

QUADRATIC EQUATIONS

ere

NG

QUADRATIC EQUATION

An equation of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$, is called a **quadratic equation** in the variable x.

For example :

 $x^2 - 3x + 2 = 0$ and $6x^2 + x - 1 = 0$ are quadratic equations in the variable x. A number α is a **root** (or **solution**) of the quadratic equation $ax^2 + bx + c = 0$ if it satisfies the equation *i.e.* if $a\alpha^2 + b\alpha + c = 0$.

For example :

When we substitute x = 2 in the quadratic equation $x^2 - 3x + 2 = 0$, we get $2^2 - 3 \times 2 + 2 = 0$ *i.e.* 4 - 6 + 2 = 0 *i.e.* 0 = 0, which is true. Therefore, 2 is a root of the quadratic equation $x^2 - 3x + 2 = 0$.

When we substitute x = 3 in the quadratic equation $x^2 - 3x + 2 = 0$, we get $3^2 - 3 \times 3 + 2 = 0$ *i.e.* 9 - 9 + 2 = 0 *i.e.* 2 = 0, which is wrong.

Therefore, 3 is not a root of the quadratic equation $x^2 - 3x + 2 = 0$.

SOLVING QUADRATIC EQUATIONS

Factorisation can be used to solve quadratic equations. The equation $x^2 - 3x + 2 = 0$ can be written as (x - 1)(x - 2) = 0. This equation can be solved by using a property of real numbers called **zero-product rule**.

Zero-product rule

If a and b are two numbers or expressions and if ab = 0, then either a = 0 or b = 0 or both a = 0 and b = 0

Using this rule, the solutions of the equation (x - 1)(x - 2) = 0 can be obtained by putting each factor equal to zero and then solving for x. Thus, we get

$$x - 1 = 0 \text{ or } x - 2 = 0$$

$$\Rightarrow$$
 $x = 1 \text{ or } x = 2.$

Hence, the solutions of the equation $x^2 - 3x + 2 = 0$ are 1 and 2.

Method to Solve a Quadratic Equation by Factorisation

Proceed as under :

* Clear all fractions and write the equation in the form $ax^2 + bx + c = 0$.

- * Factorise the left hand side into product of two linear factors.
- * Put each linear factor equal to zero and solve the resulting linear equations.

QUADRATIC EQUATIONS

Remark

The solutions (roots) may be checked by substituting in the original equation.

Example 1.

Solve : $x^2 - 2x - 15 = 0$.

Solution.

Given $x^2 - 2x - 15 = 0$ $\Rightarrow x^2 - 5x + 3x - 15 = 0$ $\Rightarrow x (x - 5) + 3 (x - 5) = 0$ $\Rightarrow (x - 5) (x + 3) = 0$ $\Rightarrow x - 5 = 0 \text{ or } x + 3 = 0$ $\Rightarrow x = 5 \text{ or } x = -3.$

Hence, the roots of the given equation are 5, -3.

Example 2.

Solve : $6x^2 + x - 1 = 0$.

Solution.

Given
$$6x^2 + x - 1 = 0$$

 $\Rightarrow 6x^2 + 3x - 2x - 1 = 0$
 $\Rightarrow 3x (2x + 1) - 1 (2x + 1) = 0$
 $\Rightarrow (2x + 1) (3x - 1) = 0$
 $\Rightarrow 2x + 1 = 0 \text{ or } 3x - 1 = 0$
 $\Rightarrow x = -\frac{1}{2} \text{ or } x = \frac{1}{3}.$

Hence, the roots of the given equation are $-\frac{1}{2}$, $\frac{1}{3}$.

Solve : $9x^2 + 6x + 1 = 0$.

Solution.

Given $9x^2 + 6x + 1 = 0$ $\Rightarrow 9x^2 + 3x + 3x + 1 = 0$ $\Rightarrow 3x (3x + 1) + 1 (3x + 1) = 0$ $\Rightarrow (3x + 1) (3x + 1) = 0$ $\Rightarrow 3x + 1 = 0 \text{ or } 3x + 1 = 0$ $\Rightarrow x = -\frac{1}{3} \text{ or } x = -\frac{1}{3}.$

Hence, the roots of the given equation are $-\frac{1}{3}$, $-\frac{1}{3}$. Note that the given quadratic equation has equal roots.

Example 4.

Solve : $2x - \frac{1}{r} = 1$.

Solution.

To clear the fractions, multiply both sides of the given equation by x. We get

$$2x^2 - 1 = x$$

$$\Rightarrow 2x^2 - x - 1 = 0$$

$$\Rightarrow 2x^2 - 2x + x - 1 = 0$$

MASTERING MATHEMATICS - VIII

 $\Rightarrow 2x (x - 1) + 1 (x - 1) = 0$ $\Rightarrow (2x + 1) (x - 1) = 0$ $\Rightarrow 2x + 1 = 0 \text{ or } x - 1 = 0$ $\Rightarrow x = -\frac{1}{2} \text{ or } x = 1.$

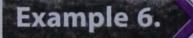
2m +

Hence, the roots of the given equation are $-\frac{1}{2}$, 1.

Example 5.

Solution.

Solve :
$$\frac{x+3}{x-1} = \frac{2x+1}{3x-5}$$
.
Given $\frac{x+3}{x-1} = \frac{2x+1}{3x-5}$
 $\Rightarrow (x+3)(3x-5) = (x-1)(2x+1)$ (By cross-multiplication)
 $\Rightarrow 3x^2 - 5x + 9x - 15 = 2x^2 + x - 2x - 1$
 $\Rightarrow 3x^2 + 4x - 15 = 2x^2 - x - 1$
 $\Rightarrow 3x^2 - 2x^2 + 4x + x - 15 + 1 = 0$
 $\Rightarrow x^2 + 5x - 14 = 0$
 $\Rightarrow x^2 + 7x - 2x - 14 = 0$
 $\Rightarrow x (x+7) - 2 (x+7) = 0$
 $\Rightarrow (x+7) (x-2) = 0$
 $\Rightarrow x + 7 = 0 \text{ or } x - 2 = 0$
 $\Rightarrow x = -7 \text{ or } x = 2$.
Hence, the roots of the given equation are -7 , 2.



Solution.

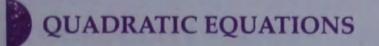
E ...

Given $\frac{x}{x-1} + \frac{x-1}{x} = \frac{5}{2}$

To clear the fractions, multiply both sides of the given equation by L.C.M. of denominators *i.e.* by 2x (x - 1). We get $2x \times x + 2 (x - 1) (x - 1) = 5x (x - 1)$ $\Rightarrow 2x^2 + 2x^2 - 4x + 2 = 5x^2 - 5x$ $\Rightarrow 4x^2 - 4x + 2 = 5x^2 - 5x$ $\Rightarrow 4x^2 - 5x^2 - 4x + 5x + 2 = 0$ $\Rightarrow -x^2 + x + 2 = 0$ $\Rightarrow x^2 - x - 2 = 0$ $\Rightarrow x^2 - 2x + x - 2 = 0$ $\Rightarrow x (x - 2) + 1 (x - 2) = 0$ $\Rightarrow (x + 1) (x - 2) = 0$ $\Rightarrow x + 1 = 0$ or x - 2 = 0 $\Rightarrow x = -1$ or x = 2. Hence, the roots of the given equation are -1, 2. Downloaded from https:// www.studiestoday.com

10

3



Exercise 18

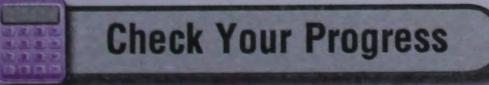
Solve the following (1 to 12) equations :

1.	(i) $x^2 - 11x + 30 = 0$	(<i>ii</i>) $4x^2 - 25 = 0$
2.	(<i>i</i>) $2x^2 - 5x = 0$	(<i>ii</i>) $x^2 - 2x = 48$
3.	(<i>i</i>) $6 + x = x^2$	(<i>ii</i>) $2x^2 - 3x + 1 = 0$
4.	(i) $3x^2 = 2x + 8$	(<i>ii</i>) $4x^2 + 15 = 16x$
5.	(<i>i</i>) $x (2x + 5) = 25$	(<i>ii</i>) $(x + 3) (x - 3) = 4$
6.	(<i>i</i>) $(2x + 3) (x - 4) = 6$	(<i>ii</i>) $(3x + 1)(2x + 3) =$
7.	(i) $4x^2 + 4x + 1 = 0$	(<i>ii</i>) $(x - 4)^2 + 5^2 = 13^2$
8.	(<i>i</i>) $21x^2 = 4(2x + 1)$	(<i>ii</i>) $\frac{2}{3}x^2 - \frac{1}{3}x - 1 = 0$
9.	(<i>i</i>) $6x + 29 = \frac{5}{x}$	(<i>ii</i>) $x + \frac{1}{x} = 2\frac{1}{2}$
10.	(i) $3x - \frac{8}{x} = 2$	(<i>ii</i>) $\frac{x}{3} + \frac{9}{x} = 4$
11.	(i) $\frac{x-1}{x+1} = \frac{2x-5}{3x-7}$	(<i>ii</i>) $\frac{1}{x+2} + \frac{1}{x} = \frac{3}{4}$
12.	(i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$	(<i>ii</i>) $\frac{x}{x+1} + \frac{x+1}{x} = 2$

Summary

- An equation of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and a \neq 0, is called a quadratic equation in the variable x.
- A number α is a root (or solution) of the equation $ax^2 + bx + c = 0$ if it satisfies it *i.e.* if

- $a\alpha^2 + b\alpha + c = 0.$
- ► A quadratic equation can be solved by factorisation and then using zero-product rule.
- To solve a quadratic equation, proceed as under :
 - (i) Clear all fractions and write the equation in the form $ax^2 + bx + c = 0$.
 - (ii) Factorise left hand side into product of two linear factors.
 - (iii) Put each linear factor equal to zero and solve the resulting linear equations.



Solve the following (1 to 3) equations :

- 1. (i) x(2x + 5) = 3(ii
- 2. (i) $4x^2 2x + \frac{1}{4} = 0$
- 3. (i) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

$$3x^2 - 4x - 4 = 0$$

$$(ii) \ 2x^2 + 7x + 6 = 0$$

(*ii*)
$$\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$$

Downloaded from https:// www.studiestoday.com