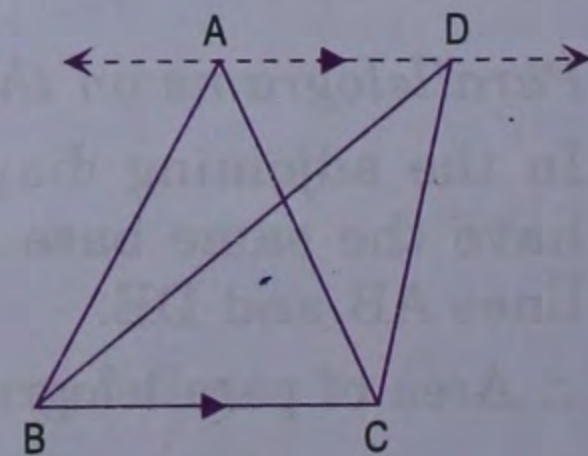
**Theorem 4**

Triangles on the same base and between the same parallel lines are equal in area.

In the adjoining diagram, $\triangle ABC$ and $\triangle DBC$ have the same base BC and are between the same parallel lines BC and AD.

\therefore Area of $\triangle ABC =$ Area of $\triangle DBC$

(Proofs of the above theorems are deferred to the next class.)

**Example 1.**

In this diagram, ABCD is a rectangle with side AB = 8 cm and AD = 5 cm.

Find

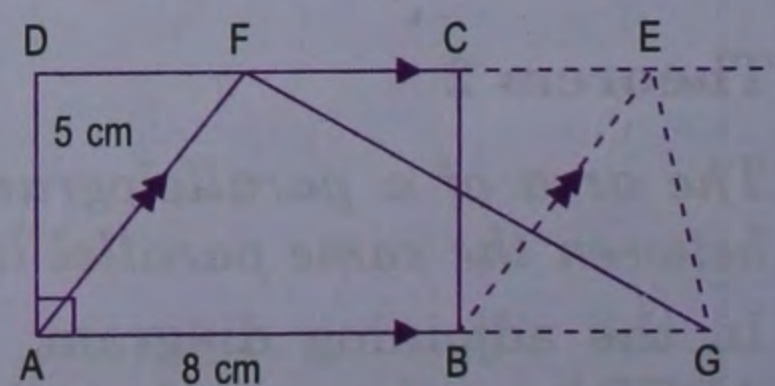
- area of rectangle ABCD
- area of parallelogram ABEF
- area of $\triangle EFG$

Solution.

(i) Area of rectangle ABCD = base \times height = $(8 \times 5) \text{ cm}^2 = 40 \text{ cm}^2$.

(ii) Area of parallelogram ABEF = area of rectangle ABCD

(on the same base AB and between same parallels AB and DE)

$$= 40 \text{ cm}^2.$$


$$(iii) \text{ Area of } \triangle EFG = \frac{1}{2} \text{ area of parallelogram ABEF}$$

(on the same base EF and between same parallels EF and AG)

$$= \left(\frac{1}{2} \times 40 \right) \text{ cm}^2 = 20 \text{ cm}^2.$$

Example 2.

ABCD is a trapezium with $AB \parallel DC$, and diagonals AC and BD intersect at O. Prove that area of $\triangle ODA$ = area of $\triangle OBC$.

Solution.

Area of $\triangle DAB$ = Area of $\triangle CAB$

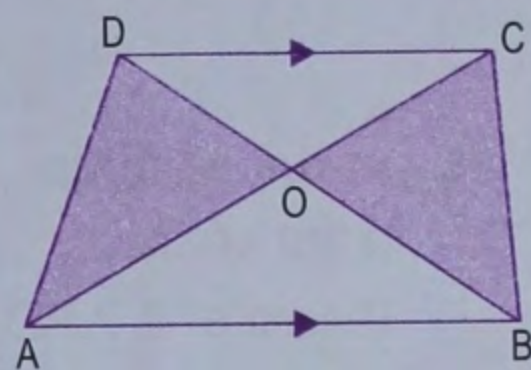
(triangles on the same base AB and between the same parallel lines AB and DC)

$$\Rightarrow \text{area of } \triangle ODA + \text{area of } \triangle OAB = \text{area of } \triangle OBC + \text{area of } \triangle OAB$$

(from figure, area axiom)

$$\Rightarrow \text{area of } \triangle ODA = \text{area of } \triangle OBC.$$

(subtracting same area of $\triangle OAB$ from both sides)

**Example 3.**

Prove that a median divides a triangle into two triangles of equal area.

Solution.

Let ABC be any triangle and AD be its median i.e. D is mid-point of BC, so $BD = DC$. From A, draw AN perpendicular to BC.

Since area of a triangle = $\frac{1}{2}$ base \times height,

$$\text{area of } \triangle ABD = \frac{1}{2} \times BD \times AN$$

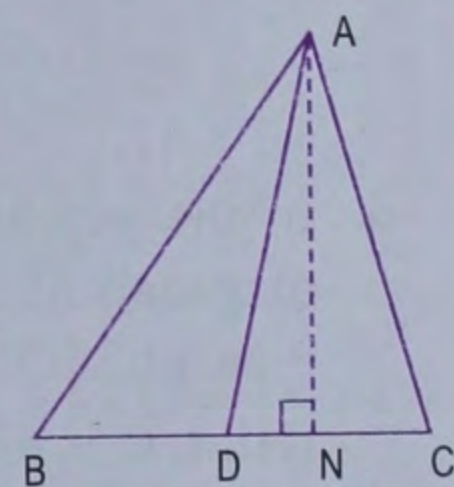
$$\text{and area of } \triangle ADC = \frac{1}{2} \times DC \times AN$$

$$= \frac{1}{2} \times BD \times AN$$

$$(\because BD = DC)$$

$$\Rightarrow \text{area of } \triangle ADC = \text{area of } \triangle ABD$$

Hence, a median of a triangle divides it into two triangles of equal area.

**Exercise 25**

1. In the adjoining figure, area of parallelogram ABCD = 37 cm^2 . Find

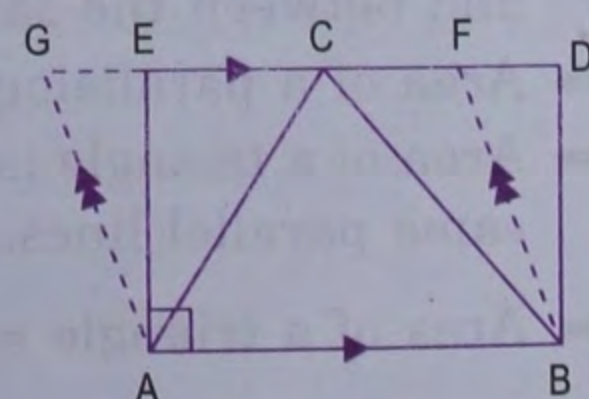
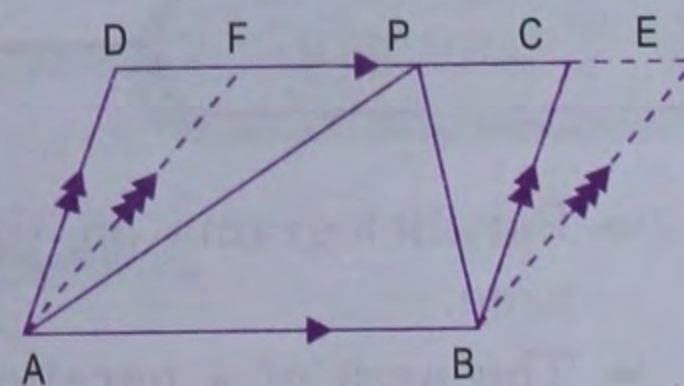
(i) area of parallelogram ABEF

(ii) area of $\triangle ABP$.

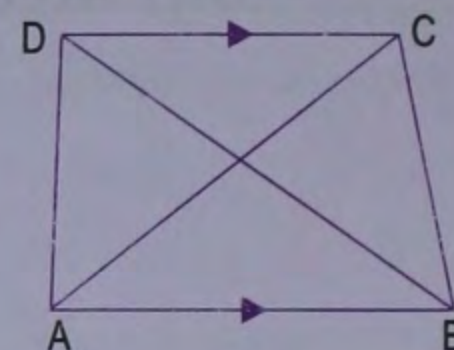
2. In the adjoining figure, area of $\triangle ABC$ = 21 square units. Find

(i) area of parallelogram ABFG

(ii) area of rectangle ABDE.



3. In the adjoining figure, $AB \parallel DC$. If area of $\triangle ABC = 19$ square units, find the area of $\triangle ABD$.

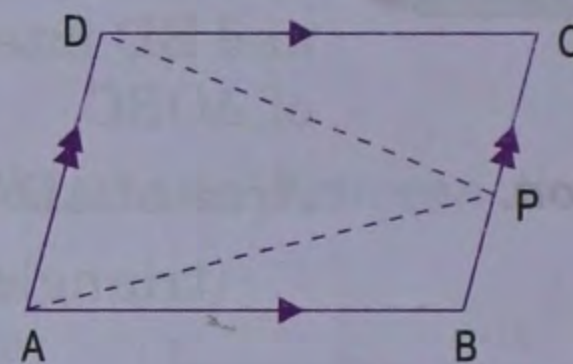


4. In the adjoining figure, ABCD is a parallelogram and P is a point on BC.

If area of $\triangle APD = 17.6 \text{ cm}^2$, find

(i) the area of parallelogram ABCD

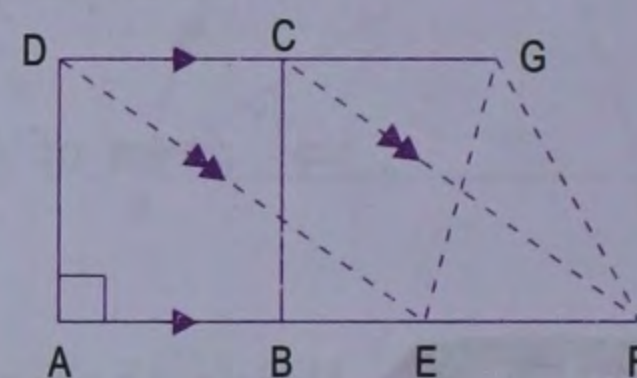
(ii) the sum of the areas of triangles ABP and DPC.



5. In the adjoining figure, ABCD is a rectangle with sides $AB = 4$ cm and $AD = 6$ cm. Find

(i) the area of parallelogram DEFC.

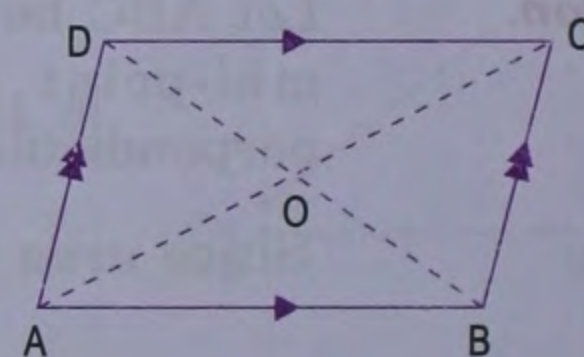
(ii) the area of $\triangle EFG$.



6. In the adjoining figure, ABCD is a parallelogram and its diagonals AC and BD intersect at O. Prove that area of $\triangle OAB =$ area of $\triangle OBC =$ area of $\triangle OCD =$ area of $\triangle OAD$.

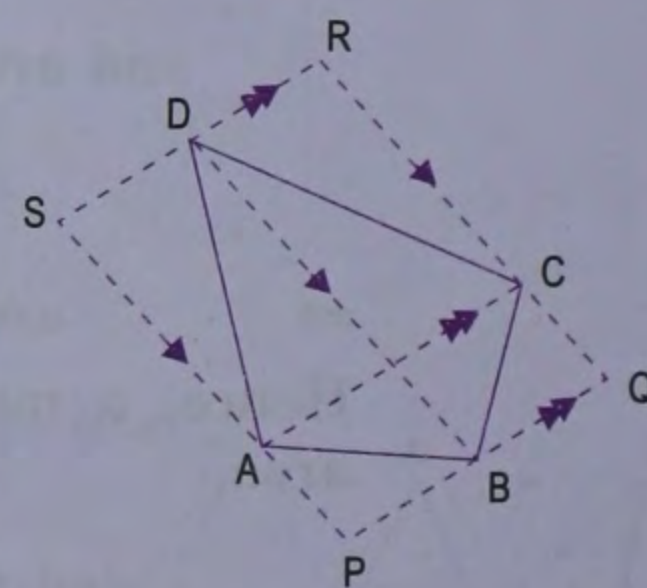
[Hint. O is mid-point of AC, by example 3,

area of $\triangle OAB = \frac{1}{2}$ area of $\triangle ABC$.]




7. In the adjoining figure, PQRS is a parallelogram formed by drawing lines parallel to the diagonals of a quadrilateral ABCD through its corners. Prove that area of parallelogram PQRS = 2 . area of quadrilateral ABCD

[Hint. Area of $\triangle ABC = \frac{1}{2}$ area of parallelogram APQC.]



Summary

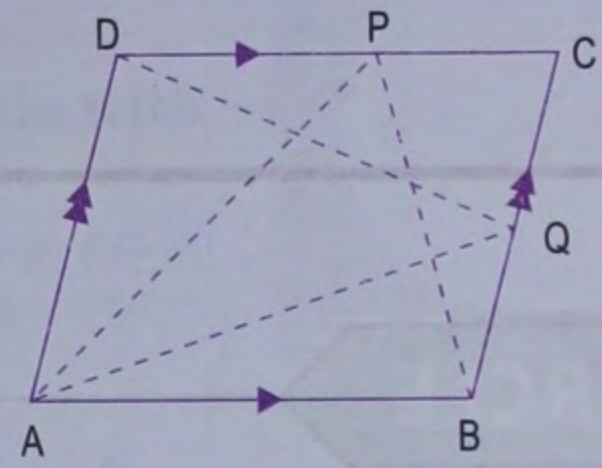
- ➔ Parallelograms on the same base and between the same parallels are equal in area.
- ➔ The area of a parallelogram is equal to the area of a rectangle on the same base and between the same parallel lines.
- ➔ Area of a parallelogram = base \times height.
- ➔ Area of a triangle is half that of a parallelogram on the same base and between the same parallel lines.
- ➔ Area of a triangle = $\frac{1}{2}$ base \times height.
- ➔ Triangles on the same base and between the same parallel lines are equal in area.


Check Your Progress

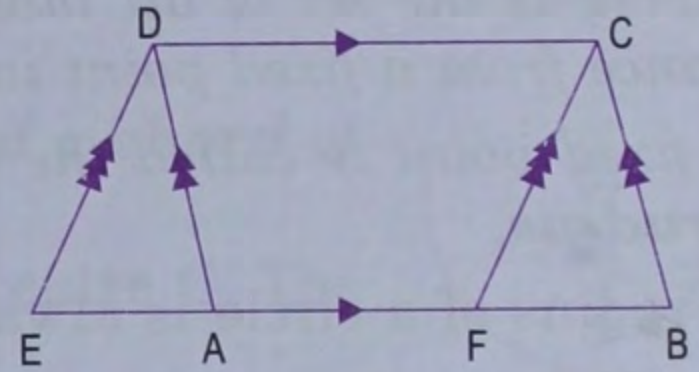
1. In the adjoining diagram, ABCD is a parallelogram. P and Q are points on the sides DC and BC respectively. Prove that

(i) area of $\triangle APB$ = area of $\triangle AQD$.

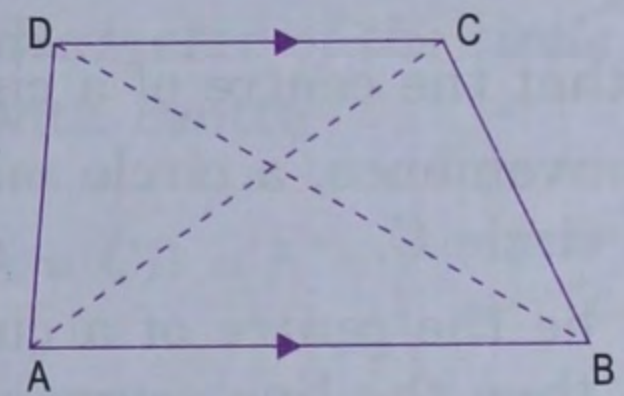
(ii) area of $\triangle APB$ = area of $\triangle ABQ$ + area of $\triangle DQC$.



2. In the adjoining figure, area of the parallelogram ABCD is 29 cm^2 . If $AB = 5.8 \text{ cm}$, find the height of the parallelogram EFCD.

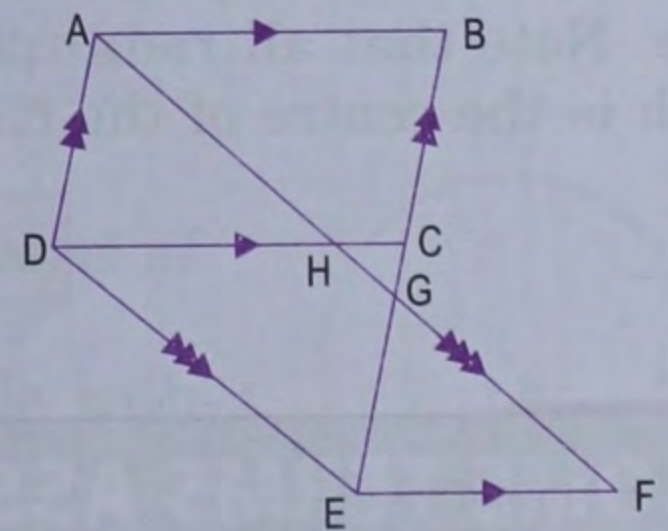


3. In the adjoining figure, $AB \parallel DC$ and area of $\triangle ABD$ is 24 sq. units. If $AB = 8$ units, find the height of $\triangle ABC$.



4. In the adjoining figure, $AB \parallel DC \parallel EF$, $DA \parallel EB$ and $DE \parallel AF$. Prove that area of parallelogram DEFH = area of parallelogram ABCD.

[Hint. Area of parallelogram ABCD
= area of parallelogram ADEG
= area of parallelogram DEFH.]



5. In the adjoining figure, DE is parallel to the side BC of $\triangle ABC$. BE and CD intersect at O. Prove that

(i) area of $\triangle BED$ = area of $\triangle CED$.

(ii) area of $\triangle BOD$ = area of $\triangle COE$.

(iii) area of $\triangle ABE$ = area of $\triangle ADC$.

