RATIO AND PROPORTION

(Including Proportion Parts)

6.1 REVIEW

Ratio	A ratio is the relationship between two quantities which expresses how many times one quantity is of the other quantity of the same kind and in the same unit. **Remember :** The two quantities must be of the same kind and in the same unit. ∴ There can be a ratio between ₹ 15 and ₹ 20, but there can be no ratio between ₹ 15 and 20 oranges.

1. The ratio between two quantities is obtained by dividing the first quantity by the second.

   e.g. if A = 36 and B = 24; then ratio between A and B = $\dfrac{36}{24}$ = $\dfrac{3}{2}$ = 3 : 2 and ratio between B and A = $\dfrac{24}{36}$ = $\dfrac{2}{3}$ = 2 : 3.

   [3 : 2 is read as : 3 *is to* 2 and 2 : 3 is read as : 2 *is to* 3]

2. The two quantities (numbers) in a ratio are called its terms. The *first term* is called the *antecedent* (means '*that which goes before*') and the *second term* is called the *consequent* (means '*that which goes after*').

3. A ratio is a pure number and has *no unit.*

4. A ratio should always be expressed in lowest terms.

   e.g. ratio between 20 and 32 = $\dfrac{20}{32}$ = $\dfrac{20+4}{32+4}$ = $\dfrac{5}{8}$ = 5 : 8

   Clearly, the ratio between two quantities is equivalent to the fraction that one quantity is of the other.

   e.g. the ratio between A and B is 3 : 2 ⇒ A = $\dfrac{3}{2}$ *times* of B.

TEST YOURSELF

1. Since, 2 kg = ... g

   The ratio between 800 g and 2 kg = ... = ...

2. Since, 2 km and 2 kg are not of the ....

   and not in the .... ; they do not form ....

6.2 COMPARING RATIOS

Example 1 :

Which ratio is greater 3 : 7 or 10 : 21 ?

Solution :

First method : (By converting each ratio into a decimal fraction).

   3 : 7 = $\dfrac{3}{7}$ = 0·4285 ....   and   10 : 21 = $\dfrac{10}{21}$ = 0·4761 ....

As 0·4761 .... is greater than 0·4285 ....; **the ratio 10 : 21 is greater.**   **(Ans.)**

47

Second method : The given ratios are 3 : 7 and 10 : 21

i.e. $\qquad \dfrac{3}{7}$ and $\dfrac{10}{21}$

or, $\qquad \dfrac{3 \times 3}{7 \times 3}$ and $\dfrac{10}{21}$ $\qquad$ (Making the denominators equal)

or, $\qquad \dfrac{9}{21}$ and $\dfrac{10}{21}$ $\qquad$ Since, $\dfrac{10}{21} > \dfrac{9}{21}$ $\qquad$ ∴ **10 : 21 is greater** $\qquad$ **(Ans.)**

Third method :

For any two ratios a : b and c : d, if :
  (i) a × d > b × c ⟹ a : b is greater than c : d.
  (ii) a × d < b × c ⟹ a : b is less than c : d.
  (iii) a × d = b × c ⟹ a : b is equal to c : d.

For given ratios 3 : 7 and 10 : 21

$\qquad 3 \times 21 = 63$ and $7 \times 10 = 70$

Since, $\qquad 3 \times 21 < 7 \times 10 \qquad \Rightarrow \qquad$ 3 : 7 is less than 10 : 21

$\qquad\qquad\qquad\qquad\qquad \Rightarrow \qquad$ **10 : 21 is greater** $\qquad\qquad$ **(Ans.)**

6.3   TO DIVIDE A GIVEN QUANTITY IN A GIVEN RATIO

Example 2 :

   Divide ₹ 832 into two parts in the ratio 4 : 9.

Solution :

   Since, $\qquad 4 + 9 = 13$

∴ $\qquad$ **1st part** $= \dfrac{4}{13}$ of the whole $= \dfrac{4}{13} \times$ ₹ 832 = ₹ **256** $\qquad$ **(Ans.)**

and, $\qquad$ **2nd part** $= \dfrac{9}{13}$ of the whole $= \dfrac{9}{13} \times$ ₹ 832 = ₹ **576** $\qquad$ **(Ans.)**

Alternative method

   Since, the two parts are in the ratio 4 : 9

   Let the parts be ₹ 4x and ₹ 9x

$\Rightarrow \qquad\qquad 4x + 9x = 832$

$\Rightarrow \qquad\qquad\quad 13x = 832$ and $x = 64$

∴ $\qquad$ **1st part** = ₹ 4x = ₹ 4 × 64 = ₹ **256** $\qquad$ **(Ans.)**

and, $\qquad$ **2nd part** = ₹ 9x = ₹ 9 × 64 = ₹ **576** $\qquad$ **(Ans.)**

Example 3 :

   Two numbers are in the ratio 5 : 4. If 3 is subtracted from the first and 2 is subtracted from the second, they become in the ratio 6 : 5. Find the numbers.

Solution :

   Since, the numbers are in the ratio 5 : 4.

   Let the numbers be 5x and 4x.

Given : $\qquad \dfrac{5x - 3}{4x - 2} = \dfrac{6}{5} \Rightarrow 25x - 15 = 24x - 12 \Rightarrow x = 3$

∴ $\qquad$ **Numbers** = 5x and 4x

$\qquad\qquad\qquad\qquad = 5 \times 3$ and $4 \times 3 =$ **15 and 12** $\qquad\qquad$ **(Ans.)**

48

Example 4 :

The ratio of the number of boys to the number of girls in a school of 450 pupil is 4 : 5. When some new boys and girls are admitted, the number of boys increases by 25 and ratio of boys to girls changes to 9 : 13. Calculate the number of new girls admitted.

Solution :

No. of boys in the school = $\frac{4}{9} \times 450 = 200$ [∵ 4 + 5 = 9]

and, no. of girls in the school = $\frac{5}{9} \times 450 = 250$

Let the number of new girls admitted be x, then new no. of boys = 200 + 25 = 225 and new no. of girls = 250 + x.

Given : $\frac{225}{250 + x} = \frac{9}{13}$ ⇒   2250 + 9x = 2925   [By cross multiplication]

⇒   9x = 2925 − 2250 = 675

⇒ $x = \frac{675}{9} = 75$

∴   **75 new girls are admitted.**   **(Ans.)**

6.4 CONTINUED RATIO

Example 5 :

If A : B = 4 : 5 and B : C = 6 : 7, find the continued ratio A : B : C.

Solution :

Step 1 : In both the given ratios, make the value of common term one (unity).

Since, in given ratios A : B = 4 : 5 and B : C = 6 : 7; B is common.

∴ Make the value of B one (unity).

For this,   A : B = 4 : 5   ⇒ $A : B = \frac{4}{5} : 1$ [Dividing each term by 5]

and,   B : C = 6 : 7   ⇒ $B : C = 1 : \frac{7}{6}$ [Dividing each term by 6]

Step 2 : Combine the two ratios, both having value of B one (unity).

Combining $A : B = \frac{4}{5} : 1$ and $B : C = 1 : \frac{7}{6}$, we get :

$$A : B : C = \frac{4}{5} : 1 : \frac{7}{6}$$

$$= \frac{4}{5} \times 30 : 1 \times 30 : \frac{7}{6} \times 30 = \mathbf{24 : 30 : 35}$$ **(Ans.)**

Alternative method :

If each term of a ratio be divided or multiplied by the same non-zero number, the value of the ratio remains unchanged.

In ratios A : **B** = 4 : **5** and **B** : C = **6** : 7; B is common and the two values of B in the given ratios are 5 and 6.

Since, L.C.M. of 5 and 6 is 30; make B in each ratio equal to 30. For this :

A : B = 4 : 5 = 4 × 6 : 5 × 6 = 24 : **30** and

B : C = 6 : 7 = 6 × 5 : 7 × 5 = **30** : 35

∴   **A : B : C = 24 : 30 : 35**   **(Ans.)**

Example 6 :

If $A : B = 8 : 9$ and $B : C = 15 : 16$; find $A : C$.

Solution :

$$A : B = 8 : 9 \Rightarrow \frac{A}{B} = \frac{8}{9} \text{ and } B : C = 15 : 16 \Rightarrow \frac{B}{C} = \frac{15}{16}$$

$$\therefore \quad \frac{A}{B} \times \frac{B}{C} = \frac{8}{9} \times \frac{15}{16} \Rightarrow \frac{A}{C} = \frac{5}{6} \Rightarrow A : C = 5 : 6 \qquad \textbf{(Ans.)}$$

Example 7 :

If $2A = 3B = 4C$; find $A : B : C$.

Solution :

Let $\quad 2A = 3B = 4C = K$

$\Rightarrow \quad 2A = K, 3B = K$ and $4C = K$ *i.e.* $A = \dfrac{K}{2}$, $B = \dfrac{K}{3}$ and $C = \dfrac{K}{4}$

$$\therefore \quad A : B : C = \frac{K}{2} : \frac{K}{3} : \frac{K}{4}$$

$$= \frac{K}{2} \times 12 : \frac{K}{3} \times 12 : \frac{K}{4} \times 12 \qquad \text{[L.C.M. of 2, 3 and 4 = 12]}$$

$$= 6 : 4 : 3 \qquad \textbf{(Ans.)}$$

EXERCISE 6 (A)

1. Which ratio is greater ?

    (i) $3 : 5$ or $7 : 11$ (ii) $8 : 15$ or $11 : 20$

    (iii) $2\dfrac{1}{2} : 3\dfrac{1}{3}$ or $3.5 : 4.5$

2. A field is 60 m long and 40 m wide. Find the ratio between its :

    (i) breadth and length;

    (ii) length and perimeter.

3. The monthly income and the monthly expenditure of a man are ₹ 15,000 and ₹ 10,500. Find the ratio between his monthly:

    (i) income and expenditure.

    (ii) expenditure and savings.

    (iii) savings and income.

4. The ratio of the number of boys to the number of girls in a school of 672 students is $5 : 7$. When some new boys and girls are admitted, the number of girls increases by 8 and the ratio of boys to girls changes to $3 : 4$. Calculate the number of new boys admitted.

5. The weights of Mr. Gupta and Mrs. Gupta are in the ratio $7 : 8$ and their combined (total) weight is 120 kg. After taking a dieting course, the weight of Mr. Gupta reduces by 6 kg and the ratio between their weights changes to $5 : 6$. Find the reduction of weight of Mrs. Gupta due to this dieting course.

6. If $A : B = 3 : 4$ and $B : C = 6 : 7$, find :

    (i) $A : B : C$ (ii) $A : C$

7. If $A : B = 4 : 9$ and $A : C = 2 : 3$, find :

    (i) $B : C$ (ii) $A : B : C$

8. If $A : C = 5 : 8$ and $B : C = 5 : 6$, find :

    (i) $A : B$ (ii) $A : B : C$

9. If $3A = 4B = 6C$, find $A : B : C$.

10. Divide 282 into two parts such that the eighth part of the first and the fifth part of the second are in the ratio $4 : 3$.

11. A school has 1260 students. The ratio of the number of girls to the number of boys is $2 : 3$. If 180 new students join the school, the ratio changes to $5 : 7$. How many new boys joined the school ?

12. Divide ₹ 4,864 into three parts such that the second part is five times the first and the ratio of the second part to the third part is $3 : 4$.

6.5 PROPORTION

When four quantities are such that the ratio of the first to the second is same as the ratio of the third to the fourth, the quantities are said to be in **proportion**.

e.g.   the ratio between 14 and 30 $= 14 : 30 = \dfrac{14}{30} = \dfrac{7}{15}$

and,   the ratio between 63 and 135 = $63 : 135 = \dfrac{63}{135} = \dfrac{7}{15}$

Since, the two ratios 14 : 30 and 63 : 135 are same, so 14, 30, 63 and 135 are proportional numbers and we write 14 : 30 = 63 : 135 or 14 : 30 : : 63 : 135.

> The double colon (: :) is used in place of sign of equality (=)

> 1. In general, the four quantities a, b, c and d are in proportion if a : b : : c : d.
> 2. The four quantities which form a proportion, are called its **terms.**
>   In proportion a : b : : c : d ; a, b, c, and d are its *first, second, third* and *fourth* terms respectively. The fourth term is also called the *fourth proportional* to the numbers a, b and c.
> 3. In a proportion first and fourth terms are called **extremes** , whereas second and third terms are called **means.**
> 4. Also, *product of extremes = product of means.*

Example 8 :

Find the fourth proportional to the numbers 4, 22 and 6.

Solution :

Let the required fourth proportional be x.

Then, $4 : 22 : : 6 : x \Rightarrow 4x = 22 \times 6$ [product of extremes = product of means]

$$\Rightarrow x = \dfrac{22 \times 6}{4} = 33 \qquad \text{(Ans.)}$$

6.6   CONTINUED PROPORTION

Three quantities of same kind are said to be in *continued proportion,* if the ratio of the first to the second is same as the ratio of the second to the third.

i.e. if a, b and c are three quantities of same kind such that a : b = b : c; then a, b and c are in *continued proportion.*

> 1. The second quantity is called the **mean proportion** between the first and the second.
> 2. The third quantity is called the **third proportional** to the first and the second quantity.

Example 9 :

Find the mean proportion between 2 and 32.

Solution :

Let the required mean proportion be x.

Then, $2 : x : : x : 32 \Rightarrow x \times x = 2 \times 32$

$$\Rightarrow x^2 = 64 \qquad \Rightarrow x = 8 \qquad \text{(Ans.)}$$

> It is evident from example 9, given above, that *the mean proportion between two numbers is equal to the positive square root of their product.*
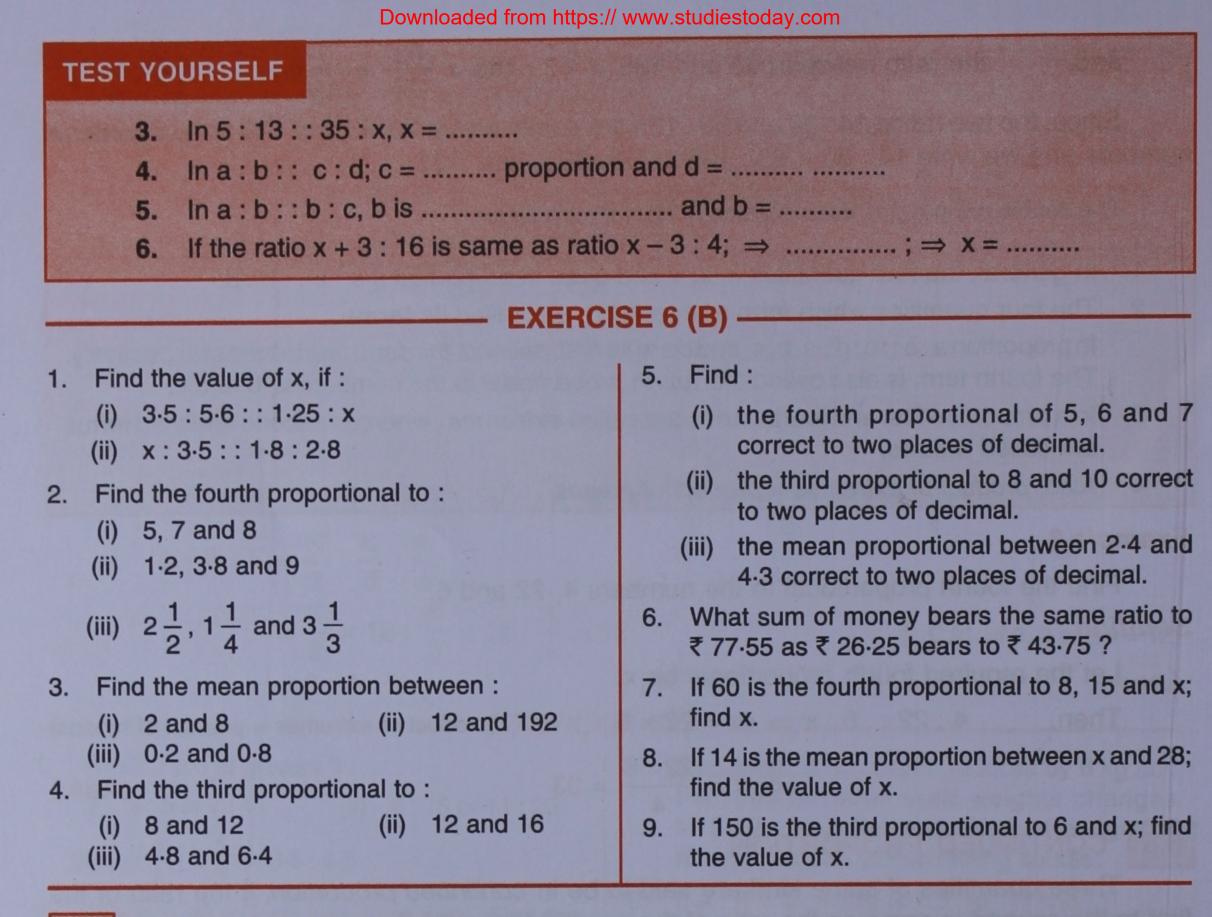
Example 10 :

Find the third proportional to 2 and 32.

Solution :

Let the required third proportional be x.

Then, $2 : 32 = 32 : x \Rightarrow 2x = 32 \times 32$

$$\Rightarrow x = \dfrac{32 \times 32}{2} = 512 \qquad \text{(Ans.)}$$

3. In $5 : 13 : : 35 : x$, $x =$ ....
4. In $a : b : : c : d$; $c =$ .... proportion and $d =$ ....
5. In $a : b : : b : c$, b is .... and $b =$ ....
6. If the ratio $x + 3 : 16$ is same as ratio $x - 3 : 4$; $\Rightarrow$ .... ; $\Rightarrow$ $x =$ ....

EXERCISE 6 (B)

1. Find the value of x, if :
   (i) $3.5 : 5.6 : : 1.25 : x$
   (ii) $x : 3.5 : : 1.8 : 2.8$

2. Find the fourth proportional to :
   (i) 5, 7 and 8
   (ii) 1.2, 3.8 and 9
   (iii) $2\frac{1}{2}, 1\frac{1}{4}$ and $3\frac{1}{3}$

3. Find the mean proportion between :
   (i) 2 and 8   (ii) 12 and 192
   (iii) 0.2 and 0.8

4. Find the third proportional to :
   (i) 8 and 12   (ii) 12 and 16
   (iii) 4.8 and 6.4

5. Find :
   (i) the fourth proportional of 5, 6 and 7 correct to two places of decimal.
   (ii) the third proportional to 8 and 10 correct to two places of decimal.
   (iii) the mean proportional between 2.4 and 4.3 correct to two places of decimal.

6. What sum of money bears the same ratio to ₹ 77.55 as ₹ 26.25 bears to ₹ 43.75 ?

7. If 60 is the fourth proportional to 8, 15 and x; find x.

8. If 14 is the mean proportion between x and 28; find the value of x.

9. If 150 is the third proportional to 6 and x; find the value of x.

6.7 **DIRECT PROPORTION**

If two quantities are so related to each other that an increase (or decrease) in first quantity causes a proportional increase (or decrease) in the second quantity, the two quantities are said to *vary directly*.

Example 11 :

If 5 pens cost ₹ 8, what is the cost of 15 pens ?

Solution :

The *increase* in the number of pens will cause a proportional *increase* in the price of pens. In this case, the ratio of 5 pens to 15 pens = the ratio of cost of 5 pens to cost of 15 pens.

$\Rightarrow$ 5 pens : 15 pens : : cost of 5 pens : cost of 15 pens

$\Rightarrow$ $5 : 15 : : 8 : x$ [Let the cost of 15 pens = ₹ x]

$\Rightarrow$ $5x = 15 \times 8$

$\Rightarrow$ $x = \dfrac{15 \times 8}{5} = 24$

$\therefore$ **Cost of 15 pens = ₹ 24**   **(Ans.)**

Alternative method :

Let x and y represent two quantities in **direct proportion**, then for all corresponding values of x and y, the ratio $\dfrac{x}{y}$ is always the same. That is, if y_1 and y_2 are the values of y corresponding to the values x_1 and x_2 of x, then $\dfrac{x_1}{x_2} = \dfrac{y_1}{y_2}$.

Consider the example 11, give above, in which $x_1 = 5$ pens, $x_2 = 15$ pens and $y_1 = ₹ 8$; then to find y_2.

Since, it is the case of direct proportion :

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} \Rightarrow \frac{5 \text{ pens}}{15 \text{ pens}} = \frac{₹ 8}{y_2} \Rightarrow y_2 = ₹ 24$$

6.8   INVERSE PROPORTION

If two quantities are so related to each other that an *increase* (or *decrease*) in first quantity causes a proportional *decrease* (or *increase*) in the second quantity; then the two quantities are said to *vary inversely.*

Example 12 :

If 9 men can do a piece of work in 16 days, how many men would do it in 24 days ?

Solution :

Here, *more* the number of men, proportinally *less* is the number of days required to finish the same work and vice-versa.

i.e. the two quantities vary inversely.

Thus, if x be the number of men required to finish the work in 24 days;

the inverse ratio of 16 days to 24 days = the ratio of 9 men to x men

i.e. $24 : 16 = 9 : x$

$\Rightarrow$ $24x = 16 \times 9 \Rightarrow x = \dfrac{16 \times 9}{24} = 6$

∴ **6 men would finish the work in 24 days**   (Ans.)

Alternative method :

Let x and y represent two quantities in inverse proportion, then for all corresponding values of x and y, the product xy is always the same.

That is, if y_1 and y_2 are the values of y corresponding to the values x_1, x_2 of x, then $x_1 y_1 = x_2 y_2$. Consider the example 12, given above; in which $x_1 = 16$ days, $x_2 = 24$ days and $y_1 = 9$ men; then to find y_2.

Since, it is the case of inverse proportion :

$x_1 y_1 = x_2 y_2 \Rightarrow 16 \times 9 = 24 \times y_2 \Rightarrow$ **$y_2 = 6$ men**

Example 13 :

If 50 pencils can be bought for ₹ 20, how many can be bought for ₹ 32 ?

Solution :

Here, $x_1 = ₹ 20$, $x_2 = ₹ 32$ and $y_1 = 50$ pencils. To find the number of pencils to be bought for ₹ 32 *i.e.* $y_2 = ?$

Since, it is the case of direct proportion, therefore,

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} \Rightarrow \frac{₹ 20}{₹ 32} = \frac{50 \text{ pencils}}{y_2 \text{ pencils}}$$

$$\Rightarrow \quad y_2 = 80 \text{ pencils}$$ (Ans.)

Example 14 :

A fort had provisions for 300 men for 90 days. After 20 days, 50 more men came to the fort. How long will the remaining food last at the same rate ?

Solution :

The remaining food would last for 300 men for 90 – 20 = 70 days.

Since, 50 more men join them therefore, the remaining food would last in less time.

Let x_1 = 300 men, x_2 = 350 men, y_1 = 70 days, then find y_2.

Since, it is the case of inverse proportion

$$x_1y_1 = x_2y_2 \implies 300 \text{ men} \times 70 \text{ days} = 350 \text{ men} \times y_2 \text{ days}$$

$$\implies y_2 = 60 \text{ days} \qquad \text{(Ans.)}$$

Example 15 :

The scale of a map is given as 1 : 3000000. Two cities are 4 cm apart on the map. Find the actual distance between them.

Solution :

The scale of a map is given as 1 : 3000000, means; if two cities are 1 cm apart on the map then the actual distance between the two cities is 3000000 cm = $\dfrac{3000000}{100}$ m = $\dfrac{30000}{1000}$ km = 30 km.

Since, more is the distance between two cities on the map, proportionally more is the actual distance between them, it is the case of direct proportionality.

$$\therefore \quad \dfrac{x_1}{x_2} = \dfrac{y_1}{y_2} \implies \dfrac{1}{4} = \dfrac{30 \text{ km}}{y_2}$$

$$\implies y_2 = 120 \text{ km}$$

x_1 = 1 cm, x_2 = 4 cm

y_1 = 30 km, then

y_2 = ?

Example 16 :

6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take, if 5 pipes of the same type are used to fill the same tank ?

Solution :

Since, lesser is the number of pipes used, proportionally more will be the time required to fill the tank. Therefore, this is a case of inverse proportion.

Let x_1 = 6 pipes, x_2 = 5 pipes and y_1 = 1 hour 20 minutes = 80 minutes

$\therefore$ **To find y_2**

$$x_1y_1 = x_2y_2 \implies 6 \text{ pipes} \times 80 \text{ minutes} = 5 \text{ pipes} \times y_2 \text{ minutes}$$

$$\implies y_2 = \dfrac{6 \times 80}{5} \text{ minutes} = 96 \text{ minutes} = \textbf{1 hour 36 minutes} \qquad \text{(Ans.)}$$

EXERCISE 6 (C)

1. What would be the cost of 23 metres of cloth, if 5 metres of cloth costs ₹ 48 ?

2. Mohan bought 8 oranges for ₹ 4·80. If John has ₹ 7.20, how many oranges, more than Mohan, can he buy ?

3. If 18 men can do a piece of work in 18 days, how many men would do it in 27 days ?

4. If 30 men can reap a field in 14 days, in how many days will 20 men reap it ?

5. How many men must be engaged to do a piece of work in 16 days which 24 men can do in 40 days ?

6. A fort had provisions for 600 men for 180 days. After 40 days, 100 men left the fort. How long will the food last at the same rate ?

7. A fort is provisioned for 42 days; after 10 days, a reinforcement of 200 men arrives and the food will now last only for 24 days. How many men were there in the fort ?

8. A man can complete his work in 24 days by working 5 hours a day. How many days will he take to complete the same work; working 8 hours a day ?

9. Cost of 6 identical articles is ₹ (2x + 3) and the

cost of 10 similar articles is ₹ (4x − 5). Find the value of x.

10. Six men can do a certain piece of work in 5x + 6 days and 11 men can do the same work in 3x days. Find the value of x.

11. A vertical pole of height 5·60 m casts a shadow of length 3·20 m on the horizontal earth surface. Under the similar conditions and at the same time, find :
    (i) the length of the shadow cast by another vertical pole of height 10·5 m.
    (ii) the vertical height of the pole which casts a shadow of length 5 m.

12. A road map represents an actual distance of 18 km by 1 cm in it. If a boy drives on a road for 216 km, what would be the corresponding distance covered in the map ?

13. A photograph of bacteria, enlarged 50,000 times, attains a length of 5 cm. Find the actual length of the bacteria.

    If the photograph of the same bacteria is enlarged 20,000 times only, find its enlarged length.

14. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days ?

15. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same ?

6.10 PROPORTIONAL PARTS

The division of a given quantity into parts which shall be proportional to certain given numbers are called **proportional parts**.

Example 17 :

Divide ₹ 315 among three persons in proportion to 2 : 3 : 4.

Solution :

$$\text{First person's share} = \frac{2}{9} \times ₹ 315 = ₹ 70, \qquad [\because 2 + 3 + 4 = 9]$$

$$\text{second person's share} = \frac{3}{9} \times ₹ 315 = ₹ 105$$

$$\text{and,} \qquad \text{third person's share} = \frac{4}{9} \times ₹ 315 = ₹ 140$$

∴ **The required three shares are ₹ 70, ₹ 105 and ₹ 140** (Ans.)

The proportion a : b : c indicates three ratios, namely the ratio of a to b, of b to c and of a to c.

Example 18 :

Divide ₹ 7,040 among A, B and C with their shares in proportion to $1\frac{1}{2} : 1\frac{2}{3} : 2\frac{1}{6}$.

Solution :

$$1\frac{1}{2} : 1\frac{2}{3} : 2\frac{1}{6} = \frac{3}{2} : \frac{5}{3} : \frac{13}{6}$$

$$= \frac{3}{2} \times 6 : \frac{5}{3} \times 6 : \frac{13}{6} \times 6 = 9 : 10 : 13$$

Since, 9 + 10 + 13 = 32   ∴ $\text{A's share} = \frac{9}{32} \times ₹ 7,040 = ₹ \mathbf{1,980}$

$$\text{B's share} = \frac{10}{32} \times ₹ 7,040 = ₹ \mathbf{2,200} \text{ and}$$

$$\text{C's share} = \frac{13}{32} \times ₹ 7,040 = ₹ \mathbf{2,860} \qquad \textbf{(Ans.)}$$

55

Example 19 :

A certain sum of money is divided into three parts in the ratio 5 : 8 : 11. If the second part is ₹ 424; find :

    (i) the total sum of money.   (ii) the other two parts.

Solution :

(i) According to the given ratio 5 : 8 : 11,

    If the second part = ₹ 8,   the total sum = ₹ (5 + 8 + 11) = ₹ 24

$\Rightarrow$ If the second part = Re 1,   the total sum = ₹ $\frac{24}{8}$ = ₹ 3

$\Rightarrow$ If the second part = ₹ 424,   **the total sum = ₹ 3 × 424 = ₹ 1,272**   (Ans.)

(ii) Now, 5 + 8 + 11 = 24

$\Rightarrow$ **First part** = $\frac{5}{24}$ × ₹ 1,272 = **₹ 265**   (Ans.)

and, **third part** = $\frac{11}{24}$ × ₹ 1,272 = **₹ 583**   (Ans.)

Algebraic method :

(i) Since, the ratio between the three parts = 5 : 8 : 11

    Let the three parts be ₹ 5x, ₹ 8x and ₹ 11x

    Given, the second part = ₹ 424 $\Rightarrow$ 8x = 424 and x = $\frac{424}{8}$ = 53

$\therefore$ **The total sum = ₹ (5x + 8x + 11x) = ₹ 24x = ₹ 24 × 53 = ₹ 1,272**   (Ans.)

(ii) **First part = ₹ 5x = ₹ 5 × 53 = ₹ 265**   (Ans.)

    Third part = ₹ 11x = ₹ 11 × 53 = ₹ 583   (Ans.)

Example 20 :

Divide ₹ 4,250 into three parts such that the second is 80% of the first and the ratio between the first and the third is 5 : 8.

Solution :

    Ratio between the first and the third = 5 : 8

$\Rightarrow$ If first part is ₹ 5, the third part is ₹ 8

$\therefore$ The second part = 80% of ₹ 5 = $\frac{80}{100}$ × ₹ 5 = ₹ 4

$\therefore$ The ratio between the three parts = 5 : 4 : 8

Since, 5 + 4 + 8 = 17 $\therefore$ **First part** = $\frac{5}{17}$ × ₹ 4,250 = **₹ 1,250**   (Ans.)

    Second part = $\frac{4}{17}$ × ₹ 4,250 = **₹ 1,000**   (Ans.)

And,   **Third part** = $\frac{8}{17}$ × ₹ 4,250 = **₹ 2,000**   (Ans.)

Algebraic method :

Since the ratio between the first and the third parts = 5 : 8

$\therefore$ If the first part be ₹ 5x, the third part = ₹ 8x

Given; the second part = 80% of ₹ 5x = $\frac{80}{100} \times ₹ 5x = ₹ 4x$

Now, $5x + 4x + 8x = 4250 \Rightarrow 17x = 4250$ and $x = 250$

∴   **First part = ₹ 5x = ₹ 5 × 250 = ₹ 1,250**   **(Ans.)**

           Second part = ₹ 4x = ₹ 4 × 250 = ₹ 1,000   **(Ans.)**

And,   **Third part = ₹ 8x = ₹ 8 × 250 = ₹ 2,000**   **(Ans.)**

Example 21 :

A purse contains ₹ 5, ₹ 2 and ₹ 1 coins in the ratio 1 : 3 : 5 by number. If the total value of all the coins be ₹ 272; find the number of each type of coins.

Solution :

Since,   the ratio between ₹ 5, ₹ 2 and ₹ 1 coins (by number) = 1 : 3 : 5

∴   The ratio between these coins (by value) = 1 × 5 : 3 × 2 : 5 × 1

                                       = 5 : 6 : 5

$5 + 6 + 5 = 16 \Rightarrow$ Value of ₹ 5 coins = $\frac{5}{16} \times ₹ 272 = ₹ 85$,

                  value of ₹ 2 coins = $\frac{6}{16} \times ₹ 272 = ₹ 102$

and,   value of ₹ 1 coins = $\frac{5}{16} \times ₹ 272 = ₹ 85$

∴ **The numbers of ₹ 5, ₹ 2 and ₹ 1 coins**

            = $\frac{85}{5}, \frac{102}{2}$ and $\frac{85}{1}$ **= 17, 51 and 85**   **(Ans.)**

Algebraic method :

Given, the ratio (by number) between ₹ 5, ₹ 2 and ₹ 1 coins = 1 : 3 : 5

Let the number of three types of coins be x, 3x and 5x.

∴   The value of x coins of ₹ 5 each = x × ₹ 5 = ₹ 5x,

          the value of 3x coins of ₹ 2 each = 3x × ₹ 2x = ₹ 6x

and,   the value of 5x coins of ₹ 1 each = 5x × ₹ 1 = ₹ 5x

                  $5x + 6x + 5x = 272$

                        $16x = 272$ and $x = \frac{272}{16} = 17$

⇒

∴   The **number of ₹ 5 coins = x = 17**

            the **number of ₹ 2 coins = 3x = 3 × 17 = 51**

and,   the **number of ₹ 1 coins = 5x = 5 × 17 = 85**   **(Ans.)**

EXERCISE 6 (D)

1. Divide 4840 into three parts proportional to 5 : 7 : 10.

2. Divide 770 into four parts proportional to 2 : 3 : 4 : 5.

3. Divide ₹ 4,260 into three parts proportional to $2\frac{1}{2} : 1\frac{2}{3} : 1\frac{3}{4}$.

4. A : B = 3 : 4 and B : C = 3 : 5. Find A : B : C and then divide 1845 among A, B and C in proportion as obtained above.

5. Divide ₹ 5,680 among A, B and C so that A : B = 5 : 6 and B : C = 8 : 9.

6. Divide ₹ 3,240 among A, B and C such that 3A = 4B = 6C.

7. Three numbers A, B and C are in the ratio 4 : 6 : 7. If B = 5·40; find A and C.

8. The daily wages of A, B and C are in the ratio 7 : 6 : 10. If C earns ₹ 80 per day; find the per day earnings of A and B.

9. Concrete consists of $1\frac{1}{2}$ parts of lime, 4 parts of gravel and $2\frac{1}{2}$ parts of sand. Out of 480 kg of concrete; how much is lime ?

10. Divide ₹ 2,800 among three persons A, B and

C. If the shares of A and B are in the ratio 12 : 13 and C's share is 40% of the total of shares of A and B; find the shares of A, B and C.

C's share = 40% of (12 + 13) = $\dfrac{40}{100} \times 25 = 10$

∴ Shares of A, B and C are in the ratio 12 : 13 : 10.

11. A purse contains ₹ 1, 50 paise and 25 paise coins by number in the ratio 3 : 8 : 16. If the total value of all the coins be ₹ 605; find the number of each type of coins.

ANSWERS

TEST YOURSELF

1. 2000, $\dfrac{800}{2000}$, $\dfrac{2}{5}$, 2 : 5   2. same type, same unit, a ratio   3. 91   4. 3rd, 4th proportion

5. mean proportion, $\sqrt{a \times c}$ 6. $\dfrac{x+3}{16} = \dfrac{x-3}{4}$; 5

EXERCISE 6(A)

1. (i) 7 : 11 (ii) 11 : 20 (iii) 3·5 : 4·5 2. (i) 2 : 3 (ii) 3 : 10   3. (i) 10:7 (ii) 7:3 (iii) 3:10   4. 20 5. 4 kg
6. (i) 9 : 12 : 14 (ii) 9 : 14   7. (i) 3 : 2 (ii) 4 : 9 : 6 8. (i) 3 : 4 (ii) 15 : 20 : 24 9. 4 : 3 : 2 10. 192 and 90
11. 84 12. ₹ 384, ₹ 1,920 and ₹ 2,560

EXERCISE 6(B)

1. (i) 2 (ii) 2·25 2. (i) 11·2 (ii) 28·5 (iii) $1\frac{2}{3}$ 3. (i) 4 (ii) 48 (iii) 0·4   4. (i) 18 (ii) $21\frac{1}{3}$ (iii) $8\frac{8}{15}$
5. (i) 8·40 (ii) 12·50 (iii) 3·21 6. ₹ 46·53   7. 32   8. 7   9. 30

EXERCISE 6(C)

1. ₹ 220·80   2. 4   3. 12 men   4. 21 days   5. 60 men   6. 168 days   7. 600   8. 15 days   9. x = 15
10. x = 12   11. (i) 6 m (ii) 8·75 m   12. 12 cm   13. 0·0001 cm; 2 cm   14. 49   15. 40 minutes

EXERCISE 6(D)

1. 1100, 1540 and 2200   2. 110, 165, 220 and 275   3. ₹ 1,800; ₹ 1,200 and ₹ 1,260
4. A : B : C = 9 : 12 : 20; A = 405, B = 540 and C = 900   5. ₹ 1,600; ₹ 1,920 and ₹ 2,160 6. ₹ 1,440;
₹ 1080 and ₹ 720   7. A = 3·6 and C = 6·3   8. A = ₹ 56 and B = ₹ 48   9. 90 kg   10. A = ₹ 960,
B = ₹ 1,040 and C = ₹ 800   11. One ₹ coins = 165, 50 paise coins = 440 and 25 paise coins = 880