SIMULTANEOUS EQUATIONS

(With Problems)

19.1 INTRODUCTION

1. Simultaneous Equations	Two equations, each containing two same variables (unknowns), are called **simultaneous equations.** *e.g.* (i) 3x + 2y = 7 and x – y = 8 (Here; x and y are variables) (ii) 5a – 4b = 18 and 2a + b = 4 (Here; a and b are variables) and so on.
2. Solution	Consider the linear equation in two variables to be 3x – 4y = 7. Any pair of values of x and y which satisfy this equation is called its solution. For example, if x = 5 and y = 2; then : $3x - 4y = 7 \Rightarrow 3 \times 5 - 4 \times 2 = 7$ $\Rightarrow 15 - 8 = 7$; *which is true.* ∴ **x = 5 and y = 2 is a solution of 3x – 4y = 7.** Now, if x = 9 and y = 5 : $3x - 4y = 7 \Rightarrow 3 \times 9 - 4 \times 5 = 7$; *which is true* ∴ **x = 9 and y = 5 is also a solution of equation 3x – 4y = 7** On trying more, we shall be able to get an infinite number of solutions of the equation 3x – 4y = 7. In the same way, it can easily be shown that every linear equation in two variables has an infinite number of solutions.

Is x = 3 and y = 2 a solution of equation x + 2y = 7 ?

$x + 2y = 7 \Rightarrow 3 + 2 \times 2 = 7$

$\Rightarrow 3 + 4 = 7$; *which is true.*

Is x = 3 and y = 2 a solution of equation 3x – 4y = 1 ?

$3x - 4y = 1 \Rightarrow 3 \times 3 - 4 \times 2 = 1$

$\Rightarrow 9 - 8 = 1$; *which is true.*

∴ x = 3 and y = 2 is a solution of simultaneous equations

x + 2y = 7 and 3x – 4y = 1.

Now, if we try to get more pairs of values of x and y to satisfy the equations x + 2y = 7 and 3x – 4y = 1, we find it impossible.

∴ **The solution of two simultaneous equations is unique.**

19.2 SOLVING SIMULTANEOUS EQUATIONS ALGEBRAICALLY

To solve simultaneous equations means; to find the values of variables used in them.

i.e. (i) To solve the equations 3x – 2y = 8 and x + 4y = 10 means to find the values of variables x and y.

(ii) To solve the equations 8m + 7n = 5 and 2m – 5n = 15 means to find the values of variables m and n.

There are mainly two methods of solving simultaneous equations algebraically :

(i) Substitution method (ii) Addition or subtraction method

Example 1 :

Solve equations $x + 3y = 3$ and $4x - 5y = 29$ by substitution method.

Solution :

Steps : 1. Take one of the two given equations (Generally, a simpler equation is taken)

Let us take equation $x + 3y = 3$

2. Find the value of one variable (x or y) in terms of the other variable.

(Here we find x in terms of y)

$\therefore\ x + 3y = 3 \Rightarrow x = 3 - 3y$

3. Substitute the value of x in the other equation.

Substituting in equation $4x - 5y = 29$, we get :

$$4(3 - 3y) - 5y = 29$$

$$\Rightarrow 12 - 12y - 5y = 29$$

$$\Rightarrow -17y = 29 - 12 = 17$$

$$\therefore y = \frac{17}{-17} = -1$$

4. Substitute $y = -1$ in the result of step 2.

i.e. $x = 3 - 3y = 3 - 3(-1) = 6$

$\therefore$ **The solution of the given equations is $x = 6$ and $y = -1$** **(Ans.)**

If instead of finding x in terms of y (step 1), we find y in terms of x and substitute in the other equation, then also the solution of the equations will be the same. Let us try :

$$x + 3y = 3 \Rightarrow 3y = 3 - x \Rightarrow y = \frac{3-x}{3}$$

Substituting in $4x - 5y = 29$; we get :

$$4x - 5\left(\frac{3-x}{3}\right) = 29$$

$$\Rightarrow \frac{12x - 15 + 5x}{3} = 29$$

$$\Rightarrow 17x - 15 = 29 \times 3 = 87$$

$$\Rightarrow 17x = 87 + 15 = 102 \qquad \Rightarrow x = \frac{102}{17} = 6$$

$$\therefore y = \frac{3-x}{3} = \frac{3-6}{3} = \frac{-3}{3} = -1$$

$\therefore$ **Required solution is : $x = 6$ and $y = -1$** **(Ans.)**

Example 2 :

Solve equations $x + 3y = 3$ and $4x - 5y = 29$ by addition or subtraction method.

Solution :

Steps : 1. Multiply one or both the equations, if necessary, by a suitable number or numbers so that either the coefficients of x or coefficients of y in both the equations are numerically equal.

By inspection we find, that if the first equation x + 3y = 3 be multiplied by 4, the coefficients of x in both the equations will be the same.

Multiplying x + 3y = 3 by 4, we get 4x + 12y = 12.

2. Add or subtract one equation from the other so that the terms with equal numerical coefficients, cancel mutually. Then solve the resulting equation.

Thus,

$$\begin{array}{rl} 4x + 12y &= 12 \\ 4x - 5y &= 29 \\ - \quad + & \quad - \\ \hline 17y &= -17 \end{array} \qquad \text{[Subtracting]}$$

$$\Rightarrow \quad y = -\frac{17}{17} = -1$$

3. Substitute y = –1 in any of the two given equations.

Substituting y = – 1, in x + 3y = 3, we get :

$$x + 3(-1) = 3$$

$$\Rightarrow \quad x - 3 = 3$$

$$\Rightarrow \quad x = 3 + 3 = 6$$

∴ **x = 6 and y = –1** **(Ans.)**

Example 3 :

Solve by substitution method : 3x – 2y = –7 and 5x – 3y = 13.

Solution :

Steps : 1. $3x - 2y = -7$

2. $3x = 2y - 7 \quad \Rightarrow \quad x = \frac{2y-7}{3}$

3. $5x - 3y = 13 \quad \Rightarrow \quad 5\left(\frac{2y-7}{3}\right) - 3y = 13$

i.e. $\frac{10y - 35 - 9y}{3} = 13 \quad \Rightarrow \quad y - 35 = 39$

$\Rightarrow \quad y = 39 + 35 = 74$

4. $x = \frac{2y-7}{3} = \frac{2 \times 74 - 7}{3} = 47$

∴ **x = 47 and y = 74** **(Ans.)**

Example 4 :

Solve by addition or subtraction method : 9x + 4y = 5 and 4x – 5y = 9.

Solution :

Steps : 1. Multiply the first equation by 5 and second equation by 4.

$$\begin{array}{rl} 45x + 20y &= 25 \\ 16x - 20y &= 36 \end{array} \qquad \begin{array}{r} [(9x + 4y = 5) \times 5] \\ [(4x - 5y = 9) \times 4] \end{array}$$

2. $$\begin{array}{rl} 61x &= 61 \\ \hline \end{array} \qquad \text{[Adding]}$$

$$\Rightarrow \quad x = \frac{61}{61} = 1$$

3. Substituting x = 1 in equation 9x + 4y = 5, we get :

$$9 \times 1 + 4y = 5 \quad \Rightarrow \quad 4y = 5 - 9 = -4$$

$$\Rightarrow \quad y = \frac{-4}{4} = -1$$

∴ **x = 1 and y = –1** **(Ans.)**

> When students are not asked to solve the given simultaneous equations by a particular method, the solution is generally done by addition or subtraction method.

EXERCISE 19 (A)

Solve each pair of equations by substitution method :

1. $x + y = 11$, $x - y = -3$
2. $x + 5y = 18$, $3x + 2y = 41$
3. $x + y = 0$, $y - x = 6$
4. $x - 4y = -8$, $x - 2y = 0$
5. $4a - b = 10$, $2a + 3b = 12$
6. $2a + 3b = 6$, $3a + 5b = 15$

Solve each pair of equations by additon or subtraction method :

7. $2x - y = 9$, $3x - 7y = 19$
8. $8x = 5y$, $13x = 8y + 1$
9. $x + 2y = 11$, $2x - y = 2$
10. $3x - 7y = 35$, $2x + 5y = 4$

Solve each pair of simultaneous equations :

11. $4x - 3y = 8$, $3x - 4y = -1$
12. $8a - 7b = 1$, $4a = 3b + 5$
13. $5x - 6y = 8$, $7y - 15x = 9$
14. $3x + 2y = -1$, $6y = 5(1 - x)$
15. $a = b + 2$, $2a - b = 7$
16. $3x + 5(y + 2) = 1$, $3x + 8y = 0$

19.3 MORE EXAMPLES

Example 5 :

Solve : $\frac{x}{4} - \frac{y}{6} = 3$ and $\frac{x}{2} - y = -2$.

Solution :

$$\frac{x}{4} - \frac{y}{6} = 3 \quad \Rightarrow \quad \frac{3x - 2y}{12} = 3 \quad \Rightarrow \quad 3x - 2y = 36 \quad \text{............... I}$$

$$\frac{x}{2} - y = -2 \quad \Rightarrow \quad \frac{x - 2y}{2} = -2 \quad \Rightarrow \quad x - 2y = -4 \quad \text{............. II}$$

In both the equations, coefficients of y are equal.

∴ On subtracting I from II, we get :

$$\begin{array}{lrl} & 3x - 2y &= 36 \\ & x - 2y &= -4 \\ \text{(change signs)} & - \quad + & \quad + \\ \hline & 2x &= 40 \\ \Rightarrow & x &= 20 \end{array}$$

Substituting x = 20 in eq. II; we get :

$$20 - 2y = -4$$

$$\Rightarrow \quad 20 + 4 = 2y$$

$$\Rightarrow \quad 2y = 24$$

$$\Rightarrow \quad y = \frac{24}{2} = 12$$

∴ **x = 20 and y = 12** **(Ans.)**

When variable(s) x or y or both are given in the denominator(s)

Example 6 :

Solve : $4x + \frac{6}{y} = 15$ and $6x - \frac{8}{y} = 14$.

Solution :

Multiplying first equation by 4 and second equation by 3, we get :

$$16x + \frac{24}{y} = 60$$

$$18x - \frac{24}{y} = 42$$

On adding : $34x = 102$

$$\Rightarrow \quad x = \frac{102}{34} = 3$$

Substituting x = 3 in first equation, we get :

$$4 \times 3 + \frac{6}{y} = 15 \Rightarrow \frac{6}{y} = 15 - 12 = 3 \Rightarrow 6 = 3y \quad \therefore y = \frac{6}{3} = 2$$

$\therefore$ **x = 3 and y = 2** **(Ans.)**

Example 7 :

Solve : $\frac{3}{x} - \frac{4}{y} = 1$ and $\frac{2}{x} - \frac{3}{y} = 0.$

Solution :

Multiplying first equation by 2 and second equation by 3, we get :

$$\frac{6}{x} - \frac{8}{y} = 2$$

$$\frac{6}{x} - \frac{9}{y} = 0$$

$$- \quad + \quad = -$$

[Subtracting]

$$\frac{1}{y} = 2 \quad \Rightarrow \quad 2y = 1 \text{ or } y = \frac{1}{2}$$

Substituting $y = \frac{1}{2}$ in second equation, we get ;

$$\frac{2}{x} - \frac{3}{\frac{1}{2}} = 0 \quad \Rightarrow \quad \frac{2}{x} = 6 \quad \Rightarrow 6x = 2$$

$$\Rightarrow \quad x = \frac{2}{6} = \frac{1}{3} \quad \therefore x = \frac{1}{3} \text{ and } y = \frac{1}{2}$$ **(Ans.)**

TEST YOURSELF

1. 5x + 5y = 35 and 3x – 3y = 18 ⇒ x + y = ... and x – y = ...
2. x + y = a and x – y = b ⇒ 2x = ..., x = ..., 2y = ... and y = ...
3. 5y – 3x = 2xy ⇒ $\frac{5}{x} - \frac{3}{y}$ = ...
4. The solution of equations ax + by = 0 and bx – ay = 0 is ....

EXERCISE 19 (B)

Solve the following pairs of simultaneous equations :

1. $\frac{x}{3} = \frac{y}{2}$; $\frac{2x}{3} - \frac{y}{2} = 2$
2. $\frac{x}{2} - \frac{y}{3} = 2$; $\frac{x}{5} + \frac{y}{3} = 15$
3. $\frac{x}{3} + \frac{x+y}{6} = 3$; $\frac{y}{3} - \frac{x-y}{2} = 6$
4. $\frac{a}{4} - \frac{b}{3} = 0$; $\frac{3a+8}{5} = \frac{2b-1}{2}$
5. $\frac{x-1}{2} + \frac{y+1}{5} = 4\frac{1}{5}$; $\frac{x+y}{3} = y - 1$
6. $\frac{1}{x} + \frac{1}{y} = 5$; $\frac{1}{x} - \frac{1}{y} = 1$
7. $\frac{3}{a} + \frac{4}{b} = 2$; $\frac{9}{a} - \frac{4}{b} = 2$
8. $\frac{8}{x} - \frac{9}{y} = 1$; $\frac{10}{x} + \frac{6}{y} = 7$
9. $\frac{6}{x} - \frac{2}{y} = 1$; $\frac{9}{x} - \frac{6}{y} = 0$
10. $3x + \frac{1}{y} = 13$; $\frac{2}{y} - x = 5$
11. $4x + \frac{3}{y} = 1$; $3x - \frac{2}{y} = 5$
12. $y - \frac{3}{x} = 8$; $2y + \frac{7}{x} = 3$
13. $\frac{x+1}{y+1} = 2$; $\frac{2x+1}{2y+1} = \frac{1}{3}$
14. $\frac{x-1}{3} + \frac{y+2}{2} = 3$; $\frac{1-x}{6} - \frac{y-4}{2} = \frac{1}{2}$

19.4 PROBLEMS BASED ON SIMULTANEOUS EQUATIONS

To solve problems with two unknows (variables) generally, simultaneous equations are used.

The following steps must be adopted :

1. Represent the two unknowns (variables) by variables, say letters x and y.
2. According to the given problem, set two equations in the form of x and y.
3. Solve the equations.

Example 8 :

Find two numbers, whose sum is 21 and difference is 9.

Solution :

Let the two numbers be x and y [Step 1]

$\therefore\ x + y = 21$ and $x - y = 9$ [Step 2]

On solving the two equations, we get $x = 15$ and $y = 6$ [Step 3]

$\therefore$ **Required numbers are 15 and 6** **(Ans.)**

1. Age problems :

Example 9 :

The sum of the ages of a father and his son is 55 years. After 16 years, the father will be twice as old as his son. Find their present ages.

Solution :

Let the present ages of father and son be x years and y years respectively.

$\therefore\ x + y = 55$ I

After 16 years : Father's age will be x + 16 years

and, son's age will be y + 16 years.

Given : $x + 16 = 2(y + 16) = 2y + 32$

$\Rightarrow \quad x - 2y = 32 - 16 = 16$ II

On solving equations I and II, we get : $x = 42$ and $y = 13$

$\therefore$ **Father's present age = 42 years and son's present age = 13 years** **(Ans.)**

2. Two digit numbers :

Consider a two digit number, say 37. In this number, the digit at ten's place is 3, the digit at unit's place is 7, the expanded notation of it is $10 \times 3 + 7$ and the sum of its digits = 3 + 7.
Similarly if in a two digit number the digit at ten's place is x and the digit at unit's place is y then the number = $10x + y$ and sum of its digits = $x + y$.

Example 10 :

A number consists of two digits whose sum is 8. When 18 is added to the number, its digits are reversed. Find the number.

Solution :

Let the digit at ten's place be x and the digit at unit's place be y.

$\therefore$ The number = $10x + y$, the sum of the digits = $x + y$ and the number on reversing the digits = $10y + x$.

Given : $x + y = 8$ I

and, $10x + y + 18 = 10y + x$ [On adding 18, the digits reverse]

$\Rightarrow \quad 10x + y - x - 10y = -18$

$\Rightarrow \quad 9x - 9y = -18$

$\Rightarrow \quad x - y = -2$ .... II [Dividing each term by 9]

On solving equations I and II, we get : $x = 3$ and $y = 5$

$\therefore$ **The required two digit number** = $10x + y = 10 \times 3 + 5 = 30 + 5 =$ **35** **(Ans.)**

3. Problems on fractions :

Example 11 :

If the numerator of a fraction is increased by 2 and the denominator by 1, it becomes $\frac{5}{8}$. If the numerator and the denominator of the same fraction are each increased by 1, the fraction becomes $\frac{1}{2}$. Find the fraction.

Solution :

Let the fraction be $\frac{x}{y}$.

On increasing the numerator by 2 and the denominator by 1, we get : $\frac{x+2}{y+1}$.

Given : $\frac{x+2}{y+1} = \frac{5}{8} \Rightarrow 8x + 16 = 5y + 5$

$\Rightarrow \quad 8x - 5y = -11$ I

On increasing numerator and denominator both by 1, we get : $\frac{x+1}{y+1}$

Given : $\frac{x+1}{y+1} = \frac{1}{2} \Rightarrow 2x + 2 = y + 1$

$\Rightarrow \quad 2x - y = -1$ II

On solving equations I and II, we get : $x = 3$ and $y = 7$.

$\therefore$ **The required fraction** $= \frac{x}{y} = \frac{3}{7}$ **(Ans.)**

4. Streams and Boats :

If the speed of a boat in still water = x km/hr and the speed of current (stream) = y km/hr then :
(i) speed of boat downstream (*i.e.* in the direction of current) = (x + y) km/hr
(ii) speed of boat upstream (*i.e.* in the direction opposite to the current) = (x – y) km/hr.

Example 12 :

A boat goes 24 km downstream in 3 hours and 16 km upstream in 4 hours. Find :
(i) speed of the boat in still water (ii) speed of the current.

Solution :

Let the speed of the boat in still water = x km/hr
and, the speed of current = y km/hr
$\therefore$ Speed of the boat downstream = (x + y) km/hr
and, speed of the boat upstream = (x – y) km/hr

Since, $\text{Speed} = \frac{\text{distance}}{\text{time}}$

$\therefore \quad x + y = \frac{24}{3} \quad \Rightarrow \quad x + y = 8$ I

and, $\quad x - y = \frac{16}{4} \quad \Rightarrow \quad x - y = 4$ II

On solving I and II, we get : $x = 6$ and $y = 2$

$\therefore$ (i) **Speed of the boat in still water = 6 km/hr** **(Ans.)**

(ii) **Speed of the current = 2 km/hr** **(Ans.)**

5. Miscellaneous Problems :

Example 13 :

Seven nuts and 8 bolts weigh 326 gms. Eleven nuts and 10 bolts weigh 448 gms. Find the weight of :
(i) one nut and one bolt. (ii) twelve nuts and 12 bolts.

Solution :

Let the weight of one nut be x grams and the weight of one bolt be y grams.

$\therefore$ Weight of 7 nuts and 8 bolts = $7x + 8y = 326$ .... I (Given)

and, weight of 11 nuts and 10 bolts = $11x + 10y = 448$ .... II (Given)

On solving equations I and II, we get : $x = 18$ and $y = 25$

$\therefore$ (i) **Weight of one nut and one bolt** = $x + y$ = 18 gms + 25 gm
= **43 gms** **(Ans.)**

$\therefore$ (ii) **Weight of 12 nuts and 12 bolts** = $12x + 12y = (12 \times 18 + 12 \times 25)$ gm
= **516 gms** **(Ans.)**

Example 14 :

A man travels for x hours at 6 km/hr and then for y hours at 10 km/hr. Altogether he goes 58 km in 8 hrs. Find x and y.

Solution :

Since, distance = speed × time

∴ distance travelled in x hours = 6x km

and, distance travelled in y hours = 10y km

Given : Total distance travelled = 58 km ⇒ $6x + 10y = 58$ I

and, total time taken = 8 hours ⇒ $x + y = 8$ II

On solving I and II, we get : $x = 5{\cdot}5$ and $y = 2{\cdot}5$ **(Ans.)**

Example 15 :

The distance between two places A and B is 90 km. Two cars start together from A and B. If both the cars go in the same direction, they meet after 9 hours and if they go in opposite directions, they meet after $1\frac{2}{7}$ hours. Find their speeds. (Assume that car A has greater speed).

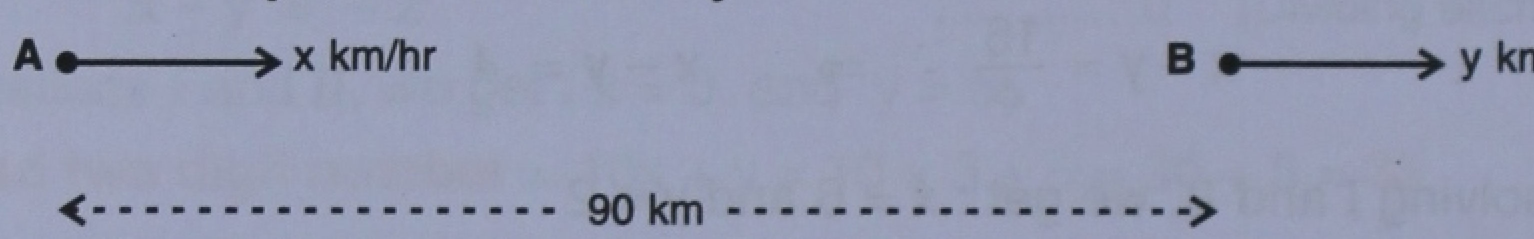
Solution :

Let the speed of car A = x km/hr and that of car B = y km/hr

When the cars go in the same direction :

Distance covered by A in 9 hours = 9x km [Since, distance = time × speed]

and, distance covered by B in 9 hours = 9y km



Clearly, to meet, car A has to travel 90 km more than car B.

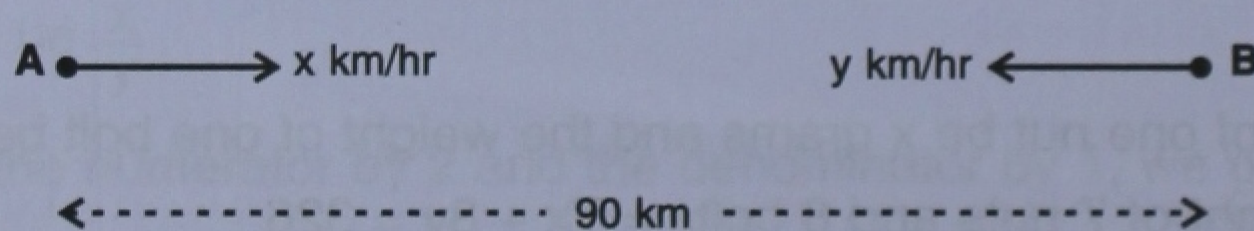
i.e. Distance covered by A – Distance covered by B = 90 km

⇒ $9x - 9y = 90$ *i.e.* $x - y = 10$ I

When the cars go in the opposite directions :

Distance covered by A in $1\frac{2}{7}$ hrs = $\frac{9}{7}x$ km

and, distance covered by B in $1\frac{2}{7}$ hrs = $\frac{9}{7}y$ km



Clearly, to meet;

Distance covered by A + Distance covered by B = 90 km

⇒ $$\frac{9}{7}x + \frac{9}{7}y = 90$$

⇒ $$\frac{9}{7}(x + y) = 90 \quad i.e. \quad x + y = 90 \times \frac{7}{9}$$

and, $x + y = 70$ II

On solving equations I and II, we get : $x = 40$ and $y = 30$

$\therefore$ **Speed of car A = 40 km/hr and speed of car B = 30 km/hr** **(Ans.)**

Example 16 :

Divide ₹ 700 into two parts such that 40% of one part exceeds 60% of the other part by 80.

Solution :

Let the two parts be ₹ x and ₹ y

$\therefore \quad x + y = 700$ I

And, $\quad$ 40% of x – 60% of y = 80

$$\Rightarrow \frac{40}{100}x - \frac{60}{100}y = 80$$

$$\Rightarrow \frac{2x}{5} - \frac{3y}{5} = 80 \quad i.e. \quad 2x - 3y = 400 \quad .... \text{II}$$

On solving equations I and II, we get :

$$x = 500 \text{ and } y = 200$$

$\therefore$ **The two parts are ₹ 500 and ₹ 200(Ans.)**

Alternative method :

Let the two parts be ₹ x and ₹ (700 – x)

$\therefore$ 40% of x – 60% of (700 – x) = 80

$$\Rightarrow \frac{40x}{100} - \frac{60(700-x)}{100} = 80$$

$$\Rightarrow \frac{2x}{5} - \frac{3(700-x)}{5} = 80$$

$$\Rightarrow 2x - 2100 + 3x = 400 \quad \Rightarrow 5x = 2500 \text{ and } x = 500$$

$\therefore$ **Required two parts** = ₹ x and ₹ (700 – x)

= ₹ 500 and ₹ (700 – 500)

= **₹ 500 and ₹ 200** **(Ans.)**

TEST YOURSELF

5. The difference between two numbers x and y is 6, then .... or ....
6. A is 58 years old and B is 56 years old, the difference between their ages is ....; the difference between their ages 15 years ago was .... and the differnce between their ages 9 years hence will be ....
7. The two digit number 10x + y increases after reversing its digits, the difference between its digits is .... And, if on reversing the digits it decreases, the difference between its digits is ....
8. For positive values of x and y if $\frac{x}{y} > 1$, then $\frac{y}{x}$ is ...., and if $\frac{x}{y} < 1$, then $\frac{y}{x}$ is ....
9. Speed of a boat downstream is 20 km h^{-1} and its speed upstream is 4 km h^{-1}, the speed of stream = ...
10. The cost of 6 apples and 5 bananas is ₹ 115, the cost of 5 apples and 6 bananas is ₹ 105; then the cost of 1 apple + 1 banana is ....

EXERCISE 19 (C)

1. Find two numbers such that twice of the first added to the second gives 21, and twice the second added to the first gives 27.
2. The difference of two numbers is 4. Three times the smaller number is 4 more than twice the larger number. Find the numbers.
3. Three times one number is equal to five times the other. If the sum of two numbers is 80; find the numbers.
4. Find two numbers, which differ by 7, such that twice the greater added to five times the smaller gives 42.
5. The sum of the ages of a father and his son is 48 years. Six years ago, the father's age was five times the age of his son. Find their present ages.
6. A is 25 years older than B. In 15 years, A will be twice of B. Find the present ages of A and B.
7. A man is 24 years older than his son. 12 years ago, he was five times as old as his son. Find the present ages of both.
8. Five years ago, the age of a father was thrice the age of his son. In five years time, the age of the father will be twice the age of his son at that time. Find their present ages.
9. The sum of the digits of a two digit number is 6 and its ten's digit is twice its unit digit. Find the number.
10. A certain number of two digits is three times the sum of its digits, and if 45 be added to it, its digits are reversed. Find the number.
11. A certain number of two digits is seven times the sum of its digits and if 27 be subtracted from it, its digits will be reversed. Find the number.
12. In a two digit number, the digit in the units place is 6 less than twice the digit in the tens place. When the digits are reversed, the number increases by 9. Find the number.
13. Find a fraction which reduces to $\frac{2}{3}$ if the numerator and the denominator are each increased by 1, and reduces to $\frac{3}{5}$ if the numerator and the denominator are each reduced by 2.
14. Find the fraction such that it becomes $\frac{1}{2}$ if 1 is added to the numerator, and $\frac{1}{3}$ if 1 is added to the denominator.
15. The numerator of a fraction is 5 less than the denominator. If the numerator is increased by 2, the fraction reduces to $\frac{2}{3}$. Find the original fraction.
16. A boat can go 72 km downstream in 4 hours and 36 km upstream in 3 hours. Calculate the speed of : (i) boat in still water, (ii) stream.
17. A boat can go 65 km upstream in 5 hours and 69 km downstream in 3 hours. Find the speed of : (i) boat in still water; (ii) stream.
18. Four knives and 6 forks cost ₹ 66; 5 knives and 4 forks cost ₹ 72. Find the cost of one knife and one fork.
19. The cost of 3 horses and 5 cows is ₹ 20,500, and the cost of 2 horses and 3 cows is ₹ 13,400. Find the cost of one horse and one cow. And also, find the cost of 5 horses and 4 cows.
20. The price of 2 kg tea-leaves and 5 kg sugar is ₹ 125; and the price of 3 kg tea-leaves and 8 kg sugar is ₹ 192. Find the price of 4 kg tea-leaves and 15 kg sugar.
21. A man walks for x hours at 4 km/hr and then for y hours at 3 km/hr. If he walks altogether 29 km in 8 hours, find the values of x and y.
22. One day, Manisha covered a distance of 24 km at x km/hr and another 40 km at y km/hr. Next day, she covered 18 km at x km/hr and another 48 km at y km/hr. If each day she took 9 hours to complete her journey find the values of x and y.
23. The distance between two stations A and B is 60 km. Two cars start together from A and B. If both the cars go in the same direction, they meet after 3 hours and if they go in the opposite directions, they meet after 30 minutes. Assuming that car A has greater speed and both the cars have uniform speeds, find their speeds.
24. Divide 900 into two parts such that 60% of one part exceeds 30% of the other by 270.
25. Divide 900 into two parts such that 60% of one part is equal to 30% of the other.

ANSWERS

TEST YOURSELF

1. 7; 6 **2.** a + b; $\frac{a+b}{2}$, a – b, $\frac{a-b}{2}$ **3.** 2 **4.** x = 0; y = 0 **5.** x – y = 6; y – x = 6

6. 2 years; 2 years; 2 years **7.** y – x; x – y **8.** < 1 ; > 1 **9.** 8 km h^{-1} **10.** ₹ $\frac{115+105}{11}$; ₹ 20

EXERCISE 19(A)

1. 4 and 7 **2.** 13 and 1 **3.** – 3 and 3 **4.** 8 and 4 **5.** 3 and 2 **6.** – 15 and 12 **7.** 4 and – 1 **8.** 5 and 8 **9.** 3 and 4 **10.** 7 and – 2 **11.** 5 and 4 **12.** 8 and 9 **13.** – 2 and – 3 **14.** – 2 and 2·5 **15.** 5 and 3 **16.** – 8 and 3

EXERCISE 19(B)

1. 6 and 4 **2.** $24\frac{2}{7}$ and $30\frac{3}{7}$ **3.** 3 and 9 **4.** 14 and 10·5 **5.** 7 and 5 **6.** $\frac{1}{3}$ and $\frac{1}{2}$ **7.** 3 and 4

8. 2 and 3 **9.** 3 and 2 **10.** 3 and $\frac{1}{4}$ **11.** 1 and – 1 **12.** – 1 and 5 **13.** x = $\frac{-3}{5}$ and y = $\frac{-4}{5}$

14. x = 4 and y = 2

EXERCISE 19(C)

1. 5 and 11 **2.** 16 and 12 **3.** 50 and 30 **4.** 11 and 4 **5.** 36 years and 12 years **6.** 35 years and 10 years **7.** 42 years and 18 years **8.** 35 years and 15 years **9.** 42 **10.** 27 **11.** 63 **12.** 78 **13.** $\frac{11}{17}$ **14.** $\frac{3}{8}$ **15.** $\frac{4}{9}$ **16.** (i) 15 km/hr (ii) 3 km/hr **17.** (i) 18 km/hr (ii) 5 km/hr **18.** ₹ 12 and ₹ 3 **19.** ₹ 5,500 ; ₹ 800 and ₹ 30,700 **20.** ₹ 295 **21.** x = 5 and y = 3 **22.** x = 6 and y = 8 **23.** A = 70 km/hr, B = 50 km/hr **24.** 600 and 300 **25.** 300 and 600.