

Chapter 7

Module 1

PERCENTAGE

PERCENT

The word percent means 'per hundred' or 'out of hundred'.
Percent is denoted by the symbol %.

Introduction

Thus, 76% means 76 out of hundred = $\frac{76}{100}$.

39.4% means 39.4 out of hundred = $\frac{39.4}{100}$.

So, the symbol % stands for one-hundredth = $\frac{1}{100}$.

PERCENTAGE

Definition

Percentage is the numerator of a fraction with denominator 100.

In the fraction $\frac{r}{100}$, percentage = r . It is written as $r\%$.



Remarks

- If the denominator of a fraction is not 100, then convert it into an equivalent fraction with denominator 100.

For example, consider the fraction $\frac{27}{40}$. It can be written as

$$\frac{27}{40} = \frac{27}{40} \times \frac{100}{100} = \frac{27}{2} \times \frac{5}{100} = \frac{135}{100} = \frac{67.5}{100}$$

which is a fraction with denominator 100, therefore, percentage = 67.5.

- In practice, the word percent and percentage both are used synonymously.

To remember
In solving problems on percentages, remember the following :

- To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and reduce the fraction to simplest form.
- To convert a fraction into a percentage, multiply the fraction by 100 and put the % sign.
- To convert a percentage into a ratio, first convert the given percentage into a fraction and then write it as a ratio.
- To convert a ratio into a percentage, first write the ratio as a fraction and then convert it into a percentage.

- To convert a percentage into a decimal, remove the % sign and move the decimal point two places to the left.
- To convert a decimal into a percentage, move the decimal point two places to the right (adding zeros if necessary) and put the % sign.
- To find certain percentage of a given quantity :

$$x\% \text{ of a given quantity} = \frac{x}{100} \times \text{given quantity}$$

- Expressing one quantity as a percentage of another quantity :

$$\text{To express } x \text{ as a percentage of } y, \text{ percentage} = \left(\frac{x}{y} \times 100 \right) \%$$

Both quantities must be of same kind (in same units).

- If certain percent of a quantity is given, then to find the quantity :

Let $x\%$ of a quantity be y , then

$$\frac{x}{100} \text{ of the quantity} = y \Rightarrow \text{required quantity} = \frac{y}{x} \times 100$$

$$\text{Thus, if } x\% \text{ of a quantity is } y, \text{ then quantity} = \frac{y}{x} \times 100$$

- Percentage increase/decrease in a quantity :

$$\text{Percentage increase} = \left(\frac{\text{increase in quantity}}{\text{original quantity}} \times 100 \right) \%$$

$$\text{Percentage decrease} = \left(\frac{\text{decrease in quantity}}{\text{original quantity}} \times 100 \right) \%$$

- If a quantity increases by $x\%$, then

new quantity = original quantity + increase in the quantity

= original quantity + $x\%$ of original quantity

= original quantity + $\frac{x}{100}$ of original quantity

= $\left(1 + \frac{x}{100} \right)$ of original quantity

Thus, if a quantity increases by $x\%$, then

new quantity = $\left(1 + \frac{x}{100} \right)$ of original quantity

- If a quantity decreases by $x\%$, then

new quantity = original quantity – decrease in the quantity

= original quantity – $x\%$ of original quantity

= original quantity – $\frac{x}{100}$ of original quantity

= $\left(1 - \frac{x}{100} \right)$ of original quantity

Thus, if a quantity decreases by $x\%$, then

new quantity = $\left(1 - \frac{x}{100} \right)$ of original quantity

Example 1.

- (i) Convert $2\frac{1}{12}\%$ into fraction.
 (ii) Convert $1\frac{11}{16}$ into percentage.
 (iii) Convert 21 : 80 into percentage.
 (iv) Express $2\frac{7}{8}\%$ as a decimal.

Solution.

- (i) $2\frac{1}{12}\% = \frac{25}{12}\% = \frac{25}{12} \times \frac{1}{100} = \frac{1}{48}$.
 (ii) $1\frac{11}{16} = \frac{27}{16} = \left(\frac{27}{16} \times 100\right)\% = \frac{27 \times 25}{4}\% = \frac{675}{4}\% = 168.75\%$.
 (iii) $21 : 80 = \frac{21}{80} = \left(\frac{21}{80} \times 100\right)\% = \frac{105}{4}\% = 26.25\%$.
 (iv) $2\frac{7}{8}\% = 2.875\% = 0.02875$.

Example 2.

Convert $\frac{5}{24}$ into a percentage correct to four significant figures.

Solution.

$$\begin{aligned} \frac{5}{24} &= \left(\frac{5}{24} \times 100\right)\% = \frac{125}{6}\% = 20.833\% \\ &= 20.83\%, \text{ correct to four significant figures.} \end{aligned}$$

Example 3.

- (i) Find $3\frac{1}{8}\%$ of 75 kg.
 (ii) What percent is 15 paise of 2 rupees 70 paise?

Solution.

$$\begin{aligned} \text{(i) } 3\frac{1}{8}\% \text{ of } 75 \text{ kg} &= \frac{25}{8}\% \text{ of } 75 \text{ kg} = \frac{25}{100} \text{ of } 75 \text{ kg} = \left(\frac{25}{800} \times 75\right) \text{ kg} \\ &= \frac{75}{32} \text{ kg} = 2\frac{11}{32} \text{ kg.} \end{aligned}$$

$$\text{(ii) } 2 \text{ rupees } 70 \text{ paise} = (2 \times 100 + 70) \text{ paise} = 270 \text{ paise}$$

$$\therefore \text{ Required percentage} = \left(\frac{15}{270} \times 100\right)\% = \frac{50}{9}\% = 5\frac{5}{9}\%$$

Example 4.

- (i) If 9.5% of a number is 76, find the number.
 (ii) Increase the number 240 by 15%.
 (iii) Decrease the number 275 by 8%.

Solution.

- (i) Let the required number be x .

According to the given condition, 9.5% of $x = 76$

$$\Rightarrow \frac{9.5}{100} \times x = 76 \Rightarrow \frac{95}{1000} \times x = 76 \Rightarrow x = \frac{76 \times 1000}{95} = 800$$

Hence, the required number is 800.

$$\begin{aligned} \text{(ii) New number} &= \left(1 + \frac{15}{100}\right) \times 240 && \left(1 + \frac{x}{100}\right) \text{ of original} \\ &= \frac{115}{100} \times 240 = \frac{23}{20} \times 240 = 276. \end{aligned}$$

$$\begin{aligned} \text{(iii) New number} &= \left(1 - \frac{8}{100}\right) \times 275 && \left(1 - \frac{x}{100}\right) \text{ of original} \\ &= \frac{92}{100} \times 275 = \frac{23}{25} \times 275 = 253. \end{aligned}$$

Example 5. On a rainy day, only 36 students out of 45 came to a class. What percent were absent?

Solution.

Total number of students = 45

Number of students absent = $45 - 36 = 9$

\therefore Percentage of absent students = $\left(\frac{9}{45} \times 100\right)\% = 20\%$.

Example 6. A time interval of 5 minutes 30 seconds is wrongly estimated at 5 minutes 31.5 seconds. What is the percentage error?

Solution.

Actual time interval = 5 min 30 sec

$= (5 \times 60 + 30) \text{ sec} = 330 \text{ sec}$

Estimated time interval = 5 min 31.5 sec

$= (5 \times 60 + 31.5) \text{ sec} = 331.5 \text{ sec}$

\therefore Error in time interval = $(331.5 - 330) \text{ sec} = 1.5 \text{ sec}$

\therefore Percentage error = $\left(\frac{1.5}{330} \times 100\right)\% = \frac{15}{33}\% = \frac{5}{11}\%$.

Example 7. In an election, there were only two candidates. The winner polled 53% votes and won by 9600 votes. Find the total number of votes polled.

Solution.

There are only two candidates.

Since the winner got 53% votes, therefore, the loser got

$(100 - 53)\% \text{ votes} = 47\% \text{ votes}$

\therefore the difference in percentage of votes = $(53 - 47)\% = 6\%$

As the winner won by 9600 votes, so difference in votes = 9600

\therefore 6% of total votes polled = 9600

Let total votes polled be x , then

$$6\% \text{ of } x = 9600 \quad \Rightarrow \quad \frac{6}{100} \times x = 9600$$

$$\Rightarrow x = \frac{9600 \times 100}{6} = 160000.$$

Hence, the total number of votes polled was 160000.

Example 8. In an exam, 27% students failed in Maths, 24% students failed in English and 20% students failed in both the subjects.

- Find the percentage of students who failed in any of the subjects.
- Find the percentage of students who passed in both the subjects.
- If 414 students passed in both the subjects, find the total number of students.

Solution.

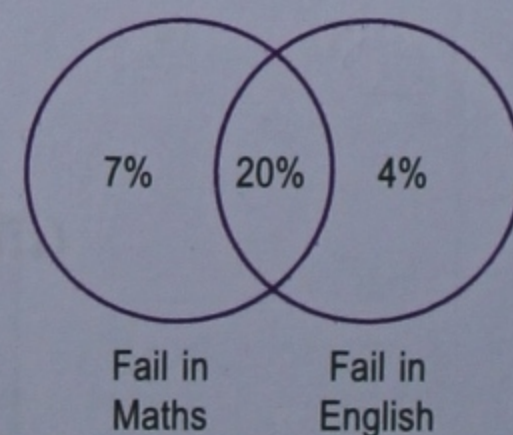
- Students failed in Maths = 27%,
students failed in English = 24%,
students failed in both Maths and English = 20%

\therefore students failed only in Maths = $27\% - 20\%$
 $= 7\%$,

students failed only in English = $24\% - 20\%$
 $= 4\%$

From the adjoining Venn diagram, we see that the percentage of students who failed in any of the subjects

$$= 20\% + 7\% + 4\% = 31\%.$$



(ii) The percentage of students who passed in both the subjects
 $= (100 - 31)\% = 69\%$.

(iii) Let the total number of students be x .

Since 69% students passed in both the subjects and the number of students who passed in both the subjects is 414,

$$\therefore 69\% \text{ of } x = 414 \Rightarrow \frac{69}{100} \times x = 414 \Rightarrow x = \frac{414 \times 100}{69} = 600.$$

Hence, the total number of students is 600.

Example 9.

On increasing the price of a cycle by 14%, it becomes ₹ 1425. What was its original price?

Solution.

Let the original price of the cycle be ₹ x .

Since the price has been increased by 14%, we get

$$\text{new price} = \left(1 + \frac{14}{100}\right) \text{ of original price}$$

$$\Rightarrow 1425 = \frac{114}{100} \times x \Rightarrow x = \frac{1425 \times 100}{114} = 1250$$

Hence, the original price of the cycle is ₹ 1250.

Example 10.

On reducing the price of a shirt by 8%, it becomes ₹ 423.20. What was its original price?

Solution.

Let the original price of the shirt be ₹ x .

Since the price has been reduced by 8%, we get

$$\text{new price} = \left(1 - \frac{8}{100}\right) \text{ of the original price}$$

$$\Rightarrow 423.20 = \frac{92}{100} \times x \Rightarrow x = \frac{423.20 \times 100}{92} = \frac{42320}{92} \Rightarrow x = 460$$

Hence, the original price of the shirt is ₹ 460.

Example 11.

Rice is costlier than wheat by 25%. By what percent is wheat cheaper than rice?

Solution.

Don't fall into the trap—'Rice is costlier by 25%, so wheat is cheaper by 25%'—this is wrong.

Let the price of certain quantity of wheat be ₹ 100.

Since rice is costlier than wheat by 25%, the price of same quantity of rice is ₹ 125.

Thus, the price of wheat is ₹ 25 less than the price of rice (₹ 125).

We want to find what percent is ₹ 25 of ₹ 125.

$$\text{Percentage} = \left(\frac{25}{125} \times 100\right)\% = 20\%.$$

Hence, wheat is 20% cheaper than rice.

Example 12.

Wheat is cheaper than rice by 25%. By what percent is rice costlier than wheat?

Solution.

Let the price of certain quantity of rice be ₹ 100.

Since wheat is cheaper than rice by 25%, the price of same quantity of wheat is ₹ 75.

Thus, the price of rice is ₹ 25 more than the price of wheat (₹ 75).

We want to find what percent is ₹ 25 of ₹ 75

$$\text{Percentage} = \left(\frac{25}{75} \times 100 \right) \% = \frac{100}{3} \% = 33\frac{1}{3} \%.$$

Hence, rice is $33\frac{1}{3} \%$ costlier than wheat.

Example 13.

If the price of diesel is raised by 20% today, by what percent it should be reduced tomorrow to bring it to the previous level.

Solution.

Let the price of certain quantity of diesel be ₹ 100.

Since the price of diesel is raised by 20%, its new price is ₹ 120.

As the price of the diesel is to be reduced to previous level, its price after reduction should be ₹ 100.

∴ the reduction in price is ₹ 20 (on ₹ 120).

We want to find what percent is ₹ 20 of ₹ 120.

$$\text{Percentage} = \left(\frac{20}{120} \times 100 \right) \% = \frac{50}{3} \% = 16\frac{2}{3} \%$$

Hence, the price should be reduced by $16\frac{2}{3} \%.$

Example 14.

If the price of sugar is raised by 25%, find by how much percent a housewife should reduce the consumption of sugar so as not to increase her expenditure.

Solution.

Let the original consumption of sugar be x kg.

Since the price of the sugar is raised by 25%,

$$\text{new price} = \left(1 + \frac{25}{100} \right) \text{ of the original price} = \frac{5}{4} \text{ of the original price.}$$

∴ In order to keep the same expenditure on sugar, the housewife has to reduce the consumption of sugar to $\frac{4}{5}$ of the original consumption.

$$\therefore \text{New consumption of sugar} = \left(\frac{4}{5} \times x \right) \text{ kg} = \frac{4}{5}x \text{ kg}$$

$$\therefore \text{Reduction in the consumption of sugar} = \left(x - \frac{4}{5}x \right) \text{ kg} = \frac{1}{5}x \text{ kg}$$

∴ Percentage of reduction in the consumption of sugar

$$= \left(\frac{\frac{1}{5}x}{x} \times 100 \right) \% = 20\%.$$

Example 15.

A student secured 32% marks in an examination and failed by 12 marks. Another student secured 42% marks and got 28 marks more than the minimum marks required to pass. Find the maximum marks and the pass percentage of marks.

Solution.

Let the maximum marks be x .

$$\text{In the first case, pass marks} = 32\% \text{ of } x + 12 = \frac{32}{100} \times x + 12$$

$$\text{In the second case, pass marks} = 42\% \text{ of } x - 28 = \frac{42}{100} \times x - 28$$

Since pass marks are same, $\frac{42}{100}x - 28 = \frac{32}{100}x + 12$

$$\Rightarrow \frac{42}{100}x - \frac{32}{100}x = 12 + 28$$

$$\Rightarrow \frac{10}{100}x = 40 \Rightarrow \frac{1}{10}x = 40 \Rightarrow x = 40 \times 10 = 400.$$

\therefore Maximum marks = 400.

$$\text{Pass marks} = 32\% \text{ of } 400 + 12$$

$$= \frac{32}{100} \times 400 + 12 = 128 + 12 = 140$$

\therefore Percentage of pass marks = $\left(\frac{140}{400} \times 100\right)\% = 35\%$.

Example 16.

2% of the people of a village died due to an epidemic. A panic set in, during which 14% of the remaining people left the village. If the population is then reduced to 2107, what was it originally?

Solution.

Let the original population of the village be x .

Since 2% people died due to an epidemic,

$$\text{the people who survived} = \left(1 - \frac{2}{100}\right)x = \frac{98}{100}x = \frac{49}{50}x.$$

Further, as 14% of the remaining people left the village due to panic,

$$\text{people left in the village} = \left(1 - \frac{14}{100}\right) \text{ of } \frac{49}{50}x = \frac{86}{100} \times \frac{49}{50}x = \frac{43 \times 49}{50 \times 50}x$$

According to the given condition,

$$\frac{43 \times 49}{50 \times 50}x = 2107 \Rightarrow x = \frac{2107 \times 50 \times 50}{43 \times 49} = 2500$$

Hence, the original population of the village was 2500.

Exercise 7

1. Express the following percentages as fractions :

(i) 356%

(ii) $2\frac{1}{2}\%$

(iii) $16\frac{2}{3}\%$

(iv) 0.04%

2. Express the following fractions as percentages :

(i) $\frac{3}{2}$

(ii) $\frac{9}{20}$

(iii) $1\frac{1}{4}$

(iv) $2\frac{1}{3}$

3. Express the following fractions as decimals. Then express the decimals as percentages :

(i) $\frac{3}{4}$

(ii) $\frac{5}{8}$

(iii) $\frac{3}{16}$

(iv) $\frac{7}{80}$

4. Express the following fractions as decimals correct to four decimal places. Then express the decimals as percentages :

(i) $\frac{2}{3}$

(ii) $\frac{5}{6}$

(iii) $\frac{4}{7}$

(iv) $2\frac{2}{9}$

5. Express the following ratios as percentages :

(i) 17 : 20

(ii) 13 : 18

(iii) 93 : 80


6. Express the following percentages as decimals :
- (i) 20% (ii) 2% (iii) $3\frac{1}{4}\%$ (iv) 0.07%
7. Find the value of :
- (i) 27% of ₹ 50 (ii) $10\frac{2}{3}\%$ of 15 m (iii) $6\frac{1}{4}\%$ of 25 kg (iv) 0.8% of 390.
8. What percent is :
- (i) 10 of 40 (ii) 300 grams of 2 kg
(iii) ₹ 7.50 of ₹ 6 (iv) $\frac{1}{3}$ of $\frac{2}{9}$?
9. What percent of :
- (i) 25 is 5 (ii) 50 kg is 65 kg (iii) ₹ 9 is ₹ 4?
10. (i) If 6% of a number is 36, find the number.
(ii) If $16\frac{2}{3}\%$ of a number is 25, find the number.
(iii) If 13.25% of a number is 159, find the number.
11. (i) Increase the number 60 by 30%.
(ii) Decrease the number 750 by 10%.
12. (i) What number when increased by 15% becomes 299?
(ii) On decreasing a number by 18%, it becomes 697. Find the number.
13. Jaya saves 23% of her monthly salary. If her expenditure per month is ₹ 9702, find her salary.
14. Mr. Khanna spent 83% of his salary and saved ₹ 1870. Calculate his monthly salary.
15. In a school, 38% of the students are girls. If the number of boys is 1023, find the total strength of the school.
16. The price of an article increases from ₹ 960 to ₹ 1080. Find the percentage increase in the price.
17. Three candidates in a school election got 146, 294 and 360 votes each. What percentage of the votes did the winner receive?
18. In a straight contest, the loser polled 42% votes and lost by 14400 votes. Find the total number of votes polled. If the total number of eligible voters was 1 lakh, find what percentage of voters did not vote.
19. Out of 8000 candidates, 60% were boys. If 80% of the boys and 90% of the girls passed the exam, find the number of candidates who failed.
20. Anshul scored 98 out of 100 in Maths, 92 out of 100 in Science, 114 out of 150 in Hindi and 44 out of 50 in History.
(i) In which subject is his performance best?
(ii) In which subject is his performance worst?
(iii) What is his aggregate percentage?
21. In an exam, $\frac{1}{4}$ of the students failed both in English and Maths, 35% students failed in Maths and 30% failed in English.
(i) Find the percentage of students who failed in any of the subjects.
(ii) Find the percentage of students who passed in both the subjects.
(iii) If the number of students who failed only in English was 25, find the total number of students.

22. On increasing the price of an article by 16%, it becomes ₹ 1479. What was its original price?
23. Pratibha reduced her weight by 15%. If now she weighs 59.5 kg, what was her earlier weight?
24. In a sale, a shop reduces all its prices by 15%. Calculate :
 - (i) the cost of an article which was originally priced at ₹ 40.
 - (ii) the original price of an article which was sold for ₹ 20.40.
25. Increase a price of ₹ 200 by 10% and then decrease the new price by 10%. Is the final price same as the original one?
26. A's salary is 20% higher than that of B. How much percent is B's salary lower than that of A?
27. A's salary is 20% lower than that of B. How much percent is B's salary higher than that of A?
28. The price of milk is increased by 20%. Find by how much percent a housewife should reduce the consumption of milk so as not to increase her expenditure?
29. Chandani purchased some parrots. 20% flew away and 5% died. Of the remaining, 45% were sold. Now 33 parrots remain. How many parrots had Chandani purchased?
30. A candidate who gets 36% marks in an examination fails by 24 marks but another candidate, who gets 43% marks, gets 18 more marks than the minimum pass marks. Find the maximum marks and the percentage of pass marks.

Summary

- ➔ Percent means per hundred or out of hundred. The symbol % stands for percent *i.e.* $\frac{1}{100}$.
- ➔ Percentage is the numerator of a fraction whose denominator is 100.
- ➔ To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and simplify.
- ➔ To convert a fraction into a percentage, multiply the fraction by 100 and put the % sign.
- ➔ To convert a percentage into a decimal, remove the % sign and move the decimal point two places to the left.
- ➔ To convert a decimal into a percentage, move the decimal point two places to the right (adding zeros if necessary) and put the % sign.
- ➔ $x\%$ of a given quantity = $\frac{x}{100} \times$ given quantity
- ➔ To express x as a percentage of y , percentage = $\left(\frac{x}{y} \times 100\right)\%$
Both quantities must be of same kind (in same units).
- ➔ If $x\%$ of a given quantity is y , then quantity = $\frac{y}{x} \times 100$.
- ➔ Percentage increase/decrease in a quantity = $\left(\frac{\text{change in quantity}}{\text{original quantity}} \times 100\right)\%$
- ➔ If a quantity increases by $x\%$, then
new quantity = $\left(1 + \frac{x}{100}\right)$ of original quantity.

- If a quantity decreases by $x\%$, then
 new quantity = $\left(1 - \frac{x}{100}\right)$ of original quantity.



Check Your Progress

- Convert $\frac{7}{24}$ into a percentage correct to four significant figures.
- Express 750 ml as a percentage of 4 litres.
- What percent of a day is half an hour?
- A lunch interval of half an hour is 5% of total office hours. Calculate
 - the total office hours
 - the working hours.
- In a straight student election, Rahul got 66% of the votes polled. If the defeated student got 187 votes, find the total number of votes polled.
- 36 kg of gun powder contains 27 kg nitre, 5.4 kg charcoal and the rest sulphur. Find the percentage content of sulphur in the gun powder.
- In an examination, 82% students passed in Maths and 75% passed in Science. 8% students failed in both subjects.
 - Find the percentage of students who failed in any of the subjects.
 - If 390 students passed in both subjects, then how many appeared in the examination?
- 75% of the students in a class passed an exam. If 2 more students had passed the exam, 80% would have been successful. How many students are there in the class?
 [Hint. Let the total number of students in the class be x , then $\frac{75}{100} \times x + 2 = \frac{80}{100} \times x$.]
- A man earning ₹ 9000 per month found that he could just buy a record player if he saved 4% of his monthly salary for one year. What was the price of the record player?
- The monthly salary of a school teacher in 2007 was ₹ 12000. It increased by 10% in 2008 and again by 10% in 2009. What is his salary in 2009?
- Sandy's height increased by 20% last year and by 15% this year. What is the total percent increase in 2 years?
- Price of a commodity decreased by 10% last year and increased by 20% this year. Find the percentage change in two years.
- In a class, section A has 42 boys out of a total of 75 students, section B has 60% girls in a total student strength of 80 and section C has two-third boys among 45 students. Find the aggregate percentage of boys of the class.
- A man gave 35% of his money to his elder son, 45% of the remaining money to his younger son and the rest to his wife. The wife received ₹ 64350. How much money he had in the beginning?
- The price of a computer has been reduced by 10%. By what percent this new value should be increased to restore it to the original value?
- The length and the breadth of a rectangle are 10 cm and 8 cm. If its length increases by 15% and breadth by 20%, find the percentage increase in its area.
- Two sides of rectangle are 20 cm and 10 cm. They are increased in the ratio 2 : 3 and 3 : 7. Find the percentage increase in its area.