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CHAPTER 20

QUADRATIC EQUATION

20.1 INTRODUCTION

An equation, which contains one unknown (variable) with highest power 2, is called a ***quadratic equation.***

In other words, a quadratic equation is a second degree equation in one variable.

e.g. (i) $x^2 - 5x + 7 = 0$ (ii) $5x^2 - 7x = 0$ (iii) $3x^2 - 8 = 0$ and so on.

The standard form of a quadratic equation is $ax^2 + bx + c = 0$; where a, b and c are all real numbers and $a \neq 0$.

Remember :

1. In quadratic equation $ax^2 + bx + c = 0$; the term ax^2 contains the second power of the unknown and the term bx contains the first power of the unknown. The term c which does not contain x is called the ***absolute term.***
2. A quadratic equation, which contains both the first and the second powers of the unknown quantity, is called a **complete quadratic equation.**

   e.g. (i) $x^2 + 5x - 6 = 0$ (ii) $3x^2 - 7x + 8 = 0$ and so on.
3. A quadratic equation, which contains only the second power of the unknown quantity, is called an **incomplete quadratic equation.**

   e.g. (i) $x^2 - 16 = 0$ (ii) $5x^2 = 20$ (iii) $7x^2 - 8 = 0$ and so on.
4. A quadratic equation gives **two** values of the unknown (variable) and both these values are called the **roots of the equation.**

TEST YOURSELF

1. $5x + 7 = 0$ is a ..... equation containing only ..... variable.
2. $5x + 7y = 8$ is a ..... equation containing ..... variables.
3. $4x^2 - 7x + 3 = 0$ is a ..... equation containing only ..... variable.
4. The degree of equation $4x^2 - 7x + 3 = 0$ is .....; and it always gives ..... values of its variable x.
5. $4x^2 - 1 = 0$ is a ..... equation and so it will give ..... values of its variable.
6. $3x^2 - 5x = 0$ is a ..... equation.
7. $(3x - 5)(4x + 3) = 0 \Rightarrow$ ..... $= 0$, or ..... $= 0 \Rightarrow x =$ ..... or $x =$ .....

20.2 SOLVING A QUADRATIC EQUATION

Steps :

1. Express the equation in the form $ax^2 + bx + c = 0$.
2. Factorise the expression on the left hand side of the resulting equation.
3. Put each of the factors equal to zero and solve.

Example 1 :

Solve : $2x^2 - 3 = 5x$

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Solution :

Step 1 : Converting into standard form :

$$2x^2 - 3 = 5x$$

$\Rightarrow$ $2x^2 - 5x - 3 = 0$ [Standard form]

Step 2 : Factorising left hand side :

$2x^2 - 6x + x - 3 = 0 \Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

Step 3 : Putting each of the factors equal to zero :

i.e. $x - 3 = 0$ or $2x + 1 = 0$

$\Rightarrow x = 3$ or $2x = -1$ [Solving equations]

$\Rightarrow$ $\mathbf{x = 3}$ or $\mathbf{x = -\frac{1}{2}}$ **(Ans.)**

Example 2 :

(i) $x^2 = 25$ (ii) $x + \frac{1}{x} = 2\frac{1}{2}$

Solution :

(i) $\mathbf{x^2 - 25 = 0} \Rightarrow (x - 5)(x + 5) = 0$ [Step 2]

$\Rightarrow x - 5 = 0$ or $x + 5 = 0$ [Step 3]

$\Rightarrow \mathbf{x = 5}$ or $\mathbf{x = -5}$ **(Ans.)**

(ii) $\mathbf{x + \frac{1}{x} = 2\frac{1}{2}} \Rightarrow \frac{x^2 + 1}{x} = \frac{5}{2}$

$\Rightarrow 2x^2 + 2 = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$ [Step 1]

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 2)(2x - 1) = 0$ [Step 2]

$\Rightarrow x - 2 = 0$ or $2x - 1 = 0$ [Step 3]

$\Rightarrow \mathbf{x = 2}$ or $\mathbf{x = \frac{1}{2}}$ **(Ans.)**

Example 3 :

(i) $x^2 - 5x = 0$ (ii) $\frac{3x - 7}{2x - 5} = \frac{x + 1}{x - 1}$

Solution :

(i) $\mathbf{x^2 - 5x = 0} \Rightarrow x(x - 5) = 0$ [Step 2]

$\Rightarrow x = 0$ or $x - 5 = 0$ [Step 3]

$\Rightarrow \mathbf{x = 0}$ or $\mathbf{x = 5}$ **(Ans.)**

(ii) $\mathbf{\frac{3x - 7}{2x - 5} = \frac{x + 1}{x - 1}} \Rightarrow (3x - 7)(x - 1) = (2x - 5)(x + 1)$ [By cross-multiplying]

$\Rightarrow 3x^2 - 3x - 7x + 7 = 2x^2 + 2x - 5x - 5$

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$\Rightarrow 3x^2 - 10x + 7 - 2x^2 + 3x + 5 = 0$

$\Rightarrow x^2 - 7x + 12 = 0$ [Step 1]

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$\Rightarrow (x - 4)(x - 3) = 0$ [Step 2]

$\Rightarrow x - 4 = 0$ or $x - 3 = 0$ [Step 3]

$\Rightarrow$ **$x = 4$ or $x = 3$** **(Ans.)**

Example 4 :

Solve : $\frac{1}{x} + \frac{1}{2+x} = \frac{3}{4}$

Solution :

$\frac{1}{x} + \frac{1}{2+x} = \frac{3}{4} \Rightarrow \frac{2+x+x}{x(2+x)} = \frac{3}{4}$

$\Rightarrow \frac{2+2x}{2x+x^2} = \frac{3}{4}$

$\Rightarrow 3(2x + x^2) = 4(2 + 2x)$

$\Rightarrow 6x + 3x^2 = 8 + 8x$ *i.e.* $3x^2 - 2x - 8 = 0$

$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$ *i.e.* $3x(x - 2) + 4(x - 2) = 0$

$\Rightarrow (x - 2)(3x + 4) = 0$ *i.e.* $x - 2 = 0$ or $3x + 4 = 0$

$\Rightarrow$ **$x = 2$ or $x = -\frac{4}{3}$** **(Ans.)**

TEST YOURSELF

8. $x(x - 3) = 0 \Rightarrow$ ..... $= 0$ or ..... $= 0 \Rightarrow x =$ ..... or $x =$ .....
9. $4x^2 - 25 = 0 \Rightarrow$ ..... $= 0 \Rightarrow$ ..... $= 0$ or ..... $= 0 \Rightarrow x =$ ..... or $x =$ .....
10. $x = \frac{9}{x} \Rightarrow x^2 =$ ..... and $x =$ .....
11. $(x - 3)^2 = 36 \Rightarrow x - 3 =$ .....$\Rightarrow x - 3 =$ ..... or $x - 3 =$ .....
$\Rightarrow x =$ ..... or $x =$ .....
12. $x(x - 5) = 6 \Rightarrow$ .....
.....

EXERCISE 20 (A)

Solve :

1. $(2x - 3)(x + 2) = 0$
2. $x(x - 1) = 42$
3. $8x^2 + x = 6 - x$
4. $\frac{5x-1}{3} = \frac{6}{x}$
5. $x^2 - 64 = 0$
6. $2x^2 - 3x + 1 = 0$
7. $x - \frac{10}{x-3} = 0$
8. $2y^2 = 12 - 5y$
9. $x(x - 2) = 9 - 2x$
10. $9x^2 = 25$
11. $a^2 + a = 90$
12. $\frac{15}{x^2} = 1 - \frac{2}{x}$
13. $6x^2 = x + 1$
14. $(2a + 1)(a + 3) + 3 = 0$
15. $(3x + 1)(2x + 3) = 3$

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16. $(x-3)^2 = 25$

17. $(x-1)^2 + (x+3)^2 = 26$

18. $\frac{x}{3} + \frac{9}{x} = 4$

19. $\frac{x}{2} + 1 = \frac{12}{x}$

20. $\frac{x+8}{3x-5} = \frac{x-8}{x+5}$

21. $\frac{1}{x+1} + \frac{1}{x-2} = \frac{1}{x-3}$

22. $\frac{7}{3x-4} - \frac{2}{x+2} = 1$

23. $\frac{2}{3x} = \frac{x-1}{2x-1}$

24. $\frac{6x-5}{x+5} = \frac{x}{3} - 5$

25. $\frac{x+7}{5} - \frac{9}{x-2} = 1$

26. $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

27. $5x + \frac{5}{x} = 26$

28. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{25}{12}$

29. $\frac{24}{18-x} - \frac{24}{18+x} = 1$

30. $(x-8)\left(\frac{960}{x} + 4\right) = 960$

20.3 PROBLEMS BASED ON QUADRATIC EQUATIONS

In solving problems based on quadratic equations, same steps are adopted as in linear equations.

Example 5 :

The square of a positive number added to one-fourth of it is equal to 17. Find the number.

Solution :

Let the required positive number be x.

$$\therefore \quad x^2 + \frac{1}{4}x = 17 \quad \Rightarrow \quad \frac{4x^2 + x}{4} = 17$$

$$\Rightarrow \quad 4x^2 + x = 68 \quad \text{i.e.} \quad 4x^2 + x - 68 = 0$$

$$\Rightarrow \quad 4x^2 + 17x - 16x - 68 = 0$$

$$\Rightarrow \quad x(4x + 17) - 4(4x + 17) = 0$$

$$\Rightarrow \quad (4x + 17)(x - 4) = 0 \quad \text{i.e.} \quad 4x + 17 = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow \quad x = -\frac{17}{4} \text{ or } x = 4$$

$\therefore$ **The required positive number = 4** **(Ans.)**

Example 6 :

The sum of a number and its reciprocal is 5·2. Find the number.

Solution :

Let the number be x, its reciprocal = $\frac{1}{x}$ $\quad \therefore \quad x + \frac{1}{x} = 5{\cdot}2$ [Given]

$$\text{i.e.} \quad \frac{x^2 + 1}{x} = \frac{26}{5} \quad \Rightarrow \quad 5x^2 + 5 = 26x \quad \left[\because 5{\cdot}2 = \frac{52}{10} = \frac{26}{5}\right]$$

$$\text{i.e.} \quad 5x^2 - 26x + 5 = 0 \quad \Rightarrow \quad 5x^2 - 25x - x + 5 = 0$$

$$\text{i.e.} \quad 5x(x - 5) - 1(x - 5) = 0 \quad \Rightarrow \quad (x - 5)(5x - 1) = 0$$

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i.e. $x - 5 = 0$ or $5x - 1 = 0 \Rightarrow x = 5$ or $x = \frac{1}{5}$

$\therefore$ **The required number is 5 or $\frac{1}{5}$.** **(Ans.)**

Example 7 :

Find two consecutive even natural numbers such that the sum of their squares is 52.

Solution :

Let the numbers be x and x + 2, $\therefore$ $x^2 + (x + 2)^2 = 52$ [Given]

$\Rightarrow$ $x^2 + x^2 + 4x + 4 - 52 = 0$

$\Rightarrow$ $2x^2 + 4x - 48 = 0$

$\Rightarrow$ $x^2 + 2x - 24 = 0$ [Dividing each term by 2]

$\Rightarrow$ $x^2 + 6x - 4x - 24 = 0$ *i.e.* $x(x + 6) - 4(x + 6) = 0$

$\Rightarrow$ $(x + 6)(x - 4) = 0$ *i.e.* $x = -6$ or $x = 4$

$\therefore$ **The required first even natural number = x = 4**

and the next even natural number = x + 2 = 4 + 2 = 6 **(Ans.)**

Example 8 :

A certain number of boys contributed equally to collect ₹ 240. Had there been 4 boys more to collect the same money, the contribution of each would have decreased by ₹ 10. Calculate the original number of boys.

Solution :

Let the original number of boys be x

$\Rightarrow$ x boys contributed = ₹ 240 *i.e* 1 boy contributed = ₹ $\frac{240}{x}$

In the second case :

Number of boys = x + 4

$\Rightarrow$ (x + 4) boys contributed = ₹ 240 *i.e* 1 boy contributed = ₹ $\frac{240}{x+4}$

According to the given statement :

$$\frac{240}{x} - \frac{240}{x+4} = 10$$

$$\Rightarrow \frac{240(x+4) - 240x}{x(x+4)} = 10$$

$\Rightarrow$ $10x(x + 4) = 240x + 960 - 240x$

$\Rightarrow$ $x^2 + 4x - 96 = 0$

$\Rightarrow$ $x^2 + 12x - 8x - 96 = 0$ *i.e.* $x(x + 12) - 8(x + 12) = 0$

$\Rightarrow$ $(x + 12)(x - 8) = 0$ *i.e.* $x = -12$ or $x = 8$

$\therefore$ **The original number of boys = 8** **(Ans.)**

Example 9 :

An express train makes a run of 240 km at a certain speed. Another train, whose speed is 12 km per hour less than the first train, takes an hour longer to make the same trip. Find the speed of the express train.

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Solution :

Let the speed of the express train = x km per hour

$\Rightarrow$ The speed of another train = (x – 12) km per hour

Since, $\text{time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ Time taken by the express train to cover 240 km $= \frac{240}{x}$ hr

And, time taken by another train to cover 240 km $= \frac{240}{x-12}$ hr

Given : Another train takes 1 hour longer than the express train

$$\Rightarrow \frac{240}{x-12} - \frac{240}{x} = 1$$

$$\Rightarrow \frac{240x - 240(x-12)}{x(x-12)} = 1$$

$$\Rightarrow x(x-12) = 240x - 240x + 2880$$

$$\Rightarrow x^2 - 12x - 2880 = 0 \quad i.e \quad x^2 - 60x + 48x - 2880 = 0$$

$$\Rightarrow x(x-60) + 48(x-60) = 0 \quad i.e \quad (x-60)(x+48) = 0$$

$$\Rightarrow x = 60 \text{ or } x = -48$$

$\therefore$ **The speed of the express train = 60 km per hour** **(Ans.)**

Example 10 :

A person bought a certain number of similar articles for ₹ 90. If he had paid rupee one more for each article, he would have got one article less for the same price. Find the price of each article.

Solution :

Let the price of each article be ₹ x

$\therefore$ No. of articles bought $= \frac{\text{Total price paid}}{\text{Price of one article}} = \frac{90}{x}$

If the price paid for each article = ₹ (x + 1)

Then, no. of articles bought $= \frac{90}{x+1}$

Given : $\frac{90}{x} - \frac{90}{x+1} = 1$

$$\Rightarrow \frac{90x + 90 - 90x}{x(x+1)} = 1$$

$$\Rightarrow x^2 + x = 90$$

$$\Rightarrow x^2 + x - 90 = 0$$

$$\Rightarrow x^2 + 10x - 9x - 90 = 0$$

$$\Rightarrow x(x+10) - 9(x+10) = 0$$

$$(x+10)(x-9) = 0$$

$$\Rightarrow x + 10 = 0 \text{ or } x - 9 = 0$$

$$x = -10 \text{ or } x = 9$$

$\Rightarrow$ **Price of each article = ₹ 9** **(Ans.)**

TEST YOURSELF

13. For three consecutive even integers, the middle one is x, then the other two integers are ..... and .....

14. If x is a natural number, the five natural numbers, each divisible by 5 can be taken as : ....., ..., ..., ... and .....

15. The product of two consecutive natural numbers is 56; the numbers are ..... and .....

16. The product of two consecutive integers is 56, the integers are ..... and ....., or ..... and .....

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EXERCISE 20 (B)

1. Find two positive numbers which have a difference of 5 and whose product is 84.
2. Two natural numbers differ by 4 and the sum of their squares is 58. Find the numbers.
3. Find two consecutive whole numbers whose product is 72.
4. Find two consecutive even natural numbers whose product is 168.
5. Find two consecutive odd whole numbers whose product is 63.
6. A number added to its reciprocal is 4·25, find the number.
7. Find two consecutive natural numbers, the sum of whose reciprocals is $\frac{11}{30}$.
8. Find two consecutive whole numbers, the difference of whose reciprocals is $\frac{1}{12}$.
9. Divide 8 into two parts such that the sum of their squares is 34.
10. A sum of ₹ 120 was distributed among a certain number of boys. If there had been 4 boys less, each would have received a rupee more. Find the number of boys.
11. ₹ 9,000 were divided equally among a certain number of persons. Had there been 20 persons more, each would have got ö 160 less. Find the original number of persons.
12. A party of tourists booked a hotel for ₹ 1,200. Three of the members failed to pay. As a result, each of the others had to pay ₹ 20 more. How many tourists were there in the party ?
13. A man covers a distance of 200 km travelling with a uniform speed of x km per hour. The distance could have been covered in 2 hours less, had the speed been $(x + 5)$ km per hour. Calculate the value of x.
14. A train travels 300 km at a uniform speed. If the speed had been 5 km an hour more, the journey would have taken two hours less. Find the speed of the train.

EXERCISE 20 (C)

1. Solve :

   (i) $2x - \frac{1}{x} = 1$

   (ii) $\frac{2x+1}{x+3} = \frac{3x-5}{x-1}$

   (iii) $x(x - 1) = 30$

   (iv) $3x^2 = 2(x + 4)$

   (v) $(x + 1)(x - 1) = 120$

   (vi) $x + \frac{1}{x} = 8{\cdot}125$

   (vii) $\frac{x^2}{4} = \frac{x}{2} + 12$

   (viii) $(3x + 1)(2x + 3) = 3$

   (ix) $4(x - 2)^2 - x^2 = (x + 4)^2$

   (x) $\frac{150}{x-5} - \frac{150}{x} = 1$

   (xi) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

2. Find two consecutive odd natural numbers such that the sum of their squares is 130.
3. ₹ 1,200 was divided equally among a certain number of children. Had there been 5 children more, each would have received ₹ 8 less. Taking the original number of children x; find :
   (i) money received by each child in the first case.
   (ii) money received by each child in the second case.
   (iii) the original number of children
4. A train covers a distance of 480 km at a speed of x km/hr. If its speed is reduced to (x – 20) km/hr, it takes 4 hours more to cover the same distance. Find the value of x.
5. Divide 20 into two parts so that the sum of their reciprocals is $\frac{4}{15}$.
6. Think of a positive number, say x. From the square of this number subtract three-times of itself. If the result is 10, find x.
7. Two pipes take x minutes and (x + 3) minutes respectively to fill a cistern. If together they fill the cistern in $3\frac{1}{13}$ minutes, find the value of x.
8. The speed of a boat in still water is 11 km/hr and the speed of the stream is x km/hr. Find in terms of x, the speed of boat upstream and the speed of boat downstream.

   If the boat takes $2\frac{3}{4}$ hours to go 12 km upstream and then return, find the value of x.

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9. A tradesman finds that by selling a bicycle for ₹ 75, which he had bought for ₹ x, he gained x%. Find the value of x.

Given : C.P. = ₹ x

and profit = x% of ₹ x = ₹ $\frac{x^2}{100}$

$\therefore$ C.P. + Profit = S.P. $\Rightarrow x + \frac{x^2}{100} = 75$

10. By selling an article for ₹ 16, a man loses as much percent as its cost price. Find the cost price of the article.

Let : C.P. = ₹ x,

therefore, loss = x% of ₹ x = ₹ $\frac{x^2}{100}$

$\therefore$ C.P. – Loss = S.P. $\Rightarrow x - \frac{x^2}{100} = 16$

ANSWERS

TEST YOURSELF

1. linear ; one **2.** linear ; two **3.** quadratic ; one **4.** 2 ; two **5.** quadratic ; two **6.** quadratic

7. $3x - 5$; $4x + 3$; $\frac{5}{3}; -\frac{3}{4}$ **8.** x, x – 3, 0 ; 3 **9.** (2x – 5) (2x + 5), 2x – 5, 2x + 5, $\frac{5}{2}; -\frac{5}{2}$ **10.** 9, ± 3

11. ± 6; 6, –6, 9 or – 3 **12.** $x^2 - 5x - 6 = 0 \Rightarrow x^2 - 6x + x - 6 = 0 \Rightarrow (x - 6)(x + 1) = 0 \Rightarrow x = 6$ or $x = -1$

13. x – 2; x + 2 **14.** 5x, 5x + 5; 5x + 10; 5x + 15; 5x + 20 **15.** 7 ; 8 **16.** 7 ; 8 or –8 ; –7

EXERCISE 20(A)

1. $\frac{3}{2}$ or – 2 **2.** 7 or –6 **3.** –1 or $\frac{3}{4}$ **4.** $-\frac{9}{5}$ or 2 **5.** 8 or – 8 **6.** 1 or $\frac{1}{2}$ **7.** –2 or 5 **8.** $\frac{3}{2}$ or – 4

9. 3 or –3 **10.** $\frac{5}{3}$ or $-\frac{5}{3}$ **11.** – 10 or 9 **12.** –3 or 5 **13.** $\frac{1}{2}$ or $-\frac{1}{3}$ **14.** – 2 or $-\frac{3}{2}$ **15.** 0 or $-\frac{11}{6}$

16. 8 or – 2 **17.** 2 or – 4 **18.** 3 or 9 **19.** 4 or – 6 **20.** 0 or 21 **21.** 5 or 1 **22.** 3 or $-\frac{10}{3}$ **23.** 2 or $\frac{1}{3}$

24. 30 or – 2 **25.** 7 or – 7 **26.** 2 or – 1 **27.** 5 or $\frac{1}{5}$ **28.** – 4 or 3 **29.** 6 or – 54 **30.** 48 or – 40

EXERCISE 20(B)

1. 7 and 12 **2.** 7 and 3 **3.** 8 and 9 **4.** 12 and 14 **5.** 7 and 9 **6.** 4 or $\frac{1}{4}$ **7.** 5 and 6 **8.** 3 and 4

9. 3 and 5 **10.** 24 **11.** 25 **12.** 15 **13.** 20 **14.** 25 km/hr

EXERCISE 20(C)

1. (i) 1 or $-\frac{1}{2}$ (ii) – 7 or 2 (iii) 6 or – 5 (iv) 2 or $-\frac{4}{3}$ (v) 11 or – 11 (vi) 8 or $\frac{1}{8}$ (vii) 8 or – 6

(viii) 0 or $-\frac{11}{6}$ (ix) 0 or 12 (x) 30 or – 25 (xi) 5 or $\frac{5}{2}$ **2.** 7 and 9 **3.** (i) ₹ $\frac{1{,}200}{x}$ (ii) ₹ $\frac{1{,}200}{x+5}$

(iii) Equation : $\frac{1200}{x} - \frac{1200}{x+5} = 8$. Original no. = 25 **4.** 60 km/hr **5.** 5 and 15 **6.** 5 **7.** 5

8. (11 – x) km/hr and (11 + x) km/hr ; x = 5 **9.** 50 **10.** ₹ 20 or ₹ 80

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