CHAPTER 17

FORMULAE

(Including change of subject and substitution)

17.1 REVIEW

1. Formula

A formula is an equation, which shows the relationship between two or more quantities.

e.g. Speed $= \frac{\text{Distance}}{\text{Time}}$ shows the relationship between Speed, Distance and Time; so it is a formula.

[Formulae is plural of formula.]

2. Framing a formula

To express a given statement in the form of an equation is called **framing a formula.**

Statement :	**Corresponding Formula :**
1. The sum of two numbers x and y is 40.	$x + y = 40$
2. A man has a total of ₹ M consisting of x coins of ₹ 2 each, y coins of one rupee each and z coins of 50 paise each.	$M = ₹\left(2x + y + \frac{50z}{100}\right)$ $= ₹\left(2x + y + \frac{z}{2}\right)$
3. The ages of two brothers are x years and (x + 5) years. After 10 years, the younger will be half of his brother's age.	$x + 10 = \frac{1}{2}(x + 5 + 10)$ $\Rightarrow \quad x + 10 = \frac{1}{2}(x + 15)$

TEST YOURSELF

1. If x kg of rice is bought at ₹ m per kg and whole of it is sold at ₹ n per kg, then total cost price of the rice = ₹ .....; total S.P. of the rice = ₹.....; profit made ₹ ..... and profit % .....
2. The total value in ₹ P, of x coins of ₹ 5 each, y coins of ₹ 2 each, z coins of ₹ 1 each and r coins of 50 paise each is ₹ P = ...
3. The force (F) applied on a body is equal to the product of its mass (m) and acceleration (a) produced in the body. ∴ F = ...
4. A boy runs for m hours at x km/hr and for n hours at y km/h, distance run by him in m hours = ... km, distance run by him in n hours = ... km and total distance run in (m + n) hours = ... km.
   ∴ Formula for his average speed = ... km/hr.

Example 1 :

A labourer is engaged for 50 days on the condition that he receives ₹ 25 for each day he works and gives ₹ 10 for each day he is absent. If he works for x days, form a formula to find his total wages, ₹ W, for 50 days.

Solution :

Given, the labourer works for x days, so he remains absent for (50 – x) days.

Since, for each day's work, he gets ₹ 25

∴ For x days' work, he gets ₹ 25x

Since, for being absent for one day, he gives ₹ 10

∴ For being absent (50 – x) days, he gives ₹ 10 (50 – x)

∴ **His total wages *i.e.* ₹ W** = ₹ 25x – ₹ 10 (50 – x)

= ₹ (25x – 500 + 10x)

= **₹ (35x – 500)** **(Ans.)**

EXERCISE 17 (A)

1. Make a formula for each of the following statements :
   (i) "The reciprocal of the focal length f, is equal to the sum of the reciprocals of the object distance u and image distance v."
   (ii) "The number of diagonals d, that can be drawn from one vertex of an n-sided polygon to all the other vertices is equal to the number of sides less 3."
   (iii) "The distance s metres, which a falling body covers in time t seconds, is 4.8 times the square of the time t."
   (iv) "The mean 'M' of the five quantities a, b, c, d and e is equal to their sum divided by the number of quantities."
2. A bus is carrying x children. If each of y children pays ₹ 2.50 and each of the remaining pays ₹ 5.25, find the total collection 'C' in rupees.
3. A worker is engaged for 100 days, on the condition that he will be paid ₹ x per day for each day he works and will be charged ₹ y per day for each day he is absent. Frame a formula to find his earning 'E' in rupees, if he works for 'd' days only.
4. A shopkeeper buys x kg sugar for ₹ y and sells it at ₹ z per kg. Find the expression for his (i) profit (ii) profit per cent.
5. A shopkeeper buys *a* kg of rice at ₹ x per kg and another *b* kg of rice at ₹ *y* per kg. If he sells the mixture at ₹ *z* per kg; find the expression for his (i) total profit (ii) profit per cent.
6. A man buys *m* articles at ₹ *x* each and another *n* articles for ₹ *y*. If he sells all the articles at ₹ *z* per article, frame an equation to find his profit.
7. During a certain month, a firm posted x letters with ₹ 5 stamps on each and another y letters with ₹ 3.50 stamps on each. Obtain an expression to find the total money, ₹ M, spent on postage.
8. Find the total money 'M' in rupees, in a purse, if the purse contains 2*x* notes of ₹ 5 each, 3*y* notes of ₹ 2 each, 6*z* coins of ₹ 1 each and 8*r* coins of 50 paise each.
9. A worker in a factory is paid ₹ x per hour for the normal work and double this rate for the overtime work. Find the total earning 'E; in rupees of a worker who works for 20 hours out of which y hours is the overtime.
10. (i) Find the number of hours 'h' in x days and y hours.
    (ii) Frame a formula for finding the number of students 'n' that may be seated in a class room in which there are S single seats and D double seats.

17.2 CHANGE OF THE SUBJECT OF A FORMULA

The *subject* of a formula is the *variable* which is expressed in *terms of other variables.*

e.g. in formula $A = \frac{1}{2}$ bh; A is expressed in terms of variables b and h; hence A is the subject of formula.

The same formula can be re–written as :

(i) $b = \frac{2A}{h}$, **b *is the subject of formula.***

(ii) $h = \frac{2A}{b}$, **h *is the subject of formula.***

TEST YOURSELF

5. $A = \frac{50}{x} \Rightarrow Ax =$ ..... and $x =$ .....; here x *is the subject of formula.*

6. $F = \frac{mv^2}{gr} \Rightarrow Fgr =$ ..... and $g =$ .....; here g *is the subject of formula.*

7. $I = \frac{E}{R+r} \Rightarrow IR +$ ..... $= E \Rightarrow Ir = E$..... and $r =$ .....

8. $a = \sqrt{\frac{b}{c}} \Rightarrow a^2 =$ ....., $\Rightarrow a^2c =$ ..... and $c =$ .....

9. $l = a + (n-1)d \Rightarrow l = a + nd - d \Rightarrow l - a + d =$ ..... and $n =$ .....

10. $\frac{1}{c} = \frac{ab}{a+b} \Rightarrow cab =$ ..... $\Rightarrow cab - b =$ ....., $\Rightarrow b($.....$) =$ ..... and $b =$ .....

***Example 2* :**

Given $S = \frac{n}{2}(a + l)$; make l, the subject of formula.

***Solution* :**

$S = \frac{n}{2}(a + l) \Rightarrow 2s = na + nl$

$\Rightarrow 2s - na = nl \quad \Rightarrow l = \frac{2s - na}{n}$ **(Ans.)**

***Example 3* :**

Given $T = 2\pi\sqrt{\frac{l}{g}}$; make g, the subject of formula.

***Solution* :**

Squaring both the sides of the given formula,

we get, $T^2 = 4\pi^2 . \frac{l}{g}$

$\Rightarrow T^2 g = 4\pi^2 l \quad \Rightarrow g = \frac{4\pi^2 l}{T^2}$ **(Ans.)**

EXERCISE 17 (B)

Change the subject of formulae for the indicated letter :

1. $F = \frac{9}{5}C + 32$; for C
2. $A = P(1 + rt)$; for t
3. $I = \frac{nE}{R+nr}$; for n
4. $I = \frac{nE}{nR+r}$; for r
5. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$; for u
6. $A = 2\pi r(r + h)$; for h
7. $m = 4\sqrt{\frac{a}{b+c}}$; for b
8. $z = \frac{a-b}{4b}$; for b
9. $S = \frac{n}{2}[2a + (n-1)d]$; for d

10. $T = \frac{1}{r}\sqrt{\frac{T}{\pi d}}$; for d

11. $\frac{a+1}{b} + 2 = \frac{3a}{b}$; for a

12. $m = \frac{xy - z}{x - 1}$; for x

13. $\frac{x - y}{x + y} = z$; for y

14. $\frac{P - 2l}{2} = b$; for l.

15. $a = \sqrt{\frac{x + b}{x - b}}$, for x

16. $F = \frac{mv - mu}{t}$; for u.

17. $V = \pi(R^2 - r^2)h$; for r.

18. $s = u + \frac{1}{2}a(2t - 1)$; for a

19. $s = u + \frac{1}{2}a(2t - 1)$; for t

20. $a = \sqrt{\frac{15x + 16y}{x + y}}$; for y

17.3 SUBSTITUTION

We know that area (A) of a rectangle is equal to the product of its length (l) and breadth (b) *i.e.* $\mathbf{A = l \times b}$.

Using this formula, the area of any rectangle can be obtained if its length and breadth are known. Also, the length can be found if its area and breadth are known, and the breadth, if its area and length are known.

The process of finding an unknown quantity of a formula, when each of the other quantities are known, is called **substitution.**

TEST YOURSELF

11. In $A = l \times b$;

(i) if $l = 30$ cm, and $b = 25$ cm; A = ... cm^2 = ... cm^2

(ii) if $A = 132 m^2$ and $l = 12$ m; b = ... m = ... m

(iii) if $A = 360\ cm^2$ and $b = 40$ cm; l = ... cm = ... cm

12. In $A = \frac{22}{7} r^2$

(i) if r = 14 cm, A = ... cm^2 = ... cm^2

(ii) if $A = 154\ m^2$, r^2 = ... = ...and r = ...m

***Example 4* :**

Given : $s = ut - \frac{1}{2} gt^2$

(i) Make g, the subject of the formula.

(ii) Find the value of g, if t = 2.5, s = 45 and u = 50.

***Solution* :**

(i) $s = ut - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 = ut - s \Rightarrow g = \frac{2(ut - s)}{t^2}$ **(Ans.)**

(ii) $g = \frac{2(50 \times 2{\cdot}5 - 45)}{(2{\cdot}5)^2}$

$= \frac{2(125 - 45)}{6{\cdot}25} = \frac{2 \times 80 \times 100}{625} = 25{\cdot}6$ **(Ans.)**

***Example 5* :**

Given : $2M + 3N = \frac{3N - M}{2P}$. Find N, if P = – 2 and M = 1.

Solution :

Ist. Method : (By changing the subject of formula)

$$2M + 3N = \frac{3N - M}{2P} \Rightarrow 4PM + 6PN = 3N - M$$

$$\Rightarrow 4PM + M = 3N - 6PN$$

$$\Rightarrow 4PM + M = N(3 - 6P)$$

$$\therefore N = \frac{4PM + M}{3 - 6P}$$

Now, $$N = \frac{4 \times -2 \times 1 + 1}{3 - 6 \times -2} = \frac{-8 + 1}{3 + 12} = \frac{-7}{15}$$ **(Ans.)**

Alternative method : (By direct substitution)

$$2M + 3N = \frac{3N - M}{2P} \Rightarrow 2 \times 1 + 3N = \frac{3N - 1}{2 \times -2}$$

$$\Rightarrow -4(2 + 3N) = 3N - 1$$

$$\Rightarrow -8 - 12N = 3N - 1$$

$$\Rightarrow -12N - 3N = -1 + 8 \Rightarrow N = -\frac{7}{15}$$ **(Ans.)**

If in the question, students are not asked to change the subject of formula, they can use any of the methods shown in example 5 above.

Example 6 :

A formula for changing temperature from degree Fahrenheit (F) to degree Celsius (C) is given by $F = \frac{9}{5}C + 32$.

(i) Express C in terms of F.

(ii) Find C, if F = 104

(iii) Is it possible for C and F to have the same value ? If yes, find the value.

Solution :

(i) $F = \frac{9}{5}C + 32 \Rightarrow \frac{9}{5}C = F - 32$

$$\Rightarrow C = \frac{5}{9}(F - 32)$$ **(Ans.)**

(ii) $C = \frac{5}{9}(104 - 32) = \frac{5}{9} \times 72 = 5 \times 8 = 40$ $[\because F = 104]$ **(Ans.)**

(iii) When F = C,

$$F = \frac{9}{5}C + 32 \Rightarrow C = \frac{9}{5}C + 32$$

$$\Rightarrow 5C = 9C + 160$$

i.e. $9C - 5C = -160 \Rightarrow 4C = -160 \Rightarrow C = \frac{-160}{4} = -40$

$\therefore$ **It is possible for C and F to have same value.** **(Ans.)**

Required value = C = F = – 40 **(Ans.)**

EXERCISE 17 (C)

1. The area A of the four walls of a room of length l, breadth b and height h is given by the formula

$$A = 2(l + b)h.$$

Find l, when $A = 54m^2$, $b = 4m$ and $h = 3m$

2. The formula for the volume of a cylinder is $v = \pi r^2 h$, where $\pi = \frac{22}{7}$.

(i) find r, when h = 14 and v = 396;

(ii) find h, when v = 770 and r = 3.5.

3. Given $l = a + (n - 1)d$; find n if $l = 31$, $a = 15$ and $d = 2$.

4. Given $t = 2\pi\sqrt{\frac{l}{g}}$, find l if $t = 1.1$, $g = 32$ and $\pi = \sqrt{10}$

5. If $s = \frac{n}{2}[2a + (n - 1)d]$; find :

(i) a, when s = 185, n = 10 and d = 3.

(ii) d, when a = 8, s = 380 and n = 10.

6. If $a = \frac{bc}{b+c}$; find c when $a = 11\frac{1}{9}$ and $b = 25$.

7. If $y = mx + c$; find :

(i) m, when y = 7, x = 5 and c = –3

(ii) x, when y = – 8, m = –1 and c = 2.

8. Nine-fifth of the temperature in centrigrade (C) of a body plus 32° is equal to the temperature of the same body in Fahrenheit (F).

(i) Making F as the subject, form a formula for the statement, given above.

(ii) Find F, when C = 55°

(iii) Make C as the subject of formula.

(iv) Find C, when F = 86°

9. Frame a formula for the statement "the reciprocal of focal length f is equal to the sum of reciprocals of the object distance u and the image distance v."

(i) Make v the subject of formula, obtained above.

(ii) Find v, when f = 15cm and u = 25cm.

10. Given $2a = \frac{x+y}{x-y}$. Find x, if y = 15 and a = 3.

11. Given $7abc - 2ab + 4bc - 2ac + 2a^2 = 58$, a = 1 and b = 2; find c.

12. If $\frac{7qr - 3pq - 7rp}{15p - 7pr + 8qr} = 1$, p = 15 and q = –5; find r.

ANSWERS

TEST YOURSELF

1. mx ; nx ; (nx – mx) ; $\frac{nx-mx}{mx} \times 100\% = \frac{n-m}{m} \times 100\%$ **2.** ₹$(5x + 2y + z + \frac{r}{2})$ **3.** ma

4. mx ; ny ; (mx + ny) ; $\frac{mx+ny}{m+n}$ **5.** 50 ; $\frac{50}{A}$ **6.** mv^2 ; $\frac{mv^2}{Fr}$ **7.** Ir ; –IR ; $\frac{E-IR}{I}$

8. $\frac{b}{c}$; b ; $\frac{b}{a^2}$ **9.** nd ; $\frac{l-a+d}{d}$ **10.** a + b ; a ; ca – 1 ; $\frac{a}{ca-1}$ **11.** (i) 30 × 25 ; 750

(ii) (132 ÷ 12) ; 11 (iii) (360 ÷ 40) ; 9 **12.** (i) $\frac{22}{7} \times (14)^2$; 616 (ii) $154 \times \frac{7}{22}$; 49 ; 7

EXERCISE 17(A)

1. (i) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (ii) d = n – 3 (iii) $s = 4{\cdot}8\ t^2$ (iv) $M = \frac{a+b+c+d+e}{5}$ **2.** $C' = 5{\cdot}25x - 2{\cdot}75y$

3. E = dx – (100 – d)y **4.** (i) zx – y (ii) $\frac{(zx-y)}{y} \times 100\%$ **5.** (i) (a + b) z – (ax + by)

(ii) $\frac{(a+b)z-(ax+by)}{ax+by} \times 100\%$ **6.** (m + n) z – (mx + y) **7.** M = ₹ (5x + 3·50y)

8. M = ₹ (10x + 6y + 6z + 4r) **9.** E = x(20 – y) + 2xy **10.** (i) h = 24x + y (ii) n = S + 2D

EXERCISE 17(B)

1. $\frac{5}{9}(F-32)$ **2.** $\frac{A-P}{P.r}$ **3.** $\frac{IR}{E-Ir}$ **4.** $\frac{nE-nIR}{I}$ **5.** $\frac{vf}{f-v}$ **6.** $\frac{A-2\pi r^2}{2\pi r}$ **7.** $\frac{16a-m^2c}{m^2}$ **8.** $\frac{a}{1+4z}$

9. $\frac{2s-2an}{n(n-1)}$ **10.** $\frac{1}{\pi r^2 T}$ **11.** $\frac{1+2b}{2}$ **12.** $\frac{m-z}{m-y}$ **13.** $\frac{x(1-z)}{(1+z)}$ **14.** $\frac{P-2b}{2}$ **15.** $\frac{b(a^2+1)}{a^2-1}$

16. $\frac{mv-Ft}{m}$ **17.** $\sqrt{\frac{\pi R^2 h-v}{\pi h}}$ **18.** $a=\frac{2(s-u)}{2t-1}$ **19.** $t=\frac{2s-2u+a}{2a}$ **20.** $y=\frac{(a^2-15)x}{16-a^2}$

EXERCISE 17(C)

1. 5 m **2.** (i) 3 (ii) 20 **3.** 9 **4.** 0·968 **5.** (i) 5 (ii) $6\frac{2}{3}$ **6.** 20 **7.** (i) 2 (ii) 10 **8.** (i) $F=\frac{9}{5}C+32$ (ii) 131° (iii) $C=\frac{5}{9}(F-32)$ (iv) 30° **9.** $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ (i) $v=\frac{fu}{u-f}$ (ii) 37·5 **10.** 21 **11.** c = 3 **12.** r = 0