CHAPTER 24

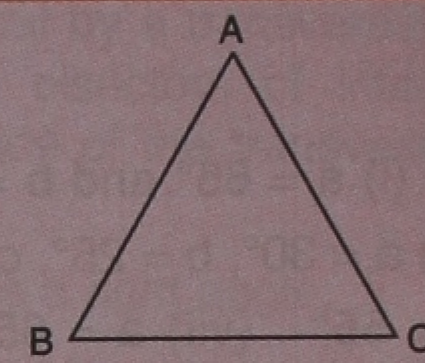
TRIANGLES

24.1 REVIEW

Triangle

A **triangle** is a plane and closed figure, bounded by three straight line segments.

The figure alongside shows a triangle ABC. Symbolically triangle ABC is written as Δ ABC, where the symbol Δ is read as 'triangle'.



(i) The three straight line segments forming a triangle are called sides of the triangle.
Thus, AB, BC and CA are the three sides of Δ ABC.

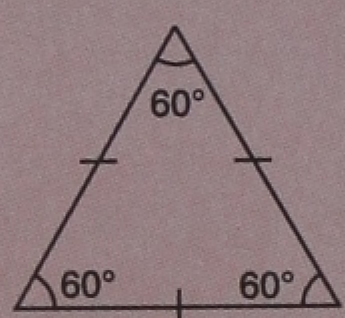
(ii) The point at which two sides of a triangle meet is called vertex of the triangle.
Thus, Δ ABC has three vertices A, B and C.

> Vertices is the plural of vertex.

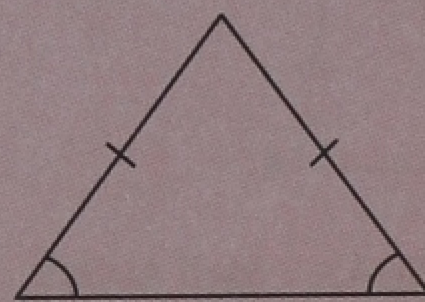
Types of triangles

(a) *According to their sides :*

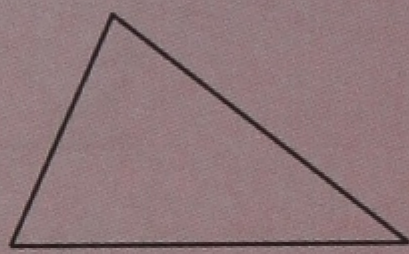
(i) *Equilateral Triangle* (ii) *Isosceles Triangle* (iii) *Scalene Triangle*



All the sides are equal. Each angle is 60°.



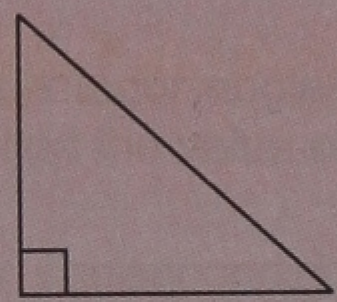
Two sides are equal. Angles opposite to equal sides are equal.



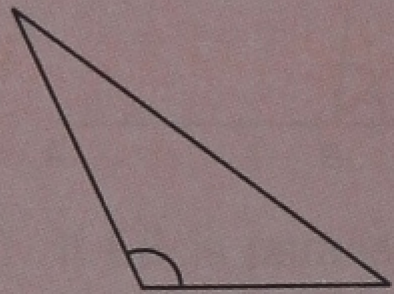
No sides equal. No angles equal.

(b) *According to their angles :*

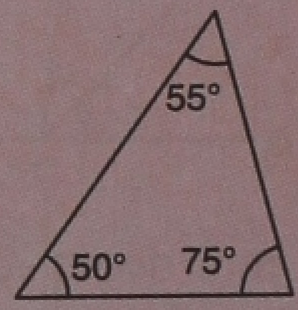
(i) Right-angled Δ (ii) Obtuse-angled Δ (iii) Acute-angled Δ



One angle must be 90°.



One angle must be obtuse *i.e.* greater than 90° and less than 180°.



Each angle is acute *i.e.* less than 90°.

1. Axioms

The self evident truths which are accepted without any proof are called **axioms.** *e.g.*

(i) If x is greater than y, then y is less than x.

(ii) If a and b both are equal to c, then $a = b$.

2. Theorem

A proposition that requires proof is called a **theorem.**

3. Proof

The course of reasoning which establishes the truth or falsity of a statement is called a **proof.**

4. Corollary

A proposition, whose truth can easily be deducted from a preceding theorem is called its **corollary.**

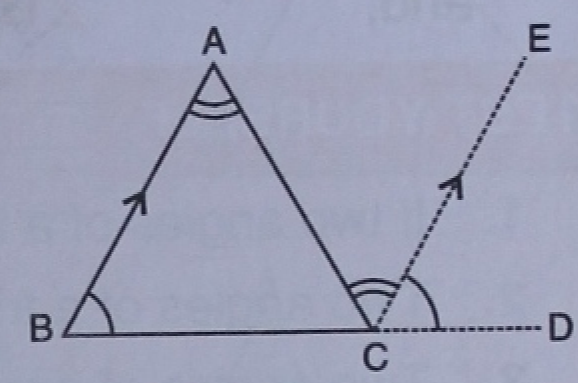
24.2 PROVING A THEOREM

Giving the general enunciation, set out the work in the following order :

1. Draw the figure.
2. Using the letters of your figure, state what is given.
3. State what is required to be proved.
4. State the construction, if necessary.
5. State the proof giving the statement and reasons separately, using suitable abbreviated references.

Theorem 1

The sum of the angles of a triangle is equal to two right angles. (*i.e.* 180°).



Given : A triangle ABC.

To Prove : $\angle BAC + \angle ABC + \angle BCA = 180°$

i.e. $\angle A + \angle B + \angle C = 180°$

Construction : Produce the side BC upto point D and draw CE parallel to BA.

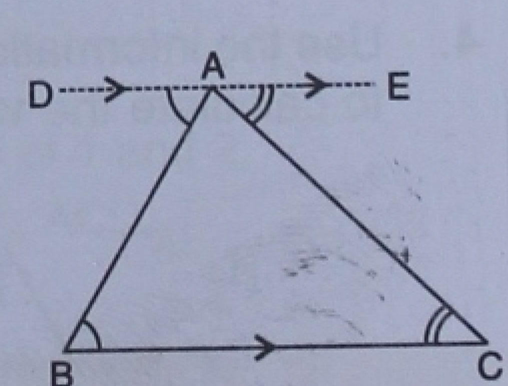
Proof :

Statement :	*Reason :*
1. Since, AB // EC and BCD is transversal, $\therefore \angle ABC = \angle ECD$	Corresponding angles
2. Again, AB // EC and AC is transversal, $\therefore \angle BAC = \angle ACE$	Alternate angles.
3. $\angle ABC + \angle BAC = \angle ECD + \angle ACE$	Adding results of 1 and 2.
4. $\angle ABC + \angle BAC + \angle BCA = \angle ECD + \angle ACE + \angle BCA$	Adding ∠BCA on both the sides.
5. But, $\angle ECD + \angle ACE + \angle BCA$ = Straight line angle BCD = 180°	Measure of straight line angle is 180°.
6. $\therefore$ **$\angle ABC + \angle BAC + \angle BCA = 180°$**	From the results of 4 and 5.

Hence Proved.

Alternative method :

Construction : Through vertex A of the Δ ABC, draw DE parallel to base BC.



Proof :

Statement :	*Reason :*
1. Since, DE // BC and AB is transversal, $\therefore \angle ABC = \angle DAB$	Alternate angles.
2. Again, DE // BC and AC is transversal, $\therefore \angle BCA = \angle CAE$	Alternate angles.
3. $\angle ABC + \angle BCA = \angle DAB + \angle CAE$	Adding results of 1 and 2.
4. $\angle ABC + \angle BCA + \angle BAC = \angle DAB + \angle CAE + \angle BAC$	Adding ∠BAC on both the sides.
5. But, $\angle DAB + \angle CAE + \angle BAC$ = Straight line $\angle DAE = 180°$	Measure of straight line angle is 180°.
6. $\therefore$ **$\angle ABC + \angle BCA + \angle BAC = 180°$**	From the results of 4 and 5.

Hence Proved.

Example 1 :

In triangle ABC; $6\angle A = 4\angle B = 3\angle C$. Find each angle of the triangle.

Solution :

Let $6\angle A = 4\angle B = 3\angle C = x$

$\Rightarrow \quad \angle A = \frac{x}{6}, \ \angle B = \frac{x}{4}$ and $\angle C = \frac{x}{3}$

$\Rightarrow \quad \frac{x}{6} + \frac{x}{4} + \frac{x}{3} = 180°$ $\quad [\because \angle A + \angle B + \angle C = 180°]$

On solving, we get : $x = 240°$

$\therefore \quad \angle A = \frac{x}{6} = \frac{240°}{6} = 40°, \ \angle B = \frac{x}{4} = \frac{240°}{4} = 60°$

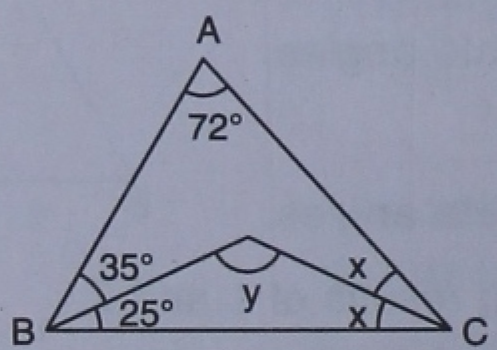
and, $\quad \angle C = \frac{x}{3} = \frac{240°}{3} = 80°$ **(Ans.)**

TEST YOURSELF

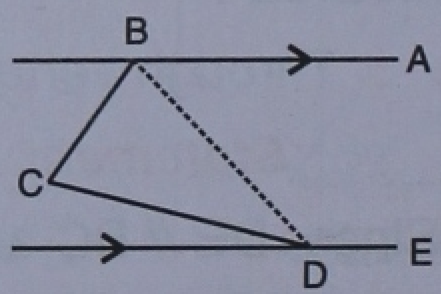
1. If two angles of a triangle are 52° and 88°, its third angle = ...
2. Two angles of a triangle are 75° each, its third angle = ...
3. Two angles of a triangle are equal and its third angle is 80°, then each of the equal angles is ....
4. Angles of a triangle are in the ratio 4 : 5 : 3.
   The largest angle of the triangle = ... = ...
   and its smallest angle is .... = ...
5. The angles of a triangle are in the ratio 3 : 7 : 5; the largest angle = ... = ...; the sum of its other two angles = ... and the special name of this triangles is ....

EXERCISE 24 (A)

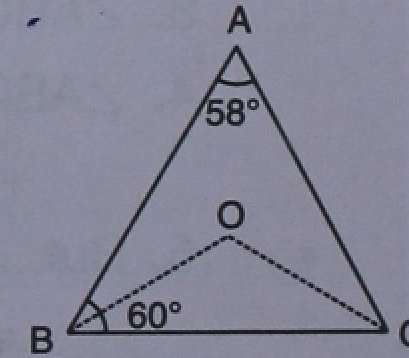
1. The angles of a triangle are $(3x)°$, $(2x - 7)°$ and $(4x - 11)°$. Find the value of x and measure of each angle of the triangle.
2. One angle of a triangle is 78° and the other two angles are in the ratio 7 : 10. Calculate the two unknown angles of the triangle.
3. In a triangle ABC, $\angle A = 2\angle B = 3\angle C$. Find each angle of the triangle.
4. Use the informations given in the figure below to calculate the values of x and y.



5. (i) In Δ ABC, $\angle A = 2x + 15°$, $\angle B = 3x - 5°$ and $\angle C = 4x + 35°$. Find each angle of the triangle.
   (ii) In Δ ABC, $\angle A = x + 15°$, $\angle B = x$ and $\angle C = 2x - 35°$. Find each angle of the triangle and then assign a special name to the triangle.
   (iii) In Δ ABC, $\angle A = x + 20°$, $\angle B = 2(x - 10°)$ and $\angle C = \frac{3}{2}x$. Show that the triangle is equilateral.
6. From the figure, given below, find the value of $\angle ABC + \angle BCD + \angle CDE$.



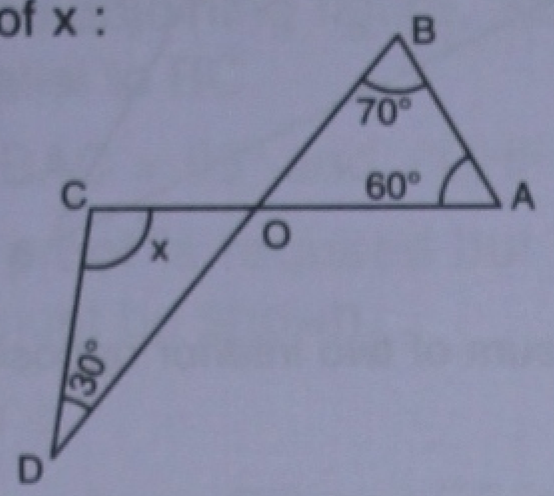
7. In triangle ABC, the bisectors of angles B and C meet at P. Prove that : $\angle BPC = 90° + \frac{1}{2}\angle A$.
8. The adjoining figure, shows a triangle ABC in which $\angle A = 58°$, $\angle B = 60°$ and bisectors of angles B and C meet at O. Find the measure of the angle BOC.



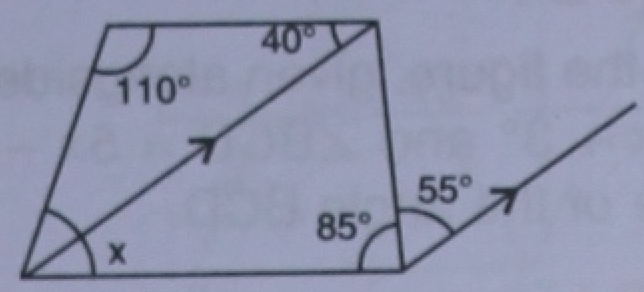
9. In triangle ABC, $\angle B = 3\angle A - 25°$ and $\angle A = \angle C$. Find angles A and B.

10. From each of the following figures, find the value of x :

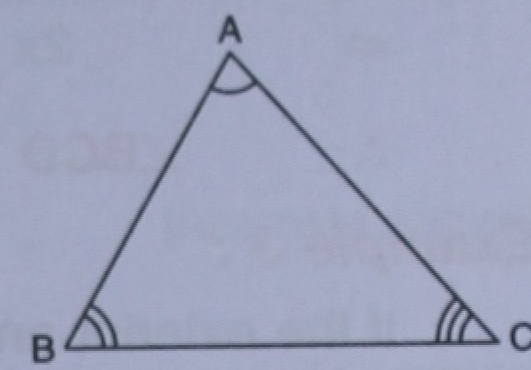
(i)



(ii)



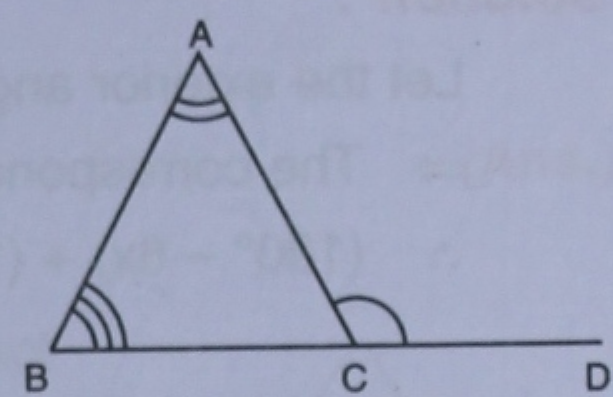
24.3 INTERIOR AND EXTERIOR ANGLES OF A TRIANGLE



1. Interior angles of a triangle :

In triangle ABC; ∠BAC (*i.e.* ∠A), ∠ABC (*i.e.* ∠B) and ∠BCA (*i.e.* ∠C) are called its interior angles as they lie inside the triangle.

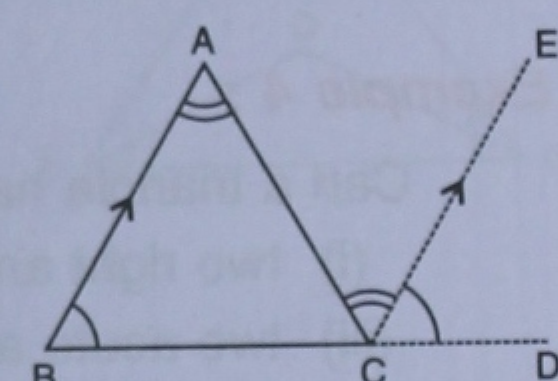
2. Exterior angles of a triangle :



In Δ ABC; if side BC is produced upto any point D, then the angle ACD is called the exterior angle of the triangle.

Since, the interior angles A and B of Δ ABC are opposite to the exterior angle ACD, they are called interior opposite angles of the angle ACD.

Theorem 2



If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Given : A triangle ABC whose side BC is produced upto point D.

To Prove : Exterior ∠ACD = ∠A + ∠B

Construction : Draw CE parallel to BA.

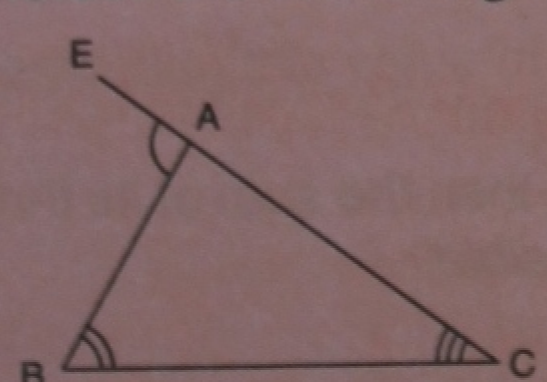
Proof :

Statement :	*Reason :*
1. Since, CE // BA and BCD is transversal, ∴ ∠ECD = ∠B	Corresponding angles
2. Since, CE // BA and AC is transversal, ∴ ∠ACE = ∠A	Alternate angles
3. ∠ECD + ∠ACE = ∠B + ∠A ⇒ **∠ACD = ∠A + ∠B**	Adding the results of 1 and 2. ∠ECD + ∠ACE = ∠ACD.

Hence Proved.

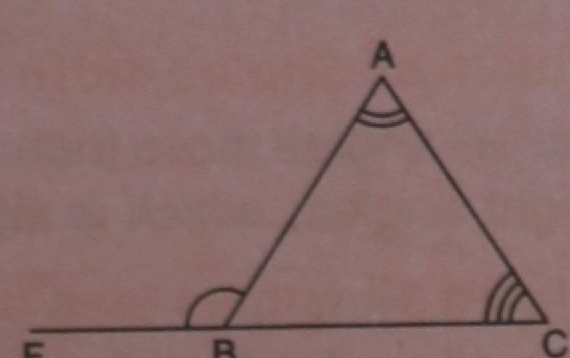
Consider the following figures :

(i)



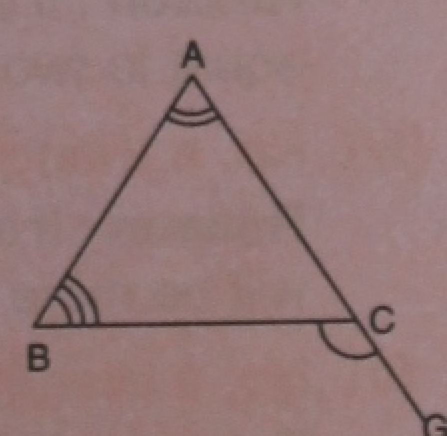
[∠BAE = ∠B + ∠C]

(ii)



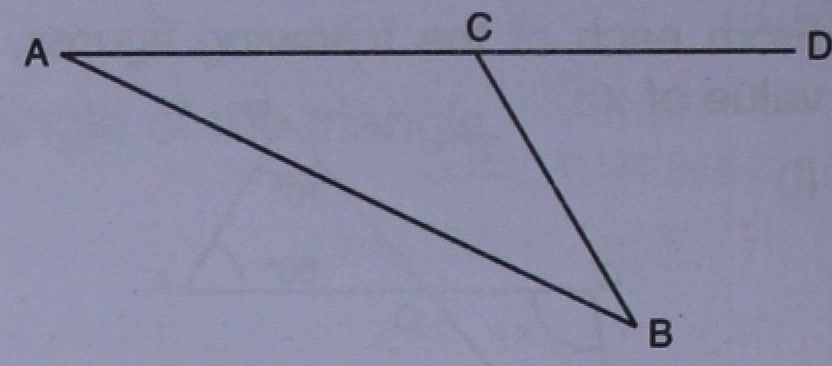
[∠ABF = ∠A + ∠C]

(iii)



[∠BCG = ∠A + ∠B]

Example 2 :



In the figure, given alongside, $\angle A = x + 8°$, $\angle B = 2x + 3°$ and $\angle BCD = 5x - 11°$, find the measure of the angle BCD.

Solution :

$\therefore \quad \angle BCD = \angle A + \angle B$ [$\because$ Exterior angle = sum of two interior opposite angles.]

$\Rightarrow \quad 5x - 11° = (x + 8°) + (2x + 3°)$

$\Rightarrow \quad 5x - 11° = 3x + 11°$

$\Rightarrow \quad 2x = 22°$ and, $x = \frac{22°}{2} = 11°$

$\therefore \quad$ **$\angle BCD$** $= 5x - 11° = 5 \times 11° - 11° =$ **$44°$** **(Ans.)**

Example 3 :

If the exterior angles of a triangle are in the ratio 6 : 7 : 5, find each exterior angle.

Solution :

Let the exterior angles be 6x, 7x and 5x

$\Rightarrow$ The corresponding interior angles are $180° - 6x$, $180° - 7x$ and $180° - 5x$

$\therefore \quad (180° - 6x) + (180° - 7x) + (180° - 5x) = 180°$ [$\because$ Sum of interior angles of a $\Delta = 180°$]

On solving, we get : $x = 20°$

$\therefore$ **The exterior angles are :** 6x, 7x and 5x= $6 \times 20°$, $7 \times 20°$ and $5 \times 20°$

= **120°, 140° and 100°** **(Ans.)**

Example 4 :

Can a triangle have :

(i) two right angles ?
(ii) two obtuse angles ?
(iii) two acute angles ?
(iv) all angles more than 60° ?
(v) all angles less than 60° ?

Give reason in each case.

Solution :

(i) A triangle cannot have two right angles. **(Ans.)**

Reason : If a triangle has two right angles, then the sum of its three angles will be more than two right angles *i.e.* more than 180°; *which is impossible*.

(ii) No, a triangle cannot have two obtuse angles. **(Ans.)**

Reason : If a triangle has two obtuse angles, then the sum of its three angles will be more than two right angles; *which is impossible*.

(iii) Yes, a triangle can have two acute angles. **(Ans.)**

Reason : If a triangle has two acute angles, then the sum of its three angles can be equal to two right angles.

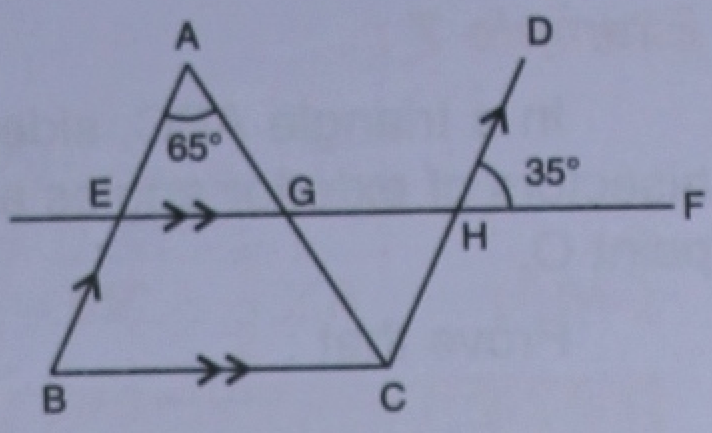
(iv) No, a triangle cannot have all angles more than 60°. **(Ans.)**

Reason : If a triangle has all angles more than 60°, then the sum of its three angles will be more than two right angles; *which is impossible*.

(v) No, a triangle cannot have all angles less than 60°. **(Ans.)**

Reason : If a triangle has all angles less than 60°, then the sum of its three angles will be less than two right angles; *which is impossible*.

Example 5 :



In the adjoining figure, AB is parallel to CD and EF is parallel to BC.

If $\angle BAC = 65°$ and $\angle DHF = 35°$, find $\angle AGH$.

No proof is required but the essential steps of working must be shown.

Solution :

Since, lines CD and EF intersect each other at point H,

$\therefore \quad \angle CHG = \angle DHF = 35°$ [Vertically opposite angles]

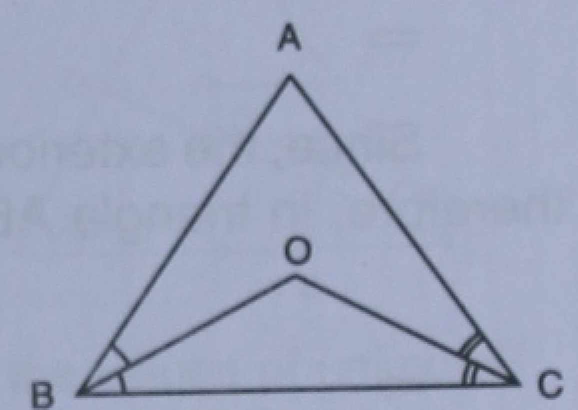
Since, AB is parallel to CD and AC is transversal,

$\therefore \quad \angle HCG = \angle BAC = 65°$ [Alternate angles]

Since, the exterior angle of a triangle is equal to the sum of its interior opposite angles, therefore in triangle CHG,

Ext. $\angle AGH = \angle HCG + \angle CHG = 65° + 35° = 100°$ **(Ans.)**

Example 6 :



In triangle ABC, the bisectors of angle B and angle C intersect each other at point O.

Prove that : $\angle BOC = 90° + \frac{1}{2}\angle A$

Solution :

In the figure, BO bisects $\angle B$, $\therefore \angle OBC = \frac{1}{2}\angle B$

and, CO bisects $\angle C$, $\therefore \angle OCB = \frac{1}{2}\angle C$.

In Δ OBC, $\angle OBC + \angle OCB + \angle BOC = 180°$ [Sum of angles of a $\Delta = 180°$]

$\Rightarrow \quad \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - \left(\frac{1}{2}\angle B + \frac{1}{2}\angle C\right)$ I

In Δ ABC, $\angle A + \angle B + \angle C = 180°$ [Sum of angles of a Δ]

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90°$ [Dividing each term by 2]

$\Rightarrow \quad \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{1}{2}A$

Substituting this value of $\frac{1}{2}\angle B + \frac{1}{2}\angle C$ in equation I, we get :

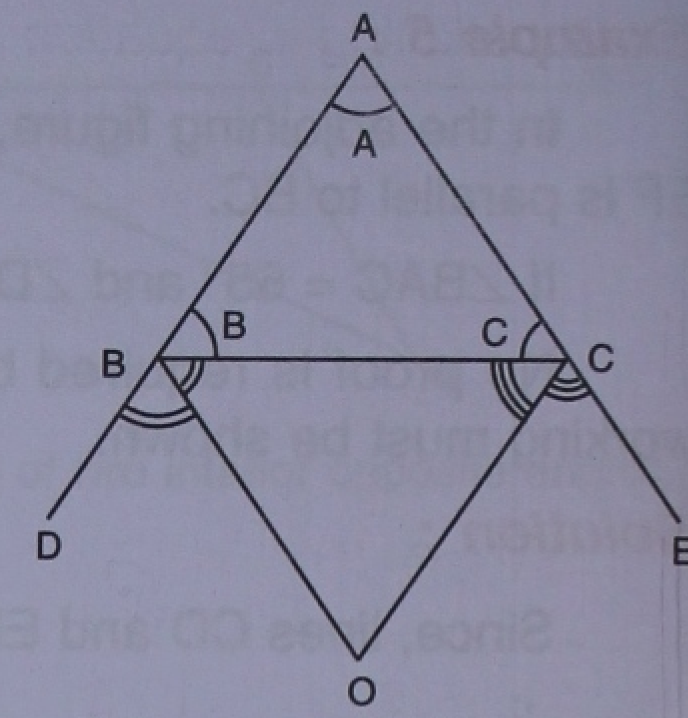
$\angle BOC = 180° - \left(90° - \frac{1}{2}\angle A\right)$

$= 180° - 90° + \frac{1}{2}\angle A = 90° + \frac{1}{2}\angle A$

Hence Proved.

Example 7 :

In a triangle ABC, sides AB and AC are produced. The bisectors of exterior angles at B and C intersect each other at point O.



Prove that :

$\angle BOC = 90° - \frac{1}{2}\angle A$

Solution :

In the figure, sides AB and AC are produced upto points D and E respectively.

BO bisects exterior angle DBC $\Rightarrow \angle OBC = \frac{1}{2}\angle DBC$

and, CO bisects exterior angle ECB $\Rightarrow \angle OCB = \frac{1}{2}\angle ECB$

Since, the sum of the angles of a triangle = 180°

In Δ BOC, $\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \quad \frac{1}{2}\angle DBC + \frac{1}{2}\angle ECB + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - \frac{1}{2}(\angle DBC + \angle ECB)$ I

Since, the exterior angle of a triangle is equal to the sum of its two interior opposite angles, therefore, in triangle ABC,

$\angle DBC = \angle A + \angle C$ and $\angle ECB = \angle A + \angle B$

$\Rightarrow \quad \angle DBC + \angle ECB = \angle A + \angle C + \angle A + \angle B$

$= \angle A + 180°$ [In ΔABC, $\angle A + \angle B + \angle C = 180°$]

$\Rightarrow \quad \frac{1}{2}(\angle DBC + \angle ECB) = \frac{1}{2}\angle A + 90°$

Substituting in equation I, we get :

$\angle BOC = 180° - \left(\frac{1}{2}\angle A + 90°\right)$

$= 180° - \frac{1}{2}\angle A - 90° = 90° - \frac{1}{2}\angle A$

Hence Proved.

Alternative method :

Since, ABD is a straight line,

$\angle B + \angle DBC = 180° \Rightarrow \angle DBC = 180° - \angle B$

Since, ACE is a straight line,

$\angle C + \angle ECB = 180° \Rightarrow \angle ECB = 180° - \angle C$

$\therefore \quad \angle DBC + \angle CEB = 180° - \angle B + 180° - \angle C$

$= 360° - (\angle B + \angle C)$

$= 360° - (180° - \angle A)$ [$\because \angle A + \angle B + \angle C = 180°$]

$= 360° - 180° + \angle A$

$= 180° + \angle A$

Substituting in equation I, we get :

$$\angle BOC = 180° - \frac{1}{2}(180° + \angle A)$$

$$= 180° - 90° - \frac{1}{2}\angle A = 90° - \frac{1}{2}\angle A \qquad \textbf{(Ans.)}$$

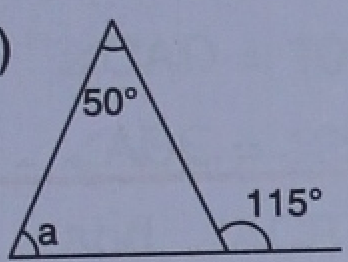
TEST YOURSELF

6. The angles of a triangle ABC are in the ratio $\angle A : \angle B : \angle C = 5 : 8 : 7$; the angle B = ... = ... and the exterior angle at vertex A is .... = ...
7. In case of a Δ ABC, exterior angle at A is bigger than the exterior angle at B; then the interior angle at A is .... than the interior angle at B.
8. In Δ ABC, the bisectors of exterior angles at vertices B and C intersect each other at point O. If $\angle A = 70$; $\angle BOC$ = ... = ...
9. In Δ ABC, $\angle B = 80°$. The bisectors of the interior angles at A and at C, intersect each other at point I. $\angle CIA$ = ... = ...

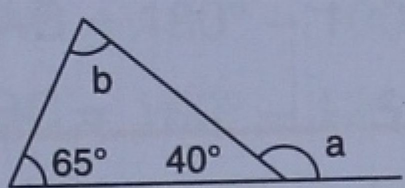
EXERCISE 24 (B)

1. Find, giving reasons, the unknown angles marked by letters a, b, etc.

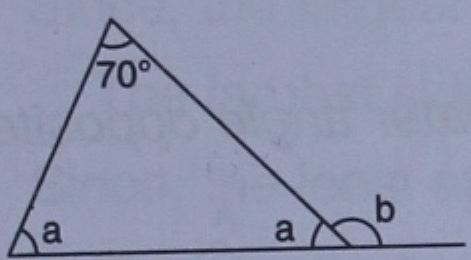
(i)



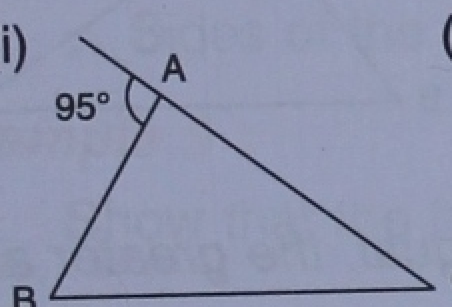
(ii)



(iii)

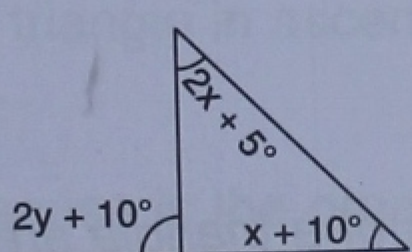


2. (i)



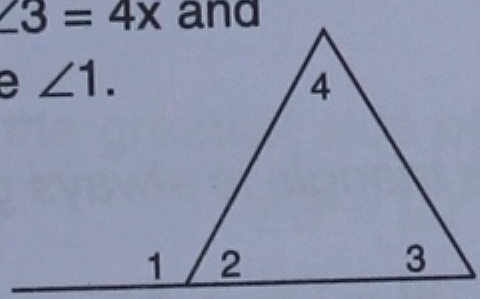
If $\angle B : \angle C = 3 : 2$, find $\angle B$ and $\angle C$.

(ii)



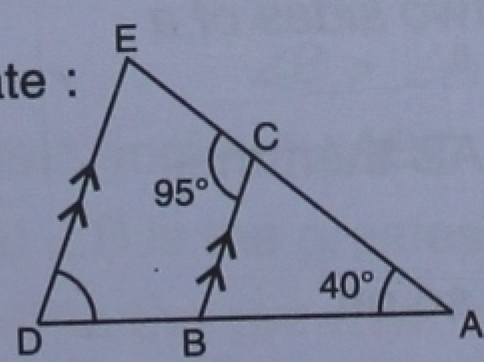
If y = 40, calculate the value of x.

3. Given : $\angle 1 = 9x - 15°$; $\angle 3 = 4x$ and $\angle 4 = 3x + 15°$. Calculate $\angle 1$.

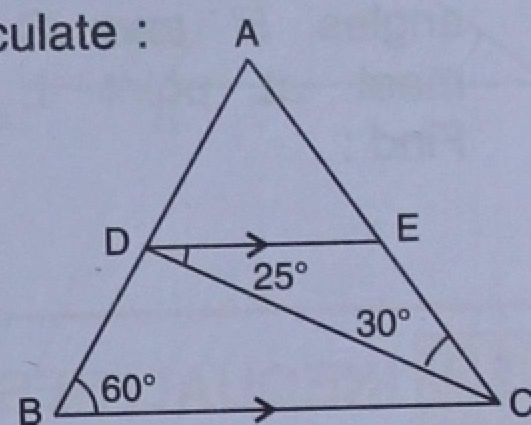


4. In the figure of question number 3, if $\angle 1 = 8x - 6°$, $\angle 3 = 3x + 4°$ and $\angle 4 = 4x + 2°$, calculate : $\angle 4$ and $\angle 2$.

5. Giving reasons, calculate :
   (i) $\angle ABC$
   (ii) $\angle ADE$

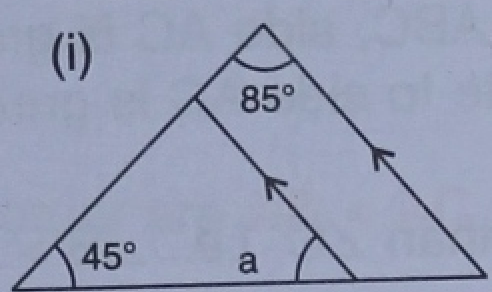


6. Giving reasons, calculate :
   (i) $\angle DEA$
   (ii) $\angle EDA$
   (iii) $\angle A$

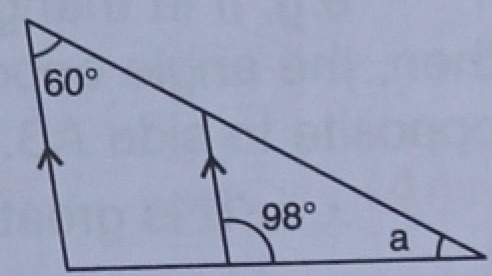


7. Find, giving reasons, the values of unknown angles marked by letter a.

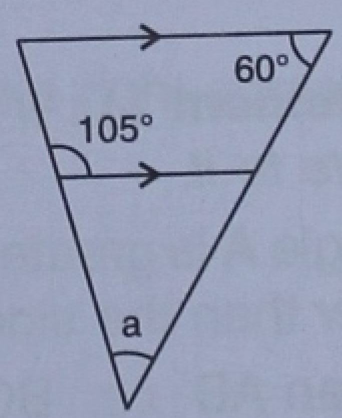
(i)



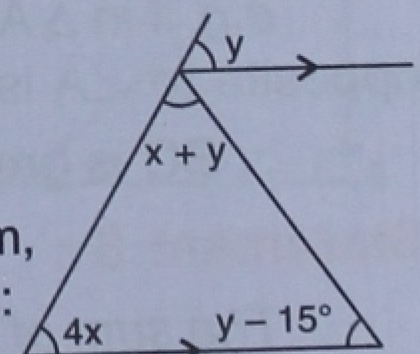
(ii)



(iii)

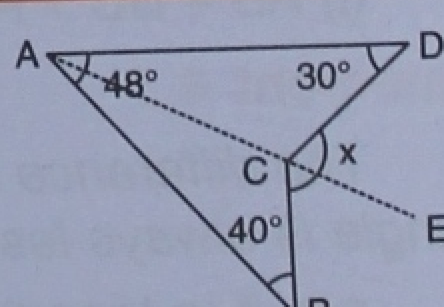


8. From the following diagram, find the values of x and y :



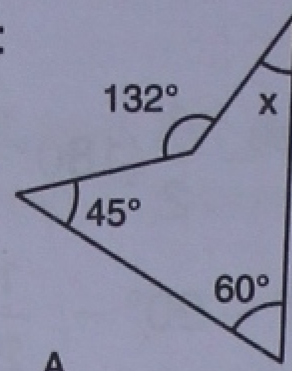
$4x = y$ and $4x + (x + y) + (y - 15°) = 180°$

9. Find the value of x from the adjoining diagram :

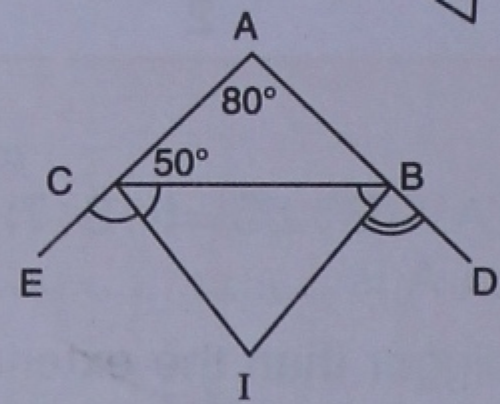


Ext. $\angle BCE$ + ext. $\angle DCE = 40° + 48° + 30°$
$\Rightarrow x = 118°$

10. Use the given figure to find x :



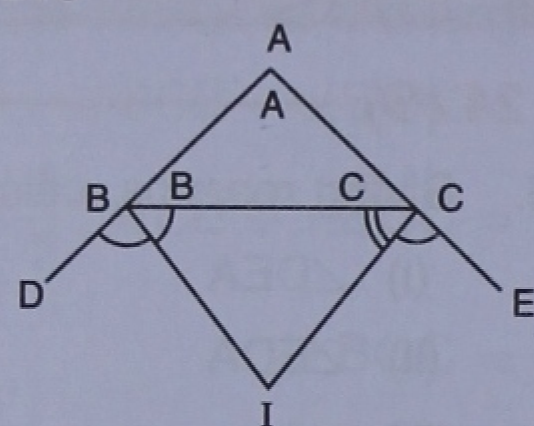
11. The figure, given alongside, shows a triangle ABC whose sides AB and AC are produced upto points D and E respectively. The bisectors of exterior angles so formed, intersect each other at point I. If ∠BAC = 80° and ∠ACB = 50°, find :



(i) ∠ECB (ii) ∠DBC (iii) ∠ICB

(iv) ∠IBC (v) ∠BIC

12. In the given figure, the bisectors of angles B and C meet at point I. Find :



(i) ∠DBC in terms of ∠A and ∠C

(ii) ∠ECB in terms of ∠A and ∠B

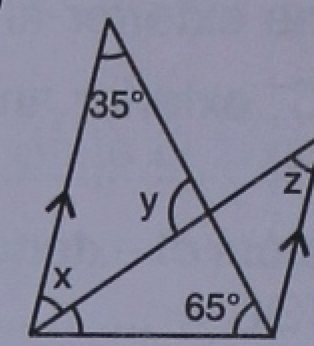
(iii) ∠IBC in terms of ∠A and ∠C

(iv) ∠ICB in terms of ∠A and ∠B.

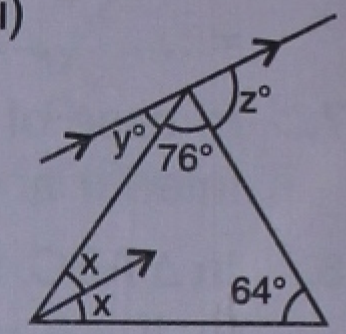
Now show that : $\angle BIC = 90° - \frac{1}{2}\angle A$

13. From each of the following figures, find the values of letters x, y and z.

(i)



(ii)



14. An angle of a triangle is 68° and the other two differ by 16°. Find the other two angles.

15. An angle of a triangle is 98° and the larger of the other two angles is 14° less than five times the smaller. Find the other two angles.

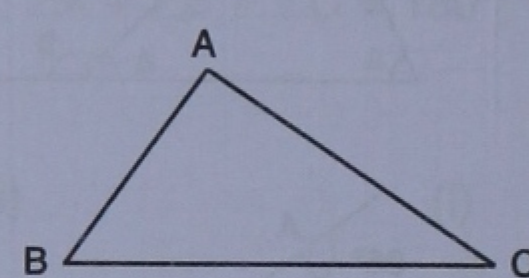
24.4 INEQUALITIES

Statement 1 :

If two sides of a triangle are unequal, the greater side has greater angle opposite to it.

e.g. if in triangle ABC, side AC is greater than side AB then, the angle opposite to side AC is greater than the angle opposite to side AB.

∴ ∠B is greater than ∠C *i.e.* ∠B > ∠C.

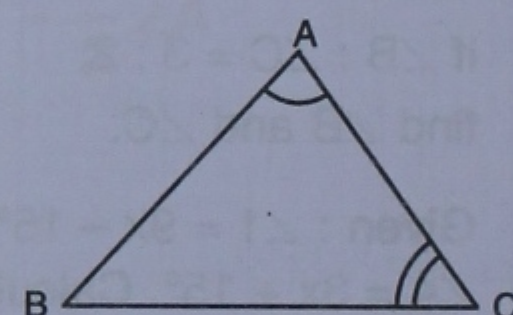


Statement 2 :

(Converse of statement 1) : *If two angles of a triangle are unequal, the greater angle has greater side opposite to it.*

e.g. if in Δ ABC; angle A is greater than angle C then, side opposite to ∠A is *greater* than the side opposite to ∠C.

∴ BC is greater than AB *i.e.* BC > AB.



Statement 3 :

The sum of the lengths of any two sides of a triangle is always greater than the third side.

i.e. in Δ ABC :

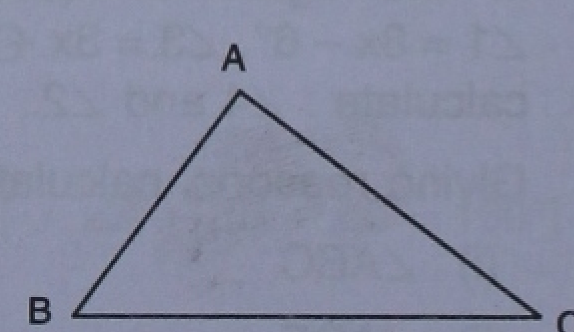
(i) AB + BC > AC (ii) BC + AC > AB and (iii) AB + AC > BC.

Statement 4 :

The difference between the lengths of any two sides of a triangle is always less than the third side.

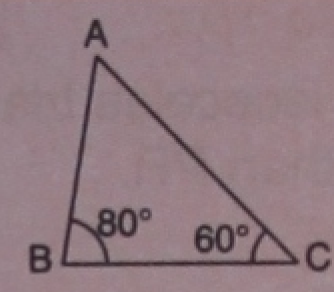
e.g. if in triangle ABC, AC is greater than AB then their difference AC – AB < BC

Similarly, BC – AB < AC (if BC > AB) and so on.

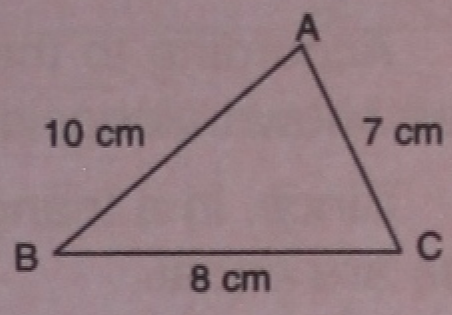


TEST YOURSELF

10. In the adjoining figure, the greatest angle = ... and the smallest angle = ... Therefore, the largest side = ... and the shortest side = ...



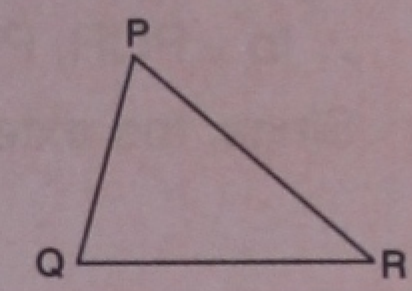
11. In the adjoining figure, the largest side = ... and the shortest side = ... Therefore, the greatest angle = ... and the smallest angle = ...



12. Fill in the blanks with symbol > or < :

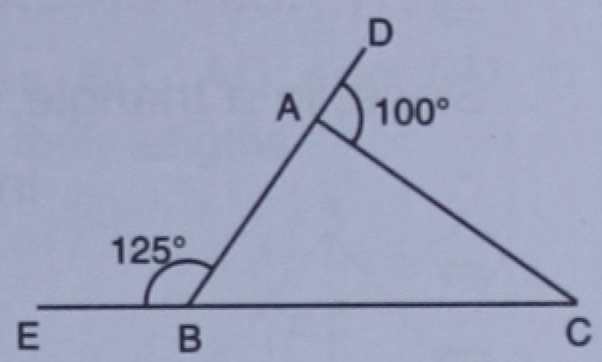
(i) In the adjoining figure; PQ + QR .... PR.

(ii) If PR > PQ, then PR – PQ .... QR.



Example 8 :

In the adjoining figure, the exterior ∠CAD = 100° and the exterior ∠ABE = 125°. Write the sides of triangle ABC, in ascending order of their lengths.



Solution :

∠CAD = 100° ⇒ ∠CAB = 180° – 100° = 80°,

∠ABE = 125° ⇒ ∠ABC = 180° – 125° = 55°

and, ∠ACB = 180° – (80° + 55°) = 45°

Since, the largest angle of the triangle ABC is ∠A = 80°

∴ The largest side = side BC

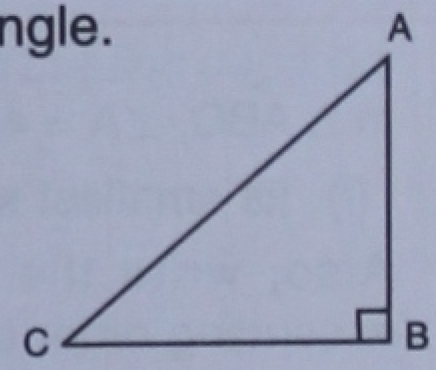
Since, the least angle of the Δ ABC is ∠C = 45°

∴ The smallest side = side AB

∴ Sides of the given triangle in ascending order are **AB < AC < BC** **(Ans.)**

Example 9 :

Show that the hypotenuse is the greatest side in a right-angled triangle.



Solution :

Given, a triangle ABC in which ∠B = 90° *i.e.* AC is the hypotenuse.

To show that :

Hypotenuse AC is the greatest side of right triangle ABC.

Since, ∠B = 90° and ∠A + ∠B + ∠C = 180°

∴ ∠A + ∠C = 90°

⇒ ∠A < 90° and ∠C < 90°

i.e. ∠A < ∠B and ∠C < ∠B

or, ∠B > ∠A and ∠B > ∠C

⇒ ∠B is the greatest angle of ΔABC

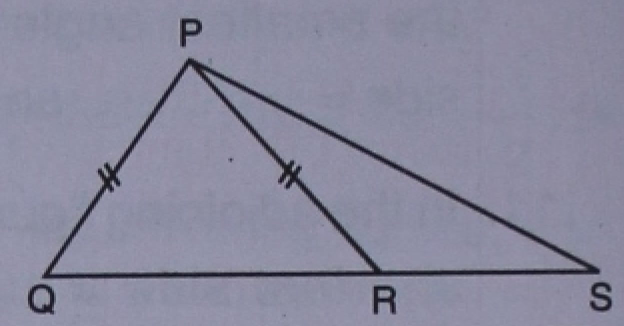
⇒ Side opposite to ∠B is the greatest side of the triangle ABC

⇒ **Hypotenuse AC is the greatest side of right triangle ABC.**

Example 10 :

In isosceles triangle PQR, PQ = PR and S is a point on QR produced. Show that PS is greater than PR.

Solution :



According to the given statement, the figure will be as shown alongside :

Since, in a triangle, the angles opposite to equal sides are equal.

$\therefore$ In $\angle PQR$, PQ = PR $\Rightarrow \angle PQR = \angle PRQ$ I

Since, the exterior angle of a triangle is equal to the sum of its interior opposite angles.

$\therefore$ In ΔPRS, $\angle PRQ = \angle RPS + \angle S$

$\Rightarrow \angle PRQ > \angle S$

$\Rightarrow \angle PQR > \angle S$ [$\because \angle PQR = \angle PRQ$]

Since, in a triangle, the side opposite to the greater angle is greater

$\therefore$ In ΔPQS, $\angle PQR > \angle S$

$\Rightarrow$ PS > PQ

$\Rightarrow$ **PS > PR** [$\because$ PQ = PR]

TEST YOURSELF

13. Two sides of a triangle are 8 cm and 5 cm, the third side can not be less than .... = ... and it can not be greater than .... = ...

14. In Δ ABC, $\angle A : \angle B : \angle C = 8 : 15 : 10$. The largest side of the triangle is ....

15. The largest side of a right angled triangle is called its ....

16. In Δ ABC, side BC > side AC, then $\angle B$ is .... $\angle A$.

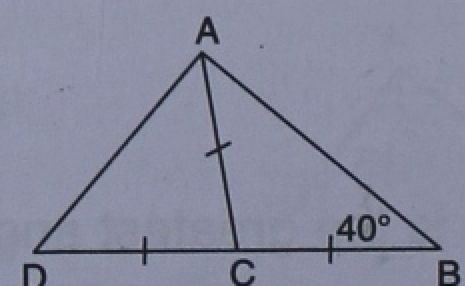
17. In quadrilateral ABCD, AB + BC is greater than .... and AD + DC is greater than ....

EXERCISE 24 (C)

1. In Δ ABC, $\angle A = 40°$ and $\angle B = 80°$. Name :
   (i) its smallest side, (ii) its largest side.
   Also, write the sides of the triangle in ascending order of their lengths.
2. (i) In triangle ABC, $\angle A = \angle B = 52°$, write the name of its largest side.
   (ii) In triangle ABC, $\angle C = 120°$, write the name of its largest side.
3. In triangle PQR, $\angle P : \angle Q : \angle R = 5 : 6 : 7$. Without finding the angles of the triangle, name its
   (i) smallest side, (ii) largest side.
4. State, giving reasons, whether it is possible to construct a triangle or not with its sides equal to:
   (i) 5 cm, 7 cm and 4 cm
   (ii) 3·6 cm, 6 cm and 2·4 cm
   (iii) 8 cm, 15 cm and 19 cm

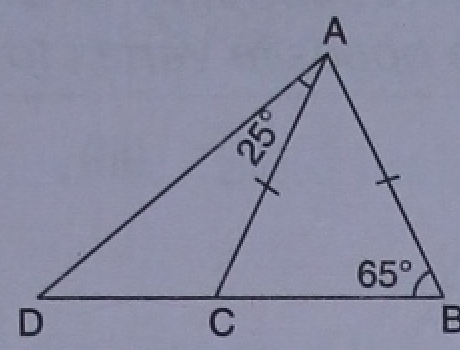
It is possible to construct a triangle, if the sum of lengths of every pair of two sides of the triangle is more than the third side and their difference is less than the third side.

5. Use the information given in the adjoining figure to prove that :

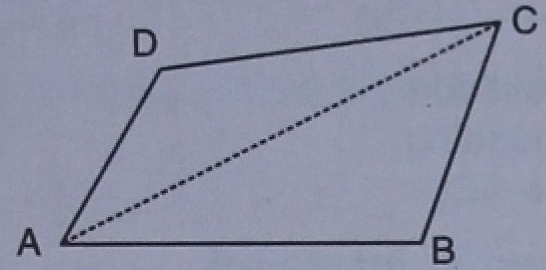


(i) AB > BC (ii) AD > AC

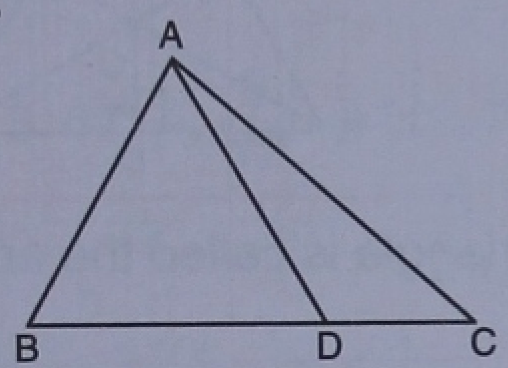
6. Use the information given in the following figure to show that :
   (i) AB > BC
   (ii) AD > AB
   (iii) BD is the largest side of the triangle ABD.



7. In quadrilateral ABCD, given below, prove that :
   (i) AD + DC > AC
   (ii) AB + BC > AC
   (iii) AB + BC + DC + AD > 2 AC

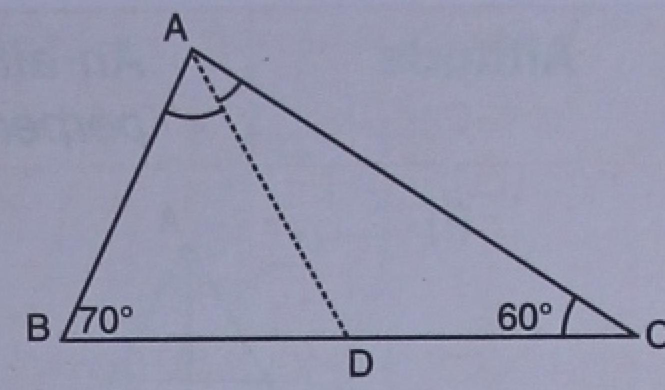


8. In triangle ABC, D is any point in side BC, show that :



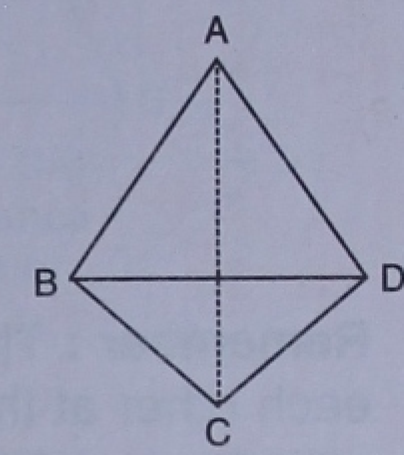
   (i) AB + BD > AD
   (ii) AC + CD > AD
   (iii) AB + BC + AC > 2AD

9. The given figure shows a triangle ABC, in which ∠B = 70°, ∠C = 60° and AD bisects ∠BAD. By finding all the ańgles of triangle ABD and all the angles of triangle ACD, show that :
   (i) AB > AD
   (ii) AC > AD



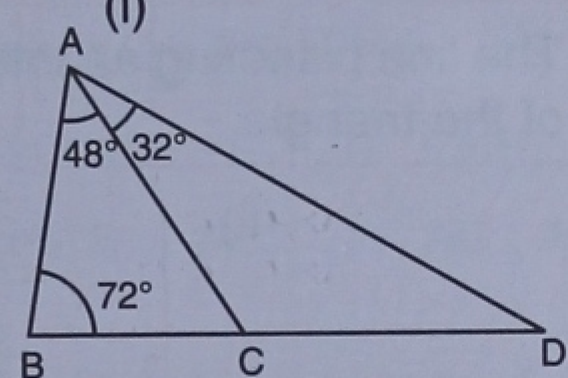
10. The given figure shows a quadrilateral ABCD. Show that :
   (i) AB + BC > AC
   (ii) BC + CD > BD
   (iii) CD + DA > AC and
   (iv) DA + AB > BD

   Now add the results of parts (i), (ii), (iii) and (iv) to show :
   AB + BC + CD + DA > AC + BD.

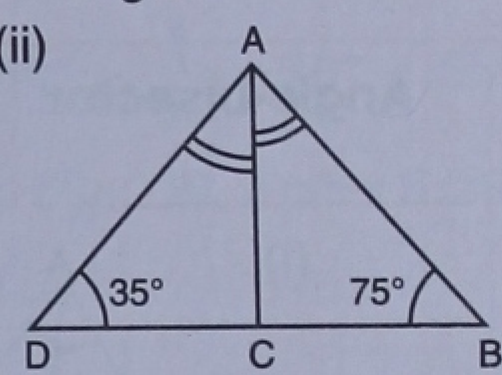


11. Arrange the sides BC, AC and CD in ascending order of their lengths :

(i)



(ii)



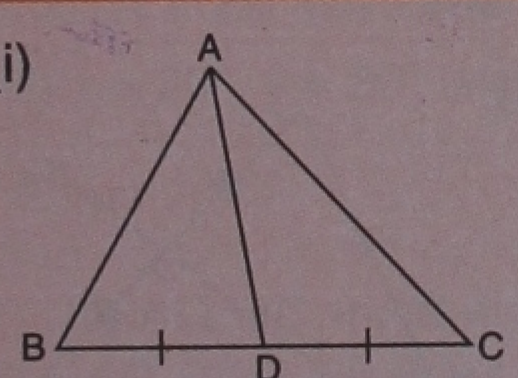
12. In triangle ABC, the bisector of ∠A meets opposite side BC at point D. Prove that : AB > BD.

13. In a right-angled triangle, the hypotenuse is the largest side, prove it.

24.5 IMPORTANT

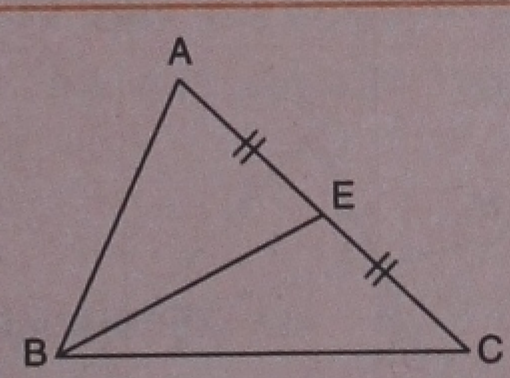
Median	*The median of a triangle,* corresponding to any side of it, *is the line segment joining the mid-point of that side with the opposite vertex.*

(i)



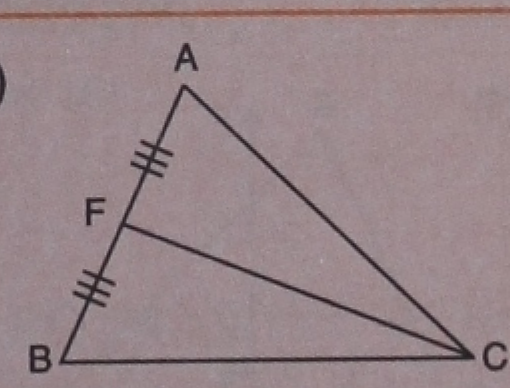
[AD is median corresponding to side BC]

(ii)



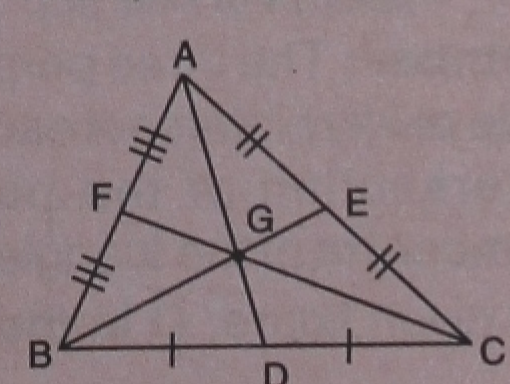
[BE is median corresponding to side AC]

(iii)



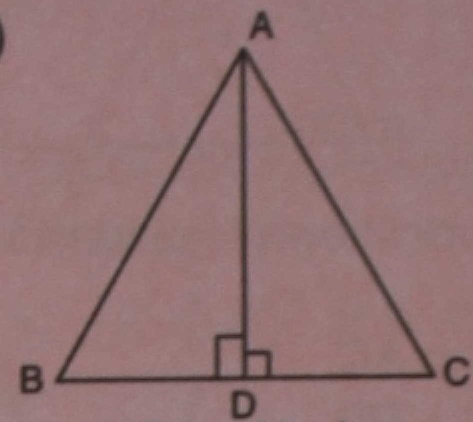
[CF is median corresponding to side AB]

Remember : The three medians of a triangle always intersect each other at the same point. This point of intersection of the medians is called centroid of the triangle.
In the given figure, **G is the centroid of Δ ABC.**



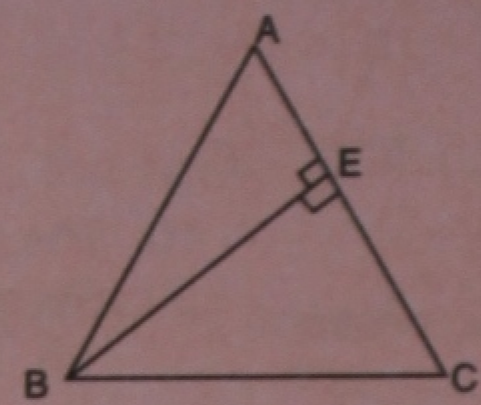
Altitude	*An altitude of a triangle*, corresponding to any side, *is the length of perpendicular drawn from the opposite vertex to that side.*

(i)



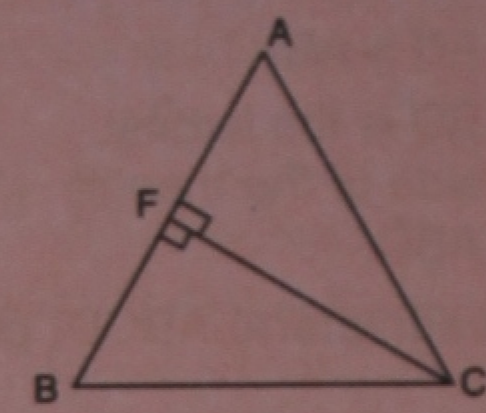
[AD is altitude corresponding to side BC]

(ii)



[BE is altitude corresponding to side AC]

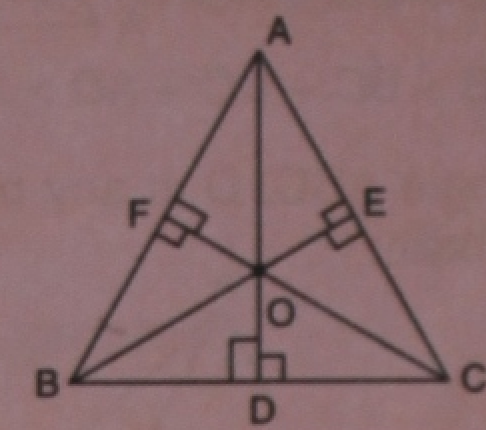
(iii)



[CF is altitude corresponding to side AB]

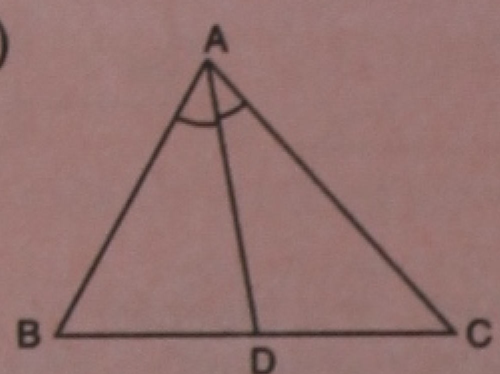
Remember : The three altitudes of a triangle always intersect each other at the same point. This point of intersection of the altitudes is called **orthocentre.**

In the given figure, **O is the orthocentre of Δ ABC.**



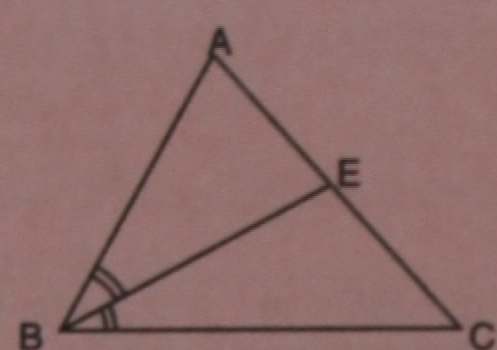
Angle-bisector	The line bisecting an interior angle of a triangle is called the angle-bisector of the triangle.

(i)



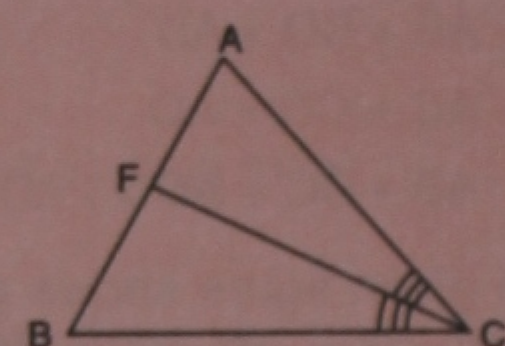
[AD is bisector of ∠BAC]

(ii)



[BE is bisector of ∠ABC]

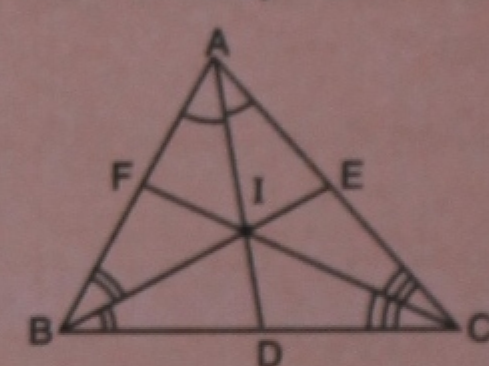
(iii)



[CF is bisector of ∠ACB]

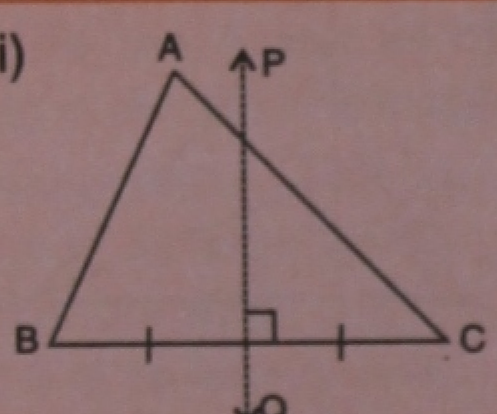
Remember : The three angle bisectors of a triangle always intersect each other at the same point. This point of intersection of the angle bisectors is called **incentre**.

In the given figure, **I is the incentre of Δ ABC**



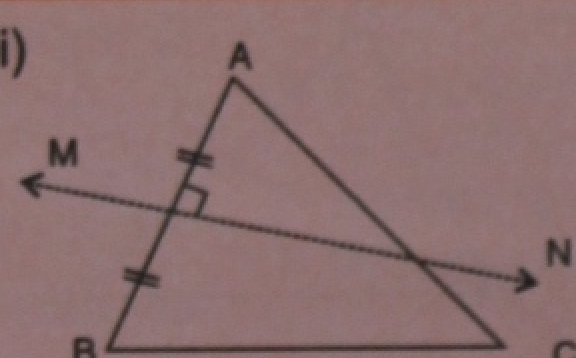
Perpendicular -bisector	*The line bisecting a side of a triangle and perpendicular to this side is called perpendicular bisector of the side of the triangle.*

(i)



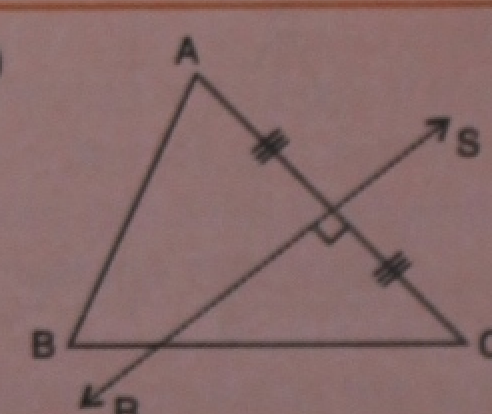
[PQ is perpendicular bisector of side BC]

(ii)



[MN is perpendicular bisector of side AB]

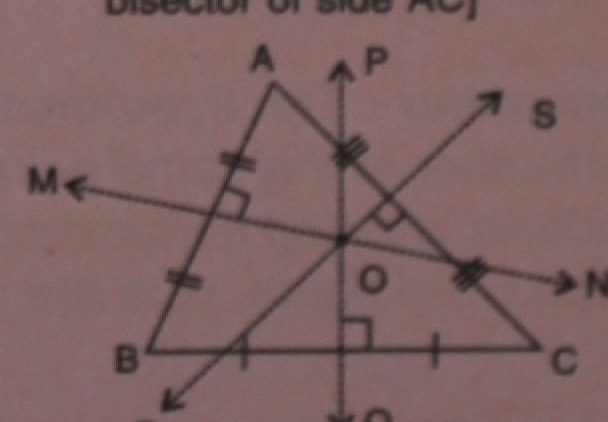
(iii)



[RS is perpendicular bisector of side AC]

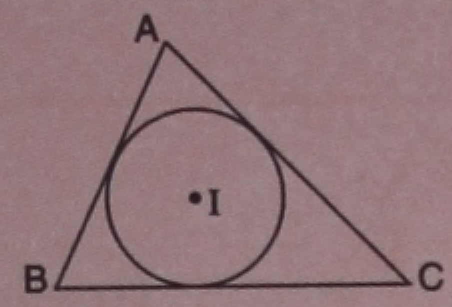
Remember : The three perpendicular bisectors of the sides of a triangle always intersect each other at the same point. This point of intersection of the perpendicular bisectors is called **circumcentre** of the triangle.

In the given figure, **O is the circumcentre of the triangle.**



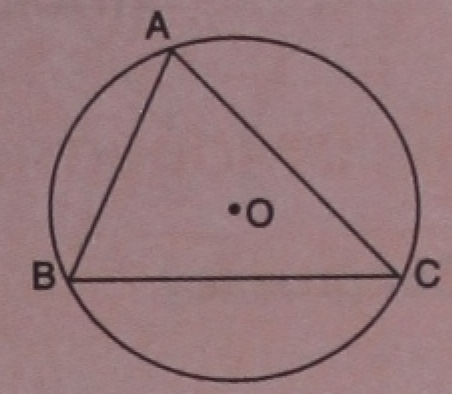
1. Incentre (I) of a triangle is the centre of the circle which touches all the three sides of the triangle.

   The circle so drawn is called **incircle** of the triangle.



2. Circumcentre (O) of a triangle is the centre of the circle which passes through all the three vertices of the triangle.

   The circle so drawn is called **circumcircle** of the triangle.



ANSWERS

TEST YOURSELF

1. 40° **2.** 30° **3.** 50° **4.** $\frac{5}{12} \times 180°$, 75°, $\frac{3}{12} \times 180°$, 45° **5.** $\frac{7}{15} \times 180°$, 84°, 96°; acute-angled triangle **6.** $\frac{8}{20} \times 180° = 72°$, $180° - \frac{5}{20} \times 180°$, 135° **7.** smaller **8.** $90° - \frac{1}{2} \times 70° = 55°$ **9.** $90° + \frac{1}{2} \times 80° = 130°$ **10.** ∠B ; ∠A ; AC ; BC **11.** AB ; AC ; ∠C ; ∠B **12.** (i) > (ii)< **13.** 8 cm – 5 cm; 3 cm, 8 cm + 5 cm; 13 cm **14.** AC **15.** hypotenuse **16.** less than **17.** AC ; AC

EXERCISE 24(A)

1. x = 22° ; 66°, 37° and 77° **2.** 42° and 60° **3.** $\left(98\frac{2}{11}\right)^{\circ}$, $\left(49\frac{1}{11}\right)^{\circ}$ and $\left(32\frac{8}{11}\right)^{\circ}$ **4.** x = 24° and y = 131° **5.** (i) 45°, 40° and 95° (ii) 65°, 50° and 65°. Isosceles triangle **6.** 360° **8.** 119° **9.** A = 41°, B = 98° **10.** (i) 100° (ii) 70°

EXERCISE 24(B)

1. (i) a = 65° (ii) a = 140° and b = 75° (iii) a = 55° and b = 125° **2.** (i) 57° and 38° (ii) 25° **3.** 120° **4.** 50° and 90° **5.** (i) 55° (ii) 55° **6.** (i) 55° (ii) 60° (iii) 65° **7.** (i) 50° (ii) 22° (iii) 45° **8.** x = 15 and y = 60 **9.** 118° **10.** 27° **11.** (i) 130° (ii) 130° (iii) 65° (iv) 65° (v) 50° **12.** (i) ∠DBC = ∠A + ∠C (ii) ∠ECB = ∠A + ∠B (iii) ∠IBC = $\frac{1}{2}$(∠A + ∠C) (iv) ∠ICB = $\frac{1}{2}$(∠A + ∠B) **13.** (i) x = 40°, y = 105° and z = 40° (ii) x = 20°, y = 20° and z = 84° **14.** 64° and 48° **15.** 66° and 16°

EXERCISE 24(C)

1. (i) BC (ii) AC, BC < AB < AC **2.** (i) AB (ii) AB **3.** (i) QR (ii) PQ **4.** (i) Yes (ii) No (iii) Yes **11.** (i) BC < AC < CD (ii) BC < AC = CD