CHAPTER 13

EXPONENTS

13.1 REVIEW

Exponent

If x is a real number and n is a natural number, we know :

$x \times x \times x \times x \times$ ..... n times $= x^n$

where x^n is called an **exponential expression** with **base** x and **exponent** (or index, or power) **n.**

x^n is read as 'x raised to the power n' or simply 'x to the power n'.

Laws of Exponents

1. **Product Law :** $a^m \times a^n = a^{m+n}$

   e.g. $3^7 \times 3^4 = 3^{7+4} = 3^{11}$, $x^8 \times x^5 = x^{8+5} = x^{13}$ and so on.

2. **Quotient Law :** $\dfrac{a^m}{a^n} = a^{m-n}$, if $m > n$

   $= \dfrac{1}{a^{n-m}}$, if $n > m$

   e.g. $\dfrac{3^7}{3^4} = 3^{7-4} = 3^3$, $\dfrac{x^5}{x^8} = \dfrac{1}{x^{8-5}} = \dfrac{1}{x^3}$ and so on.

3. **Power law :** $(a^m)^n = a^{mn}$

   e.g. $(3^7)^4 = 3^{7\times4} = 3^{28}$, $(x^8)^5 = x^{40}$ and so on.

TEST YOURSELF

1. $2 \times 2 \times 2 \times 2$ ..... 15 times = ... and is read as : .....
2. $-5 \times -5 \times -5 \times$ ..... 12 times = ... and is read as : .....
3. $a^5 \times a^7$ = ..., $a^5 \times a^{-7}$ = ..., $a^{-5} \times a^7$ = ... and $a^{-5} \times a^{-7}$ .....
4. $\dfrac{a^8}{a^5}$ = ..., $\dfrac{a^5}{a^8}$ = ..., $\dfrac{a^5}{a^{-8}}$ = ... and $\dfrac{a^8}{a^{-5}}$ = ...
5. $(a^5)^8$ = ..., $(a^8)^5$ = ..., $(a^{-8})^5$ = ... and $(a^{-8})^{-5}$ = ...
6. $3^{15} \times 3^6 \times 3^{-10}$ = ..., $5^4 \times 5^{-7} \times 5^6$ = ... and $7^2 \times 7^8 \times 7^{-6}$ = ...
7. $\dfrac{2^6 \times 2^4}{2^8}$ = ..., $\dfrac{4^6 \times 4^{-3}}{4^2}$ = ... and $\dfrac{8^5 \times 8^4}{8^{-3}}$ = ...

13.2 MORE ABOUT EXPONENTS

1. $(a \times b)^n = a^n \times b^n$

   e.g. $(a^5 \times b^{-3})^4 = (a^5)^4 \times (b^{-3})^4 = a^{20} \times b^{-12}$ and $(3^4 \times 5^{-3})^{-2} = 3^{-8} \times 5^6$

2. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

   e.g. $\left(\dfrac{a^{-3}}{b^4}\right)^6 = \dfrac{(a^{-3})^6}{(b^4)^6} = \dfrac{a^{-18}}{b^{24}}$ and $\left(\dfrac{5^7}{3^{-4}}\right)^{-3} = \dfrac{5^{-21}}{3^{12}}$

3. $a^0 = 1$; **if** $a \neq 0$

i.e. any non–zero number raised to the power zero is always equal to one (1).

e.g. $5^0 = 1, 7^0 = 1, (-8)^0 = 1, (2^{-5})^0 = 1$ and so on.

4. $a^{-m} = \frac{1}{a^m}$ **and** $\frac{1}{a^{-m}} = a^m$; **if** $a \neq 0$

e.g. $2^{-3} = \frac{1}{2^3}, \frac{1}{5^{-7}} = 5^7, \frac{2^{-3}}{3^{-5}} = \frac{3^5}{2^3}$ and so on.

5. $\sqrt[n]{a} = a^{\frac{1}{n}}$ **and** $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

e.g. $\sqrt{5} = 5^{\frac{1}{2}}$ $\sqrt[6]{5^7} = 5^{\frac{7}{6}}$ $\sqrt[3]{a^2 \times b^4} = a^{\frac{2}{3}} \times b^{\frac{4}{3}}$, etc.

Also remember that :

(i) $(-a)^m = a^m$; *if m is even* and (ii) $(-a)^m = -a^m$; *if m is odd.*

e.g. $(-5)^4 = -5 \times -5 \times -5 \times -5 = 5^4$ and $(-5)^3 = -5 \times -5 \times -5 = -5^3$

TEST YOURSELF

8. $(a^2b^{-3})^4$ = ...

9. $(b^{-4}x^2)^{-2}$ = ...

10. $(3x^2y)^2$ = ...

11. $\left(\frac{5m^2}{2n^3}\right)^3$ = ...

12. $\left(\frac{2a}{b^2}\right)^5$ = ...

13. $\left(x^{\frac{2}{3}} \cdot y^{\frac{-3}{2}}\right)^6$ = ...

14. $(a^{-2}b)^{-2}.(ab)^{-3}$ = ... = ...

15. $(125^3)^0$ = ...

16. $(-2)^5 \times (-2)^3$ = ... = ...

Example 1 :

Evaluate :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}}$

(ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}}$

(iii) $-2^4 - \left(\sqrt{3}\right)^0 \times (-2)^6 \div 4$

Solution :

(i) $4^{\frac{3}{2}} \times 125^{\frac{-2}{3}} = (2^2)^{\frac{3}{2}} \times (5^3)^{\frac{-2}{3}}$ $[4 = 2 \times 2 = 2^2, 125 = 5 \times 5 \times 5 = 5^3]$

$= 2^3 \times 5^{-2}$ $\left[2 \times \frac{3}{2} = 3 \text{ and } 3 \times \frac{-2}{3} = -2\right]$

$= \frac{8}{5^2}$ $\left[2^3 = 2 \times 2 \times 2 = 8 \text{ and } 5^{-2} = \frac{1}{5^2}\right]$

$= \frac{8}{25}$ **(Ans.)**

(ii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \div (32)^{\frac{-2}{5}} = \left(\frac{2}{3}\right)^{3\times\frac{2}{3}} \div (2^5)^{\frac{-2}{5}}$

$\left[\frac{8}{27} = \frac{2\times2\times2}{3\times3\times3} = \left(\frac{2}{3}\right)^3 \text{ and } 32 = 2\times2\times2\times2\times2 = 2^5\right]$

$\left[3\times\frac{2}{3} = 2 \text{ and } 5\times\frac{-2}{5} = -2\right]$

$= \left(\frac{2}{3}\right)^2 \div 2^{-2}$

$= \frac{2^2}{3^2} \times \frac{1}{2^{-2}}$

$\left[\frac{1}{2^{-2}} = 2^2\right]$

$= \frac{4}{9} \times 2^2$

$= \frac{4\times4}{9} = \frac{16}{9} = 1\frac{7}{9}$ **(Ans.)**

(iii) **Given expression**

$= -2^4 - 1 \times 2^6 \div 2^2$ $[(\sqrt{3})^0 = 1; (-2)^6 = 2^6 \text{ and } 4 = 2\times2 = 2^2]$

$= -2^4 - 2^4$ $[2^6 \div 2^2 = 2^{6-2} = 2^4]$

$= -16 - 16 = -32$ **(Ans.)**

Example 2 :

Simplify : $\frac{x^{m+n} \times x^{n+l} \times x^{l+m}}{(x^m \times x^n \times x^l)^2}$

Solution :

Given expression $= \frac{x^{m+n+n+l+l+m}}{x^{2m} \times x^{2n} \times x^{2l}}$

$= \frac{x^{2m+2n+2l}}{x^{2m+2n+2l}} = 1$ **(Ans.)**

Example 3 :

Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

Solution :

Given expression $= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$ **(Ans.)**

EXERCISE 13

1. Compute :

(i) $1^8 \times 3^0 \times 5^3 \times 2^2$

(ii) $(4^7)^2 \times (4^{-3})^4$

(iii) $(2^{-9} \div 2^{-11})^3$

(iv) $\left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2}$

(v) $\left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$

(vi) $(12)^{-2} \times 3^3$

(vii) $(-5)^4 \times (-5)^6 \div (-5)^9$

(viii) $\left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5$

(ix) $9^0 \times 4^{-1} \div 2^{-4}$ (x) $(625)^{-\frac{3}{4}}$

(xi) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$ (xii) $\left(\frac{1}{32}\right)^{-\frac{2}{5}}$

(xiii) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$

(xiv) $(243)^{\frac{2}{5}} \div (32)^{-\frac{2}{5}}$

(xv) $(-3)^4 - (\sqrt[4]{3})^0 \times (-2)^5 \div (64)^{\frac{2}{3}}$

(xvi) $(27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}}$

2. Simplify :

(i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$

(ii) $[(64)^{-2}]^{-3} \div [\{(-8)^2\}^3]^2$

(iii) $(2^{-3} - 2^{-4})\,(2^{-3} + 2^{-4})$

3. Evaluate :

(i) $(-5)^0$ (ii) $8^0 + 4^0 + 2^0$

(iii) $(8 + 4 + 2)^0$ (iv) $4x^0$

(v) $(4x)^0$ (vi) $[(10^3)^0]^5$

(vii) $(7x^0)^2$

(viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

4. Simplify :

(i) $\frac{a^5 b^2}{a^2 b^{-3}}$ (ii) $15y^8 \div 3y^3$

(iii) $x^{10}y^6 \div x^3y^{-2}$ (iv) $5z^{16} \div 15z^{-11}$

(v) $(36x^2)^{\frac{1}{2}}$ (vi) $(125x^{-3})^{\frac{1}{3}}$

(vii) $(2x^2y^{-3})^{-2}$ (viii) $(27x^{-3}y^6)^{\frac{2}{3}}$

(ix) $(-2x^{2/3}\,y^{-3/2})^6$

5. Simplify : $(x^{a+b})^{a-b} \cdot (x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a}$

6. Simplify : (i) $\sqrt[5]{x^{20}y^{-10}z^5} \div \frac{x^3}{y^3}$

(ii) $\left(\frac{256a^{16}}{81b^4}\right)^{\frac{-3}{4}}$

7. Simplify and express as positive indices :

(i) $(a^{-2}b)^{-2} \cdot (ab)^{-3}$ (ii) $(x^n y^{-m})^4 \times (x^3 y^{-2})^{-n}$

(iii) $\left(\frac{125a^{-3}}{y^6}\right)^{\frac{-1}{3}}$ (iv) $\left(\frac{32x^{-5}}{243y^{-5}}\right)^{\frac{-1}{5}}$

(v) $(a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$

(vi) $(xy)^{m-n} \cdot (yz)^{n-l} \cdot (zx)^{l-m}$

8. Show that :

$$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} = 1$$

9. Evaluate : $\frac{x^{5+n} \times (x^2)^{3n+1}}{x^{7n-2}}$

10. Evaluate : $\frac{a^{2n+1} \times a^{(2n+1)(2n-1)}}{a^{n(4n-1)} \times (a^2)^{2n+3}}$

11. Prove that : $(m + n)^{-1}\,(m^{-1} + n^{-1}) = (mn)^{-1}$

12. Prove that :

(i) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

(ii) $\frac{1}{1 + x^{a-b}} + \frac{1}{1 + x^{b-a}} = 1$

ANSWERS

TEST YOURSELF

1. 2^{15}, 2 raised to the power 15 **2.** $(-5)^{12}$; –5 raised to the power 12 **3.** $a^{12}, a^{-2}, a^2, a^{-12}$ **4.** $a^3, a^{-3}, a^{13}, a^{13}$ **5.** $a^{40}, a^{40}, a^{-40}, a^{40}$ **6.** $3^{11}, 5^3, 7^4$ **7.** $2^2, 4, 8^{12}$ **8.** a^8b^{-12} **9.** b^8x^{-4} **10.** $9x^4y^2$

11. $\frac{125m^6}{8n^9}$ **12.** $\frac{32a^5}{b^{10}}$ **13.** $x^4 \cdot y^{-9}$ **14.** $a^4b^{-2} \cdot a^{-3}b^{-3}$; ab^{-5} **15.** 1 **16.** $(-2)^8 = 256$

EXERCISE 13

1. (i) 500 (ii) 16 (iii) 64 (iv) $\frac{4}{9}$ (v) 10 (vi) $\frac{3}{16}$ (vii) –5 (viii) $-\frac{1}{3}$ (ix) 4 (x) $\frac{1}{125}$ (xi) $1\frac{7}{9}$ (xii) 4

(xiii) $\frac{1}{100}$ (xiv) 36 (xv) 83 (xvi) $13\frac{1}{2}$ **2.** (i) 132 (ii) 1 (iii) $\frac{3}{256}$ **3.** (i) 1 (ii) 3 (iii) 1 (iv) 4 (v) 1 (vi) 1

(vii) 49 (viii) $3\frac{62}{81}$ **4.** (i) a^3b^5 (ii) $5y^5$ (iii) x^7y^8 (iv) $\frac{1}{3}z^{27}$ (v) $6x$ (vi) $\frac{5}{x} = 5x^{-1}$

(vii) $\frac{y^6}{4x^4} = \frac{1}{4}.y^6.x^{-4}$ (viii) $\frac{9y^4}{x^2} = 9x^{-2}y^4$ (ix) $\frac{64x^4}{y^9} = 64x^4.y^{-9}$ **5.** 1 **6.** (i) xyz (ii) $\frac{27b^3}{64a^{12}} = \frac{27}{64}.a^{-12}b^3$

7. (i) $\frac{a}{b^5}$ (ii) $\frac{x^ny^{2n}}{y^{4m}} = x^n.y^{2n-4m}$ (iii) $\frac{ay^2}{5}$ (iv) $\frac{3x}{2y}$ (v) $\frac{1}{a^{2/3}.b^{1/2}}$ (vi) $\frac{x^ly^mz^n}{x^ny^lz^m} = x^{l-n}.y^{m-l}.z^{n-m}$

9. x^9 **10.** $\frac{1}{a^{n+6}}$